Rational Inattention and the Business Cycle Effects of Productivity and News Shocks

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This version: December 2021, first version: November 2020

Abstract

We solve a real business cycle model with rational inattention (an RI-RBC model). In the standard model, anticipated fluctuations in productivity fail to cause business cycle comovement. In response to news about higher future productivity, consumption rises but employment and investment fall. Introducing rational inattention helps produce comovement. Agents choose an optimal signal about the state of the economy. The optimal signal turns out to confound current with expected future productivity. Labor and investment demand rise after a news shock, causing an output expansion. Rational inattention also improves the propagation of a standard productivity shock, by inducing persistence.

Keywords: information choice, rational inattention, real business cycle model, news shocks, productivity shocks (*JEL*: D83, E32, E71).

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1 Introduction

The basic challenge for any business cycle model is to specify an impulse and a propagation mechanism that produce business cycle comovement. This challenge is difficult, as Barro and King (1984) first explained.[1] A key insight from the real business cycle model is that fluctuations in productivity generate comovement in the standard neoclassical economy. Employment, investment, output, and consumption move together after a productivity shock, as they do in the data in a business cycle expansion or contraction.[2]

However, this insight is sensitive to the timing of information in the model. In the real world, information about changes in productivity may become available some time before they occur. In the model, it matters if agents can learn in advance about changes in productivity. If agents can learn in advance, variables respond in ways inconsistent with a business cycle. Anticipated fluctuations in productivity do not cause comovement. Suppose productivity will rise in the future (while current productivity is unchanged). The news causes a wealth effect. Firms have no incentive to increase labor demand before productivity improves, while households reduce labor supply due to the wealth effect. As a result, hours worked fall. With capital predetermined and current productivity unchanged, output contracts. Lower saving due to the wealth effect causes a reduction in the capital stock over time. Investment declines while consumption rises. The model fails to produce comovement in response to news about future productivity. It predicts an output contraction after news that productivity will improve.[3]

It is convenient to model anticipated fluctuations in productivity as "news shocks about productivity" ("news shocks" for short). A shock drawn by nature in quarter t affects productivity in quarter $t+h$, where h is a strictly positive integer. The question is how the economy responds to a news shock before quarter $t+h$. In the standard neoclassical model, labor input, investment, and output fall while consumption rises. Labor input, investment, and output increase only once

[1] Much more recently, Jaimovich and Rebelo (2009), p.1097, write that "the ability to generate comovement is a natural litmus test for macroeconomic models. It is a test that most models fail."

[2] Kydland and Prescott (1982), Hansen (1985), Prescott (1986), and King, Plosser, and Rebelo (1988) are classic references on the RBC model.

[3] With a high elasticity of intertemporal substitution, the model predicts a rise in employment and investment and a fall in consumption. The substitution effect due to an increase in the real interest rate dominates the wealth effect in this case, pushing consumption down and labor supply up.

productivity improves. In the New Keynesian model, each firm commits to supply output at a fixed price, and therefore a rise in consumption exerts upward pressure on the demand for labor and investment. The response of the economy to a news shock depends on monetary policy. With optimal monetary policy the response is identical to the flexible-price neoclassical benchmark.[4]

In these models once information becomes available, agents absorb it completely. We move away from this feature in this paper, by introducing rational inattention into an otherwise standard RBC economy. We ask how this single friction changes the propagation of a news shock. Rational inattention is the idea that people cannot process all available information (available information is not internalized information) and they allocate attention optimally (Sims, 2003). In a rational inattention model, an agent chooses an optimal signal about the state of the economy, recognizing that a more informative signal requires more attention, which is costly. The agent takes actions based on the optimal signal, rather than based on perfect information or some exogenous incomplete information set. How does a news shock propagate when people have a limited ability to process information and can choose what information to absorb?

We consider a baseline RBC model. Neoclassical firms produce homogeneous output with capital and labor. There are no adjustment costs. Households have standard preferences for consumption and leisure. The perfect information equilibrium is familiar. We focus on the equilibrium when firms are subject to rational inattention and households have perfect information.[5]

The main qualitative insight from the paper is that rational inattention induces an increase in the firms' demand for labor and investment on impact of a positive news shock. The reason is that the optimal signal confounds current with expected future productivity. Thus, firms react on impact of a news shock as if productivity has already changed with some probability. The intuition for the optimality of a confounding signal is twofold. First, noise in signals due to rational inattention introduces delay in actions. Paying attention to future productivity helps reduce this delay in actions. Second, a one-dimensional signal that confounds current productivity with expected

[4] With suboptimal monetary policy a standard New Keynesian model (Smets and Wouters, 2007) produces comovement after news about future productivity, but the impulse response of employment turns negative once productivity improves. The same is true in a heterogeneous agent version of the model (we thank Christian Wolf for these observations). For a review of the literature on news shocks, see Lorenzoni (2011), Beaudry and Portier (2014), and Jaimovich (2017).

[5] We add rational inattention on the side of households in a later section of the paper.

future productivity requires less attention than a two-dimensional signal consisting of a signal on current productivity and a separate signal on future productivity. For these two reasons, a signal confounding current with expected future productivity is optimal.[6]

The main quantitative insight from the paper is that the rational inattention effect on labor and investment demand is strong enough to change the responses of employment and output on impact of a news shock from negative to positive, and the response of investment from negative to zero. The rational inattention effect on labor demand more than offsets the wealth effect on labor supply. Thus, employment and output increase on impact of a news shock. The rational inattention effect on investment demand offsets the wealth effect on saving supply. As a result, the response of investment on impact of a news shock equals zero, as opposed to a sizable negative number in the standard model. To arrive at these quantitative results, we solve a dynamic stochastic general equilibrium model with rational inattention, which is a non-trivial task.

Hence, the single assumption of rational inattention by firms makes the model predict an output expansion after news that productivity will improve. By assuming that households have perfect information, we stack the deck against us, because in this case the wealth effect that reduces the supply of labor and saving is fully operating. We also solve a version of the model with both firms and households subject to rational inattention. We find that comovement strengthens.

In addition, we ask if rational inattention improves the propagation of a standard productivity shock (a shock that affects productivity in the same period in which the shock is drawn). It has been a challenge for the RBC model to reproduce the persistence in the data. The first-order autocorrelations of employment, investment, and output growth are positive in the data but zero or negative in the baseline model.[7] We find that when firms are subject to rational inattention, the impulse responses of employment, investment, and output to a productivity shock become hump-shaped. Since the optimal signal contains noise, the firms' beliefs are anchored on the steady state and evolve slowly. As a result, employment, investment, and output respond with delay to a productivity shock. The first-order autocorrelations of employment, investment, and output growth

[6] In Lucas (1972) firms are assumed to observe a one-dimensional signal about nominal aggregate and relative demand. In the rational inattention RBC model with news shocks, firms choose to observe a one-dimensional signal about current and expected future productivity.

[7] This shortcoming of the RBC model was first noted by Cogley and Nason (1995) and Rotemberg and Woodford (1996).

in the model become positive and are approximately in line with the data. This finding holds true even though rational inattention is the only source of inertia and the marginal cost of attention is small.

The literature has explored a number of ways to obtain a model that predicts comovement in response to news about future productivity. Jaimovich and Rebelo (2009) modify the baseline RBC model by adding three assumptions: investment adjustment costs, variable capital utilization, and a new class of preferences. Investment adjustment costs and variable capital utilization produce an increase in input demand in response to a news shock, whereas the new preferences control the wealth effect on input supply.[8] Beaudry and Portier (2004, 2007) move to a multi-sector neoclassical setting. They introduce a complementarity so that higher output in one sector makes production more efficient in other sectors, leading to a rise in input demand. Another approach has been to combine nominal stickiness with suboptimal monetary policy. Lorenzoni (2009) analyzes a New Keynesian economy with a Taylor rule where noise in a public signal about productivity causes comovement.[9] By contrast, we explore how a single new assumption, rational inattention, changes the propagation of a news shock in the baseline RBC model. The assumption of rational inattention seems well suited to apply to the question if people have an incentive to be perfectly aware of the timing of productivity changes.

A vast empirical literature finds that a sizable fraction of movements in total factor productivity is forecastable.[10] Authors make different assumptions to identify shocks that move TFP a lot in the future and little, or not at all, on impact. In an influential paper, Beaudry and Portier (2006) show that two alternative identification assumptions in a vector autoregression both yield the result that news shocks cause business cycle comovement. The subsequent research pursues three additional approaches to identification. Papers that use patent data, either as a variable in a VAR or as an external instrument, find that news shocks produce comovement.[11] Papers that identify news

[8] Schmitt-Grohé and Uribe (2012) estimate a related augmented RBC model.

[9] Angeletos and La'O (2010) study a neoclassical model with strategic complementarity and dispersed information in which a similar noise shock causes comovement. On news and noise see also Blanchard, L'Huillier, and Lorenzoni (2013) and Chahrour and Jurado (2018).

[10] Beaudry and Portier (2006), Barsky and Sims (2011), Barsky, Basu, and Lee (2015), Kurmann and Sims (2019), Miranda-Agrippino, Hacioglu-Hoke, and Bluwstein (2020), Cascaldi-Garcia and Vukotić (2020), Görtz, Tsoukalas, and Zanetti (2020), Chahrour and Jurado (2021), and others.

[11] Miranda-Agrippino, Hacioglu-Hoke, and Bluwstein (2020), Cascaldi-Garcia and Vukotić (2020).

shocks using the max-share method of Francis et al. (2014) reach, to some extent, conflicting conclusions. The results depend on the details of the identification assumptions and on the sample period. Barsky and Sims (2011) and Kurmann and Sims (2019) do not find comovement after a news shock, while Görtz, Tsoukalas, and Zanetti (2020) who focus on data since the onset of the Great Moderation do find comovement.[12] Finally, Chahrour and Jurado (2021) identify a fundamental shock to TFP and report that macroeconomic variables exhibit business cycle comovement in anticipation of that shock.

Our model suggests that empirical researchers who study different sample periods can be expected to reach conflicting conclusions regarding comovement. Whether a news shock produces comovement in the model depends on macroeconomic volatility. In a low volatility environment (think of the Great Moderation), agents pay little attention to the macroeconomy and news shocks cause positive comovement of consumption and labor input. In a high volatility environment (think of the period before the Great Moderation), agents pay more attention and news shocks cause negative comovement. We illustrate this prediction of the model in an experiment in which we change the volatility of the productivity process. As we discuss, data from the U.S. Survey of Professional Forecasters support the view that agents pay less attention to the macroeconomy since the onset of the Great Moderation than before.

Turning to standard productivity shocks, the literature has explored the idea that moving away from full information rational expectations can improve the propagation mechanism relative to the baseline RBC model. Eusepi and Preston (2011) abandon rational expectations altogether, replacing it by adaptive learning. They find that the first-order autocorrelations of employment, investment, and output growth in the model become positive. We add rational inattention, a form of incomplete information rational expectations, to the baseline RBC model. Surprisingly, the single assumption of rational inattention turns out to be sufficient to bring the first-order autocorrelations of employment, investment, and output growth in the model approximately into

[12] The details of the identification assumptions are different in Barsky and Sims (2011), Kurmann and Sims (2019), and Görtz, Tsoukalas, and Zanetti (2020). Görtz, Tsoukalas, and Zanetti (2020) also show that when they use the identification assumptions of Barsky and Sims (2011) or Kurmann and Sims (2019) and focus on the data since the onset of the Great Moderation, they find that news shocks produce comovement. See also Görtz, Gunn, and Lubik (2020).

line with the data.[13]

Solving a DSGE model with rational inattention is challenging. One needs to solve attention problems (signal choice problems) of individual agents in a dynamic model. Furthermore, one needs to find a fixed point of an economy in which the optimal signal of an agent depends on the signals chosen by other agents. Several papers make progress solving attention problems of individual agents in a dynamic environment (Sims, 2003, Maćkowiak and Wiederholt, 2009, Woodford, 2009, Sims, 2010, Steiner, Stewart, and Matějka, 2017, Maćkowiak, Matějka, and Wiederholt, 2018, Afrouzi and Yang, 2020, Jurado, 2020, Miao, Wu, and Young, 2020, and Stevens, 2020).[14] Maćkowiak and Wiederholt (2015) solve a DSGE model with rational inattention where the physical environment is similar to a simple New Keynesian model (for example, there is no capital).[15] By contrast, here the physical environment is a neoclassical business cycle model. We adopt a guess-and-verify method to find the fixed point, at each iteration using the results of Maćkowiak, Matějka, and Wiederholt (2018) to solve agents' attention problems. One issue in the literature on rational inattention is how to define equilibrium. We assume that prices, which all agents take as given, adjust to guarantee market clearing.[16]

The next section defines the physical environment. Section 3 introduces rational inattention. Section 4 develops intuition for the effects of rational inattention, by considering special cases of the model. Section 5 studies the effects of productivity shocks and news about future productivity in the complete model. Section 6 considers a version of the model in which all agents, firms and households, are subject to rational inattention. Section 7 concludes and outlines further research. There is an online Appendix with supplementary material.

[13] Business cycle models face the challenge of matching the persistence in the macro data more generally, not only conditional on a productivity shock. See Sims (1998) for a general discussion, Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007) in the context of New Keynesian models, and Auclert, Rognlie, and Straub (2020) in the context of a heterogeneous agent New Keynesian model. Our finding may therefore be helpful also for model builders who allow for sources of fluctuations other than productivity.

[14] See the survey of rational inattention by Maćkowiak, Matějka, and Wiederholt (2021) for a summary of these papers.

[15] See also Ellison and Macaulay (2019) and Afrouzi and Yang (2020).

[16] In Maćkowiak and Wiederholt (2015), in each market one side of the market sets the price and the other side chooses the quantity.

2 Model – physical environment

We consider a baseline RBC model that allows for an additional factor of production ("an entrepreneurial input") in fixed supply. The production function is Cobb-Douglas and exhibits decreasing returns to scale in the variable factors, capital and labor. We introduce a third factor in fixed supply because to formulate the attention problem of a firm we need the firm's choice of capital and labor under perfect information, not only the capital-labor ratio, to be determinate.

Time is discrete. There is a continuum of firms indexed by $i \in [0, 1]$. All firms produce the same good using an identical technology represented by the production function

$$Y_{it} = e^{a_t} K_{it-1}^{\alpha} L_{it}^{\phi} N_i^{1-\alpha-\phi}$$

where Y_{it} is output of firm i in period t, K_{it-1} is capital input, L_{it} is labor input, and e^{a_t} is total factor productivity, common to all firms. N_i is an entrepreneurial input, specific to firm i, in fixed supply. The parameters α and ϕ satisfy $\alpha \geq 0$, $\phi \geq 0$, and $\alpha + \phi < 1$.

The capital stock of firm i evolves according to the law of motion

$$K_{it} - K_{it-1} = I_{it} - \delta K_{it-1}$$

where $\delta \in (0, 1]$ is the depreciation rate. The firm maximizes the expected discounted sum of profits or dividends. The dividend of firm i in period t, D_{it}, is given by

$$D_{it} = Y_{it} - W_t L_{it} - I_{it}$$

where W_t is the wage rate. The dividends of all firms flow to a mutual fund. Households own and trade shares in the mutual fund.[17]

Total factor productivity is determined according to the law of motion

$$a_t = \rho a_{t-1} + \sigma \varepsilon_{t-h} \tag{1}$$

where ε_t follows a Gaussian white noise process with unit variance, $\rho \in (0, 1)$, $\sigma > 0$, and $h \geq 0$. A shock drawn by nature in period t affects productivity in period $t + h$. We solve the model either with $h = 0$ (a standard productivity shock) or with $h \geq 1$ (a news shock).[18]

[17] When firm i was sold to the mutual fund, the entrepreneurial input was paid the present value of its future marginal products and in return committed to supply its service without additional payments.

[18] We also consider the case when productivity is driven by two orthogonal shocks, a standard productivity shock and a news shock. See the end of Section 4.1. For ease of exposition, we abstract from long-run growth.

There is a continuum of households indexed by $j \in [0,1]$. Each household j maximizes the expected discounted sum of utility. The discount factor is $\beta \in (0,1)$. The utility function is

$$U(C_{jt}, L_{jt}) = \frac{C_{jt}^{1-\gamma} - 1}{1-\gamma} - \frac{L_{jt}^{1+\eta}}{1+\eta}$$

where C_{jt} is consumption by household j in period t, L_{jt} is hours worked, $\gamma > 0$ is the inverse of the elasticity of intertemporal substitution, and $\eta \geq 0$ is the inverse of the Frisch elasticity of labor supply. Typically, we will set $\gamma = 1$ and $\eta = 0$. The budget constraint in period t is

$$V_t Q_{jt} - V_t Q_{jt-1} = W_t L_{jt} + D_t Q_{jt-1} - C_{jt}$$

where V_t is the price of a share in the mutual fund in period t, Q_{jt} is household j's share in the mutual fund, and $D_t \equiv \int_0^1 D_{it} di$ is the dividend from the mutual fund.

Aggregate output is $Y_t \equiv \int_0^1 Y_{it} di$. Aggregate capital and investment are defined analogously. Aggregate consumption is $C_t \equiv \int_0^1 C_{jt} dj$.

In equilibrium in every period the wage adjusts so that labor demand equals labor supply, $\int_0^1 L_{it} di = \int_0^1 L_{jt} dj$, and the price of a share in the mutual fund adjusts so that asset demand equals asset supply normalized to one, $\int_0^1 Q_{jt} dj = 1$.

The non-stochastic steady state of this economy is described in Appendix A. To solve the model when firms and households have perfect information, we log-linearize their first-order conditions and the other equilibrium conditions at the non-stochastic steady state. This yields the completely standard log-linear equilibrium conditions stated in Appendix B. We refer to the solution as the perfect information equilibrium.

3 Model – rational inattention by firms

A rationally inattentive individual cannot process all available information but can decide what information to focus on. The decision-maker in firm i chooses an optimal signal about the state of the economy. He or she maximizes the expected discounted sum of profits, recognizing that a more informative signal requires more attention, which is costly.[19] This section begins by deriving the agent's objective. We then state the agent's attention problem. Finally, we define the equilibrium

[19] The optimal signal may follow a multivariate stochastic process.

in the economy in which firms are subject to rational inattention and households have perfect information.

3.1 Loss in profit from suboptimal actions

We derive an expression for the expected discounted sum of losses in profit when actions of firm i deviate from the profit-maximizing actions – the actions the firm would take if it had perfect information in every period. To obtain this expression, we compute the log-quadratic approximation to the expected discounted sum of profits at the non-stochastic steady state.

Recall that the profit of firm i in period t is given by $Y_{it} - W_t L_{it} + (1-\delta) K_{it-1} - K_{it}$. We assume that the mutual fund instructs each firm to value profits according to the marginal utility of consumption.[20] The profit function can be written in terms of log-deviations from the non-stochastic steady state:

$$C^{-\gamma} e^{-\gamma c_t} Y \left\{ e^{a_t + \alpha k_{it-1} + \phi l_{it}} - \phi e^{w_t + l_{it}} + \left(\frac{\alpha}{\beta^{-1} - 1 + \delta} \right) \left[(1-\delta) e^{k_{it-1}} - e^{k_{it}} \right] \right\}$$

where an upper-case letter without a time subscript denotes the value of a variable in the non-stochastic steady state, and a lower-case letter denotes the log-deviation of a variable from its value in the non-stochastic steady state. The term $C^{-\gamma} e^{-\gamma c_t}$ is the marginal utility of consumption.

Taking the quadratic approximation to the expected discounted sum of profits, we obtain the following expression for the expected discounted sum of losses in profit from suboptimal actions:

$$\sum_{t=0}^{\infty} \beta^t E_{i,-1} \left[\frac{1}{2} (x_t - x_t^*)' \Theta_0 (x_t - x_t^*) + (x_t - x_t^*)' \Theta_1 \left(x_{t+1} - x_{t+1}^* \right) \right] \tag{2}$$

where $x_t \equiv (k_{it}, l_{it})'$, $x_t^* \equiv (k_{it}^*, l_{it}^*)'$, the matrices Θ_0 and Θ_1 are given by

$$\Theta_0 = -C^{-\gamma} Y \begin{bmatrix} \beta\alpha (1-\alpha) & 0 \\ 0 & \phi (1-\phi) \end{bmatrix}$$

$$\Theta_1 = C^{-\gamma} Y \begin{bmatrix} 0 & \beta\alpha\phi \\ 0 & 0 \end{bmatrix}$$

and the stochastic process x_t^* satisfies the equations

$$E_t a_{t+1} - (1-\alpha) k_{it}^* + \phi E_t l_{it+1}^* = \frac{\gamma E_t (c_{t+1} - c_t)}{1 - \beta (1-\delta)} \tag{3}$$

[20] All households have the same consumption level so long as households have perfect information.

$$a_t + \alpha k^*_{it-1} - (1-\phi)\, l^*_{it} = w_t \tag{4}$$

and the initial condition $k^*_{i,-1} = k_{i,-1}$. See Appendix C. The vector x^*_t is the *profit-maximizing* input choice when the decision-maker in the firm has perfect information in every period. Equations (3)-(4) are the usual optimality conditions for capital and labor, where E_t denotes the expectation operator conditioned on the entire history up to and including period t. Equation (3) states that the profit-maximizing capital input equates the expected marginal product of capital to the cost of capital, where the latter is proportional to the expected consumption growth rate. Equation (4) states that the profit-maximizing labor input equates the marginal product of labor to the wage. The vector x_t is an *alternative* input choice. Expression (2) gives the expected discounted sum of losses in profit when the stochastic process for the firm's actions, x_t, differs – for whatever reason – from the stochastic process for the profit-maximizing actions, x^*_t. After the quadratic approximation this loss is quadratic in $x_t - x^*_t$. The interaction term $(x_t - x^*_t)'\, \Theta_1 \left(x_{t+1} - x^*_{t+1}\right)$ appears because bringing too much capital into a period raises the optimal labor input in that period.

Maćkowiak, Matějka, and Wiederholt (2018) derive analytical results for a class of dynamic rational inattention problems known as linear quadratic Gaussian pure tracking problems. In those problems, the period t payoff is a quadratic form in the contemporaneous deviation of the action vector from some target vector, where the target vector follows a Gaussian stochastic process and does not depend on the decision-maker's own past actions. It turns out that objective (2) can be written as the objective of a pure tracking problem by redefining the vectors x_t and x^*_t.

We show in Appendix C that expression (2) is equivalent to

$$\sum_{t=0}^{\infty} \beta^t E_{i,-1} \left[\frac{1}{2} (x_t - x^*_t)'\, \Theta\, (x_t - x^*_t) \right] \tag{5}$$

where $x_t \equiv (k_{it}, l_{it} - \frac{\alpha}{1-\phi} k_{it-1})'$, $x^*_t \equiv (k^*_{it}, l^*_{it} - \frac{\alpha}{1-\phi} k^*_{it-1})'$, the matrix Θ is given by

$$\Theta = -C^{-\gamma} Y \begin{bmatrix} \beta\alpha\left(1-\alpha-\frac{\alpha\phi}{1-\phi}\right) & 0 \\ 0 & \phi\,(1-\phi) \end{bmatrix}$$

and the stochastic process x^*_t satisfies

$$x^*_t = \begin{pmatrix} \frac{1}{1-\alpha-\phi} \left[E_t a_{t+1} - \phi E_t w_{t+1} - (1-\phi) \frac{\gamma E_t (c_{t+1} - c_t)}{1-\beta(1-\delta)} \right] \\ \frac{1}{1-\phi} \left(a_t - w_t\right) \end{pmatrix}. \tag{6}$$

The first entry of the vector x_t is still the capital stock to be carried into period $t+1$, k_{it}. The second entry of the vector x_t is now the labor input *for a given capital stock*, $l_{it} - [\alpha/(1-\phi)] k_{it-1}$. The target vector x_t^* is given by equation (6). Its first entry – the profit-maximizing capital stock to be carried into period $t+1$ – is proportional to the difference between expected productivity and a weighted average of the expected wage and the cost of capital, where the expectation is conditioned on the entire history up to and including period t. Its second entry – the profit-maximizing labor input for a given capital stock – is proportional to the difference between productivity and the wage. Since the matrix Θ in objective (5) is diagonal, the best response of firm i in period t given any information set $\mathcal{I}_{it}$ is the conditional expectation of x_t^*, $x_t = E(x_t^* | \mathcal{I}_{it})$. Moreover, assuming that the firm chooses (k_{it}, l_{it}) is equivalent to assuming that the firm chooses $(k_{it}, l_{it} - \frac{\alpha}{1-\phi} k_{it-1})$ so long as the firm knows its own past action k_{it-1}, which is the case if $\mathcal{I}_{it-1} \subset \mathcal{I}_{it}$.

3.2 The attention problem of a firm

In period $t = -1$, the decision-maker in firm i chooses the stochastic process for the signal to maximize the expected discounted sum of profits, (5), net of the cost of attention. In every period $t = 0, 1, 2, ...$, the decision-maker observes a realization of the optimal signal and takes actions – chooses capital and labor.

The statement of the attention problem can be simplified, without loss of generality, based on Maćkowiak, Matějka, and Wiederholt (2018). Let x_{1t}^* denote the first element and x_{2t}^* the second element of x_t^*, $x_{1t}^* = k_{it}^*$ and $x_{2t}^* = l_{it}^* - [\alpha/(1-\phi)] k_{it-1}^*$. Suppose that x_{1t}^* and x_{2t}^* each follows a finite-order ARMA process. The vector x_t^* has a first-order VAR representation

$$\xi_{t+1} = F\xi_t + v_{t+1}$$

where v_t is a Gaussian vector white noise process, F is a square matrix, and ξ_t is a vector containing x_{1t}^* and x_{2t}^* and, if appropriate, lags of x_{1t}^* and x_{2t}^* and current and lagged ε_t. The state vector ξ_t contains all information available in period t about the current and future profit-maximizing actions. The analytical results of Maćkowiak, Matějka, and Wiederholt (2018) imply that the optimal signal is a signal about the state vector ξ_t; furthermore, without loss of generality, one can restrict attention to signals that are at most two-dimensional.[21]

[21] In Maćkowiak, Matějka, and Wiederholt (2018) the optimal action x_t^* is a scalar while in this model the optimal

The decision-maker in firm i solves:

$$\max_{G,\Sigma_\psi} \sum_{t=0}^{\infty} \beta^t \left\{ E_{i,-1}\left[\frac{1}{2}\left(x_t - x_t^*\right)' \Theta \left(x_t - x_t^*\right)\right] - \lambda I\left(\xi_t; S_{it} | \mathcal{I}_{it-1}\right)\right\} \tag{7}$$

subject to

$$\xi_{t+1} = F\xi_t + v_{t+1} \tag{8}$$

$$x_t = E\left(x_t^* | \mathcal{I}_{it}\right) \tag{9}$$

$$\mathcal{I}_{it} = \mathcal{I}_{i,-1} \cup \{S_{i0}, \ldots, S_{it}\} \tag{10}$$

$$S_{it} = G'\xi_t + \psi_{it} \tag{11}$$

where

$$I\left(\xi_t; S_{it} | \mathcal{I}_{it-1}\right) = H\left(\xi_t | \mathcal{I}_{it-1}\right) - H\left(\xi_t | \mathcal{I}_{it}\right). \tag{12}$$

Expression (7) states that the decision-maker maximizes the expected discounted sum of profits net of the cost of attention. The cost of attention in any period t is proportional to mutual information $I\left(\xi_t; S_{it} | \mathcal{I}_{it-1}\right)$, where $\lambda > 0$ is the marginal cost of attention. Mutual information is defined below. The decision-maker takes as given the law of motion for the state vector (equation (8)). The agent's actions are equal to the conditional expectation of the profit-maximizing actions given the period t information set (equation (9)). The period t information set $\mathcal{I}_{it}$ consists of the sequence of signal realizations $S_{i0}, \ldots, S_{it}$ and initial information $\mathcal{I}_{i,-1}$ (equation (10)). The optimal signal is a signal about the state vector ξ_t (equation (11)), where the noise ψ_{it} follows a Gaussian vector white noise process with variance-covariance matrix Σ_ψ. The noise ψ_{it} is assumed to be independently distributed across firms.[22] The decision-maker chooses the signal weights G (the number of signals and what each signal is about) and the variance-covariance matrix of the noise Σ_ψ. Equation (12) states that mutual information (between the signal and the state vector) equals

action x_t^* is a vector, but the proof of Proposition 1 in Maćkowiak, Matějka, and Wiederholt (2018), which states that the optimal signal is a signal about the state vector, extends in a straightforward way from the case of a scalar x_t^* to the case of a vector x_t^*. To show that the optimum can be attained with a two-dimensional signal, one can follow the steps in the proof of Proposition 2 in Maćkowiak, Matějka, and Wiederholt (2018). Note also that the optimal signal is Gaussian because the objective is quadratic and the optimal action is Gaussian. See Maćkowiak, Matějka, and Wiederholt (2021).

[22] Woodford (2003) and Maćkowiak and Wiederholt (2015) make the same assumption. This assumption implies that information is dispersed: in every period, each firm i has a different conditional expectation $E\left(x_t^* | \mathcal{I}_{it}\right)$.

the difference between prior uncertainty and posterior uncertainty about the state vector in a given period. $H(\xi_t|\mathcal{I}_{i\tau})$ denotes the entropy of ξ_t conditional on $\mathcal{I}_{i\tau}$, $\tau = t-1, t$. $H(\xi_t|\mathcal{I}_{it-1})$ is the prior uncertainty, before receiving the period t signal, and $H(\xi_t|\mathcal{I}_{it})$ is the posterior uncertainty.

Both the expected discounted sum of profits and the cost of attention in expression (7) depend on conditional second moments.[23] The conditional second moments can in principle vary over time, because the decision-maker conditions on more signal realizations as time passes. To abstract from transitional dynamics in the conditional second moments, we assume that after choosing the signal process in period -1, the agent receives a sequence of signals in period -1 such that the conditional second moments are independent of time. The conditional second moments can then be computed using the steady-state Kalman filter, with state equation (8) and observation equation (11), and problem (7)-(12) can be solved numerically in a straightforward way.[24]

3.3 Equilibrium

We focus on the equilibrium when decision-makers in firms are subject to rational inattention and households have perfect information. For simplicity, until Section 6 we refer to this equilibrium as the rational inattention equilibrium.[25]

The rational inattention equilibrium can be defined as follows. In period -1, each firm solves problem (7)-(12). In every period $0, 1, 2, ...$, firms and households maximize given their information sets, and markets clear: the wage w_t adjusts so that labor demand equals labor supply, $\int_0^1 l_{it} di = \int_0^1 l_{jt} dj$, and the price of a mutual fund share v_t adjusts so that asset demand equals asset supply, $\int_0^1 q_{jt} dj = 0$.

Some details about firms' and households' maximization are helpful. Equations (9)-(11) together with the choice of G and Σ_ψ and the law of motion of the state (8) yield the input choices of firm i, k_{it} and l_{it}. Firm-level output, investment, and profit follow from $y_{it} = a_t + \alpha k_{it-1} + \phi l_{it}$, $\delta i_{it} = k_{it} - (1-\delta) k_{it-1}$, and $(D/Y) d_{it} = y_{it} - (WL/Y)(w_t + l_{it}) - (I/Y) i_{it}$, whereas aggregate

[23] There is a well-known, closed-form expression for mutual information in the Gaussian case. See Maćkowiak, Matějka, and Wiederholt (2018).

[24] We relax this assumption at the end of Section 4.1. Maćkowiak, Matějka, and Wiederholt (2018) make the same assumption. Woodford (2003) also uses the steady-state Kalman filter to compute conditional second moments in a model in which agents observe exogenously given signals.

[25] In Section 6 we add rational inattention on the side of households.

variables from $y_t = \int_0^1 y_{it} di$, $k_t = \int_0^1 k_{it} di$, $i_t = \int_0^1 i_{it} di$, and $d_t = \int_0^1 d_{it} di$.[26] Since households have perfect information, they satisfy the usual first-order conditions

$$\gamma E_t \left(c_{t+1} - c_t\right) = \beta E_t v_{t+1} - v_t + \left(1 - \beta\right) E_t d_{t+1} \tag{13}$$

and

$$w_t - \gamma c_t = \eta l_t. \tag{14}$$

Households are identical, implying that $c_{jt} = c_t$ and $l_{jt} = l_t$ for each j. Finally, the resource constraint reads[27]

$$y_t = \left(C/Y\right) c_t + \left(I/Y\right) i_t. \tag{15}$$

4 Developing intuition

How does rational inattention affect the propagation of productivity shocks and news about future productivity? To develop intuition this section studies special cases of the model. In the first special case, labor is the only variable input. In the second special case, capital is the only variable input. Section 5 analyzes the rational inattention equilibrium of the complete model.

4.1 The case with labor only

Suppose that labor is the only variable input, $\alpha = 0$. The attention problem of a firm simplifies. The firm's action (labor input choice) is one-dimensional with $x_t = l_{it}$, $x_t^* = l_{it}^* = \left[1/\left(1-\phi\right)\right]\left(a_t - w_t\right)$, and $\Theta = -C^{-\gamma} Y \phi \left(1 - \phi\right)$. Labor supply is governed by equation (14). Households live hand-to-mouth because there is no capital and all households are identical.

The perfect information equilibrium can be solved for analytically. Equating labor demand, $\int_0^1 l_{it} di = \left[1/\left(1-\phi\right)\right]\left(a_t - w_t\right)$, and labor supply, which follows from equations (14) and $c_t = y_t = a_t + \phi l_t$, yields the solution for aggregate labor input

$$l_t = \left(\frac{1-\gamma}{1-\phi+\gamma\phi+\eta}\right) a_t. \tag{16}$$

[26] These equations result from log-linearization of the production function, the law of motion of capital, the definition of profit, and the definitions of the aggregate variables. All relevant steady-state ratios appear in Appendix A.

[27] To obtain the resource constraint, we log-linearize the flow budget constraint of household j and we aggregate, imposing market clearing and plugging in the equation for the dividend from the mutual fund.

Labor input is proportional to productivity. The impulse response of labor input to a news shock is zero until productivity changes. Firms have no incentive to change labor demand until productivity changes. Similarly, households have no incentive to change labor supply in this special case of the model. The wealth effect on labor supply vanishes, because hand-to-mouth households cannot vary saving and consumption in response to a news shock.

Consider the rational inattention equilibrium. To find the fixed point where all firms are subject to rational inattention and hold correct beliefs about the law of motion of the state, we use a guess-and-verify method. We guess that in equilibrium the profit-maximizing labor input l_{it}^{*} follows a finite-order ARMA process. This yields the law of motion of the state (8). We solve the attention problem of firm i, (7)-(12) with $x_t = l_{it}$, $x_t^{*} = l_{it}^{*}$, and $\Theta = -C^{-\gamma}Y\phi(1-\phi)$. Equations (9)-(11) together with the choice of G and Σ_{ψ} and the law of motion of the state (8) yield the firm's labor input choice, l_{it}. We verify the guess for the profit-maximizing labor input l_{it}^{*} from the optimality condition $l_{it}^{*} = [1/(1-\phi)](a_t - w_t)$ where the market-clearing wage w_t follows from equations (14), $c_t = y_t = a_t + \phi l_t$, and $l_t = \int_0^1 l_{it} di$. One period in the model equals one quarter. As an example, we assume $\gamma = 0.5$, $\eta = 0$, $\phi = 0.6$, $\beta = 0.99$, $\rho = 0.9$, $\sigma = 0.01$, and $\lambda = (4/100,000)C^{-\gamma}Y$, which means that the per period marginal cost of attention is equal to $4/100,000$ of steady-state output.[28]

The upper-left panel in Figure 1 shows the impulse response of aggregate labor input l_t to a productivity shock ($h = 0$).[29] In the perfect information equilibrium, labor input is proportional to productivity and thus the impulse response peaks on impact and declines monotonically (line with points). The impulse response is weaker and hump-shaped in the rational inattention equilibrium (line with circles). This is the usual result that rational inattention produces dampening and delay due to noise in signals about the state of the economy. For a similar figure, see for instance Figure 1 in Sims (2003).

The upper-right panel in Figure 1 shows the impulse response of l_t to a news shock ($h = 4$). The shock is drawn in period 0 while productivity changes in period $h = 4$. In the perfect information equilibrium, labor input is proportional to productivity (equation (16)) and thus the impulse response is zero until productivity changes (line with points). Under rational inattention

[28] Section 5 discusses the choice of the value for the marginal cost of attention λ.

[29] In all figures, an impulse response of 1 is a 1 percent deviation from the non-stochastic steady state.

labor demand rises on impact of a news shock. The reason is that the optimal signal of firms confounds current with expected future productivity. The increase in labor demand puts upward pressure on the wage. Labor supply is still governed by equations (14) and $c_t = y_t = a_t + \phi l_t$. We find that in equilibrium labor input is positive on impact of a news shock (line with circles) and keeps rising thereafter.

To see analytically that a confounding signal is optimal, consider the following special case. Suppose that a measure zero of firms are subject to rational inattention. Since a measure one of firms have perfect information, the equilibrium employment is given by equation (16) and the equilibrium wage is $w_t = [(\gamma + \eta)/(1 - \gamma)]l_t$, implying that the profit-maximizing labor input of an individual firm is proportional to productivity: $l^*_{it} = [1/(1-\phi)](a_t - w_t) = [(1-\gamma)/(1-\phi+\gamma\phi+\eta)]a_t$. Suppose that $a_t = \rho a_{t-1} + \sigma\varepsilon_{t-1}$ (i.e., $h = 1$). The profit-maximizing labor input then has a first-order VAR representation with state vector $\xi_t = (l^*_{it}, \varepsilon_t)'$, or equivalently $\xi_t = (a_t, \varepsilon_t)'$. The optimal signal follows from Propositions 1, 2, and 5 in Maćkowiak, Matějka, and Wiederholt (2018). Proposition 1 states that the optimal signal is about the state vector, $S_{it} = G'\xi_t + \psi_{it}$. Proposition 2 states that with a one-dimensional action (here, labor input), the optimal signal is a *one-dimensional* signal about the state vector, $S_{it} = a_t + g\varepsilon_t + \psi_{it}$. Proposition 5 states that $g \neq 0$. Hence the optimal signal confounds current with expected future productivity. It turns out that this result still holds when all firms are subject to rational inattention and $h > 1$.

To gain intuition for the optimality of a confounding signal, compare rational inattention with an alternative model. Continue to assume that a measure zero of firms are subject to rational inattention (partial equilibrium) and $h = 1$. In the alternative model, a measure zero of firms solve the same attention problem *subject to the restriction* that one must obtain a two-dimensional signal consisting of a signal on current productivity and a separate signal on future productivity. A signal process of this form does not confound current with future productivity. We find that firms in the alternative model set to zero the precision of the signal on future productivity (they decide to observe only a signal on current productivity). The lower-left panel in Figure 1 reports the impulse response of labor input to a news shock by rationally inattentive firms (line with circles) and by firms in the alternative model (line with squares). Before productivity changes (in period 0), firms in the alternative model make no labor input mistake conditional on a news shock. Once productivity has changed (in period 1 and subsequent periods), firms in the alternative model make larger labor input mistakes conditional on a news shock than rationally inattentive firms. Overall, firms in the alternative model do worse than firms in the rational inattention model (the expected profit loss is larger in the alternative model than in the rational inattention model).[30]

It is a common result in rational inattention models that agents choose to receive a low-dimensional signal (i.e., a signal on a summary or index of the multi-dimensional state), because reducing the dimensionality of the signal saves on attention. In the RBC model, a one-dimensional signal that confounds current productivity with expected future productivity requires less attention than a two-dimensional signal consisting of a signal on current productivity and a separate signal on future productivity. In addition, a non-zero weight on the innovation to future productivity in the low-dimensional signal is optimal, because noise in the signal introduces delay in actions and the non-zero weight on future productivity helps reduce this delay, lowering the overall expected profit loss. With pervasive delay it is optimal to get changes into beliefs early.

In the same partial equilibrium setting as in the lower-left panel in Figure 1, let us vary the marginal cost of attention λ. The impulse response of the action under rational inattention (here, labor input) on impact of a news shock, in period 0, is non-monotonic in λ. With a λ near zero (Figure 1, lower-right panel, line with asterisks), the solution is close to the perfect information case in which the impulse response on impact is zero. With a high λ (line with diamonds), the solution is close to a "no information" model in which the impulse response in all periods is zero.

Appendix D reports additional numerical results. One result is that the more distant is the change in productivity, the weaker is the response of the action on impact of a news shock. If productivity will change in the near future, a rationally inattentive agent believes that productivity has already changed with a non-trivial probability. The short-run response of the action can then be strong (even though the perfect information response is zero). If productivity will change only in a longer run, the agent is fairly confident that productivity has not yet changed. The short-run response of the action approaches the perfect information response.

Appendix D also considers a version of the model in which productivity is driven by two orthogonal shocks, a standard productivity shock and a news shock. The result that labor input rises on

[30] In the lower-left panel in Figure 1, to make the comparison as clear as possible, we hold the amount of attention equal in the two models. For a given marginal cost λ, firms in the alternative model would choose to pay more attention but would still do worse than firms in the rational inattention model.

impact of a positive news shock is unchanged (and the impulse response of labor input to a news shock is very similar to the one reported in this subsection). In addition, in Appendix D we drop the assumption that, after choosing the signal process in period -1, the agent receives a sequence of signals in period -1 such that the conditional second moments are independent of time. The impulse response of labor input is almost identical with and without this assumption.

4.2 The case with capital only

Suppose that capital is the only variable input, $\phi = 0$. The attention problem of a firm is analogous to Section 4.1. The firm's action (capital input choice) is one-dimensional with $x_t = k_{it}$, $x_t^* = k_{it}^* = \frac{1}{1-\alpha}\left[E_t a_{t+1} - \frac{\gamma E_t(c_{t+1}-c_t)}{1-\beta(1-\delta)}\right]$, and $\Theta = -C^{-\gamma} Y \beta\alpha(1-\alpha)$. Consumption-saving behavior of households is governed by equation (13).

Assume log utility from consumption, $\gamma = 1$, and full capital depreciation, $\delta = 1$. The perfect information equilibrium can be solved for analytically: $k_t = \alpha k_{t-1} + a_t$, $k_t = i_t = y_t = c_t = d_t = v_t$. In this special case, the model can produce some positive autocorrelation in investment and output growth. However, the impulse responses of all variables to a news shock are zero until productivity changes. An increase in expected productivity creates an incentive to invest in the period before productivity improves, but this incentive is completely offset by a rise in the cost of capital.

Consider the rational inattention equilibrium. To find the fixed point, we guess that in equilibrium the profit-maximizing capital input k_{it}^* follows a finite-order ARMA process. This yields the law of motion of the state (8). We solve the attention problem of firm i, (7)-(12) with $x_t = k_{it}$, $x_t^* = k_{it}^*$, and $\Theta = -C^{-\gamma} Y \beta\alpha(1-\alpha)$. Equations (9)-(11) together with the choice of G and Σ_ψ and the law of motion of the state (8) yield the firm's capital input choice, k_{it}. Aggregating across firms produces k_t, i_t, y_t, and d_t, while the budget constraint implies that $c_t = d_t$. We verify the guess for the profit-maximizing capital input k_{it}^* from the optimality condition $k_{it}^* = [1/(1-\alpha)][E_t a_{t+1} - E_t(c_{t+1} - c_t)]$. The market-clearing mutual fund share price v_t follows from equation (13) and the solution for c_t. As an example, we assume $\gamma = 1$, $\alpha = 0.33$, $\beta = 0.99$, $\delta = 1$, $\rho = 0.9$, $\sigma = 0.01$, and $\lambda = (4/100,000)C^{-\gamma}Y$.

The top panel in Figure 2 displays the impulse response of aggregate investment i_t to a productivity shock ($h = 0$). The rational inattention equilibrium (line with circles) features more first-order autocorrelation in the growth rate of investment compared with the perfect information equilibrium (line with points). The middle panel in Figure 2 shows the impulse response of i_t to a news shock ($h = 4$). In the perfect information equilibrium, the impulse response of investment is zero until productivity changes in period 4 (line with points). In the rational inattention equilibrium, investment is positive in period 0 (line with circles) and keeps rising thereafter. Rational inattention induces an increase in investment demand on impact of a news shock. As a result, investment rises in equilibrium.

Since the attention problem of a firm is analogous to Section 4.1, the intuition for what happens to investment demand is the same as the intuition given there. The forward-looking attention choice leads investment demand to react immediately to a news shock, as if productivity has already changed with some probability. Similarly to Section 4.1, the bottom panel in Figure 2 compares the rational inattention model with the alternative model (the model with a restricted signal process of the form "a separate signal on each element of the state vector") with $h = 4$. The alternative model yields no capital input mistakes conditional on a news shock from period 0 through period $h - 1$, followed by larger mistakes than in the rational inattention model. By smoothing the action, the signal in the rational inattention model lowers the overall expected profit loss.[31]

Let us summarize Section 4. The impulse responses to productivity shocks and news shocks change significantly when firms are subject to rational inattention. Employment and investment react with delay to a productivity shock. They *rise* in response to news that productivity will improve, because the optimal signal confounds current with expected future productivity.

5 Predictions of the model

What does rational inattention imply about the business cycle effects of productivity shocks and news about future productivity? We return to the complete model with variable capital and labor, $\alpha > 0$ and $\phi > 0$, and focus on comparing the rational inattention equilibrium with the perfect

[31] We emphasized that rational inattention makes very different predictions than the alternative model in the case of news shocks ($h \geq 1$). With $h = 0$ actions based on the optimal signal are also different from actions based on the restricted signal, except when the optimal action follows an AR(1) process. How much difference there is depends on the details of the model. In this model the difference turns out to be modest. Consider the partial equilibrium analysis with $h = 0$ and the same parameter values. The profit-maximizing capital input follows an AR(2) process. The investment growth rate of rationally inattentive firms has a serial correlation of 0.61. With the restricted signal the serial correlation rises to 0.65.

information equilibrium.

We set $\gamma = 1$, $\eta = 0$, $\alpha = 0.33$, $\phi = 0.65$, $\beta = 0.99$, $\delta = 0.025$, $\rho = 0.9$, and $\sigma = 0.008$. Thus, we assume log utility from consumption and linear disutility from work, $\alpha + \phi$ close to 1, a depreciation rate of 2.5 percent per quarter, and a persistent productivity process with an innovation that has a standard deviation of 0.8 percent.[32] Below we state the value of λ.[33]

5.1 The effects of productivity shocks

Let $h = 0$. Consider the perfect information equilibrium. Figure 3 shows the impulse responses to a productivity shock (lines with points). Aggregate labor input, investment, output, and consumption move in the same direction, consistent with a business cycle. The impulse responses of labor input, investment, and output peak on impact and decline monotonically. Following common practice, we compare unconditional second moments in the model and in the data. Table 1 reports selected unconditional moments for the model (column "Perfect information") and for the quarterly post-war data from the United States.[34] The comparison is familiar. Let us focus on the persistence of growth rates. The first-order autocorrelations of employment, investment, and output growth are positive in the data but negative in the model. In the model these variables inherit the autocorrelation of exogenous productivity growth.[35]

[32] Fernald (2014) constructs a quarterly series on the growth rate of TFP adjusted for capacity utilization. Regressing Fernald's series on its own lag in the sample 1955Q1-2007Q4 yields a point estimate of -0.08, which would imply a coefficient of $1 - 0.08 = 0.92$ in equation (1). The estimated standard deviation of the error term is 0.0083. Rounding off these estimates, we arrive at $\rho = 0.9$ and $\sigma = 0.008$. One can also convert Fernald's series into a series on the log level of TFP and fit an AR(1) to that series after detrending, but the estimated coefficient depends on the detrending method.

[33] Only the ratio σ^2/λ matters for the equilibrium impulse responses, because the first term in objective (7) is linear in σ^2 and the second term is linear in λ.

[34] We use the data from Eusepi and Preston (2011). The sample period is 1955Q1-2007Q4. Productivity is defined as real GDP divided by hours worked, measured as in Francis and Ramey (2009). See Data Appendix in Eusepi and Preston (2011). The unconditional moments from the model are computed from the equilibrium MA representation of each variable.

[35] The model matches well the standard deviation of consumption, investment, and productivity relative to output, while underpredicting the volatility of hours worked. The model matches well the correlation of consumption, hours worked, and investment with output, while overstating the correlation of productivity with output. Finally, the model matches well the first-order autocorrelation of consumption growth. It turns out that rational inattention has little effect on these predictions of the model. See Table 1.

Consider the rational inattention equilibrium. Searching for the fixed point is more difficult than in Section 4, because we must consider two inputs, capital and labor, and two factor prices, the cost of capital and the wage. To find the fixed point, we guess that in equilibrium consumption c_t follows a finite-order ARMA process. With $\gamma = 1$ and $\eta = 0$, the optimality condition (14) states that the wage process w_t equals the consumption process c_t. This condition holds because households have perfect information. Therefore, a guess about consumption implies a guess about both factor prices, the cost of capital (the expected consumption growth rate) and the wage. We calculate the implied ARMA representations of the optimal inputs $x_{1t}^* = k_{it}^*$ and $x_{2t}^* = l_{it}^* - [\alpha/(1-\phi)]\, k_{it-1}^*$ from equation (6). This yields the law of motion of the state (8). We solve the attention problem of firm i, (7)-(12). Equations (9)-(11) together with the choice of G and Σ_ψ and the law of motion of the state (8) yield the firm's inputs, $k_{it} = x_{1t}$ and $l_{it} = x_{2t} + [\alpha/(1-\phi]k_{it-1}$. Aggregating across firms produces k_t, i_t, l_t, y_t, and d_t. We verify the guess for the equilibrium consumption process by solving for c_t from the resource constraint (15). The market-clearing mutual fund share price v_t follows from equation (13) and the solution for d_t and c_t.

What are the effects of rational inattention on the propagation of a productivity shock? We set $\lambda = (6/100,000)C^{-\gamma}Y$, which means that the per period marginal cost of attention is equal to $6/100,000$ of steady-state output.[36] In the rational inattention equilibrium, the impulse responses of employment, investment, and output become hump-shaped (Figure 3, lines with circles). These impulse responses are hump-shaped even though there are no adjustment costs. The first-order autocorrelations of employment, investment, and output growth become positive (Table 1, column "Rational inattention"). The model matches well the first-order autocorrelation of employment growth in the data, even though rational inattention is the only source of inertia and the marginal cost of attention is small. The model underpredicts somewhat the serial correlation of output and investment growth.

In Figure 3 note also that consumption declines somewhat when firms become subject to rational inattention. Households consume less because rationally inattentive firms underestimate productivity and produce less than in the perfect information equilibrium.

Section 4 explained the effects of rational inattention one input at a time. In this section the

[36] In the rational inattention equilibrium we can compute the expected profit loss of firm i from suboptimal actions. This is equal per period to $3/100,000$ of steady-state output, even less than the marginal cost of attention.

new feature is that rational inattention induces delay in the demand for both inputs, capital and labor, at the same time. Figure 3 shows the impulse response of the conditional expectation of productivity by firms to a productivity shock. The impulse response is hump-shaped, indicating that the firms' beliefs are anchored on the steady state and evolve slowly. The rational inattention effect turns out to be sufficient to bring the first-order autocorrelations of employment, investment, and output growth in the model approximately into line with the data.

The amount of inattention in the model, governed by the parameter λ, can be compared to survey data on expectations. Coibion and Gorodnichenko (2015) show that models with an informational friction predict a regression relationship between the average forecast error and forecast revision in a cross-section of agents. Suppose that firms in this model report their forecasts of output. Let $\hat{y}_{t+\tau|t}$ denote the period t average forecast of output in period $t+\tau$, where τ is a positive integer. The average forecast error, $y_{t+\tau} - \hat{y}_{t+\tau|t}$, is positively related to the average forecast revision, $\hat{y}_{t+\tau|t} - \hat{y}_{t+\tau|t-1}$. The regression coefficient increases in the size of the informational friction, in this model governed by the value of λ. Coibion and Gorodnichenko (2015) and Bordalo, Gennaioli, Ma, and Shleifer (2019) estimate this regression relationship using survey data on forecasts of a number of variables. Typically, these authors report coefficients in the range of 0.3-1.4.[37] We repeat their estimation using quarterly data on median forecasts of output (real GDP) from the U.S. Survey of Professional Forecasters for the period 1968Q4-2019Q4 obtained from the Federal Reserve Bank of Philadelphia. Focusing on $\tau = 3$, we estimate a regression coefficient of 0.76 with a standard error of 0.30.[38] Next, we simulate data from our model with the parameter values used in this section, including the value of λ. When we run the same regression on the simulated data, on average across the simulations we obtain a coefficient of 0.96. We conclude that the amount of inattention in the model is consistent with the survey data on expectations. It is remarkable that the first-order autocorrelations of employment, investment, and output growth in the model are approximately in line with the macro data and, at the same time, the model is consistent with the

[37] See in particular Coibion and Gorodnichenko (2015), Table 1 and Figures 1-2, and Bordalo, Gennaioli, Ma, and Shleifer (2019), Table 3.

[38] Coibion and Gorodnichenko (2015) and Bordalo, Gennaioli, Ma, and Shleifer (2019) also focus on $\tau = 3$. Both papers report results for forecasts of output *growth* but not output *level*. Some observations on forecasts of the level of output cannot be used due to base year changes in the dataset; furthermore, we remove as outliers the top 1 percent of forecast errors and revisions.

survey data on expectations.

5.2 The effects of news about future productivity

Let $h \geq 1$. We focus on $h = 2$ and $h = 4$, following the key papers on news shocks which also focus on changes in productivity a few quarters ahead ($h = 3$ in Beaudry and Portier, 2004, $h = 2$ in Jaimovich and Rebelo, 2009).

Consider the perfect information equilibrium. Figures 4 and 5 show the impulse responses with $h = 2$ and $h = 4$, respectively (lines with points). The shock is drawn in period 0 while productivity changes in period h. A news shock causes a wealth effect. Consumption and leisure are normal goods, and therefore households want to consume more (save less) and work less after a positive news shock. Firms have no incentive to increase labor demand before productivity improves, while households reduce labor supply due to the wealth effect. As a result, employment falls. With capital predetermined and current productivity unchanged, output contracts. On impact firms have no incentive to increase investment, while the wealth effect reduces desired saving by households. Investment declines while consumption rises. The model fails to produce business cycle comovement in response to news about future productivity. It predicts an output contraction after news that productivity will improve. Note also that, after decreasing on impact, employment, investment, and output keep falling between when the news arrives (period 0) and when productivity changes (period h). This is particularly clear in Figure 5 ($h = 4$). An increase in expected productivity creates an incentive to invest in the period before productivity improves (period $h - 1$), but this incentive is more than offset by a rise in the cost of capital. Employment, investment, and output increase only once productivity improves.

With a high elasticity of intertemporal substitution, the model predicts a fall in consumption and a rise in labor input and investment. The substitution effect due to an increase in the real interest rate dominates the wealth effect in this case, pushing consumption down and labor supply up. "However, no combination of parameters can generate a joint increase in consumption, investment, and employment." (Lorenzoni, 2011, p.539.)

Consider the rational inattention equilibrium (Figures 4-5, lines with circles).[39] In both figures

[39] In the economy with $h = 2$ we set $\lambda = (6/100,000)C^{-\gamma}Y$. This yields a per period expected profit loss equal to $4/100,000$ of steady-state output. In the economy with $h = 4$ we set $\lambda = (22/100,000)C^{-\gamma}Y$. The per period

employment is positive in period 0 and keeps rising thereafter. The conditional expectation of productivity by firms increases on impact, which pushes up labor demand. In general equilibrium, the desire of households to reduce labor supply is pulling employment down. It turns out that the rational inattention effect on labor demand is strong enough to *more than offset* the wealth effect on labor supply. As a result, employment rises in equilibrium.

Figures 4-5 show the impulse response of investment in general equilibrium ("RI general equilibrium") and the impulse response of investment by rationally inattentive firms of measure zero when other firms have perfect information ("RI partial equilibrium," line with asterisks). In partial equilibrium, investment is positive in period 0 and keeps rising thereafter. The conditional expectation of productivity by rationally inattentive firms increases on impact, which pushes up investment demand. In general equilibrium, the desire of households to reduce saving for a given level of output is pulling investment down. We find that the rational inattention effect on investment demand approximately offsets the wealth effect on saving supply. The response of investment on impact of a news shock is close to zero (whereas it is nearly -3 percent in the perfect information equilibrium). Note also that investment rises between period 0 and period h. This is particularly clear in Figure 5 ($h = 4$).

With capital predetermined and an increase in employment in period 0, the impulse response of output on impact of a news shock is positive. Output increases further between period 0 and period h, as employment and investment rise. The rational inattention effect on input demand induces an output expansion in response to a news shock.

Consider in more detail what affects investment in general equilibrium. Investment rises on impact of a positive news shock relative to the perfect information equilibrium. The cost of capital increases (the expected consumption growth rate rises). The profit-maximizing capital input of an individual firm falls. See the first line in equation (6). Capital is a strategic substitute. An individual firm demands less capital when other firms invest more. This general equilibrium feedback effect turns out to be very strong. The coefficient on the expected consumption growth rate in the first line of equation (6) equals -504.[40] The coefficient on the expected consumption growth rate increases in the depreciation rate, δ, and decreases in the elasticity of output with respect to

expected profit loss turns out to equal $15/100,000$ of steady-state output.

[40] Labor is also a strategic substitute. However, the general equilibrium dampening of labor demand due to a higher wage is weak. The coefficient on the wage in the second line of equation (6) equals -2.9.

labor, ϕ. In Section 4.2, with full capital depreciation and without labor input ($\delta = 1$, $\phi = 0$), this coefficient decreases in absolute value by more than two orders of magnitude, to -1.5, implying that the strategic substitutability is much weaker. The impulse response of equilibrium investment on impact of a news shock is positive in this case.

To summarize, rational inattention induces an increase in the demand for labor and investment in response to news that productivity will improve. The rational inattention effect on labor demand more than offsets the wealth effect on labor supply. Thus, employment and output rise on impact. The rational inattention effect on investment demand offsets the wealth effect on saving supply. As a result, the response of investment on impact equals zero, as opposed to a sizable negative number in the perfect information equilibrium.

In Figures 4-5 note also that consumption increases somewhat when firms become subject to rational inattention. Households consume more because rationally inattentive firms overestimate productivity and produce more than in the perfect information equilibrium.

What is the optimal signal? In problem (7)-(12) the firm can in principle choose a multi-dimensional signal process, consisting of signals on elements of the state vector ξ_t, signals on linear combinations of the elements of ξ_t, or both. We find that a one-dimensional signal on all elements of the state vector is optimal. A signal on all elements of the state vector confounds current with expected future productivity.[41] Furthermore, we find that the impulse response of the optimal signal to a news shock is positive on impact (Appendix Figure 1, upper-left panel, $h = 2$). To simplify, the message to firms from a positive signal realization is: "Hire and invest, productivity is either already up or about to rise (and it is not that important precisely when productivity rises)."

As in Section 5.1, we can compare the amount of inattention in the model to the SPF data. When we run the Coibion-Gorodnichenko regression on data simulated from the economy with $h = 2$ (with $\tau = 3$), on average we obtain a coefficient of 1.17. This amount of inattention is consistent with the survey data on expectations.[42] With $h = 4$ the model needs a higher marginal cost of attention to produce an increase in employment after a positive news shock. When we

[41] We find that a univariate signal process is optimal in this model even though the optimal action follows a bivariate process (recall the definition of x_t^* in equation (6)). We write that the optimal signal is on "all elements" of the state vector bearing in mind that an element can be dropped if it can be written as a linear combination of the other elements.

[42] Recall that in the SPF data the analogous regression coefficient is 0.76 with a standard error of 0.30.

run the Coibion-Gorodnichenko regression on data simulated from the economy with $h = 4$ (with $\tau = 3$), on average we obtain a coefficient of 2.81. This amount of inattention is somewhat greater than implied by the SPF data.[43]

5.3 Changing macroeconomic volatility

The impulse responses in the model depend on how much attention agents choose to pay, and the optimal attention varies with the environment. In Sections 5.1-5.2, we set the volatility of the productivity process based on the post-war U.S. data. Specifically, we set $\sigma = 0.008$ to match the standard deviation of the quarterly growth rate of TFP adjusted for capacity utilization in the period 1955Q1-2007Q4. The TFP growth rate was less variable in the second half of this sample than in the first half, a part of the decline in macroeconomic volatility known as the Great Moderation. The standard deviation of the TFP growth rate decreased from 0.9 percent (1955Q1-1984Q4) to 0.7 percent (1985Q1-2007Q4). Let us resolve the model with $\sigma = 0.009$ (higher volatility) and again with $\sigma = 0.007$ (lower volatility). The other parameter values remain as in Section 5.2.

We find that in the lower volatility economy ($\sigma = 0.007$) the period 0 impulse response of labor input to a news shock is positive (like in the baseline with $\sigma = 0.008$). A news shock produces positive comovement of labor input and consumption on impact. In the higher volatility economy ($\sigma = 0.009$), the period 0 impulse response of labor input is negative. Here a news shock produces negative comovement of labor input and consumption on impact (Appendix Figure 1, upper-right panel, $h = 2$). The reason behind the change in the sign is intuitive. With higher volatility agents pay about 50 percent more attention to the state of the economy, and therefore the response of labor input is closer to the perfect information RBC model, than with lower volatility. This effect is strong enough to change the sign of the impulse response of employment to a news shock. Thus, the model suggests that empirical researchers who study different sample periods can be expected to reach conflicting conclusions regarding comovement.[44]

[43] It seems plausible that in the real world decision-makers in small and medium firms pay less attention to the aggregate economy than professional forecasters.

[44] Recall from the introduction that Barsky and Sims (2011) and Kurmann and Sims (2019) do not find business cycle comovement after a news shock, while Görtz, Tsoukalas, and Zanetti (2020) who focus on data since the onset of the Great Moderation find comovement. Görtz, Tsoukalas, and Zanetti (2020) also show that when they use the

The SPF data support the view that agents pay less attention to the macroeconomy since the onset of the Great Moderation than before. Coibion and Gorodnichenko (2015, Section III.A) make this point in detail. In Section 5.1, we reported the result from running the Coibion-Gorodnichenko regression on median forecasts of output from the SPF for the period 1968Q4-2019Q4 (with $\tau = 3$). We estimated a coefficient of 0.76 with a standard error of 0.30. Let's split the sample in half and rerun this regression in the two subsamples.[45] In the first half of the sample the coefficient falls to 0.48 (the standard error is 0.38). In the second half of the sample the coefficient rises to 1.21 (the standard error is 0.48). This finding is in line with the hypothesis of "less attention since the onset of the Great Moderation than before."[46]

6 Rational inattention by firms and households

We focused on the equilibrium when decision-makers in firms are subject to rational inattention and households have perfect information. To obtain comovement in response to a news shock, it seems critical to find a mechanism leading to a shift in labor demand and investment demand for a given level of productivity. Rational inattention *by firms* is such a mechanism. To illustrate in the most transparent way the effects of rational inattention by firms, we assumed that households have perfect information. We found that rational inattention by firms also improves the propagation of a standard productivity shock. In this section, we add rational inattention on the side of households. Let us first describe the attention problem of an individual household j and afterwards explain how we solve for the fixed point when all firms and all households are subject to rational inattention.[47]

Each household j chooses a signal about the state of the economy to maximize the expected

identification assumptions of Barsky and Sims (2011) or Kurmann and Sims (2019) and focus on the data since the onset of the Great Moderation, they find comovement.

[45] The SPF sample starts only in 1968 and therefore it seems reasonable to split the sample in half, rather than divide it into unequal "before" and "after" the onset of the Great Moderation subsamples. That alternative approach, however, happens to yield regression results very similar to the ones reported here.

[46] In Section 5.2 we also ran the same Coibion-Gorodnichenko regression on data simulated from the baseline rational inattention economy, obtaining a coefficient of 1.17 ($h = 2$). Repeating this regression in the model with $\sigma = 0.009$ and $\sigma = 0.007$ yields coefficients of 0.78 and 1.42, respectively.

[47] The attention problem of each firm i is essentially unchanged. Households no longer have the same consumption level in this version of the model. We assume that firm i values profits according to the marginal utility of consumption of the representative (average) household.

discounted sum of utility. The household recognizes that a more informative signal requires more attention, which is costly. Proceeding analogously to Section 3, we derive an expression for the expected discounted sum of losses in utility when actions of household j deviate from the utility-maximizing actions – the actions the household would take if it had perfect information in every period. To obtain this expression, we compute the log-quadratic approximation to the expected discounted sum of utility at the non-stochastic steady state, arriving at

$$\sum_{t=0}^{\infty} \beta^t E_{j,-1} \left[\frac{1}{2} \left(x_t - x_t^* \right)' \Theta \left(x_t - x_t^* \right) \right], \tag{17}$$

where

$$x_t = \begin{pmatrix} \omega_V \left(q_{jt} - q_{jt-1} \right) \\ \gamma \left[\omega_V \left(\frac{1}{\beta} q_{jt-1} - q_{jt} \right) + \omega_W l_{jt} \right] + \eta l_{jt} \end{pmatrix} \tag{18}$$

$$\Theta = -C^{1-\gamma} \gamma \begin{bmatrix} \left(1 - \frac{1}{1+\frac{\eta}{\omega_W \gamma}} \right) \frac{1}{\beta} & 0 \\ 0 & \frac{1}{1+\frac{\eta}{\omega_W \gamma}} \frac{1}{\gamma^2} \end{bmatrix} \tag{19}$$

and

$$x_t^* = \begin{pmatrix} z_t - (1-\beta) \sum_{s=t}^{\infty} \beta^{s-t} E_t \left[z_s \right] + \left(1 + \omega_W \frac{\gamma}{\eta} \right) \frac{1}{\gamma} \beta \sum_{s=t}^{\infty} \beta^{s-t} E_t \left[r_{s+1} \right] \\ w_t - \gamma \left(\omega_W w_t + \omega_D d_t \right) \end{pmatrix}. \tag{20}$$

Here $z_s \equiv \omega_W \left(w_s + \frac{1}{\eta} w_s \right) + \omega_D d_s$ and $r_{s+1} \equiv \beta v_{s+1} - v_s + (1-\beta) d_{s+1}$. See Appendix E.[48]

This objective has a simple interpretation. The first element of x_t is the change in asset holdings. The second element of x_t is the component of the marginal rate of substitution between consumption and leisure that is directly controlled by the household through the choice of asset holdings, q_{jt}, and hours worked, l_{jt}. The vector x_t^* is the vector of optimal choices under perfect information in period t. It is optimal to increase asset holdings when income is high relative to permanent income or when the return on saving is high. It is optimal to equate the marginal rate of substitution between consumption and leisure to the wage. When the household deviates from these optimal choices, the household loses an amount of utility determined by the matrix Θ. This matrix is diagonal, because a *sub*optimal marginal rate of substitution between consumption and leisure does not affect the optimal change in asset holdings, and a *sub*optimal change in asset holdings does not affect the optimal marginal rate of substitution between consumption and leisure.[49]

[48] The coefficients ω_V, ω_W and ω_D denote the steady-state ratios V/C, WL/C and D/C, respectively.

[49] A given change in asset holdings can be financed with different combinations of consumption and hours worked. One of these combinations equates the marginal rate of substitution between consumption and leisure to the wage.

We assume that the household chooses asset holdings, q_{jt}, and hours worked, l_{jt}, in every period t. One can also think of the household as choosing directly the vector x_t in equation (18). These two assumptions are equivalent so long as the household knows its own past action q_{jt-1}, which is the case if $\mathcal{I}_{jt-1} \subset \mathcal{I}_{jt}$, where $\mathcal{I}_{jt}$ denotes the information set of household j in period t. The fact that the matrix Θ is diagonal implies that the best response of household j in period t given any information set $\mathcal{I}_{jt}$ is the conditional expectation of x_t^*, $x_t = E\left(x_t^* | \mathcal{I}_{jt}\right)$.

We also assume that the vector x_t^* given by equation (20) has a first-order VAR representation, $\xi_{t+1} = F\xi_t + v_{t+1}$. The rational inattention problem of a household then has the exact same form as the rational inattention problem of a firm (equations (7)-(12)), and it can be solved using any solution method for linear quadratic Gaussian pure tracking problems (Maćkowiak, Matějka, and Wiederholt, 2018, Afrouzi and Yang, 2020, Miao, Wu, and Young, 2020).

Finding a fixed point of an economy in which all firms and households are subject to rational inattention and hold correct beliefs about the law of motion of the state is more difficult than what we have done so far. Now equilibrium depends on the signals chosen by firms and on the signals chosen by households. To find the fixed point, we guess that in equilibrium consumption c_t and the wage w_t each follows a finite-order ARMA process. We calculate the implied ARMA representation of the optimal inputs of firm i and we solve the firm's attention problem, as in Section 5. From the solution we obtain the firm's inputs, k_{it} and l_{it}, and the aggregate variables k_t, i_t, y_t, and d_t, again as in Section 5. Turning to the attention problem of household j, we note that the optimal choice vector x_t^* depends on the process for w_t, d_t, and v_t (the first element x_{1t}^* depends on w_t, d_t, and v_t, and the second element x_{2t}^* depends on w_t and d_t). See equation (20). The price of a mutual fund share v_t adjusts so that in equilibrium asset demand equals asset supply, $\int_0^1 q_{jt} dj = 0$. To impose this asset market clearing condition, we compute the process for v_t such that x_{1t}^* equals 0 given the guess for w_t and the solution for d_t. We also calculate the ARMA representation of x_{2t}^* implied by the guess for w_t and the solution for d_t. We then solve the household's attention problem. Since $x_{1t}^* = 0$, the perfect tracking of x_{1t}^* requires no attention and the solution to the household's attention problem has the feature that $x_{1t} = 0$, which implies that $q_{jt} = 0$ and $\int_0^1 q_{jt} dj = 0$. The household's optimal signal choice together with the equation for x_{2t} (the second line in (18)) and $q_{jt} = 0$ yield the household's hours worked, l_{jt}. We adjust the guess for consumption c_t and the wage w_t until the resource constraint, equation (15), holds and labor demand equals labor supply,

$\int_0^1 l_{it} di = \int_0^1 l_{jt} dj$.

We assume the same parameter values as in Section 5, except that the marginal cost of attention to a household, which we call μ, no longer equals 0 as is implicit there. We set $\mu = (1/100,000)\, C^{1-\gamma}$ in the economy with $h = 0$ and $\mu = (3/100,000)\, C^{1-\gamma}$ in the economy with $h = 2$, which means that the household's marginal cost of attention is equal to $1/100,000$ of steady-state consumption ($3/100,000$, respectively) per period. In equilibrium the per period expected utility loss from inattention turns out to equal $5/1,000,000$ of steady-state consumption with $h = 0$ and $8/1,000,000$ with $h = 2$. The derivation of the household's objective assumes that η is a strictly positive number, whereas $\eta = 0$ in Section 5. Therefore we now set η equal to a very small, strictly positive number (so that utility is approximately linear in hours worked). The equilibria studied in Section 5 are essentially identical whether $\eta = 0$ or η equals a very small, strictly positive number.

This appears to be the first time in the literature that a general equilibrium model is solved in which all agents are subject to rational inattention and prices, which the agents take as given, adjust so that markets clear (here, the wage adjusts to equate labor demand and supply and the price of a mutual fund share adjusts to equate asset demand and supply).[50]

Figure 6 shows the equilibrium with firms and households subject to rational inattention (lines with asterisks). The top row is the case of $h = 0$. The bottom row is the case of $h = 2$. The perfect information equilibrium (lines with points) and the equilibrium from Section 5 with rationally inattentive firms and perfectly informed households (lines with circles) are displayed for comparison.

Begin with a standard productivity shock, $h = 0$. On impact rational inattention by households reduces labor supply for a given wage, because it takes time for households to recognize that working conditions have improved. To restore equilibrium in the labor market the wage rises (the impulse response of the wage is stronger on impact when households are rationally inattentive than when they have perfect information). A higher wage depresses investment demand (the profit-maximizing capital stock is decreasing in the expected wage, see the first line of equation (6)). In equilibrium employment, investment, output, and consumption fall compared with the equilibrium from Section 5.1. Thus, rational inattention by households adds further dampening and delay to the impulse responses of these variables to a productivity shock.

[50] In Maćkowiak and Wiederholt (2015), all firms and households are also subject to rational inattention. In each market one side of the market sets the price and the other side chooses the quantity.

Next, consider a news shock, $h = 2$. Rational inattention by households has two effects in the model. It weakens the wealth effect on labor supply and saving supply, since a news shock is an instantaneous change in the present value of income and rational inattention creates a dampened and delayed reaction of consumption and leisure to this change in permanent income. In addition, under rational inattention the labor supply decision becomes forward-looking, which makes households even more willing to supply labor at a given wage on impact of a positive news shock. The payoffs from future work rise, and the optimal signal of households confounds the payoff from current work with the payoffs from future work. Both effects of households' rational inattention strengthen comovement. To restore equilibrium in the labor market the wage falls (the impulse response of the wage is weaker on impact when households are rationally inattentive than when they have perfect information). A lower wage stimulates investment demand. In equilibrium employment, investment, and output rise on impact of a news shock compared with the equilibrium from Section 5.2. In parallel, saving rises while consumption falls. These effects get reversed once productivity rises (the wage increases, investment falls, and so on, relative to the equilibrium from Section 5.2, as we have seen from the impulse responses to a productivity shock).

We conclude that rational inattention by households *strengthens* comovement after a news shock. With rationally inattentive households employment, investment, and output are *even higher* on impact of a news shock, because these households then supply more labor and save more than perfectly informed households.

7 Conclusions

Very few papers so far have solved a DSGE model with rational inattention. This paper solves a benchmark RBC model with rational inattention (RI-RBC).

The RI-RBC model generates over-reaction to news by decision-makers in firms on impact of a news shock and under-reaction once productivity actually changes, where over- and under-reaction are defined relative to the profit-maximizing actions. It is the anticipation of the under-reaction later on that makes the over-reaction early on desirable. We find that the rational inattention effect on firms' labor demand on impact of a news shock more than offsets the wealth effect on labor supply; thus, employment and output increase on impact of a news shock. We also find that the rational inattention effect on firms' investment demand on impact of a news shock offsets the wealth effect on saving supply; and that rational inattention by households strengthens comovement.

Comovement after news shocks is usually generated by introducing preferences that weaken the wealth effect on labor supply, investment adjustment costs, and variable capital utilization (Jaimovich and Rebelo, 2009), or complementarities in a multi-sector setting (Beaudry and Portier, 2007). We find it interesting that comovement emerges in the benchmark RI-RBC model. Furthermore, the rational inattention explanation for comovement after news shocks can potentially also rationalize why researchers who study different sample periods can reach conflicting conclusions regarding comovement in the data.

Hump-shaped impulse responses are usually generated by introducing adjustment costs or exogenous imperfect information. We find it interesting that a single friction (costly attention) and the agents' optimal response to that friction (rational inattention) generates both hump-shaped impulse responses and comovement after news shocks.

When we introduced rational inattention on the side of households, we assumed that they choose how much to save and how much to work. In future work, it would be worthwhile to solve the model under the assumption that households choose how much to consume and how much to work. We conjecture that one would obtain very similar results in that alternative setup. To see this, consider the response of consumption and saving to a positive news shock. An inattentive household which chooses consumption under-reacts to the optimal consumption response, and thus consumes less (saves more) than under perfect information. An inattentive household which chooses saving under-reacts to the *negative* optimal saving response, and thus also saves more (consumes less) than under perfect information. In future research, it would also be interesting to study the implications of rational inattention for how the economy responds to fiscal news shocks (news about future government spending, future taxes, or future transfers).

References

[1] Afrouzi, Hassan, and Choongryul Yang. 2020. "Dynamic Rational Inattention and the Phillips Curve." Working paper.

[2] Angeletos, George-Marios, and Jennifer La'O. 2010. "Noisy Business Cycles." *NBER Macroeconomics Annual*, 24: 319-378.

[3] Auclert, Adrien, Matthew Rognlie, and Ludwig Straub. 2020. "Micro Jumps, Macro Humps: Monetary Policy and Business Cycles in an Estimated HANK Model." Working paper.

[4] Barro, Robert J., and Robert G. King. 1984. "Time-Separable Preferences and Intertemporal-Substitution Models of Business Cycles." *Quarterly Journal of Economics,* 99(4): 817-839.

[5] Barsky, Robert B., Susanto Basu, and Keyoung Lee. 2015. "Whither News Shocks?" *NBER Macroeconomics Annual*, 29(1): 225-264.

[6] Barsky, Robert B., and Eric R. Sims. 2011. "News Shocks and Business Cycles." *Journal of Monetary Economics,* 58(3): 273-289.

[7] Beaudry, Paul, and Franck Portier. 2004. "An Exploration into Pigou's Theory of Cycles." *Journal of Monetary Economics*, 51(6): 1183-1216.

[8] Beaudry, Paul, and Franck Portier. 2006. "Stock Prices, News, and Economic Fluctuations." *American Economic Review,* 96(4): 1293-1307.

[9] Beaudry, Paul, and Franck Portier. 2007. "When Can Changes in Expectations Cause Business Cycle Fluctuations in Neo-Classical Settings?" *Journal of Economic Theory,* 135(1): 458-77.

[10] Beaudry, Paul, and Franck Portier. 2014. "News-Driven Business Cycles: Insights and Challenges." *Journal of Economic Literature*, 52(4): 993-1074.

[11] Blanchard, Olivier J., Jean-Paul L'Huillier, and Guido Lorenzoni. 2013. "News, Noise, and Fluctuations: An Empirical Exploration." *American Economic Review,* 103(7): 3045-3070.

[12] Bordalo, Pedro, Nicola Gennaioli, Yueran Ma, and Andrei Shleifer. 2019. "Overreaction in Macroeconomic Expectations." *American Economic Review*, forthcoming.

[13] Cascaldi-Garcia, Danilo, and Marija Vukotić. 2020. "Patent-Based News Shocks." *Review of Economics and Statistics*, forthcoming.

[14] Chahrour, Ryan, and Kyle Jurado. 2018. "News or Noise? The Missing Link." *American Economic Review,* 108(7): 1702-1736.

[15] Chahrour, Ryan, and Kyle Jurado. 2021. "Recoverability and Expectations-Driven Fluctuations." *Review of Economic Studies*, forthcoming.

[16] Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans. 2005. "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy." *Journal of Political Economy*, 113(1): 1-45.

[17] Cogley, Timothy, and James M. Nason. 1995. "Output Dynamics in Real-Business-Cycle Models." *American Economic Review,* 85(3): 492-511.

[18] Coibion, Olivier, and Yuriy Gorodnichenko. 2015. "Information Rigidity and the Expectation Formation Process: A Simple Framework and New Facts." *American Economic Review*, 105(8): 2644-2678.

[19] Ellison, Martin, and Alistair Macaulay. 2019. "A Rational Inattention Unemployment Trap." Working paper.

[20] Eusepi, Stefano, and Bruce Preston. 2011. "Expectations, Learning, and Business Cycle Fluctuations." *American Economic Review*, 101(6): 2844-2872.

[21] Fernald, John. 2014. "A Quarterly, Utilization-Adjusted Series on Total Factor Productivity." Federal Reserve Bank of San Francisco Working Paper 2012-19.

[22] Francis, Neville, Michael T. Owyang, Jennifer E. Roush, and Riccardo DiCecio. 2014. "A Flexible Finite-Horizon Alternative to Long-Run Restrictions with an Application to Technology Shocks." *Review of Economics and Statistics*, 96(4): 638-647.

[23] Francis, Neville, and Valery Ramey. 2009. "Measures of Per Capita Hours and Their Implications for the Technology-Hours Debate." *Journal of Money, Credit, and Banking*, 41(6): 1071-97.

[24] Görtz, Christoph, Christopher Gunn, and Thomas A. Lubik. 2020. "Is There News in Inventories?" Working paper.

[25] Görtz, Christoph, John D. Tsoukalas, and Francesco Zanetti. 2020. "News Shocks under Financial Frictions." *American Economic Journal: Macroeconomics*, forthcoming.

[26] Hansen, Gary D. 1985. "Indivisible Labor and the Business Cycle." *Journal of Monetary Economics*, 16 (3): 309-27.

[27] Jaimovich, Nir. 2017. "News Shocks." In *The New Palgrave Dictionary of Economics*. London: Palgrave Macmillan.

[28] Jaimovich, Nir, and Sergio Rebelo. 2009. "Can News about the Future Drive the Business Cycle?" *American Economic Review*, 99(4): 1097-1118.

[29] Jurado, Kyle. 2020. "Rational Inattention in the Frequency Domain." Working paper.

[30] King, Robert G., Charles I. Plosser, and Sergio T. Rebelo. 1988. "Production, Growth and Business Cycles: I. The Basic Neoclassical Model." *Journal of Monetary Economics*, 21(2/3): 195-232.

[31] Kurmann, André, and Eric R. Sims. 2019. "Revisions in Utilization-Adjusted TFP and Robust Identification of News Shocks." *Review of Economics and Statistics*, forthcoming.

[32] Kydland, Finn E., and Edward C. Prescott. 1982. "Time to Build and Aggregate Fluctuations." *Econometrica*, 50(6): 1345-70.

[33] Lorenzoni, Guido. 2009. "A Theory of Demand Shocks." *American Economic Review*, 99(5): 2050-2084.

[34] Lorenzoni, Guido. 2011. "News and Aggregate Demand Shocks." *Annual Review of Economics*, 3(1): 537-557.

[35] Lucas, Robert E. 1972. "Expectations and the Neutrality of Money." *Journal of Economic Theory*, 4(2): 103-124.

[36] Maćkowiak, Bartosz, Filip Matějka, and Mirko Wiederholt. 2018. "Dynamic Rational Inattention: Analytical Results." *Journal of Economic Theory*, 176(C): 650-692.

[37] Maćkowiak, Bartosz, Filip Matějka, and Mirko Wiederholt. 2021. "Rational Inattention: A Review." *Journal of Economic Literature*, forthcoming.

[38] Maćkowiak, Bartosz, and Mirko Wiederholt. 2009. "Optimal Sticky Prices under Rational Inattention." *American Economic Review*, 99(3), 769-803.

[39] Maćkowiak, Bartosz, and Mirko Wiederholt. 2015. "Business Cycle Dynamics under Rational Inattention." *Review of Economic Studies*, 82(4): 1502-1532.

[40] Miao, Jianjun, Jieran Wu, and Eric Young. 2020. "Multivariate Rational Inattention." Working paper.

[41] Miranda-Agrippino, Silvia, Sinem Hacioğlu-Hoke, and Kristina Bluwstein. 2020. "When Creativity Strikes: News Shocks and Business Cycle Fluctuations." Working paper.

[42] Prescott, Edward C. 1986. "Theory Ahead of Business-Cycle Measurement." *Carnegie-Rochester Conference Series on Public Policy*, 25(1): 11-44.

[43] Rotemberg, Julio J., and Michael Woodford. 1996. "Real-Business-Cycle Models and the Forecastable Movements in Output, Hours, and Consumption." *American Economic Review*, 86(1): 71-89.

[44] Schmitt-Grohé, Stephanie, and Martín Uribe. 2012. "What's News in Business Cycles." *Econometrica*, 80(6): 2733-2764.

[45] Sims, Christopher A. 1998. "Stickiness." *Carnegie-Rochester Conference Series on Public Policy,* 49(1): 317-356.

[46] Sims, Christopher A. 2003. "Implications of Rational Inattention." *Journal of Monetary Economics*, 50(3): 665-690.

[47] Sims, Christopher A. 2010. "Rational Inattention and Monetary Economics." In *Handbook of Monetary Economics*, edited by Benjamin M. Friedman and Michael Woodford, 155-181. Amsterdam: Elsevier.

[48] Smets, Frank, and Rafael Wouters. 2007. "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach." *American Economic Review*, 97(3): 586-606.

[49] Steiner, Jakub, Colin Stewart, and Filip Matějka. 2017. "Rational Inattention Dynamics: Inertia and Delay in Decision-Making." *Econometrica*, 85(2): 521-553.

[50] Stevens, Luminita. 2020. "Coarse Pricing Policies." *Review of Economic Studies*, 87(1): 420-453.

[51] Woodford, Michael. 2003. "Imperfect Common Knowledge and the Effects of Monetary Policy." In *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps*, edited by Philippe Aghion, Roman Frydman, Joseph Stiglitz, and Michael Woodford, 25-58. Princeton: Princeton University Press.

[52] Woodford, Michael. 2009. "Information-Constrained State-Dependent Pricing." *Journal of Monetary Economics*, 56(S): 100-124.

Table 1: Business cycle statistics

	Data	Model, *h = 0* Perfect information	Rational inattention
Relative standard deviation			
σ_c/σ_y	0.55	0.56	0.59
σ_l/σ_y	0.92	0.66	0.58
σ_i/σ_y	2.89	3.05	2.94
σ_a/σ_y	0.52	0.46	0.51
Correlation			
$\rho_{c,y}$	0.79	0.78	0.81
$\rho_{l,y}$	0.86	0.85	0.83
$\rho_{i,y}$	0.90	0.93	0.92
$\rho_{a,y}$	0.40	1.00	0.99
First-order serial correlation			
Δc	0.27	0.23	0.28
Δl	0.41	-0.06	0.44
Δi	0.35	-0.06	0.14
Δy	0.30	-0.05	0.13
Δa	-0.06	-0.05	-0.05

Data: United States, 1955Q1-2007Q4, from Eusepi and Preston (2011).
Model: Unconditional moments computed from the equilibrium MA representation of each variable.

Figure 1: Impulse responses with α = 0

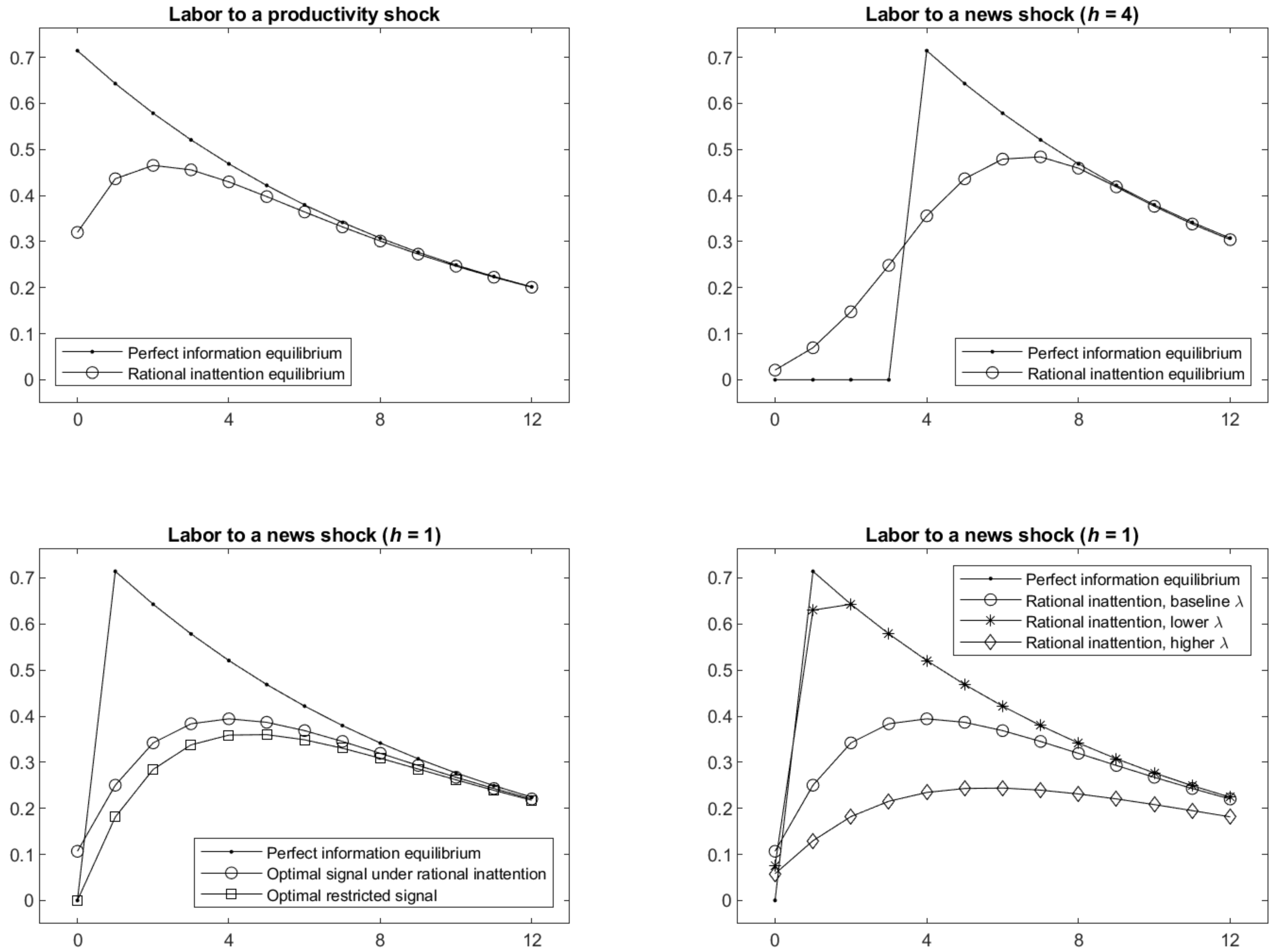

Figure 2: Impulse responses with ϕ = 0

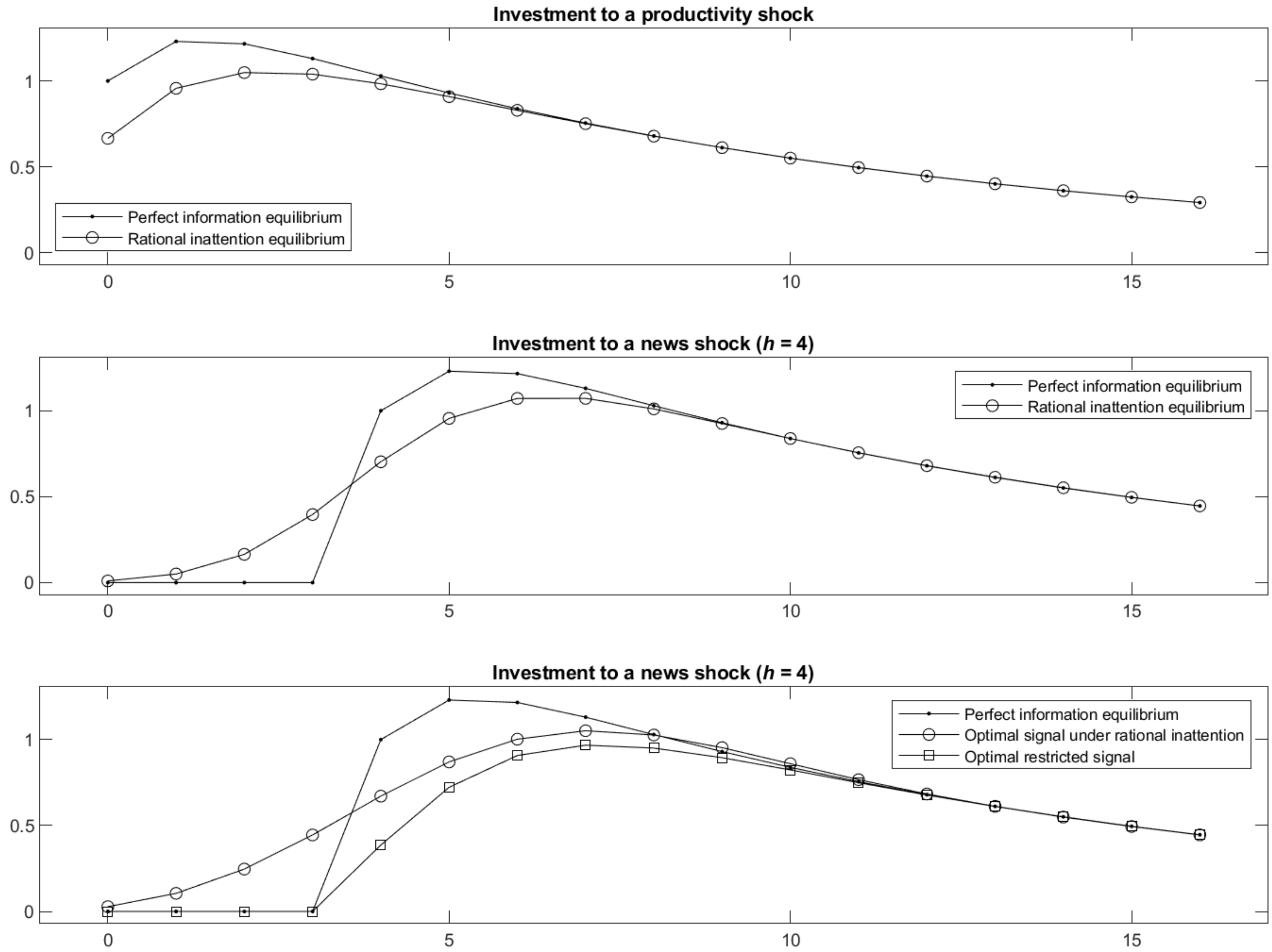

Figure 3: Impulse responses to a productivity shock

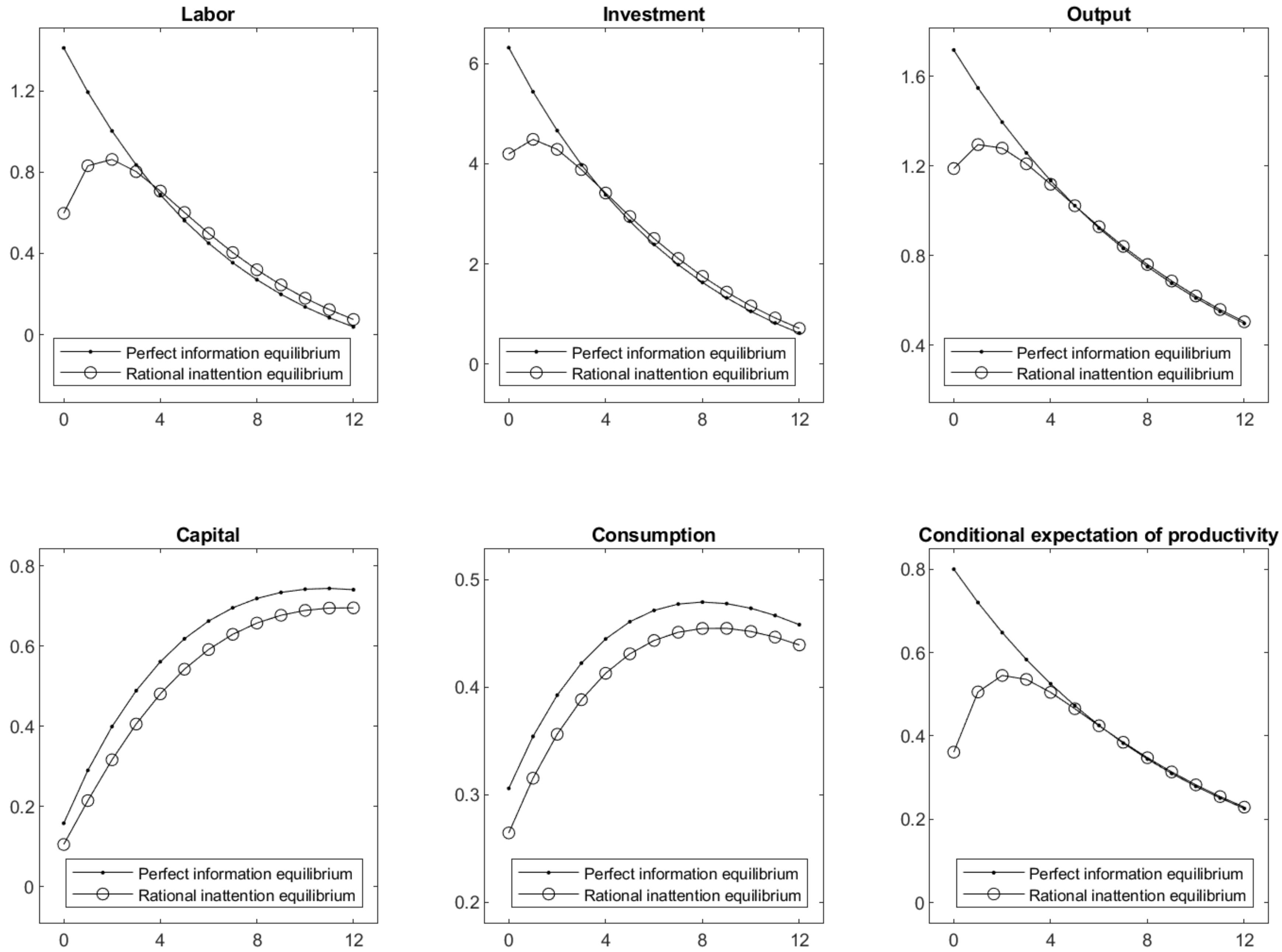

Figure 4: Impulse responses to a news shock (*h* = 2)

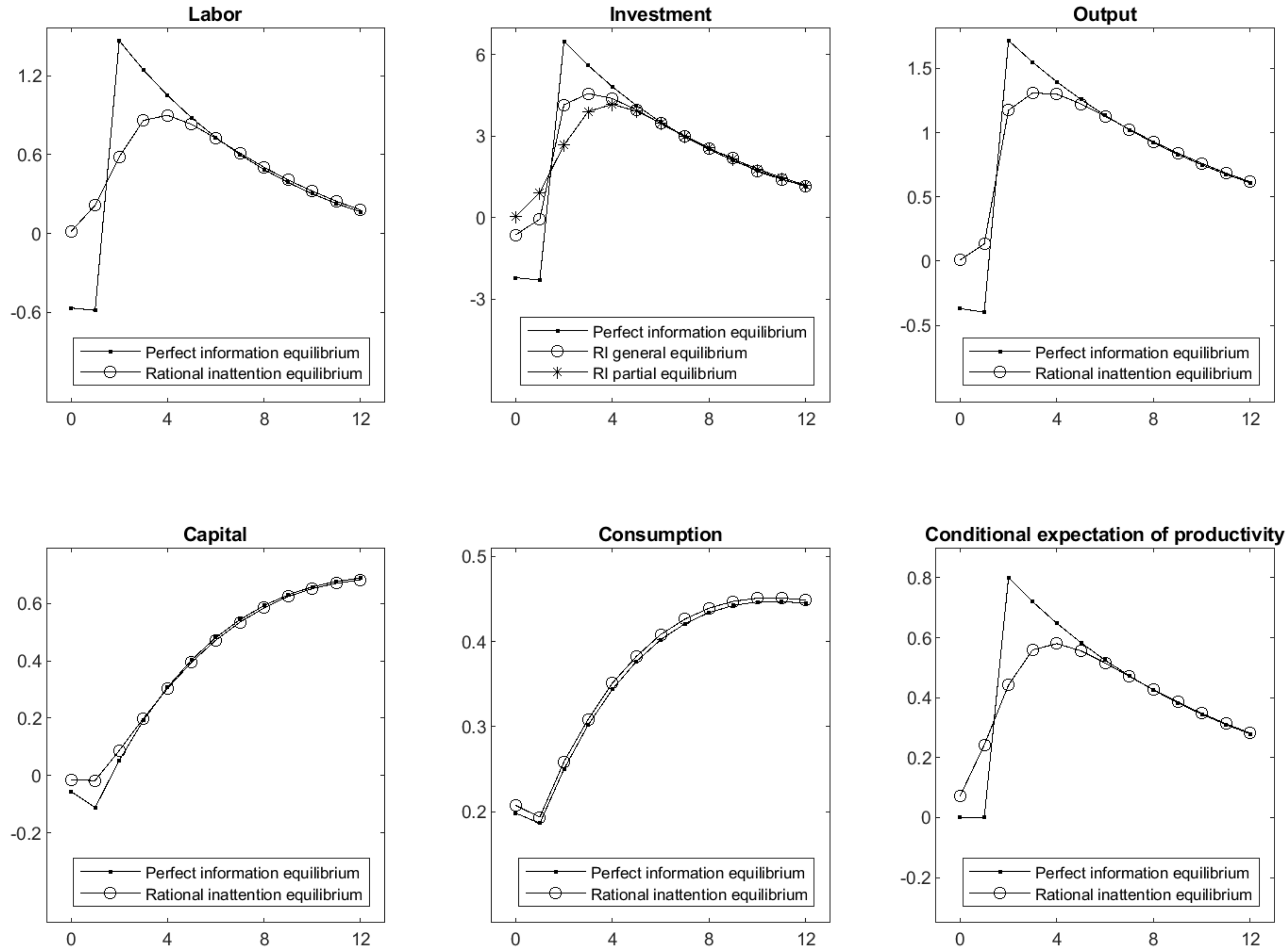

Figure 5: Impulse responses to a news shock (*h* = 4)

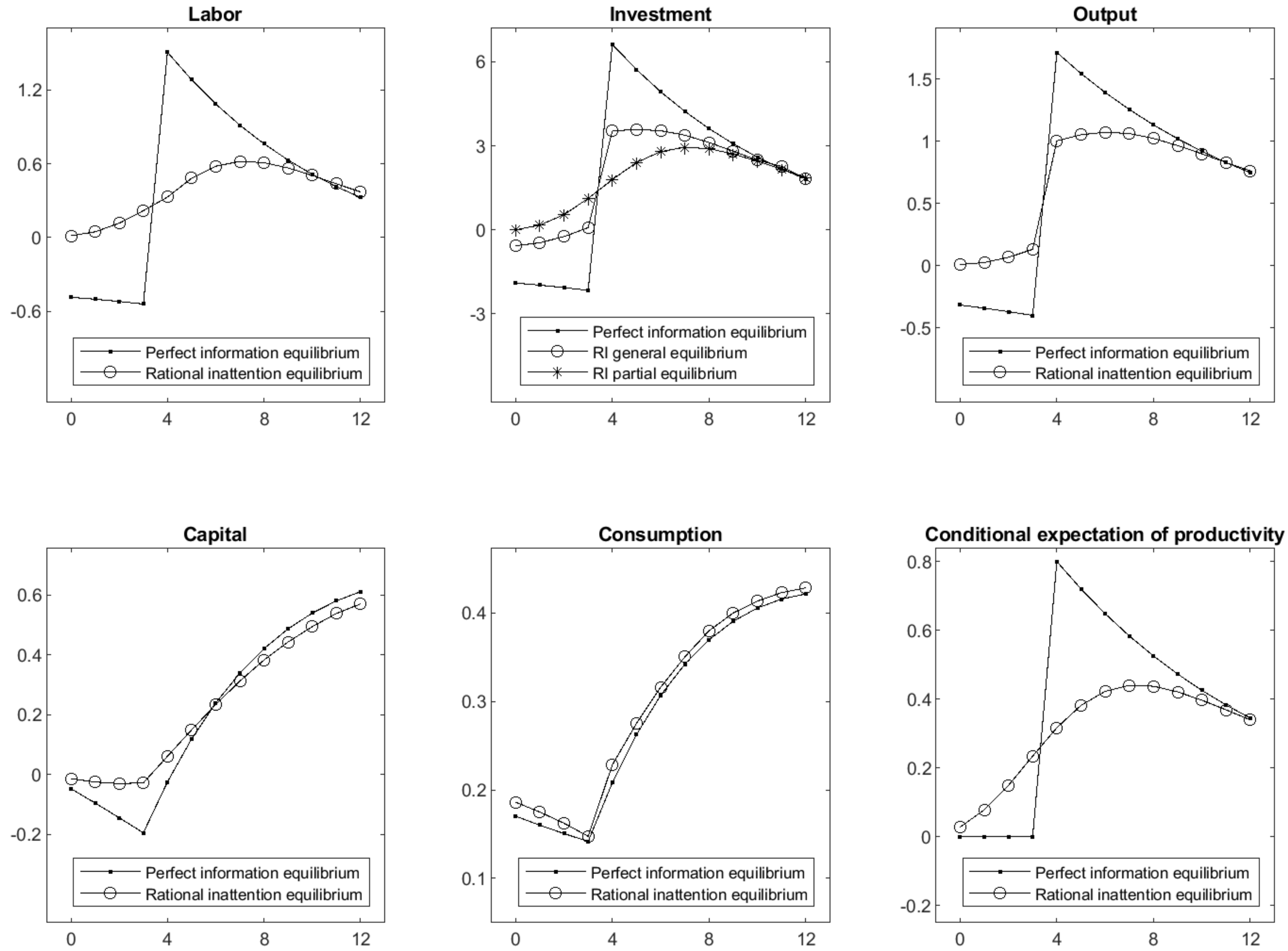

Figure 6: Impulse responses with rational inattention by firms and households