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Sample spacings for identification: the case of English auctions with absentee bidding

Marleen Marra*

Abstract

This paper presents new nonparametric identification results for ascending auctions with independent private values. The standard identification approach is infeasible in the motivating setting, because absentee bidding conceals the number of bidders. I exploit insights from the statistics literature about the stochastic ordering of adjacent sample spacings. I show how to use such sample spacings to set-identify structural features, using an incomplete model and without knowing all highest bids or the number of bidders. Applying the sample spacing method to a small sample of wine auctions, I show that it identifies informative bounds on policy-relevant counterfactuals. It turns out that Sotheby's restricts full exploitation of the exclusion principle of optimal reserve prices. As a result, sellers set sub-optimally low reserve prices. They benefit up to 13% from adopting a common reserve price rule equal to 120% of the Wine Department's pre-auction value estimate. (JEL codes: D44, C01, C46, C57)

KEYWORDS: Nonparametric set-identification, English auctions, Order statistics, Shape restrictions, Optimal reserve price

*Department of Economics, Sciences Po, 28 Rue des Saints-Péres, 75007 Paris, France. E-mail: marleen.marra@sciencespo.fr

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1 Introduction

English auctions can be notoriously secretive.[1] Motivated by the limited information observable in English auctions with absentee bidding, I show how so-called *sample spacings* (Pyke (1965, 1972)) deliver nonparametric set-identification of structural features of interest. The structural analysis of IPV English auction data typically relies on knowing the number of bidders and at least one bid order statistic (Athey and Haile (2002)). When bid data does not contain the number of bidders, the known mapping of the distribution of an order statistic from an i.i.d. sample *of known size* and its parent distribution cannot be applied. In this paper, I propose a new method that relies on the stochastic difference between adjacent order statistics, which contain previously unexplored identifying information.

The new set-identification method can be summarized in one paragraph. First, dropout values of absentee bidders are shown to bound the third-highest valuation. It is already widely known that the second-highest valuation is never more than the winning bid plus bidding increment (Haile and Tamer (2003)). Even without knowing the number of bidders, the bid vector is therefore informative about the stochastic spacing between the second- and third-highest valuation (e.g. the second-to-last spacing). The crucial final step relies on facts from the statistics literature, that properly normalized spacings from distribution functions with increasing failure rates are stochastically decreasing (Pyke (1965)). This bounds the last spacing, and also set-identifies (counterfactual) surplus and revenue. This last step is crucial because of the well-known issue in English auctions that the auction stops when the second-highest value bidder drops out.

The case of English auctions with absentee bidding is a fitting example of a setting where information revealed by sample spacings can benefit structural analysis. Absentee bidders report their maximum willingness to pay to the auctioneer who then bids on their behalf during the live auction.[2] Highest bids and number of bidders cannot be discerned from the bid vector when identities are not known, since bidders may not all place just one bid at their maximum valuation. This limited information content diverges from what is assumed known in previous English auction studies, in-

[1]See e.g. Akbarpour and Li (2019)

[2]Absentee bidding in English auctions is discussed previously in: Ginsburgh (1998), Rothkopf et al. (1990) and Thiel and Petry (1995). Lucking-reiley (2000) finds that this practice has been used since at least 1878 for stamp auctions.

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cluding Paarsch (1997) using all bidders' drop-out values and the number of bidders, Haile and Tamer (2003) and Chesher and Rosen (2015, 2017) using a vector of highest bids and the number of bidders, Song (2004) using a vector of bids that includes the second and third-highest drop-out values in some auctions, and Aradillas-López et al. (2013) using the second highest drop-out value and number of bidders (relaxing IPV). These papers are all groundbreaking in their econometric use of bid data from English auctions, but their identification strategies cannot be applied to the limited data central to this paper. For the analysis developed here, one needs to observe only: 1) a vector of bids and 2) which bids are submitted by absentee bidders. The results extend to auctions where bids from at least three different bidders can be identified.

To underscore its practical use in overcoming data limitations, I apply the method to an original dataset of fine wine auctions with absentee bidding collected at Sotheby's. Leveraging information from drop-out values of absentee bidders and the second-highest bidder, expected bidder surplus is estimated without knowing the number of bidders. Empirical results show that expected winning bidder surplus is higher for high-end wines, and lies between 75 to 125 percent of the average highest bid in the full sample. More fundamentally, this empirical application shows that the sample spacings method delivers informative bounds even in small samples with large bidding increments. This qualification matters: even when the number of bidders would be known, large bid increments would result in bounds on outcomes of interest (applying Haile and Tamer (2003)) unless they are assumed away.

This setting is also interesting from a policy angle as Sotheby's allows sellers to set a secret reserve price, but restricts it to be less than the low bound of the Wine Department's estimated value bracket. My estimates show that this results in sub-optimally low reserves, which precludes sellers from leveraging the *exclusion principle* of optimal reserve prices (see Krishna (2009), based on Myerson (1981) and Riley and Samuelson (1981)). In other words, they place too much weight on the sale probability and too little on revenues conditional on a sale. I consider a simple counterfactual policy where the reserve price is automatically set at α times the pre-auction value estimate, and show that expected seller profit increases by up to 13 percent when setting $\alpha = 1.2$. This upper bound corresponds to roughly 25.000 pounds of additional seller surplus for a single day of wine auctions, of which the auction house holds about 20 per year in its London branch alone. Also the lower bound is non-negative for any $\alpha \in [0.75, 1.2]$, indicating that it is not only the binding

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value bracket constraint that delivers low sale revenues. Even automatic reserve prices at a value of $\alpha \leq 1$ increase expected seller profit compared to the current policy where they set reserves individually.

As such, the results in this paper contribute to the structural analysis of bid data with incomplete auction models. Within that literature, Tang (2011), Aradillas-López et al. (2013), and Coey et al. (2017) are among the most related, focusing on directly (bounding) structural features of interest rather than latent value distribution. Song (2004) provides results to point-identify the value distribution using a pair of adjacent order statistics in ascending auctions. Similar to how absentee bidding reveals additional information in my motivating example, Song (2004) exploits a feature of eBay auctions to extract the third-highest valuation from the bid vector. While other papers rely on multiple order statistics (e.g. Song (2004), Mbakop (2017), and Luo and Xiao (2019)) or non-ordered measurements (e.g. Li and Vuong (1998) and Krasnokutskaya and Seim (2011)) to support identification in (auction) models, the innovation of my approach is that it relies on shape restrictions and their relation to the way adjacent order statistics are spaced out. This adds previously unexploited identifying information that helps facilitate structural analysis. Also related is recent work that uses shape properties to aid identification and estimation in auctions, notably Larsen and Zhang (2018) and Pinkse and Schurter (2019).

The paper proceeds as follows. After introducing the model in section 2, section 3 sets out the main results. An application in section 4 illustrates the method's relevance for structural analysis of incomplete bid data. Section 5 concludes.

2 Auction model

The mechanism is a standard English auction with a flexible closing rule, fixed bidding increments, and a secret reserve price. Bidders have the option to place a sealed bid ahead of the auction. Ex-ante symmetric, risk neutral bidders have unit demands and face negligible entry and bidding cost. I follow the convention to denote random variables in upper case and their realizations in lower case. All results in this paper are conditional on a vector of observed auction covariates, $\mathbf{Z}$. To economize on notation, I therefore restrict attention to conditional values. Formally, bidder i draws a valuation $V_i \sim f_{V_i}$, and $V_i = \psi(\mathbf{Z}, U_i)$, with $\psi(.)$ some function and $U_i \perp \mathbf{Z}$. Key assumptions of the auction model are:

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Assumption 1. *All n bidders symmetrically and independently draw values from a common conditional value distribution, such that:*
i) $F_{U_i}(.) = F_U(.)$, $\forall i = \{1, .., n\}$ *(exchangeability)*
ii) $F_{\mathbf{U}}(.) = F_U(.)^n$ *(independence)*
iii) $F_U(.)$ *is absolutely continuous and is defined on bounded support* $[0, \bar{u}]$ *(regularity conditions)*

This is the symmetric (conditional) IPV assumption that is the main tenet in the structural analysis of English auctions (Paarsch and Hong (2006)). Exchangeability of U_i for all bidders in the auction also guarantees that bidders' preference for live or absentee bidding is independent of their valuation. The following shape restrictions are imposed:

Assumption 2. $F_U(.)$ *satsifies:*
i) $\frac{f_U(u)}{1-F_U(u)}$ *weakly increases in* u *(increasing failure rate, IFR)*
ii) $U \leq^{disp} Y$, *where* $Y \sim^{i.i.d.} Unif[0, \bar{u}]$ *(less dispersed than uniform, LDTU)*[3]

These are mild conditions useful for identification with sample spacings. From a reliability theory perspective, the assumption rules out distribution functions that age slower than the exponential distribution (i) and that age faster than the uniform distribution on the same support (ii). As extreme dispersion as generated by the uniform distribution is never fitting (to my knowledge) to describe latent values in auction data. But also IFR is minimally restrictive, and many seminal (auction) papers previously relied on this shape restriction to generate insights without imposing a specific parametric distribution function (e.g. Myerson (1981) and Riley and Samuelson (1981) to derive a unique optimal reserve price). [4]

Furthermore, the following assumption restricts variation of the unobserved number of bidders n.

Assumption 3. *If n varies across auctions with identical covariates,* $F_U(.|n) = F_U(.|n') \; \forall n \neq n'$.

This "exogenous participation" assumption formalizes what is already implied by the combination of IPV and negligible entry cost: the absence of selective entry.

[3]Specifically, when $G(.) = Unif[0, \bar{u}]$, $F_U^{-1}(b) - F^{-1}(a) \leq G_Y^{-1}(b) - G_Y^{-1}(a)$, $\forall 0 \leq a \leq b \leq 1$.
[4]Bagnoli and Bergstrom (2005) provide references to many papers that use the slightly stronger log-concavity shape restriction in the economics literature.

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Finally, I impose intuitive behavioral assumptions that define bidding strategies in the incomplete model of Haile and Tamer (2003):

Assumption 4. *Bidders are rational and attentive.*

While these assumptions are satisfied in all symmetric separating equilibria of the button auction model of Milgrom and Weber (1982), they also allow for alternative behavior including not bidding at all or bidding less than one's valuation.

2.1 Absentee bidding

Absentee bidding is a widely adopted practice both in Egnlish auctions, as also documented by Akbarpour and Li (2019). For the data in my empirical application, Sotheby's provides the following guidance: *Absentee bids are to be executed as cheaply as permitted by other bids or reserves and in an amount up to but not exceeding the specified amounts. Bids will be rounded down to the nearest amount consistent with the bidding increment. In the case of identical (rounded) bids, the earliest submitted form will take precedence.*[5]

Auction mechanisms with absentee bidding are designed in a way that does not disadvantage absentee bidders.[6] Hence, if only one absentee bid is submitted for an item, the opening bid must start below that value. The rest of the bidding sequence will vary by institution. The following stylized sequence is based on my empirical observations that: i) the data contains various auctions with multiple absentee bids before live bidding starts and ii) after initial absentee bids the live bidding alternates with absentee bids:

Bidding sequence. *With one absentee bid, the opening bid equals a fixed share of the reserve price. With multiple absentee bids, the opening bid equals the minimum of the absentee bids. Subsequent bids follow fixed bidding increments until all but one absentee bidder drops out. Afterwards, live bids are alternated against the last remaining absentee bidder unless he has dropped out.*

[5]Source: http://www.sothebys.com/en/auctions/2014/finest-rarest-wines-l14711.html (last accessed June 11 2020).

[6]Otherwise, rational bidders would not find it optimal to place absentee bids. An auctioneer not representing (absentee) bidders truthfully would not go undetected as the distribution of winning bids for absentee bidders would be stochastically dominated by the winning bid distribution for live bidders. This goes beyond the detectability property of single deviations from truthful auctioneer behavior in Akbarpour and Li (2019) and would provide an empirical test of the protocol being truthful in practice (when the number of bidders is known).

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Crucially, the bidding sequence formalizes that absentee bidders are not (dis)advantaged relative to live bidders, which is maintained throughout. To measure this restriction against structural analysis of bid data in other IPV English auction studies, consider the two benchmark models. Paarsch (1997) assume that data is generated by the button auction model in which case all bids are assumed to be a different bidder's maximum willingness to pay. But even when relaxing the behavioral restrictions of the button auction model and using the incomplete model of Haile and Tamer (2003) for structural analysis of bid data, it is required to observe a vector of highest bids. Instead of needing to know highest bids and the number of bidders, I assume that:

Informational requirement. *The econometrician observes: i) a vector of bids, ii) which ones are absentee bids.*

Note that English auctions with absentee bidding are merely a motivating example for using sample spacings for structural analysis of bidding data. This core idea applies to any setting where drop-out values of at least two groups of bidders are observed, and those groups are ex-ante identical. Clearly, this is a relatively weak informational requirement nested in the case of observing bidder identities.

2.2 Equilibrium bidding strategies

This section describes the equilibrium bid strategies as a function of a bidder's conditional valuation, $\beta^k(u)$, for $k \in \{abs, live\}$ for respectively absentee and live bidders. I restrict attention to type-symmetric Bayes Nash equilibria in weakly undominated strategies. Recall that bidders are symmetric up to their bidding mode and conditional valuation draw. However, there is a crucial difference in the set of bidding strategies available to them. Live bidders can squat or jump bid, bid only once or bid many times incrementally.[7] The strategy available to absentee bidders is limited to the height of the bid submitted.

Lemma 1. *It is optimal for an absentee bidder with $U = u$ to bid: $\beta^{abs}(u) = u$.*

Proof. For absentee bidders, the auction is strategically equivalent to an IPV second-price sealed bid auction. It therefore follows directly from Vickrey (1961) that truthful revelation is a weakly undominated strategy for absentee bidders. □

[7]See Hasker and Sickles (2010) and the references therein.

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Lemma 2. *It is optimal for a live bidder with $U = u$ to bid: $\beta^{live}(u) \leq u$, with the added constraint that a final bid is placed if $u \geq$ standing price plus bidding increment.*

Proof. For live bidders, the auction is strategically equivalent to the English auction setting in Haile and Tamer (2003), where bidding up to one's valuation is optimal and where bidders won't let an opponent win at a price they are willing to beat. □

2.3 Resulting bounds on valuation order statistics

In this section, I leverage additional information revealed by absentee bids to bound an additional order statistic of the valuation distribution. Lemma 1 established that absentee bidders bid truthfully. However, the sequence of observed bids also include intermediate bids as the auctioneer is to determine the lowest price given competing bidders and increments, and bidder identities are unobserved. The stylized bidding sequence is used to determine which bids correspond to bidders' drop-out values and which bids correspond to such intermediate bids.

Let B denote the random variable of bids conditional on observables ($\mathbf{Z}$), b the number of submitted bids in an auction, $\Delta \geq 0$ the minimum bidding increment, and subscripts $+$ and $-$ respectively upper and lower bounds on the relevant order statistics. Using order statistic notation, $B_{b:b}$ is the highest submitted (conditional) bid, $B_{b-1:b}$ the second-highest, and B_x the highest observed drop-out value less than $B_{b-1:b}$. As used in Haile and Tamer (2003); the second-highest valuation is bounded between $B_{b-1:b}$ and $B_{b:b} + \Delta$, and the highest valuation must clearly also exceed $B_{b:b}$:

$$U_{n:n} \geq B_{b:b} \equiv U^-_{n:n} \tag{1}$$

$$U_{n-1:n} \geq B_{b-1:b} \equiv U^-_{n-1:n} \tag{2}$$

$$U_{n-1:n} \leq B_{b:b} + \Delta \equiv U^+_{n-1:n} \tag{3}$$

I add an additional relationship, using information revealed by absentee bidders' drop-out values, to bound the third-highest valuation:

$$U_{n-2:n} \geq B_x \equiv U^-_{n-2:n} \tag{4}$$

This equation holds by the definition of B_x. It is identified in any auction where a drop-out value is observed that is less than the two highest observed bids. Simply

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put, the highest two bids must be submitted by different bidders and the highest drop-out value less than the second-highest bid must be by a third bidder, delivering a lower bound on the third-highest valuation.

Characterizing B_x: examples. To illustrate, consider hypothetical bid vectors. Let A_i and L_j respectively denote the ith lowest overall bid, submitted by an absentee bidder, and the jth lowest overall bid, submitted by a live bidder:

Auction 1: A_1, A_2, A_3, L_4, L_5

Auction 2: A_1, L_2, A_3, L_4, L_5

Auction 3: A_1, L_2, A_3

In Auction 1 and Auction 2, L_4 and L_5 must be placed by different (live) bidders because nobody outbids himself, so both are (lower bounds) on drop-out values of different live bidders. A_3 is the highest other drop-out value identified, from an absentee bidder, and hence equal to B_x by definition.[8] In Auction 3, A_3 is the lowest identified drop-out value of an absentee bidder and L_2 is another drop-out value, but no lower bound on the third-highest valuation can be established.

3 The identifying power of sample spacings

The censoring problem in English auctions arises from the fact that the auction stops when the bidder with the second-highest valuation drops out. A challenge to identification of the latent value distribution and related structural features of interest is therefore that the highest valuation is never observed. In this section, I address this issue by exploiting results from the statistics literature on the stochastic spacing of order statistics. All identification results rely on the econometrician observing a large set of independent auctions.

Definition: sample spacings. Following Pyke (1965, 1972), spacings $D_{i:n}$ between two adjacent order statistics are defined as: $D_{i:n} = U_{i:n} - U_{i-1:n}$, $\forall i = 2, .., n$. Normalized spacings are defined as: $\tilde{D}_{i:n} = (n-i+1)(U_{i:n} - U_{i-1:n})$, $\forall i = 2, .., n$. Both D_i and $\tilde{D}_i$ are random variables with CDF $F_{D_{i:n}}$ and $F_{\tilde{D}_{i:n}}$ $\forall i = 2, ..., n$.

[8] Also note that in Auction 1 there must be more than one absentee bidder as there are multiple absentee bids placed before live bidding starts, and any other explanation would violate that the auctioneer acts in the best interest of the absentee bidder. According to the stylized bidding sequence, both A_1 and A_3 are drop-out values of absentee bidders. Practically, due to the fixed bidding increments and rounding of absentee bids in my data, all these drop-out values are lower bounds on the valuations of the corresponding bidders.

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To understand the usefulness of the last spacing, consider that it contains same information as the highest two conditional order statistics combined. Specifically, the density function of $D_{i:n}$ based on $U \sim^{i.i.d.} F_U(.)$ (Pyke (1965)):

$$f_{D_{i:n}}(d) = \frac{n!}{(i-2)!(n-i!)} \int_0^{\bar{u}} F_U(x)^{i-2}[1 - F_U(x+d)]^{n-i} f_U(x) f_U(x+d) dx, \quad (5)$$

which equals the density of $U_{i:n}$ conditional on the realization of $U_{i-1:n}$ in expectation over all such realizations. Seller surplus (π_S) and winning bidder revenue (π_B) at counterfactual reserve prices are of primary interest in structural auction studies. They can be expressed in terms of the last spacing and the marginal distribution of the second-highest conditional valuation:

$$\pi_S(r) = \int_0^{\bar{u}} [1 - F_{D_{n:n}}(r - u_{n-1})] \max(r, u_{n-1}) dF_{U_{n-1:n}}(u_{n-1}) \quad (6)$$

$$\pi_B(r) = \int_0^{\bar{u}} [1 - F_{D_{n:n}}(r - u_{n-1})] \{ \int_{\max(r,u_{n-1})}^{\bar{u}} u_n dF_{D_{n:n}}(u_n - u_{n-1}) \} dF_{U_{n-1:n}}(u_{n-1}) \quad (7)$$

$[1 - F_{D_{n:n}}(r - u_{n-1})]$ is the sale probability when $U_{n-1:n} = u_{n-1}$ and with reserve price r, which is equal to: $1 - F_{U_{n:n}|U_{n-1:n}}(r|u_n - 1)$. For expositional clarity of the novel identification approach, the rest of this section proceeds under the assumption that $\Delta = 0$, so that $F_{U_{n-1:n}}$ is point-identified by the distribution of the second-highest bid. This is in line with other contributions to the set-identification of auction primitives in ascending auctions (e.g. Song (2004), Aradillas-López et al. (2013), Coey et al. (2019)).

Let $F^-_{D_{n:n}}(.)$ and $F^+_{D_{n:n}}(.)$ respectively denote the lower and upper bound on the distribution of $D_{n:n}$, such that: $F^-_{D_{n:n}}(d) \leq F_{D_{n:n}}(d) \leq F^+_{D_{n:n}}(d)$, $\forall d \in [0, \bar{u}]$. By stochastic dominance, bounds on π_B and π_S are defined as:

$$\begin{aligned} \int_0^{\bar{u}} [1 - F^-_{D_{n:n}}(r - u_{n-1})] \max(r, u_{n-1}) dF_{U_{n-1:n}}(u_{n-1}) &\geq \pi_S(r) \quad (8) \\ \geq \int_0^{\bar{u}} [1 - F^+_{D_{n:n}}(r - u_{n-1})] \max(r, u_{n-1}) dF_{U_{n-1:n}}(u_{n-1}) & \end{aligned}$$

$$\begin{aligned} \int_0^{\bar{u}} [1 - F^-_{D_{n:n}}(r - u_{n-1})] \{ \int_{\max(r,u_{n-1})}^{\bar{u}} u_n dF^-_{D_{n:n}}(u_n - u_{n-1}) \} dF_{U_{n-1:n}}(u_{n-1}) &\geq \pi_B(r) \quad (9) \\ \geq \int_0^{\bar{u}} [1 - F^+_{D_{n:n}}(r - u_{n-1})] \{ \int_{\max(r,u_{n-1})}^{\bar{u}} u_n dF^+_{D_{n:n}}(u_n - u_{n-1}) \} dF_{U_{n-1:n}}(u_{n-1}) & \end{aligned}$$

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As such, the upper bound on the sale probability is derived with $F^{-}_{D_{n:n}}$; a stochastically larger last spacing means a larger probability that $U_{n:n}$ exceeds r. The upper bound on the winning bidder surplus conditional on a sale (the inner integral in 7) is also derived from $F^{-}_{D_{n:n}}$ as it translates directly into a larger $U_{n:n} - U_{n-1:n}$.

If n would be known, one could work with the marginal distribution of $F_{U_{n:n}}$ by taking it out of the (outer) integrals in (6)-(7).[9] My identification approach instead exploits the relation between ageing properties of $F_U(.)$ and the stochastic ordering of its sample spacings.

3.1 Identification of $F^{-}_{D_{(n:n)}}(.)$ and $F^{+}_{D_{(n:n)}}(.)$

Due to the censoring problem, $F^{-}_{D_{(n:n)}}(.)$ is the more challenging bound to pin down. To do so, I rely on the result by Barlow and Proschan (1966) that normalized spacings from distribution functions with increasing failure rates are stochastically decreasing. Intuitively, this is because *normalized* spacings from the exponential distribution are exchangeable random variables (Pyke (1965)), and $F_U(.)$ ages faster than the exponential in the reliability theory sense (by the shape assumption 2 ii).

Lemma 3. *$U_{n-1:n}$ and $U^{-}_{n-2:n}$ identify $F^{-}_{D_{(n:n)}}(.)$ without knowing n:*

$$F^{-}_{D_{n:n}}(d) \equiv P[2(U_{n-1:n} - U^{-}_{n-2:n}) \leq d] \ , \forall d \in [0, \bar{u}] \tag{10}$$

Proof. For all d on its support:

$$\begin{aligned} F_{D_{n:n}}(d) &= P[D_{n:n} \leq d] \text{ (by definition of CDF)} \\ &\geq P[2D_{n-1:n} \leq d] \text{ (by decreasing normalized spacings} \\ &\quad \text{due to IFR (Barlow and Proschan (1966)))} \\ &\geq P[2(U_{n-1:n} - U^{-}_{n-2:n}) \leq d] \text{ (by definition of bound)} \end{aligned}$$

□

This result alone is therefore sufficient to bound π_B and π_S from above. Lower bounds could be derived by rewriting (6)-(7) in terms of marginal value distribu-

[9] This is for example done in Aradillas-López et al. (2013). They also focus directly on overcoming the censoring problem, in an incomplete model that allows for correlated private values. Their clever identification strategy exploits observed exogenous variation in the number of bidders.

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tions, and then using that an upper bound on $F_{U_{n:n}}(.)$ is trivially identified as the distribution of the winning bid.

Sample spacings provide additional identifying information for the lower bounds on π_B and π_S. $F^+_{D_{n:n}}(.)$ is based on the fact that uniform spacings, e.g. spacings of i.i.d. draws from a uniform distribution, are exchangeable random variables (Pyke (1965)), and that $F_U(.)$ ages slower than the uniform in the reliability theory sense (by the shape assumption 2 i).

Lemma 4. *$U_{n-1:n}$ and $U_{n-2:n}$ identify $F^+_{D_{(n:n)}}(.)$ without knowing n:*

$$F^+_{D_{n:n}}(d) \equiv P[(U_{n-1:n} - U_{n-2:n}) \leq d] \ , \forall d \in [0, \bar{u}] \tag{11}$$

Proof. Let $Y \sim^{i.i.d.} G_Y(.) = Unif[0, \bar{u}]$, and $\{H_{i:n}\}_{i=1,..,n}$ its sample spacings. By assumption 2 i, $U \leq^{disp} Z$. Bartoszewicz (1986) proves that this implies the stochastic ordering: $D_{i:n} \leq^{st} H_{i:n} \ \forall i \in \{1, .., n\}$. Therefore for all d on its support:

$$\begin{aligned} F_{D_{(n:n)}}(d) &= P[D_{n:n} \leq d] \text{ (by definition of CDF)} \\ &\leq P[D_{n-1:n} \leq d] \text{ (by increasing spacings due to LDTU,} \\ &\quad \text{applying Pyke (1965) and Bartoszewicz (1986))} \end{aligned}$$

□

This bound is based on the second-to-last spacing being pinned down exactly from bidding data, such as in the eBay auction model of Song (2004) or based on it Adams (2007) and il Kim and Lee (2014), and in *any* auction model where bids are equated to drop-out values. The presented model describing traditional English auctions with absentee bidding allows for additional incompleteness of the bid vector in terms of drop-out values. The result is presented regardless to provide a richer perspective on the use of sample spacings in the structural analysis of bid data. In the empirical application, I verify a necessary condition for applicability of this upper bound, based on the empirical and simulated sale probabilities. A comparison of estimation results based on the distribution of highest bid provides additional insight into these two approaches.

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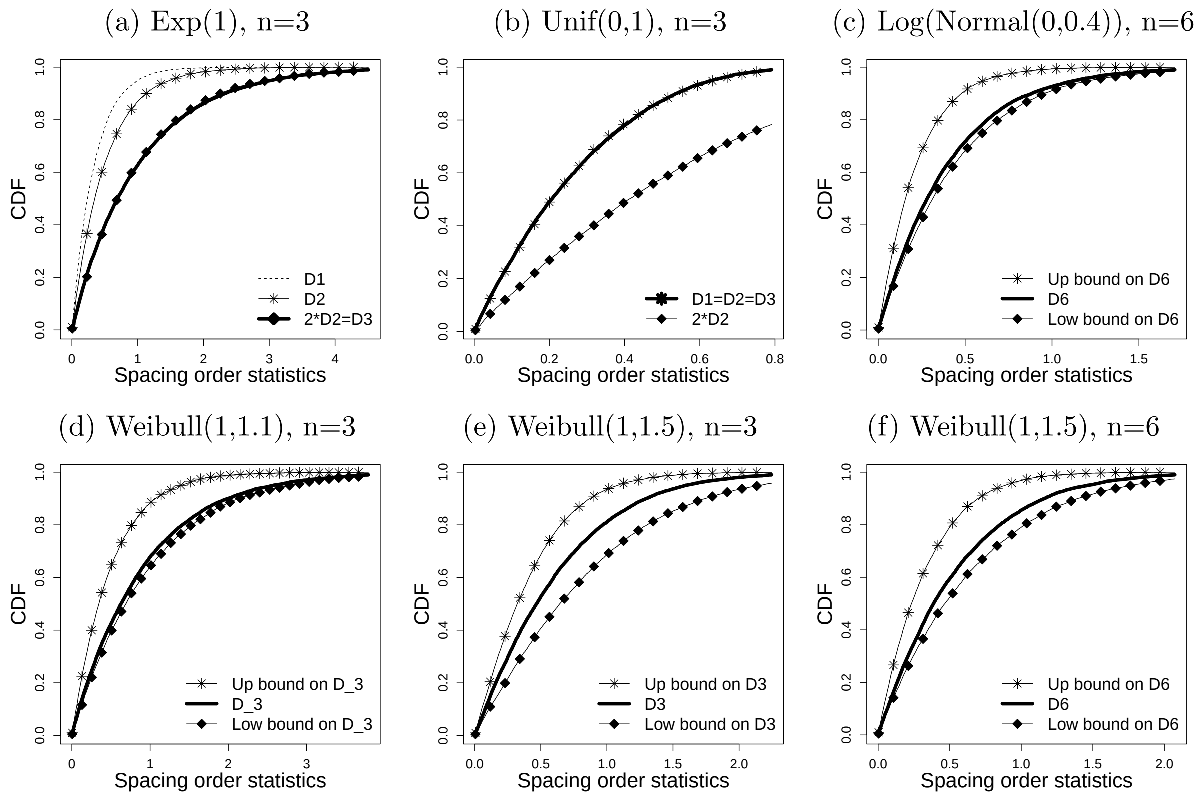

Figure 1: Informativeness of bounds on $F_{D_{n:n}}(.)$)

Based on Lemma 3 and 4 applied to different data generating processes, plotted for spacings between the 1st percentile of $D_{n-1:n}$ and the 99th percentile of $D_{n:n}$ (x-axis). Di in legends refers to $D_{i:n}$.

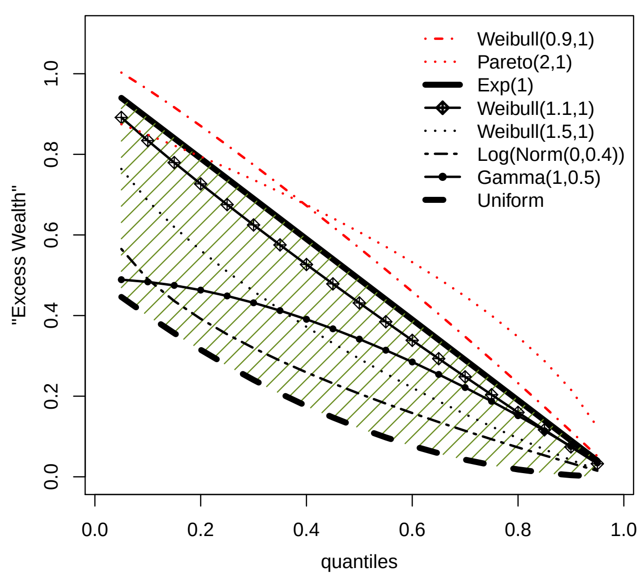

Figure 2: Illustration of excess wealth order to summarize shape restrictions

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3.2 Simulations

Figure 1 shows how Lemma 3 and 4 apply to familiar distribution functions. Plots compare the true distribution of the spacing between the highest two order statistics with its bounds implied by the difference between $U_{n-1:n}$ and $U_{n-2:n}$. Plotted are simple empirical CDF's based on 10.000 simulated sets of values, abstracting from bidding increments.[10]

Plots a and b illustrate the simple basis for the presented identification results. Plot a is based on the Exponential distribution. $F^-_{D_{n:n}}(.)$ as derived in Lemma 3 collapses to the true $F_{D_{n:n}}(.)$. For any distribution function that ages faster than the Exponential, the last spacing will therefore be stochastically dominated by $F^-_{D_{n:n}}(.)$. Plot b is based on the Uniform distribution. $F^+_{D_{n:n}}(.)$ as derived in Lemma 4 collapses to the true $F_{D_{n:n}}(.)$. For any distribution that ages slower than the Uniform, the last spacing will therefore be stochastically dominating $F^+_{D_{n:n}}(.)$.

Four other points are important to take away from these simulations. First, leveraging information revealed by sample spacings can indeed result in highly informative bounds. Second, $F^-_{D_{n:n}}(.)$ is tighter the less IFR the underlying distribution function is (plot d versus e). This simultaneously means that the lower bound is less tight in those instances. Third, the larger the number of (unobserved) bidders, the tighter $F^-_{D_{n:n}}(.)$ (plot f versus e). And fourth, IFR is sufficient but not necessary for the sample spacing method to identify $F^-_{D_{n:n}}(.)$ (plot c).

I find the excess wealth order intuitive to interpret the two shape restrictions in Assumption 2, especially because identification focuses on the upper tail of the distribution of F_U.

Definition: Excess Wealth Order. Random variable U is larger than Y in the excess wealth order ($U \geq^{EW} Y$) if and only if: $\int_{F_U^{-1}(p)}^{\infty} 1 - F_U(t)dt \geq \int_{F_Y^{-1}(p)}^{\infty} 1 - F_Y(t)dt, \forall p$. For IFR distribution functions it holds that the dispersive ordering implies the excess wealth order (**?**), so that all IFR distributions satisfying $U \geq^{EW} Y \sim Unif[a, b]$ also satisfies $U \geq^{disp} Y$.

It is easy to show that the excess wealth of $E \sim Exp(\lambda)$ at quantile p equals: $\frac{p}{\lambda}$, e.g. in Figure 2 the thick solid line with slope -1 (exponential with rate $\lambda = 1$). IFR distributions (with $F(0) = 0$) have lower excess wealth at each q. So distribution

[10] Generally, parametric distribution functions that have an increasing failure rate include the Normal, Exponential, Logistic, Extreme Value, Weibull (shape parameter ≥ 1), Gamma (shape parameter ≥ 1), and Beta (shape parameter ≥ 1)) (Bagnoli and Bergstrom (2005)).

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functions with excess wealth in between of the $Unif[0,1]$ and $Exp(1)$ distributions satisfy the two shape assumptions of this paper. This corresponds to the shaded area in the figure.

4 Application: wine auctions at Sotheby's

I apply the sample spacing method to a unique dataset covering the 884 lots from the "Finest and Rarest Wines & Vintage Port" auction on November 19th 2014 at Sotheby's London.[11] This provides a good and conservative test case as it is a relatively small dataset, with large bidding increments, and it does not contain the number of bidders. The dispersion of highest bids is large, even when normalizing by the number of bottles in the lot, and especially the upper tail is long. To reduce the impact of extremal values I therefore exclude auctions for which $V^-_{n:n}$ or $V^+_{n:n}$ exceed their 95th percentile.[12] Descriptive statistics of the remaining sample are provided in Table 1. As common in traditional English auctions: increments are high at between 2 - 16 percent of the winning bid.

The presence of absentee bidding requires bidders to be willing to announce their bids before others do, which is by itself a strong indication that bidders do not anticipate a "winners curse" and that the assumption of private values is justified. It is reasonable to expect that correlation in valuations is captured by auction observables, including the estimated value by Sotheby's Wine Department, if these specialists are unlikely to be outperformed by potential buyers in establishing the current value of the wines. The data supports this idea. The high predictive power of Sotheby's pre-auction value estimate is highlighted in Figure 3 plot a, showing its relation with the realized winning bid. Empirical tests based on Kolmogorov-Smirnov tests confirm that IPV is reasonable assumption when conditioning on Sotheby's value estimate, and that additional conditioning variables do not strengthen this conclusion.[13]

[11] I collected the data by simply registering as an online bidder, recording the complete auction, translating the video material into a dataset of bids, and adding lot descriptors from the catalogue.

[12] This is done unconditional on Z but normalizing the variables by the number of bottles. The 99th (95th) percentiles of unconditional per-bottle $V^-_{n:n}$ and $V^+_{n:n}$ are respectively: 945.83 (3356.67) and 2419.50 (6540.00) pounds.

[13] The test is done under the equilibrium assumptions that the distribution of the lowest (second-highest) bid equals the distribution of lowest (second-highest) valuation, and performed separately for 3-20 bidders. Resulting high p-values when at least conditioning on Sotheby's low value estimate indicate the unlikeliness that the resulting parent distribution is not the same, justifying the IPV framework up to the equilibrium assumptions being valid. Some additional caution in interpreting

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Table 1: Descriptive statistics for sample of fine wine auctions

	N	Mean	St. Dev.	Min	25th pct.	Median	75th pct.	Max
Opening bid (pounds)	697	846.255	847.841	50	280	520	1,150	9,000
Highest bid (pounds)	697	999.857	993.776	90	340	620	1,300	9,200
Number bottles per lot	697	8.624	4.721	1	6	8	12	36
Highest bid per bottle (pounds)	697	166.434	183.561	5	40	87.50	225	933.33
Increment at highest bid (%)	697	5.623	1.949	2.041	4.348	5.263	6.667	16.000
Is sold	697	0.902	0.297	0	1	1	1	1
Number of bids	697	4.184	2.936	2	2	3	5	25
Sotheby's low estimate (pounds)	697	917.805	980.857	80	300	550	1,200	13,500
Sotheby's high estimate (% above low estimate)	697	27.418	6.582	9.091	23.077	27.273	30.769	62.500
Special format (0.5l, magnum, double mag, ...)	697	0.063	0.243	0	0	0	0	1
Vintage†	647	1999	9.787	1929	1995	2002	2006	2012
Mixed lot (different wines)	697	0.088	0.283	0	0	0	0	1

†: The vintage is missing for non-vintage champagnes and for mixed lots of various vintages.

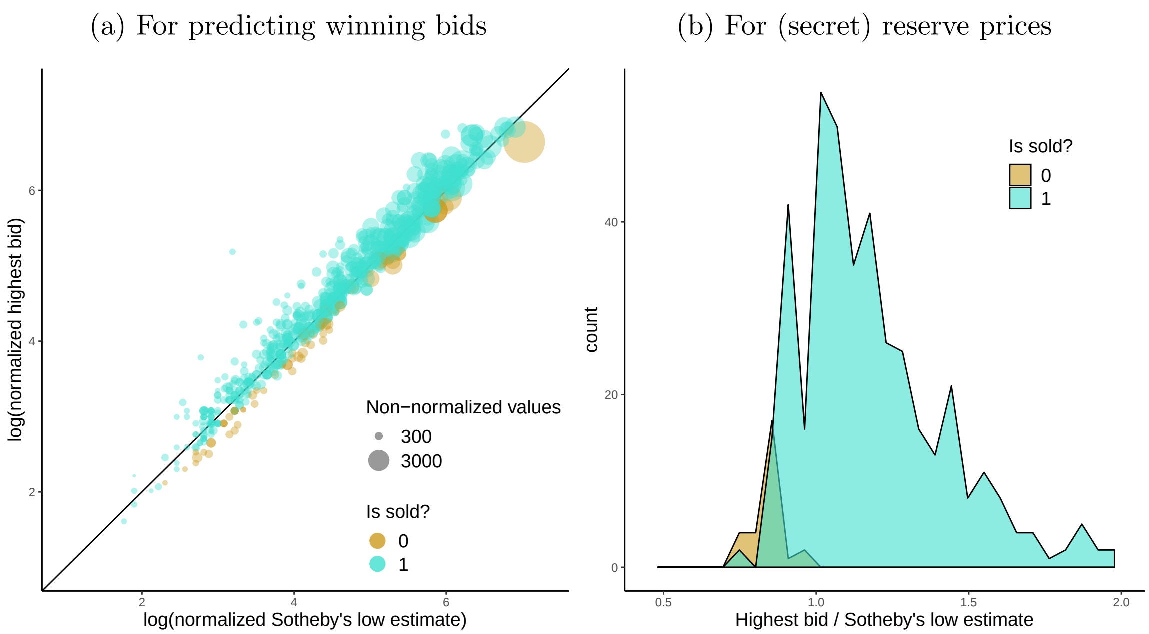

Figure 3: The role of Sotheby's pre-auction value estimate.

Plot a) normalized values are divided by the number of bottles in the lot; dot sizes reflect non-normalized values.

4.1 Nonparametric estimation

This section shows how to estimate bounds on π_S and π_B (equations 8 and 9) by applying the spacings identification approach. $\hat{\pi}_S$ and $\hat{\pi}_B$ are based on product Kernel estimators of conditional density functions ($dF_{V_{n-1:n}|Z}(.)$, $dF^{+}_{D_{n:n}|Z}$, $dF^{-}_{D_{n:n}|Z}$) and

these results is warranted: the small sample size may also be a reason that the Kolmogorov-Smirnov tests do not reject the null, which is why the table with test results is not reported here.

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conditional cumulative distribution functions ($F^{+}_{D_{n:n}|Z}(.)$, $F^{-}_{D_{n:n}|Z}(.)$), such as:

$$d\hat{F}^{+}_{D_{n:n}|Z}(d|z) = \frac{\frac{1}{h^{D-}}\sum_{t\in\mathcal{T}^-}^{K}\left(\frac{D^{t-}_{n:n}-d}{h^{D-}}\right)K\left(\frac{Z^{t-}-z}{h^{Z-}}\right)}{\sum_{t\in\mathcal{T}^-}K\left(\frac{Z^{t-}-z}{h^{Z-}}\right)} \tag{12}$$

$$\hat{F}^{-}_{D_{n:n}|Z}(d|z) = \frac{\sum_{t\in\mathcal{T}^+}L\left(\frac{D^{t+}_{n:n}-d}{h^{D+}}\right)K\left(\frac{Z^{t+}-z}{h^{Z+}}\right)}{\sum_{t\in\mathcal{T}^+}K\left(\frac{Z^{t+}-z}{h^{Z+}}\right)}, \quad \text{with: } L(x) = \int_{-\infty}^{x}K(u)\,du \tag{13}$$

$\forall(d,z)$ on their supports. The minus (plus) superscripts on $\mathcal{T}$, $D_{n:n}$, Z, and h indicate that they relate to the lower (upper) bound on the last spacing and the therefore relevant observations. $K(.)$ indicates the Epanechnikov kernel function, h the cross-validated variable-specific bandwidths as functions of the relevant sample sizes. Moreover, let $T^- = |\mathcal{T}^-|$ ($T^+ = |\mathcal{T}^+|$) denote the total number of auctions in which $D^-_{n:n}$ ($D^+_{n:n}$) is identified, with the caligraphic script denoting sets of such auctions.[14]

For a meaningful analysis of these estimators, estimated spacings CDF's are first applied to bound consumer surplus in the data (CS^t):

$$\hat{CS}^t \in [\int(b^t+x)d\hat{F}^{-}_{D_{n:n}|Z}(x|z)dx-(b^t+\Delta), \int(b^t+x)d\hat{F}^{+}_{D_{n:n}|Z}(x|z)dx-(b^t+\Delta)], \tag{14}$$

for all t sold, and 0 otherwise, and with $B_{b-1:b} = b_t$ and $B_{b:b} = b_t + \Delta$ respectively the observed second-highest and highest "winning bid" equal to b_t plus bidding increment. $\hat{CS}^t$ differs from counterfactual surplus as defined in (7) as it does not involve the sale probability: it conditions on the event of a sale given the current (unknown) reserve price. The upper bound on CS^t is also compared against one derived from the distribution of $B_{b:b}$ itself, as anticipated on page 12:

$$\hat{CS}^t \geq \int x d\hat{F}^{+}_{V_{n:n}|Z}(x|z)dx - (b^t + \Delta) \tag{15}$$

After estimating the three conditional densities, $\hat{CS}^t$ is approximated numerically on a fine grid of x, for all sold auctions. Table 2 reports estimated bounds on $\mathbb{E}[CS^t]$, both by tertile of the conditioning variable and for the whole sample. Results highlight

[14]The former requiring at least two bids. The latter requiring at least three bids, and identification of an absentee drop-out value less than the second-highest bid. Exogenous variation in the unobserved number of bidders (if there is variation) and IPV guarantee that these selection criteria are inconsequential. Uniform consistency of the PDF requires $h^{D-} \to 0$, $h^{Z-} \to 0$, and $T^- h^{D-} h^{Z-} \to \infty$ as $T^- \to \infty$, and for the CDF that $h^{Z+} \to 0$ and $T^+ h^{Z+} \to \infty$ as $T^+ \to \infty$.

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Table 2: Estimated bounds on $\mathbb{E}[CS^t]$, by tertile of Z

	1st tertile	2nd tertile	3rd tertile	all auctions
Observed:				
N (sold)	225	195	209	629
Z, mean	29.981	96.546	341.194	154.025
Winning bid	35.087	111.218	385.726	175.197
Estimated bounds on $\mathbb{E}[CS^t]$:				
Lower bound (based on $V^-_{n:n}$)	55.392	118.029	276.335	151.592
95% Confidence interval	[50.471,61.768]	[112.477,123.637]	[254.843,291.649]	[141.767,162.380]
Lower bound (based on $D^-_{n:n}$)	47.830	134.683	234.089	130.792
95% Confidence interval	[43.626,52.584]	[125.689,143.423]	[218.889,248.296]	[120.669,139.681]
Upper bound (based on $D^+_{n:n}$)	76.460	214.824	382.647	218.339
95% Confidence interval	[68.655,85.092]	[191.840,237.115]	[342.632,432.390]	[201.082,233.435]

Estimates are in pounds per bottle. Confidence intervals are calculated from 100 bootstrap samples. Z = Sotheby's low value estimate.

(a) Bounds (b) Difference up-low bound

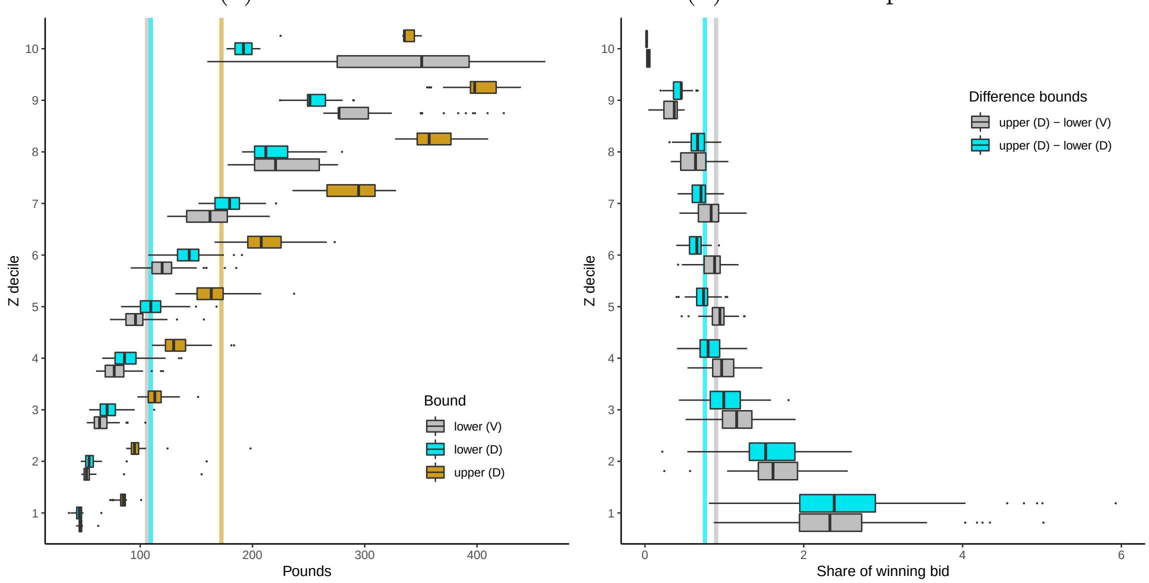

Figure 4: Heterogeneity of auction-level surplus CS^t

Plot a: point estimates of bounds, by decile of Z=Sotheby's pre-auction value estimate. Plot b: differences in point estimates bounds as a share of the winning bid, by decile of Z. Vertical lines indicate sample medians.

how informative the estimated bounds are, even in this "worst-case scenario": a small sample with large bidding increments and without knowing the number of bidders.

The (point estimate of the) lower bound on expected consumer surplus equals 75 percent of the average winning bid (87 percent when based on $V^-_{n:n}$ instead of $D^-_{n:n}$), not far removed from the upper bound at 125 percent. Estimated bounds are wider at

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the tertile level, based on only a third of the sample and about 200 observations. The middle tertile for instance has estimated bounds between 121 and 193 percent of the average winning bid, and the first tertile between 137 and 218 percent of the winning bid. Another clear result is that consumer surplus is higher for higher-end wines, with even the upper bound of the estimated surplus for the first tertile being less than the lower bound for the second tertile, and similarly for comparing the second to the third tertile. This goes through even when taking the 95% bootstrapped confidence intervals into account.

Figure 4 explores further auction-level heterogeneity of $\hat{CS}^t$. Plot a) displays heterogeneity of estimated bounds by decile of the conditioning variable. The previous result that higher-value lots deliver more consumer surplus (which is not trivially so) is reinforced. The boxplots also reveal that there is more heterogeneity for higher-end wines, insofar as this is reflected by Sotheby's pre-auction value estimate. Plot b) plots the *difference* between the point estimate of high and low bound on CS^t, in this case reported as a share of the winning bid, and plotted by decile of the winning bid. There is again a remarking auction-level variation, with the sample spacing method resulting in bounds being more informative in terms of this outcome for higher-end wines.

4.2 Policy simulation: optimal reserve price

It is an official policy of Sotheby's to allow only reserve prices at or below the low bound of the value estimate.[15] Also empirically, Figure 3 plot b shows that secret reserve prices are set at a fixed share of Sotheby's low value estimate. The highest bids are below this estimate for all unsold lots (between 75-100%). Together with the high sale probability of 90 percent, this begs the question whether and how much sellers would benefit from adopting an optimal reserve price; a counterfactual of primary interest in empirical auction studies.[16]

To bound the increase in expected seller revenue from adopting a higher reserve price, I run the following simulation exercise. I consider counterfactual reserve prices $\tilde{r}_t = \alpha Z_t$ for a range of $\alpha \in (1, 2)$, thereby relaxing the constraint on reserve prices

[15] Source: https://www.sothebys.com/en/glossary (last accessed June 11 2020).

[16] Including in: Paarsch (1997), Haile and Tamer (2003), Tang (2011), Aradillas-López et al. (2013), Coey et al. (2017, 2019), either first estimating the latent value distribution or identifying this structural feature directly (within bounds) as in this paper.

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insofar as the current policy is binding. To evaluate the impact on sellers, I estimate bounds on π_S by applying the results from Lemma's 3 and 4 to (8). With reserve prices being secret, the highest bid of the bid vector is the floor of the counterfactual bid realizations and the trade-off of increasing the price conditional on a sale with a lower sale probability drives the results.

Specifically, the expectation over $U_{n-1:n}$ in (8) is done over realizations of $B_{b:b}$ in the data (not excluding unsold lots). Suggestive evidence that the upper bound $F^+_{D_{n:n}}(.)$ in Lemma 4 applies is that the simulated lower bound on the sale probability indeed is lower than the empirical sale probability, at 93.7 percent versus 94.6 percent. All counterfactual results are simulated only for auctions in which $D^+_{n:n}$ is identified (e.g. where B_x is identified), to make sure the lower and upper bounds relate to the same primitive. $\hat{F}^+_{D_{n:n}}(.)$ and $F^-_{D_{n:n}}(.)$ are estimated as described in the previous section, with the addition that the larger cross-validated bandwidths from the upper bound are used in both estimators to guarantee that the bounds don't cross in the even smaller sample (167 observations).

The benchmark policy is that Sotheby's determines Z and sellers choose a reserve price less than Z, so this counterfactual sheds light on two questions: 1) whether sellers on the whole would be better off when adopting the simple policy of a standardized and reserve price rule, and 2) whether sellers and Sotheby's gain from allowing the reserve price (rule) to exceed Z.

Resulting $\hat{\pi}_S(r)$ are plotted in Figure 5. Despite not point-identifying it, the simulations reveal interesting facts. Moving from a system where sellers set their secret reserve individually between 75-100% of Z to one where there is a common reserve price rule *at any level* between 75-100% of Z is clearly beneficial to sellers. This suggests that even conditional on the reserve price constraint $r \leq Z$, individual sellers set suboptimal reserve prices. They would gain at least 2.5 percent by setting a common reserve price rule $\alpha \in [0.75, 1]$, and the upper bound for such a policy is estimated at 6 percent gain.

The real kicker comes for higher values of α meaning that reserve prices are set sub-optimally low, and that the reserve price constraint is indeed binding. The policy to set $\tilde{r}_t = 1.2z_t$ increases seller revenue up to 13 percent.

Recall that Sotheby's provides a value *bracket* for each lot, and it is notable that $\alpha = 1.2$ would set the secret reserve near the upper end of this bracket (the median upper bound is 27% higher than Z). In other words, by restricting $\alpha \leq 1$,

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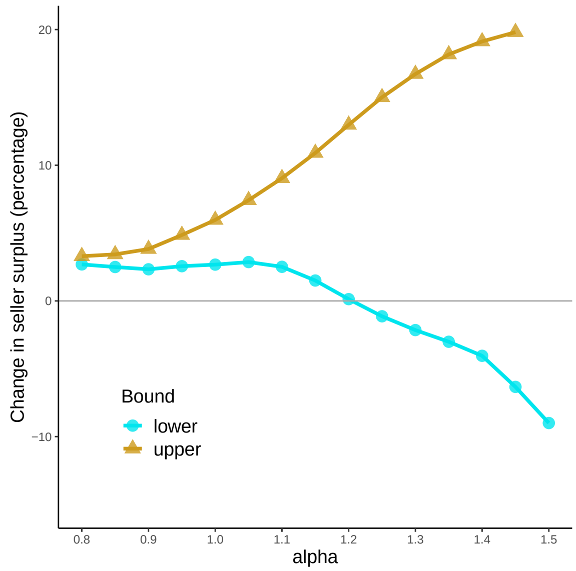

Figure 5: Policy simulation: $\pi_S^{\Delta}(\tilde{r})$ with reserve price rule $r_t = \alpha z_t$

Sotheby's does not fully allow for reserve prices to satisfy the *exclusion principle* of optimal reserve prices (Krishna (2009), based on Myerson (1981) and Riley and Samuelson (1981)). It places too much weight on the sale probability and too little on the expected revenue conditional on a sale. It is even suboptimal for Sotheby's themselves as most of their income comes from commissions.At higher values of α, the lower bound on $\hat{\pi}_S(r)$ becomes negative so can't be said with certainty that higher levels are beneficial.

In terms of monetary values and extrapolating to the whole sample, the wine auctions would generate up to 25,694 pounds in additional seller surplus when setting a common reserve price rule of $\tilde{r}_t = 1.2z_t$. This is for just one day of auctions; Sotheby's holds over 20 of these "Finest and Rarest Wine auctions" per year.

5 Conclusion

This paper proposes a new approach to identify policy counterfactuals from limited Enlisch auction data. It shows previously unexplored identifying power of the *spacing* of order statistics in combination with weak shape restrictions. Simulations furthermore highlight the simple identification approach based on ageing properties of the extremal cases of exponentially and uniformly distributed latent values. A particu-

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lar benefit of the approach is that it provides a feasible solution to not knowing the number of bidders, as in the motivating example of English auctions with absentee bidding. While the results apply more generally, this setting is exploited to identify additional information about the drop-out value of one absentee bidder such that the distribution of the second-to-last spacing is bounded. Combined with results from the statistics literature about the stochastic ordering of adjacent spacings, the paper shows that this delivers an upper bound on the last spacing and hence overcomes the censuring problem of English auctions.

The method is applied to a new dataset of fine wine auctions in which the number of bidders is unknown. Results highlight that even in small samples with large bidding increments, sample spacings allow for the estimation of informative bounds on structural features of interest. For example, consumer surplus is estimated to be between 75-125 percent of the average highest bid. It also turns out that surplus is higher for higher-end wines. Structural estimates are used to evaluate the benefit of relaxing Sotheby's policy that the reserve price has to be lower than the pre-auction value estimate. Results show that the restriction is indeed binding: sellers benefit by up to 13 percent from setting a reserve price equal to 120 percent of the value estimate. This suggests that the current policy limits sellers to fully leverage the exclusion principle of optimal reserve prices.

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