Price Discrimination with Redistributive Concerns

Daniel M A Barreto, Alexis Ghersengorin, Victor Augias

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Daniel M. A. Barreto[†] Alexis Ghersengorin[‡] Victor Augias[§]

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Abstract

Consumer data can be used to sort consumers into different market segments, allowing a monopolist to charge different prices at each segment. We study consumer-optimal segmentations with redistributive concerns, i.e., that prioritize poorer consumers. Such segmentations are efficient but may grant additional profits to the monopolist, compared to consumer-optimal segmentations with no redistributive concerns. We characterize the markets for which this is the case and provide a procedure for constructing optimal segmentations given a strong redistributive motive. For the remaining markets, we show that the optimal segmentation is surprisingly simple: it generates one segment with a discount price and one segment with the same price that would be charged if there were no segmentation.

JEL classification codes: D42, D83, D39.

Keywords: Third-degree price discrimination; information design; redistribution; inequality; welfare.

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[†]Sciences Po, e-mail: daniel.barreto@sciencespo.fr.

[‡]Paris School of Economics and Sciences Po, e-mail: alexis.ghersengorin@psemail.eu.

[§]Sciences Po and CNRS, e-mail: victor.augias@sciencespo.fr.

1 Introduction

Consumers are continuously leaving traces of their identities on the internet, be it through social media activity, search-engine utilization, online-purchasing and so on. The vast amount of consumer data that is generated and collected has acquired the status of a highly-valued good, as it allows firms to tailor advertisements and prices to different consumers. In practice, the availability of consumer data **segments** consumers: observing that a given consumer has certain characteristics allows firms to fine-tune how they interact with people that share those characteristics. Adjusting how coarse-grained the information available about consumers is impacts how they will be segmented, what sort of digital market interactions they will have and what prices they will pay. This suggests room for regulatory oversight.

As shown by Bergemann, Brooks, and Morris (2015), consumer segmentation and price discrimination can induce a wide range of welfare outcomes. It can not only be used to increase social surplus—by creating segments with prices that allow more consumers to buy—, but can also be performed in a way that ensures that all created surplus accrues to consumers — that is, that maximizes consumer surplus. This is done by creating segments that pool together consumers with high and low willingness to pay, thus allowing higher willingness to pay consumers to benefit from lower prices. However, an important aspect of price discrimination that remains overlooked by the literature is its **distributive effect**: since different consumers pay different prices, this practice defines how surplus is distributed *across* consumers, raising questions about how it can benefit poorer consumers relative to richer ones. Indeed, if willingness to pay and wealth are positively related, segmentations that maximize total consumer surplus tend to benefit richer consumers.

In this paper we provide a normative analysis of the distributive impacts of market segmentation. Our aim is to study how this practice impacts different consumers and how it should be performed under the objective of increasing consumer welfare while prioritizing poorer consumers. Our results draw qualitative characteristics of segmentations that achieve this goal, which can be used to inform future regulation. Importantly, our analysis also shows that the prioritization of poorer consumers can be inconsistent with the maximization of total consumer surplus: raising the surplus of poorer consumers may only be possible while granting additional profits to the producer, at the expense of richer consumers.

We consider a setting in which a monopolist sells a good on a market composed of heterogeneous consumers, each of whom can consume at most one unit and is characterized by their willingness to pay for the good. A social planner can provide information about consumers' willingness to pay to the monopolist. The information provision strategy effectively divides the aggregate pool of consumers into different **segments**, each of which can be priced differently by the monopolist. The social planner's objective is to maximize a weighted sum of consumers' surplus. As in Dworczak, Kominers, and Akbarpour (2021), we consider weights that are decreasing on the consumer's willingness to pay, capturing the notion of a redistributive motive under the assumption that consumers with higher willingness to pay are on average richer than those with lower willigness to pay.

We first establish that optimal segmentations are Pareto efficient, such that satisfying a redistributive objective does not come at the expense of social surplus. Bergemann et al. (2015) show that, in the absence of redistributive concerns, consumer-optimal segmentations do not strictly benefit the monopolist: all of the surplus created by the segmentation accrues to consumers. In contrast, we show that once redistributive preferences are considered, consumer-optimal segmentations may imply additional profits to the monopolist. This happens because increasing the surplus of poor consumers is done by pooling them with even poorer consumers, such that they can benefit from lower prices. In doing so, richer consumers become more representative in other segments, which might increase the price they pay. We characterize the set of markets for which this is the case and denote them as rent markets. For no-rent markets, on the contrary, *any* redistributive objective can be met while still maximizing total consumer surplus. In this case, our analysis selects one among the many consumer-optimal segmentations established by Bergemann et al. (2015). These insights are illustrated through a three-type example in section 3.

Our analysis also provides insights on how to construct optimal segmentations. We show that, in no-rent markets, consumer-optimal segmentations with redistributive concerns exhibit a stunningly simple form, simply dividing consumers into two segments: one where the price is the same that would be charged under no segmentation and one with a discount price. In rent markets, we show that consumer-optimal segmentations under sufficiently strong redistributive preferences divide consumers into contiguous segments based on their willingness to pay, having consumers with the same willingness to pay belong to at most two different segments. This allows us to construct a procedure that generates consumer-optimal segmentations under strong redistributive preferences, which is discussed in section 4.2.

Related Literature. Third-degree price discrimination and its welfare effects are the subject of an extensive literature. Early analysis (Pigou, 1920; Robinson, 1933) and subsequent development (Schmalensee, 1981; Varian, 1985) considered exogenously fixed market segmentations and studied conditions under which such segmentations would increase or decrease total surplus.

This literature has recently undergone a transformation, prompted by both technical innovations in microeconomic theory and the change in character of the practice of price discrimination brought about by the ascent of digital markets. Recent developments incorporate an information design approach to study the welfare impacts of third-degree price discrimination over *all possible* market segmentations, rather than taking a segmentation as exogenously fixed. Bergemann et al. (2015) analyze a setting with a monopolist selling a single good and characterize attainable pairs of consumer and producer surplus, showing that any distribution of total surplus over consumers and producer that guarantee at least the uniform-price profit for the producer is attainable. In particular, they show that there are typically many consumer-optimal segmentations of a given market. Their analysis has been extended to multi-product settings by Haghpanah and Siegel (2022a,b) and to imperfect competition settings by Elliott, Galeotti, Koh, and Li (2021) and Ali, Lewis, and Vasserman (2022). Hidir and Vellodi (2020) study market segmentation in a setting where the monopolist can offer one from a continuum of goods to each consumer, such that consumers, upon disclosing their information, face a trade-off between being offered their best option and having to pay a fine-tuned price. Finally, Roesler and Szentes (2017) and Ravid, Roesler, and Szentes (2022) study the inverse problem of information design to a buyer who is uncertain about the value of a good. Our paper differs from these by focusing on how surplus is distributed *across* consumers, and by studying consumer-optimal segmentations when different consumers are assigned different welfare weights. We show that, once distributional preferences are taken into account, optimal segmentations might not coincide with consumer-optimal segmentations under uniform welfare weights. When they do, our analysis selects one among the many direct consumer-optimal segmentations established in Bergemann et al. (2015).

Our paper also dialogues with a recent literature on mechanism design and redistribution, most notably with Dworczak et al. (2021) and Akbarpour, Dworczak, and Kominers (2020), who study the design of allocation mechanisms under redistributive concerns; and Pai and Strack (2022), who study the optimal taxation of a good with a negative externality when agents differ on their utility for the good, disutility for the externality and marginal value for money. A key difference in the results obtained in these papers and ours is that, in their settings, redistributive mechanisms are not pareto-efficient: redistribution implies some loss in social surplus. This is not the case in our paper, where optimal redistributive segmentations always maximize total surplus.

Finally, our paper dialogues with Dube and Misra (2022), who study experimentally the welfare implications of personalized pricing implemented through machine learning. The authors find a negative impact of personalized pricing on total consumer surplus, but note that a majority of consumers benefit from price reductions under personalization, pointing that under some inequality-averse weighted welfare functions, data-enabled price personalization might increase welfare. Their paper shows experimentally how the implementation of market segmentations aimed at maximizing profits might generate, as a by-product, the redistribution of surplus among consumers. Our paper, on the other hand, shows theoretically how consumer-optimal redistributive segmentations might grant additional profits for the firm.

2 Model

A monopolist (he) sells a good to a continuum of mass one of buyers, each of whom can consume at most one unit. We normalize the marginal cost of production of the good to zero. The consumers privately observe their type v, which represents their willingness to pay for the good, and which can take K possible values $\{v_1, \dots, v_K\} \equiv V$, where:

$$0 < v_1 < \cdots < v_K.$$

A *market* μ is a distribution over the valuations and we denote the set of all possible markets:

$$M \equiv \Delta(V) = \left\{ \mu \in \mathbb{R}^K \;\middle|\; \sum_{k=1}^{K} \mu_k = 1 \text{ and } \mu_k \geq 0 \text{ for all } k \in \{1, \dots, K\} \right\}.$$

Price v_k is **optimal for market** $\mu \in M$ if it maximizes the expected revenue of the monopolist when facing market μ[1], that is:

$$v_k \sum_{i=k}^{K} \mu_i \geq v_j \sum_{i=j}^{K} \mu_i, \quad \forall j \in \{1, \dots, K\}.$$

Let M_k denote the set of markets where price v_k is optimal:

$$M_k = \left\{ \mu \in M \;\middle|\; v_k \in \arg\max_{v_i \in V} v_i \sum_{j=i}^{K} \mu_j \right\}.$$

In the remaining of the paper we will hold an aggregate market fixed and denote it by $\mu^\star \in M$.

Segmentation.

The consumers' types are perfectly observed by a social planner (she) who can **segment** consumers, that is, sort consumers into different (sub-)markets. The set

[1]Note that we can restrict the action set of the monopolist to be equal to V, since any price $p \notin V$ is dominated by some $v \in V$.

of possible segmentations of a given aggregate market $\mu^\star$ is:

$$\Sigma(\mu^\star) \equiv \left\{ \sigma \in \Delta(M) \,\middle|\, \sum_{\mu \in \text{supp}(\sigma)} \mu \sigma(\mu) = \mu^\star, |\text{supp}(\sigma)| < \infty \right\}.^{2}$$

Formally, a segmentation is a probability distribution on M which averages to the aggregate market $\mu^\star$. The requirement that the different segments generated by a segmentation average to the aggregate market ensures that the segmentation simply sorts existing consumers into different groups, without fundamentally altering the aggregate composition of consumers in a market. This requirement is akin to the Bayes Plausibility condition that is typically used in the Bayesian Persuasion literature (Kamenica and Gentzkow, 2011).

Given a segmentation σ, the monopolist can price differently at each segment μ in the support of σ. As will become clear in section 4, segments with more than one optimal price play a key role in our results, such that we focus on the following pricing rule $p : M \to V$ applied by the monopolist:

$$p(\mu) = \min \left\{ \underset{k \in \{1,\dots,K\}}{\arg\max}\, v_k \sum_{i=k}^{K} \mu_i \right\}.$$

That is, the monopolist charges at each segment the smallest among the optimal prices in that segment. This pricing rule makes the objective of the social-planner (which is stated below) upper semicontinuous and ensures the existence of an optimal segmentation[3].

We can therefore write the *utility* of a consumer of type v_k in market μ as:

$$U_k(\mu) \equiv \max\{0, v_k - p(\mu)\}.$$

Social objective.

The social planner's objective is to maximize a weighted sum of consumers' surplus, with positive weights $\boldsymbol{\lambda} \in \mathbb{R}_+^K$, where each dimension λ_k of vector $\boldsymbol{\lambda}$ represents

[2]Where supp(x) is the support of a distribution x.

[3]Although technically important, this pricing rule does not impact our results qualitatively. Indeed, any joint distribution of consumers and prices that can be induced by the social planner under this pricing rule could be approximated arbitrarily well by a social planner facing a monopolist who selects among optimal prices in some other way.

the welfare weight attributed to consumers of type v_k. For a given market μ, the weighted consumer surplus of such market is given by:

$$W(\mu) \equiv \sum_{k=1}^{K} \lambda_k \, \mu_k \, U_k(\mu).$$

Hence, for any aggregate market $\mu^\star$, the social planner's objective is given by the following well-defined maximization program, whose value is denoted $V(\mu^\star)$:

$$\max_{\sigma \in \Sigma(\mu^\star)} \quad \sum_{\mu \in \text{supp}(\sigma)} \sigma(\mu) W(\mu). \tag{S}$$

Given an aggregate market $\mu^\star$, a segmentation $\sigma \in \Sigma(\mu^\star)$ is **optimal** if

$$\sum_{\mu \in \text{supp}(\sigma)} \sigma(\mu) W(\mu) = V(\mu^\star).$$

We focus on welfare weights that are decreasing on the consumer's willingness to pay, such that $\lambda_k \geq \lambda_{k'}$ for any $k < k' \leq K-1$, and say that the social planner has **redistributive preferences** if the inequality holds strictly for some $k, k' \in \{1, \ldots, K\}$. Under the assumption that consumers with lower willingness to pay are on average poorer than consumers with higher willingness to pay, this amounts to attributing a greater weight to surplus accruing to poorer consumers[4].

Efficiency.

Every consumer has a value for the good that is strictly greater than the marginal cost of production. Hence, social surplus is maximized when every consumer buys the good.

We say that a market μ is **efficient** if every consumer can buy the good, that is, if the lowest optimal price for the seller at that market allows everyone to consume: $p(\mu) = \min\ \text{supp}(\mu)$. A segmentation σ is **efficient** if it is only supported on efficient markets.

[4] We follow here the approach by Dworczak et al. (2021).

Informational Rents.
We denote the profit of the monopolist at a given market μ as:

$$\pi(\mu) = p(\mu) \sum_{k \in C(p(\mu))} \mu_k$$

where $C(p) = \{k \in \{1,\ldots,K\} | v_k \geq p\}$ is the set of consumer types that buy the good given a price p. Similarly, the profit of the monopolist under a given segmentation σ is denoted as:

$$\Pi(\sigma) = \sum_{\mu \in \text{supp}(\sigma)} \sigma(\mu)\pi(\mu)$$

We know that a segmentation σ can only weakly increase the profit of a monopolist, such that, $\Pi(\sigma) \geq \pi(\mu^\star)$, $\forall \sigma \in \Sigma(\mu^\star)$. We say that a segmentation σ grants a **rent** to the monopolist whenever this inequality holds strictly.

Uniformly Weighted Consumer-Optimal Segmentations.
If $\lambda_k = \lambda'_k > 0$ for all $k, k' \in \{1,\ldots,K\}$, program (S) maximizes total (or average) consumer surplus over all possible segmentations. A segmentation that solves this optimization problem is named **uniformly weighted consumer-optimal**. As shown in Bergemann et al. (2015), uniformly weighted consumer-optimal segmentations are (i) efficient—and hence achieve the maximum feasible social surplus[5]—, and (ii) do not grant the monopolist any rent.

Typically, for an interior aggregate market $\mu^\star$, there exists infinitely many uniformly weighted consumer-optimal segmentations. In section 4.3, we characterize the set of aggregate markets for which consumer-optimal segmentations *with redistributive preferences* are also uniformly weighted consumer-optimal, thus providing a natural way to select among these segmentations for such markets.

[5]For a given market μ, the maximum feasible social surplus is given by

$$s(\mu) = \sum_k \mu_k v_k.$$

Note that a segmentation of μ achieves $s(\mu)$ if and only if it is efficient.

Discussion of the Model

Information Provision as Segmentation.
In digital markets, information provision about consumers often occurs through the assignment of *labels* to different consumers. Indeed, one could think of a model in which the social planner adopts a labeling strategy $\Psi : V \to \Delta(L)$, where L is the set of labels that she distributes. The meaning of each label is then pinned down by the social planner's strategy, and the monopolist optimally chooses different prices for consumers with different labels.

Such a model is equivalent to ours. Indeed, any segmentation $\sigma \in \Sigma(\mu^\star)$ can be implemented by some labeling strategy Ψ, and any labeling strategy Ψ implements some segmentation $\sigma \in \Sigma(\mu^\star)$. The approach of working directly in the space of feasible distributions over distributions of types, rather than in the space of distributions of signals, is standard in the information design literature (Kamenica, 2019).

Continuum of Consumers.
While we consider a setting with a continuum of consumers, our model is equivalent to one in which there is a discrete number of consumers, with types independently distributed according to $\mu^\star$. Under this interpretation, the social planner commits ex-ante to an information structure σ to inform the monopolist, which defines the distribution of posterior beliefs μ that the monopolist will form upon facing each consumer.

3 Three-Value Case

In this section, we illustrate our model and some of the results from the following sections in the simple three-value case.

Setup.
Let's consider three types, $V = \{v_1 = 1, v_2 = 2, v_3 = 3\}$. We can conveniently depict the set of markets M as the two-dimension probability simplex (see Mas-Colell, Whinston, and Green, 1995, p.169). It is depicted in figure 1, where each vertex of

the simplex represents a degenerate market on a value $v \in V$, denoted by the Dirac measure δ_v.

In the left panel of figure 1 are drawn the three different price regions M_1, M_2 and M_3. The points in each of the regions correspond to the markets for which each of the different prices $\{1,2,3\}$ are optimal for the monopolist[6]. The border between two adjacent regions represents markets for which there are more than one optimal price. Given pricing rule p, the price charged in such markets is the lowest amongst the optimal.

In the right panel, an aggregate market $\mu^\star = (0.3, 0.4, 0.3)$ is represented, which is in the interior of the region M_2, meaning that v_2 is a strictly optimal price for $\mu^\star$. Two possible segmentations are depicted: the one in green dashed lines, that segments $\mu^\star$ into the three degenerate markets (thus implementing first-degree price discrimination); and the one in black dotted lines, that segments $\mu^\star$ into three segments: μ', containing types all three types and being priced v_1; μ'', containing only types v_2 and v_3 and being priced v_2; and μ''', containing all three types and being priced v_3.

Any splitting of $\mu^\star$ into a set of points $S \subset M$ represents a feasible segmentation, as long as $\mu^\star \in co(S)$[7]. A segmentation is optimal given weights $(\lambda_1, \lambda_2, \lambda_3)$, with $\lambda_1 \geq \lambda_2 \geq \lambda_3$, if it maximizes the sum of weighted consumer surplus over all segments generated. Note that consumers of type v_1 never get any consumer surplus (since the monopolist never charges a price lower than their willingness to pay), such that the optimal segmentation trades-off surplus obtained by types v_2 and v_3. We will focus, without loss of generality, on direct segmentations, i.e. segmentations in which there is not more than one segment with a given price.

General Properties of Optimal Segmentations.

A first step for finding the optimal segmentation of $\mu^\star$ is to observe that any optimal segmentation must be efficient. To see that, consider the black dotted segmentation in the right panel of figure 1. Both μ' and μ'' are efficient, since all the consumers in these segments are able to buy the good. The remaining segment

[6] Formally, for any k, $M_k = \text{cl}(p^{-1}(v_k))$, where $\text{cl}(S)$ denotes the topological closure of a generic set S.

[7] For any set S, $co(S)$ denotes the convex hull of S

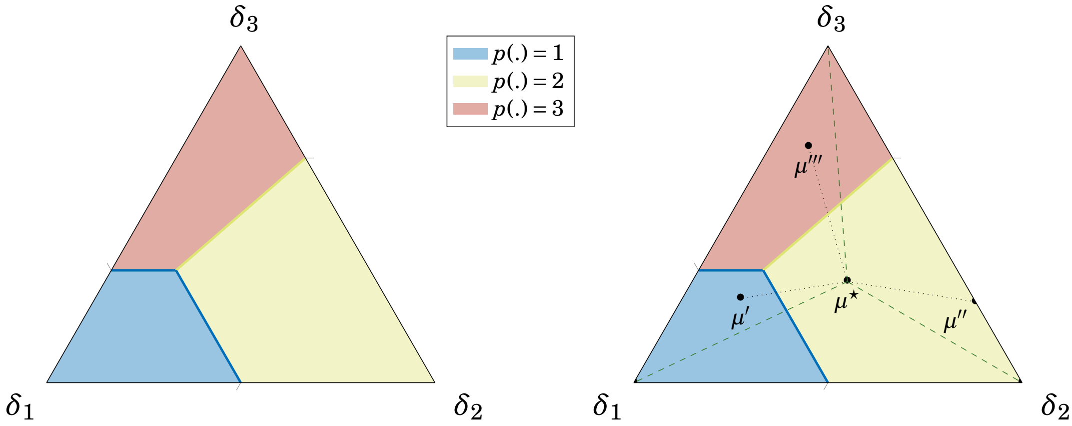

Figure 1: The Simplex representing M and two feasible segmentations.

μ''', however, is not efficient, as it contains some consumers with type v_1 and v_2 who are not able to consume under that segment's price. One could solve that by re-segmenting μ''' in the following way: creating a segment μ_b''' containing all of the types v_1 and v_2 and some of the types v_3 that used to belong to μ''', and another segment δ_3 containing only the remaining types v_3. Note that the amount of type v_3 in μ_b''' can be adjusted to ensure that this segment will have price v_1. That way, both of the resulting segments will be efficient. Furthermore, this re-segmentation of μ''' *unambiguously* increases consumer welfare, since it has no impact on the welfare of consumers in μ' and μ'' and (weakly) increases the surplus of every consumer previously belonging to μ'''.

Indeed, a welfare-increasing segmentation can be performed to any inefficient market. This narrows down the search for an optimal segmentation, as we know that it must be supported *only* on efficient segments. The left panel of figure 2 depicts, in orange, the efficient markets. These are: the degenerate market δ_3; the set of markets in region M_2 that have no consumer with value 1; and the entire region M_1.

We can further note that, in an optimal segmentation, the segment with price v_1 must not belong to the interior of region M_1. To see that, consider the right panel of figure 2. In it are depicted two segmentations: σ_a, which splits $\mu^\star$ into μ_a and μ', and σ_b, which splits $\mu^\star$ into μ_b and μ'. Segmentation σ_b is always preferred over σ_a for two reasons. First, μ_b has a higher share of types v_2 and v_3 than μ_a.

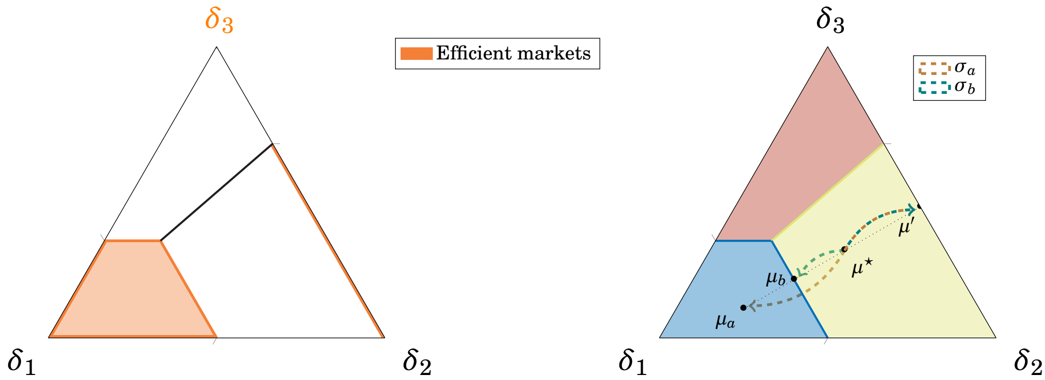

Figure 2: Efficient Markets and Segmentations.

Since these are the only two types that are extracting surplus on the segment whose price is v_1, having a higher share of them increases the social planner's objective. Second, μ_b is "closer" to $\mu^\star$, which means that $\sigma_b(\mu_b) > \sigma_a(\mu_a)$. That means that segmentation σ_b is able to include a bigger mass of consumers in the segment where they will extract the largest surplus, thus also increasing the social planner's objective.

The argument outlined above illustrates how every segmentation generating a segment on the interior of region M_1 must be dominated by some segmentation that instead generates a segment on the boundary of regions M_1 and M_2. This amounts to saying that any optimal segmentation must include a segment in which the monopolist is indifferent between charging price v_1 or charging some other price. The intuition for that is simple: if the monopolist strictly prefers to charge price v_1 in that segment, then there's still room for "fitting" other types in that segment in a Pareto improving way.

Uniformly Weighted Consumer-Optimal Segmentations.

We begin by considering the case where $\lambda_1 = \lambda_2 = \lambda_3$. The left panel of figure 3 depicts three different segmentations, σ_a, σ_b and σ_c, each of them generating one segment with price v_1 and one segment with price v_2. All of these three segmentations are uniformly weighted consumer-optimal. This follows from the fact that i) they maximize total (consumer + producer) surplus, since they are all efficient, and ii) the monopolist does not get any of the surplus that is created from the segmentation [8].

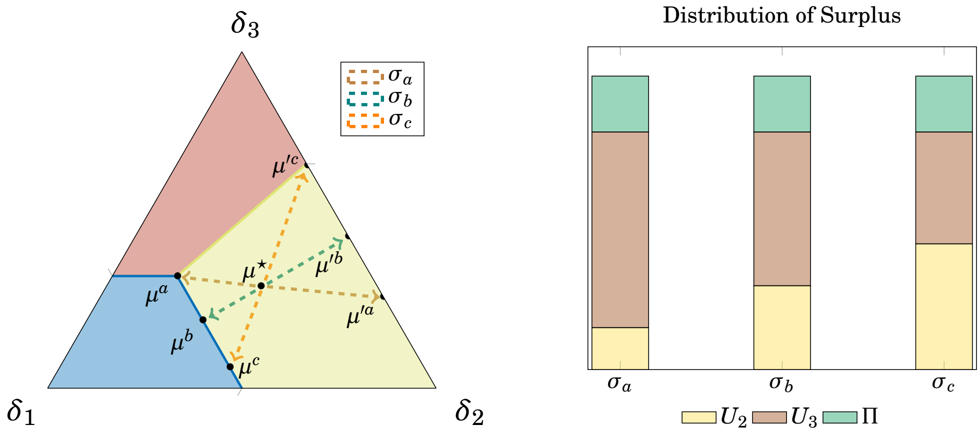

Figure 3: Uniformly Weighted Consumer-Optimal Segmentations.

Indeed, there are uncountably many uniformly weighted consumer-optimal segmentations of $\mu^{\star}$. All of these are equivalent in that they maximize total consumer surplus, but they are not equivalent in how they distribute such surplus *across* consumers. This can be seen in the right panel of figure 3: while the three segmentations of the left panel induce the same profit for the monopolist and the same total consumer surplus, σ_c induces greater surplus for consumers of type v_2 than the other segmentations. This is so because, among the segments priced at v_1, μ_c is the one that includes the most consumers of type v_2, who can then benefit from a low price.

Consumer-Optimal Segmentations under Redistributive Preferences. Let's now consider the case when $\lambda_2 > \lambda_3$. Among the segmentations depicted in the left panel of figure 3, segmentation σ_c is now preferred over σ_a and σ_b. But is it optimal? One way of increasing the surplus of consumers of type v_2 further is to exchange consumers between the two segments generated by σ_c: by

[8] One way of seeing this is as follows: A decision-maker strictly benefits from observing a piece of information if, as a result of this observation, she is able to make better decisions than she would have made absent this information. In our setting, this amounts to the monopolist being able to, as a result of the segmentation, choose *different* prices than the uniform price, at markets in which these different prices are *strictly* preferred over the uniform price. Since price v_2 belongs to the set of optimal prices in every segment generated by the segmentations in figure 3, the monopolist does not strictly benefit from them.

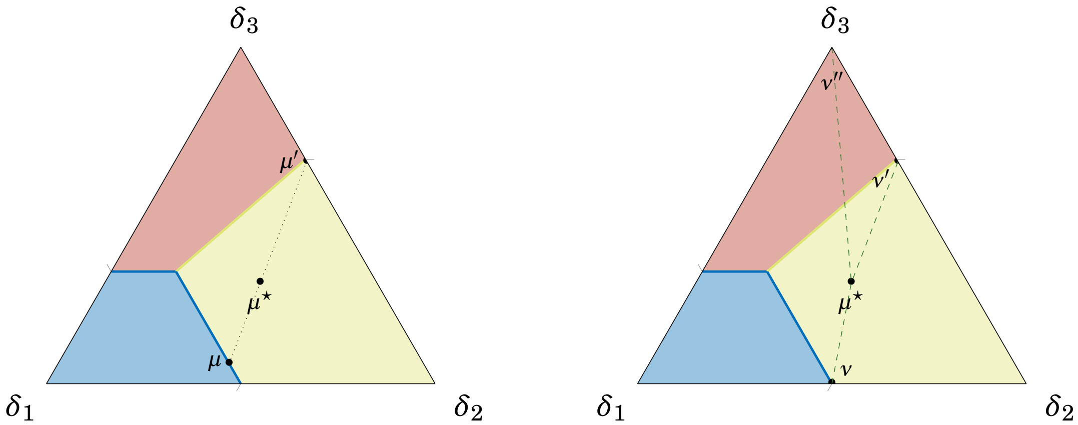

Figure 4: Optimal Segmentations with Redistributive Preferences.

exchanging the remaining consumers of type v_3 that are present in μ^c against some of the consumers of type v_2 present in μ'^c, one can increase the amount of types v_2 that pay a low price. While this exchange increases the surplus of types v_2, it dramatically decreases the surplus of types v_3, since now there are sufficiently many of them in segment μ'^c for the monopolist to want to increase the price charged at that segment. This would lead to a segmentation that is no longer uniformly weighted consumer-optimal: the price increase in segment μ'^c would cause some of the surplus that was previously captured by consumers of type v_3 to now be granted to the monopolist instead. The result below establishes when this exchange is desirable from the social planner's perspective.

Result 1. *Let* $\mu^\star = (0.3, 0.4, 0.3)$*:*

1. *for* $\frac{\lambda_2}{\lambda_3} < \frac{v_3+v_2-v_1}{v_2-v_1}$*, the consumer-optimal segmentation under redistributive preferences is also uniformly weighted consumer-optimal and generates two segments: one containing types* $\{v_1, v_2, v_3\}$ *and the other one only containing types* $\{v_2, v_3\}$*. This segmentation is represented in the left panel of figure 4;*

2. *for* $\frac{\lambda_2}{\lambda_3} > \frac{v_3+v_2-v_1}{v_2-v_1}$*, the consumer-optimal segmentation under redistributive preferences is* ***not*** *uniformly weighted consumer-optimal and generates three segments: the first containing types* $\{v_1, v_2\}$*, the second containing types* $\{v_2, v_3\}$ *and the third containing only types* $\{v_3\}$*. This segmentation is represented in the right panel of figure 4.*

An important consequence of this result is that if the social planner's preferences are sufficiently redistributive, meaning that λ_2 is sufficiently greater than λ_3, the optimal segmentation might give a *rent* (i.e. an additional profit) to the monopolist. By packing more consumers with lower types together, the social planner also makes higher types more distinguishable, thus allowing the monopolist to raise their prices. The above example illustrates the main argument of the paper: while market segmentation can redistribute surplus without any loss of efficiency, sometimes raising the surplus of poorer consumers can only be done if some of the surplus from richer consumers is granted to the monopolist.

However, not every aggregate market requires the granting of rents to the monopolist in order to satisfy redistributive objectives. Consider for instance the aggregate market $\mu^\star = (0.2, 0.65, 0.15)$, represented in the left panel of figure 5. The optimal segmentation of this market given **any** preferences $\lambda_2 \geq \lambda_3$ is the one depicted in the figure: it always generates a segment with $\{v_1, v_2\}$ and another one with $\{v_2, v_3\}$, and this segmentation is always uniformly weighted consumer-optimal. On this aggregate market, satisfying a redistributive objective never requires granting rents to the monopolist because it contains sufficiently many consumers of type v_2, such that even after pooling as many as possible of them with types v_1 in segment μ, there are still sufficiently many types v_2 left to ensure that types v_3 will not be over-represented in segment μ'.

The result below characterizes the set of aggregate markets that, under a sufficiently strong redistributive motive, would require granting rents to the monopolist. We denote this set as the **rent region**.

Result 2. *The rent region is:*

$$\text{Int}\Big(\text{co}\big(\{\delta_3, \mu^{123}, \mu^{12}, \mu^{23}\}\big)\Big).^{9}$$

This result is illustrated in the right panel of figure 5, where the rent region is depicted in orange. Equivalently, the complement of this set denotes the aggregate markets for which any redistributive objective can be met without granting rents to the monopolist — that is, while maximizing total consumer surplus—. We call this set the **no-rent region**. The following section generalizes the insights presented

[9] $\text{Int}(S)$ and $\text{co}(S)$ are respectively the *interior* and the *convex hull* of the set S.

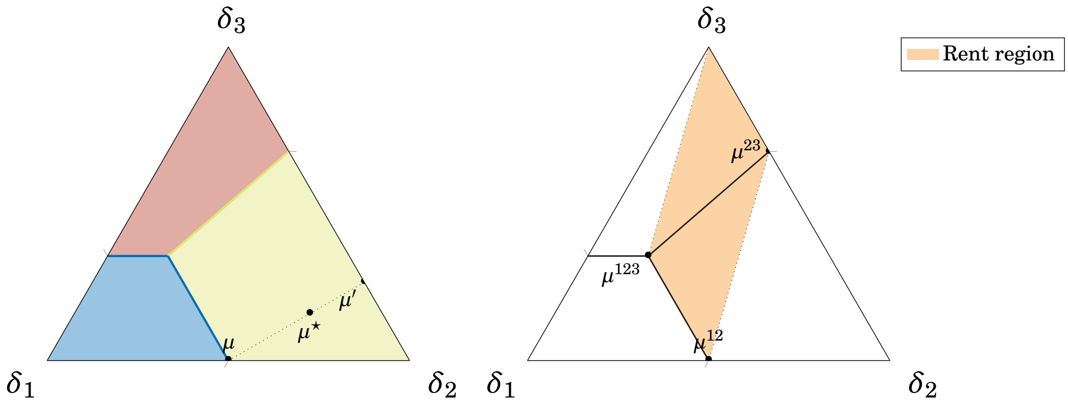

Figure 5: Rent Region.

through this example. Section 4.1 generalizes the fact that optimal segmentations are efficient and include discount segments supported at markets at which the monopolist is indifferent between more than one price, while section 4.2 establishes properties of optimal segmentations when the redistributive motive is sufficiently strong and shows how to construct optimal segmentations in this case. Finally, section 4.3 characterizes generally the no-rent and rent regions and shows that optimal segmentations for markets belonging to the no-rent region exhibit a very simple form, with only one discount segment and one uniform price segment.

4 Optimal Segmentations

We now turn to the analysis of the general case. In section 4.1 we derive general properties of optimal segmentations — that is, characteristics that are present in optimal segmentations given any decreasing welfare weights $\boldsymbol{\lambda}$. Section 4.2 then constructs optimal segmentations under strongly redistributive preferences: when the weight assigned to lower types is sufficiently larger than the weight assigned to higher types. Finally, section 4.3 characterizes the set of aggregate markets for which satisfying a redistributive objective might require granting additional profits to the monopolist.

4.1 General Properties

Efficient segmentations. Our first result echoes our analysis of efficiency in the three-value case and establishes that i) we can always restrict ourselves to efficient segmentations—as long as the weights are non-negative; ii) if the weights are all strictly positive (i.e. if $\lambda_K > 0$ under our assumption of decreasing weights), only efficient segmentations can be optimal.

Proposition 1. *For any aggregate market $\mu^\star$ and any weights $\boldsymbol{\lambda} \in \mathbb{R}^K_+$ (not necessarily decreasing), there exists an efficient optimal segmentation of $\mu^\star$. Furthermore, if every weight is strictly positive ($\boldsymbol{\lambda} \in \mathbb{R}^K_{++}$), any optimal segmentation is efficient.*

This result is a direct consequence of Proposition 1 in Haghpanah and Siegel (2022b)—which itself follows from the proof of Theorem 1 in Bergemann et al. (2015). Indeed, their result states that any inefficient market can be segmented in a Pareto improving manner, that is, in a way that weakly increases the surplus of all consumers. This implies that, as long as the social planner does not assign a negative weight to any consumer, there must be an efficient optimal segmentation. As a consequence, the social planner's redistributive objective never comes at the expense of efficiency.

Direct segmentations. A segmentation σ is **direct** if all segments in σ have different prices, that is, if for any $\mu, \mu' \in \text{supp}(\sigma)$, $p(\mu) \neq p(\mu')$. Our next lemma shows that it is without loss of generality to focus on direct segmentations.

Lemma 1. *For any aggregate market $\mu^\star$ and any segmentation $\sigma \in \Sigma(\mu^\star)$, there exists a direct segmentation $\sigma' \in \Sigma(\mu^\star)$ such that,*

$$\sum_{\mu \in \text{supp}(\sigma)} \sigma(\mu) W(\mu) = \sum_{\mu \in \text{supp}(\sigma')} \sigma'(\mu) W(\mu).$$

We further show that there always exists an optimal and direct segmentation that is only supported on the boundaries of price regions $\{M_k\}_k$. For this, denote for any aggregate market $\mu^\star$, $I(\mu^\star) \equiv \{k \mid v_k \in \text{supp}(\mu^\star)\}$, the set of indices k such that v_k is in the support of $\mu^\star$.

Lemma 2. *For any aggregate market $\mu^\star$ that is not efficient, there exists an optimal direct segmentation supported on boundaries of sets $\{M_k\}_{k\in I(\mu^\star)}$.*

4.2 Strongly Redistributive Social Preferences

In this section, we derive some characteristics of the optimal segmentation when the social planner's preferences are *strongly redistributive*, that is, when the weights $\boldsymbol{\lambda}$ are strongly decreasing on the type v.

Definition 1. *The weights $\boldsymbol{\lambda}$ are κ-**strongly redistributive** if, for any $k < k' \leq K-1$, $\frac{\lambda_k}{\lambda_{k'}} \geq \kappa$.*

That is, a social planner exhibits κ-strongly redistributive preferences (κ-SRP) if the weight she assigns to a consumer of type v_k is at least κ times larger than the weight she assigns to any consumer of type greater than v_k.

Before stating the main result of this section, let us formally define the *dominance* ordering between any two sets.

Definition 2. *Let $X, Y \subset \mathbb{R}$, X **dominates**[10] Y, denoted $X \geqslant_D Y$, if for any $x \in X$ and any $y \in Y$, $x \geq y$.*

Proposition 2. *For any aggregate market $\mu^\star$ in the interior of M, there exists $\underline{\kappa}$ such that if $\boldsymbol{\lambda}$'s are $\underline{\kappa}$-strongly redistributive, then for any optimal direct segmentation $\sigma \in \Sigma(\mu^\star)$ and any markets $\mu, \mu' \in \operatorname{supp}(\sigma), \mu \neq \mu'$: either $\operatorname{supp}(\mu) \geqslant_D \operatorname{supp}(\mu')$ or $\operatorname{supp}(\mu') \geqslant_D \operatorname{supp}(\mu)$.*

The result stated above establishes that, when the social planner's preferences exhibit a sufficiently strong taste for redistribution, optimal segmentations divide the type space V into contiguous overlapping intervals, with the overlap between any two segments being composed of at most one type. The following corollary is a direct consequence of proposition 2:

Corollary 1. *For any aggregate market $\mu^\star$ in the interior of M, there exists $\underline{\kappa}$ such that if $\boldsymbol{\lambda}$'s are $\underline{\kappa}$-strongly redistributive, then for any optimal direct segmentation $\sigma \in \Sigma(\mu^\star)$, any market $\mu \in \operatorname{supp}(\sigma)$ and any k such that $\min\{\operatorname{supp}(\mu)\} < v_k < \max\{\operatorname{supp}(\mu)\}$: $\sigma(\mu)\mu_k = \mu_k^\star$.*

[10] Note that this definition of dominance is stronger than the notion of dominance in the strong set order (Topkis, 1998).

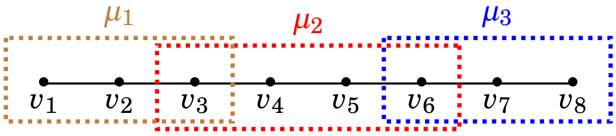

Figure 6: Structure of optimal segmentations under strong redistributive preferences.

The above result states that any segment μ belonging to a segmentation that is optimal under strong redistributive preferences contains *all* of the consumers with types strictly in-between $\min\{supp(\mu)\}$ and $\max\{supp(\mu)\}$. Together with proposition 2, it implies that, under κ-SRP optimal segmentations, every consumer type v will belong to *at most* two segments: either it will belong to the interior of the support of a segment μ, such that all consumers of this type have surplus $v - \min(supp(\mu))$, or it will be the boundary type between two segments μ and μ', such that a fraction of these consumers (those belonging to segment μ) gets surplus $v - \min(supp(\mu))$ and the rest gets no surplus. The structure of optimal segmentations under strong redistributive preferences is illustrated in figure 6.

These results, along with proposition 1, completely pin down the κ-SRP optimal direct segmentation. One can construct it by employing the following procedure, presented as follows through steps:

- **Step i)** Start by creating a segment — call it μ_a — with all consumers of type v_1.

- **Step ii)** Proceed to including in μ_a, successively, all consumers of type v_2, then all of the types v_3, and so on. From proposition 1 we know that μ_a must be efficient, meaning that we must have $p(\mu_a) = v_1$. As such, the process of inclusion of types higher than v_1 must be halted at the point in which adding a new consumer in μ_a would result in v_1 no longer being an optimal price in this segment. We denote as $v_{(a|b)}$ the type that was being included when the process was halted.

- **Step iii)** Create a new segment — call it μ_b — with all of the remaining types $v_{(a|b)}$.

- **Step iv)** Proceed to including in μ_b, successively, all of consumers of type

$v_{(a|b)+1}$, then all of the types $v_{(a|b)+2}$, and so on. Halt this process at the point in which adding a new consumer in μ_b would result in $v_{(a|b)}$ no longer being an optimal price in this segment. We denote as $v_{(b|c)}$ the type that was being included when the process was halted.

- **Step v)** Create a new segment with all of the remaining types $v_{(b|c)}$. Repeat the process described in the last steps until every consumer has been allocated to a segment.

4.3 Optimal Segmentations and Informational Rents

This section explores the question of when does an optimal segmentation maximize total consumer surplus or, conversely, when it grants a rent for the monopolist.

Say that an aggregate market $\mu^\star$ belongs to the **rent region** if there exists some $\underline{\kappa}$ such that if the social planner has $\underline{\kappa}$-strongly redistributive preferences, the optimal segmentation grants a rent to the monopolist. Conversely, denote **no-rent region** the set of aggregate markets for which any optimal segmentation with redistributive preferences also maximizes total consumer surplus.

Before we characterize the rent and no-rent regions, we define a particular segmentation, which we will call σ^{NR}:

Definition 3. *Let $\mu^\star$ be an aggregate market with uniform price v_u. Call σ^{NR} the segmentation that splits $\mu^\star$ into two segments μ^s and μ^r, with:*

$$\mu^s = \left(\frac{\mu_1^\star}{\sigma}, \frac{\mu_2^\star}{\sigma}, \dots, \mu_u^s, 0, \dots, 0\right),$$
$$\mu^r = \left(0, 0, \dots, \mu_u^r, \frac{\mu_{u+1}^\star}{1-\sigma}, \dots, \frac{\mu_K^\star}{1-\sigma}\right),$$

where $\mu_u^s = \frac{v_1}{v_u}$, $\mu_u^r = \frac{\mu_u^\star - \sigma\mu_u^s}{1-\sigma}$ and $\sigma = \frac{v_u \sum_{i=1}^{u-1} \mu_i^\star}{v_u - v_1}$.

Segmentation σ^{NR} is very simple and generates only two segments: one pooling all the consumers who would not buy the good on the unsegmented market (those with type lower than v_u) and another one pooling all the consumers who would buy the good on the unsegmented market (those with type higher than v_u). Under

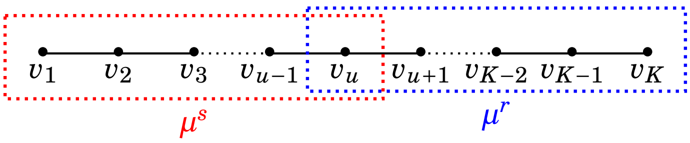

Figure 7: Segmentation σ^{NR}.

segmentation σ^{NR}, the only consumer type that gets assigned to two different segments is v_u.

Proposition 3. *An aggregate market $\mu^\star$ belongs to the no-rent region if and only if σ^{NR} is an efficient segmentation of $\mu^\star$.*

Proposition 3 establishes a simple criterion that defines whether an aggregate market belongs to the no-rent region: it suffices to check if, under σ^{NR}, $p(\mu^s) = v_1$ and $p(\mu^r) = v_u$. Whenever this is not true, the aggregate market belongs to the rent region.

Corollary 2. *Consider an aggregate market $\mu^\star$. If σ^{NR} is not an efficient segmentation of $\mu^\star$, then there exists $\underline{\kappa}$ such that, if welfare weights $\boldsymbol{\lambda}$ are $\underline{\kappa}$-strongly redistributive, any optimal segmentation grants a rent to the monopolist.*

The intuition for the results above is as follows. A market belongs to the no-rent region if, given any redistributive preferences, its optimal segmentation maximizes total consumer surplus. On one hand, we know from proposition 2 that, under strong redistributive preferences, optimal segmentations divide the type space into overlapping intervals, with the overlap between two segments being comprised of at most one type. On the other hand, we have as a necessary and sufficient condition for total consumer surplus to be maximized that the segmentation is i) efficient and ii) the uniform price v_u is an optimal price at *every* segment generated by this segmentation. Condition i) ensures that total surplus is maximized, while condition ii) ensures that producer surplus is kept at it's uniform price level, meaning that all of the surplus created by the segmentation goes to consumers. Since condition ii) can only be satisfied if type v_u belongs in the support of all segments, we get that the conditions for optimality under strong redistributive preferences and for total consumer surplus to be maximized can only be simultaneously met by a segmentation that only generates two segments, with the overlap in the support of both segments being comprised of v_u.

Such a segmentation indeed maximizes total consumer surplus if it is efficient and if v_u is an optimal price in both segments. This is the case if v_1 and v_u are both optimal optimal prices on the lower segment, and if v_u is an optimal price in the upper segment. Segmentation σ^{NR} is the *only* segmentation that can potentially satisfy all of these conditions at once, as it includes in the lower segment the exact proportion of types v_u that would make the monopolist indifferent between charging a price of v_1 or v_u. As such, segmentation σ^{NR} maximizes total consumer surplus if and only if it is efficient.

Corollary 3. *If an aggregate market $\mu^\star$ belongs to the no-rent region, then σ^{NR} is its only direct consumer-optimal segmentation under any redistributive preferences.*

This result establishes that, for markets in the no-rent region, optimal segmentations have an extremely simple structure: they only generate a discount segment with price v_1, pooling all the types who would not consume under the uniform price and some of the types v_u, and a residual segment with price v_u, containing all of the remaining consumers. Furthermore, this segmentation must be optimal under *any* decreasing welfare weights $\boldsymbol{\lambda}$. As such, this result selects for the markets belonging to the no-rent region one among the many uniformly weighted consumer-optimal segmentations that were outlined in Bergemann et al. (2015).

Due to the structure of segmentation σ^{NR}, all of the surplus that is generated by the segmentation is given to consumers with types below or equal to v_u, all of which get the maximum surplus they could potentially get. Since it is impossible to raise the surplus of any type below v_u, and impossible to raise the surplus of types above v_u without redistributing from lower to higher types, this segmentation must be optimal whenever the weights assigned to different consumers are (weakly) decreasing on the type.

The results in this section establish that there are essentially two types of markets: those for which redistribution can be done only within consumers, while keeping total consumer surplus maximal, and those for which increasing the surplus of lower types past a certain point necessarily decreases the total pie of surplus accruing to consumers and grants additional profits to the monopolist.

Appendix

Appendix A Proof of section 4.1

Proof of lemma 1. Let $\sigma \in \Sigma$ and suppose that there exist $\mu, \mu' \in \text{supp}(\sigma)$ with $p(\mu) = p(\mu')$. Consider the following market:

$$\tilde{\mu} = \frac{\sigma(\mu)}{\sigma(\mu)+\sigma(\mu')}x + \frac{\sigma(\mu')}{\sigma(\mu)+\sigma(\mu')}x'.$$

By the convexity of $X_{p(\mu)}$, $p(\tilde{\mu}) = p(\mu)$. Define σ' in the following way: $\sigma'(\tilde{\mu}) = \sigma(\mu)+\sigma(\mu')$, $\sigma'(\mu) = \sigma'(\mu') = 0$ and $\sigma' = \sigma$ otherwise. Is it easy to check that $\sum_{\mu\in\text{supp}(\sigma)} \sigma(\mu)W(\mu) = \sum_{\mu\in\text{supp}(\sigma')} \sigma'(\mu)W(\mu)$. We can iterate this operation as many times as the number of pairs $\nu, \nu' \in \text{supp}(\sigma')$ such that $p(\nu) = p(\nu')$ to finally obtain the desired conclusion. □

Proof of lemma 2. Let $\mu^\star$ be an inefficient aggregate market, hence for any optimal segmentation $\sigma \in \Sigma(\mu^\star)$, $|\text{supp}(\sigma)| \geq 2$. Let σ be a direct and optimal segmentation of $\mu^\star$ and $\mu \in \text{supp}(\sigma)$ such that μ is in the interior of $X_{p(\mu)}$. Let ν be any other market in the support of σ. Consider the market:

$$\xi = \frac{\sigma(\mu)}{\sigma(\mu)+\sigma(\nu)}\mu + \frac{\sigma(\nu)}{\sigma(\mu)+\sigma(\nu)}\nu.$$

Because $\mu^\star$ is inefficient, it is without loss of generality to assume that ξ is also inefficient.

Denote $\bar{\mu}$ (resp. $\bar{\nu}$) the projection of ξ on the boundary of the simplex M in direction of μ (resp. ν). For σ to be optimal, the segmentation of ξ between μ with probability $\frac{\sigma(\mu)}{\sigma(\mu)+\sigma(\nu)}$ and ν with probability $\frac{\sigma(\nu)}{\sigma(\mu)+\sigma(\nu)}$ must be optimal. In particular, it must be optimal among any segmentation on $[\bar{\mu}, \bar{\nu}]$.

There exists a one-to-one mapping $f : [\bar{\mu}, \bar{\nu}] \to [0,1]$ such that for any $\gamma \in [\bar{\mu}, \bar{\nu}]$, $\gamma = f(\gamma)\bar{\mu} + (1-f(\gamma))\bar{\nu}$. Thus, the set $[\bar{\mu}, \bar{\nu}]$ can be seen as all the distributions on a binary set of states of the world $\{\bar{\mu}, \bar{v}\}$, where for any $\gamma \in [\bar{\mu}, \bar{v}]$, $f(\gamma)$ is the probability of $\bar{\mu}$.

Therefore, the maximization program,

$$\max_{\sigma} \sum_{\gamma \in \text{supp}(\sigma)} \sigma(\gamma) W(\gamma) \tag{$\bar{S}$}$$

$$\text{s.t. } \sigma \in \Sigma^{[\bar{\mu},\bar{v}]}(\xi) \equiv \left\{ \sigma \in \Delta([\bar{\mu}, \bar{v}]) \;\middle|\; \sum_{\gamma \in \text{supp}(\sigma)} \sigma(\gamma)\gamma = \xi, \; \text{supp}(\sigma) < \infty \right\},$$

is a bayesian persuasion problem (Kamenica and Gentzkow, 2011), with a binary state of the world and a finite number of actions. Hence, applying theorem 1 in Lipnowski and Mathevet (2017), there exists an optimal segmentation only supported on extreme points of sets $M \in \mathscr{M}^{[\bar{\mu},\bar{v}]} \equiv \left\{ M_k \cap [\bar{\mu}, \bar{v}] \mid k \in \{1, \dots, K\} \text{ and } M_k \cap [\bar{\mu}, \bar{v}] \neq \emptyset \right\}$. It happens that for any $M \in \mathscr{M}^{[\bar{\mu},\bar{v}]}$, so that $M = M_k \cap [\bar{\mu}, \bar{v}]$ for some k, if γ is an extreme point of M, then it is on the boundary of (M_k).

Let (μ', v') with respective probabilities $(\alpha, 1-\alpha)$ be a solution to ($\bar{S}$) where μ' and v' are extreme points of some $M \in \mathscr{M}^{[\bar{\mu},\bar{v}]}$. We now consider the segmentation $\bar{\sigma}$ such that $\bar{\sigma}(\gamma) = \sigma(\gamma)$ for all $\gamma \in \text{supp}(\sigma) \setminus \{\mu, v\}$, $\bar{\sigma}(\mu') = (\sigma(\mu) + \sigma(v))\alpha$, $\bar{\sigma}(v') = (\sigma(\mu) + \sigma(v))(1-\alpha)$, and $\bar{\sigma} = 0$ otherwise. One can easily check that $\bar{\sigma} \in \Sigma(\mu^\star)$. If $\bar{sigma}$ is not direct, that is, there exists $\gamma \in \text{supp}(\bar{\sigma})$ such that (w.l.o.g.) $p(\gamma) = p(\mu')$, then construct a direct segmentation $\bar{\bar{\sigma}}$ following the same process as in the proof of lemma 1. Then, if $\bar{\bar{\sigma}}$ is not only supported on boundaries of sets $\{M_k\}_{k \in I(\mu^\star)}$, reiterate the same process as above, until you reach the desired conclusion. □

Appendix B Proofs of Section 4.2.

Proof of proposition 2. Fix an aggregate market $\mu^\star$ and let $\sigma \in \Sigma(\mu^\star)$ be optimal and direct. Suppose by contradiction that there exist $\mu, \mu' \in \text{supp}(\sigma)$ such that $v_a := \min\{\text{supp}(\mu)\} < \max\{\text{supp}(\mu')\} =: v_d$ and $v_b := \min\{\text{supp}(\mu')\} < \max\{\text{supp}(\mu)\} =: v_c$. Assume further, without loss of generality, that $\min\{\text{supp}(\mu)\} < \min\{\text{supp}(\mu')\}$.

Define $\bar{\mu} := \frac{\sigma(\mu)}{\sigma(\mu)+\sigma(\mu')}\mu + \frac{\sigma(\mu')}{\sigma(\mu)+\sigma(\mu')}\mu'$. A consequence of σ being optimal is that $V(\bar{\mu}) = \frac{\sigma(\mu)}{\sigma(\mu)+\sigma(\mu')}W(\mu) + \frac{\sigma(\mu')}{\sigma(\mu)+\sigma(\mu')}W(\mu')$. The proof consists in showing that we can improve on this splitting of $\bar{\mu}$ and thus obtains a contradiction.

Define, for small $\epsilon > 0$, $\check{\mu}, \check{\mu}'$ as follows:

$$\check{\mu}_k = \begin{cases} \mu_k + \epsilon & \text{if } k = b \\ \mu_k - \epsilon & \text{if } k = c \\ \mu_k & \text{otherwise.} \end{cases}$$

$$\check{\mu}'_k = \begin{cases} \mu'_k - \frac{\sigma(\mu)}{\sigma(\mu')}\epsilon & \text{if } k = b \\ \mu'_k + \frac{\sigma(\mu)}{\sigma(\mu')}\epsilon & \text{if } k = c \\ \mu'_k & \text{otherwise.} \end{cases}$$

By construction, $\bar{\mu} = \frac{\sigma(\mu)}{\sigma(\mu)+\sigma(\mu')}\check{\mu} + \frac{\sigma(\mu')}{\sigma(\mu)+\sigma(\mu')}\check{\mu}'$. Note that v_a is still an optimal price for $\check{\mu}$. Indeed, for any $v_a \le v_k \le v_b$, the profit made by fixing price v_k is equal in markets μ and $\check{\mu}$ and for any $v_b < v_k \le v_c$ the profit made by fixing price v_k is strictly lower in $\check{\mu}$ than in μ. On the contrary, $\phi(\check{\mu}') \ge \phi(\mu')$ and it is possible that the inequality holds strictly. In any case, it must be that $\phi(\check{\mu}') = v_e$ for $b \le e \le d$. Denote $\alpha := \frac{\sigma(\mu)}{\sigma(\mu)+\sigma(\mu')}$, hence $\frac{\sigma(\mu)}{\sigma(\mu')} = \frac{\alpha}{1-\alpha}$.

$$\alpha W(\check{\mu}) + (1-\alpha)W(\check{\mu}') - \big(\alpha W(\mu) + (1-\alpha)W(\mu')\big) \tag{1}$$

$$= \alpha\big(W(\check{\mu}) - W(\mu)\big) + (1-\alpha)\big(W(\check{\mu}') - W(\mu')\big) \tag{2}$$

$$= \alpha\epsilon\big(\lambda_b(v_b - v_a) - \lambda_c(v_c - v_a)\big) \tag{3}$$

$$+ (1-\alpha)\big(-\sum_{k>e} \lambda_k \mu'_k (v_e - v_b) - \sum_{b<k\le e} \lambda_k \mu'_k (v_k - v_b) + \lambda_c \frac{\alpha}{1-\alpha}\epsilon(v_c - v_e)\big) \tag{4}$$

$$= \alpha\epsilon\lambda_b(v_b - v_a) - \alpha\epsilon\lambda_c(v_e - v_a) - (1-\alpha)\big(\sum_{k>e} \lambda_k \mu'_k (v_e - v_b) + \sum_{b<k\le e} \lambda_k \mu'_k (v_k - v_b)\big) \tag{5}$$

$$> \alpha\epsilon\lambda_b(v_b - v_a) - \alpha\epsilon\lambda_{b+1}(v_e - v_a) - (1-\alpha)\big(\sum_{k>e} \lambda_{b+1} \mu'_k (v_e - v_b) + \sum_{b<k\le e} \lambda_{b+1} \mu'_k (v_k - v_b)\big) \tag{6}$$

$$= \alpha\epsilon\lambda_b(v_b - v_a) - \lambda_{b+1}\Big[\alpha\epsilon(v_e - v_a) - (1-\alpha)\big(\sum_{k>e} \mu'_k (v_e - v_b) + \sum_{b<k\le e} \mu'_k (v_k - v_b)\big)\Big] \tag{7}$$

Finally,

$$(7) \ge 0 \iff \frac{\lambda_b}{\lambda_{b+1}} \ge \kappa$$

where

$$\kappa = \frac{\alpha\epsilon(v_e - v_a) - (1-\alpha)\big(\sum_{k>e}\mu'_k(v_e - v_b) + \sum_{b<k\leq e}\mu'_k(v_k - v_b)\big)}{\alpha\epsilon(v_b - v_a)}$$

which ends the proof. □

Appendix C Proofs of Section 4.3.

Proof of proposition 3. As argued in the core of the text, all markets with uniform price v_u belonging to no-rent region must be optimally segmented by splitting μ^* between $\mu^s = (\frac{\mu_1^*}{\sigma}, \frac{\mu_2^*}{\sigma}, \ldots, \mu_u^s, 0, \ldots, 0)$ and $\mu^r = (0, 0, \ldots, \mu_u^r, \frac{\mu_{u+1}^*}{1-\sigma}, \ldots, \frac{\mu_K^*}{1-\sigma})$. Such a segmentation indeed gives no rents to the monopolist if v_u is an optimal price in both μ^s and μ^r. That is, if:

$$v_1 = v_u\mu_u^s \geq v_j(\sum_{i=j}^{u-1}\frac{\mu_i^*}{\sigma} + \mu_u^s) \qquad \forall\, 2 \leq j \leq u-1 \tag{NR-s}$$

$$v_u \geq v_j(\sum_{i=j}^{K}\frac{\mu_i^*}{1-\sigma}) \qquad \forall\, u+1 \leq j \leq K \tag{NR-r}$$

As such, any optimal segmentation under strong redistributive preferences that maximizes consumer surplus must have $\mu_u^s = \frac{v_1}{v_u}$, $\sigma = \frac{v_u}{v_u - v_1}\sum_{i=1}^{u-1}\mu_i^*$ and $\mu_u^r = \frac{\mu_u^* v_u - \sum_{i=1}^{u}\mu_i^* v_1}{\sum_{i=u}^{K}\mu_i^* v_u - v_1}$, which pins down segmentation σ^{NR}. Conditions (NR-s) and (NR-r) are satisfied whenever σ^{NR} is efficient, which concludes the proof.

It is also interesting to note that conditions (NR-s) and (NR-r) define the no-rent region inside M_u as a convex polytope. Indeed, we can rearrange both conditions and get:

$$0 \geq -\alpha(j)\sum_{i=1}^{j-1}\mu_i^* + (1-\alpha(j))\sum_{i=j}^{u-1}\mu_i^* \qquad \forall\, 2 \leq j \leq u-1 \tag{NR-s}$$

$$-\frac{v_1}{v_j(v_u - v_1)} \geq -\beta(j)\sum_{i=u}^{j-1}\mu_i^* + \big(1-\beta(j)\big)\sum_{i=j}^{K}\mu_i^* \qquad \forall\, u+1 \leq j \leq K \tag{NR-r}$$

for $\alpha(j) = \frac{v_1(v_u - v_j)}{v_j(v_u - v_1)}$ and $\beta(j) = \frac{v_u^2}{v_j(v_u - v_1)}$.

The conditions expressed above define $K-2$ half-spaces in $\mathbb{R}^K$. The no-rent region in M_u is thus given by the closed polytope defined by the intersection of such half-spaces. We can represent such polytope as follows:

$$NRR_u = \{\mu \in M_u : A\mu \leq z\},$$

with

$$A = \begin{bmatrix} S & O_S \\ O_R & R \end{bmatrix} \in \mathbb{R}^{K-2\times K} \text{ and } z = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ -\frac{v_1}{v_{u+1}(v_u - v_1)} \\ \vdots \\ -\frac{v_1}{v_K(v_u - v_1)} \end{bmatrix} \in \mathbb{R}^{K-2}$$

where O_S and O_R are null matrices with, respectively, dimensions $(u-2)\times(u-1)$ and $(K-u)\times(K+1-u)$, and

$$S = \begin{bmatrix} -\alpha(2) & 1-\alpha(2) & \cdots & 1-\alpha(2) & 1-\alpha(2) \\ -\alpha(3) & -\alpha(3) & \cdots & 1-\alpha(3) & 1-\alpha(3) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -\alpha(u-2) & -\alpha(u-2) & \cdots & 1-\alpha(u-2) & 1-\alpha(u-2) \\ -\alpha(u-1) & -\alpha(u-1) & \cdots & -\alpha(u-1) & 1-\alpha(u-1) \end{bmatrix} \in \mathbb{R}^{(u-2)\times(u-1)},$$

$$\text{R}= \begin{bmatrix} -\beta(u+1) & 1-\beta(u+1) & \cdots & 1-\beta(u+1) & 1-\beta(u+1) \\ -\beta(u+2) & -\beta(u+2) & \cdots & 1-\beta(u+2) & 1-\beta(u+2) \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ -\beta(K-1) & -\beta(K-1) & \cdots & 1-\beta(K-1) & 1-\beta(K-1) \\ -\beta(K) & -\beta(K) & \cdots & -\beta(K) & 1-\beta(K) \end{bmatrix} \in \mathbb{R}^{(K-u)\times(K+1-u)}$$

for $\alpha(j) = \frac{v_1(v_u - v_j)}{v_j(v_u - v_1)}$ and $\beta(j) = \frac{v_u^2}{v_j(v_u - v_1)}$.

□

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