Are Bartik Regressions Always Robust to Heterogeneous Treatment Effects?*

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July 21, 2022

Abstract

Bartik regressions use locations' differential exposure to nationwide sector-level shocks as an instrument to estimate the effect of a location-level treatment on an outcome. We show that under parallel-trends assumptions, Bartik regressions may estimate weighted sums of location-and-period-specific treatment effects, with some negative weights. Accordingly, they may not be robust to heterogeneous effects across locations or periods. We provide simple diagnostic tools researchers may use to assess the robustness of their regression. Finally, we propose alternative correlated-random-coefficient estimators that are more robust to heterogeneous effects than Bartik regressions. We use our results to revisit two empirical applications.

*We are very grateful to Xavier D'Haultfœuille, Paul Goldsmith-Pinkham, Peter Hull, Michal Kolesár, Félix Pasquier, Isaac Sorkin, and seminar participants at the Sao Paulo School of Economics for their helpful comments.

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1 Introduction

The "Bartik instrument", also known as the "shift-share instrument", is a popular method to estimate the effect of a treatment on an outcome. It has first been proposed by Bartik (1991). Since then it has been applied in many fields, including labor (see Altonji & Card 1991, Card 2001, 2009), international trade (see Autor et al. 2013, 2020), and finance (see Greenstone et al. 2020).

To fix ideas, in this introduction we describe our paper in the context of the canonical setting considered by Bartik (1991), but our results apply to any Bartik design. Assume one wants to estimate how the evolution of employment affects the evolution of wages. One could construct a data set with wages and employment evolutions at, say, the commuting-zone (CZ) level, and regress the former evolution on the latter. However, employment growth may be endogenous. For example, it may be correlated with other CZ-level shocks that could have an impact on wage growth. Instead, a Bartik two-stage least squares (2SLS) regression uses a weighted average of the nationwide employment growth in each industry, with weights equal to the share that each industry accounts for in the employment of a CZ, to instrument for CZs' employment growth. Intuitively, this strategy may solve the aforementioned endogeneity problem, because the variation of the Bartik instrument across CZs comes from their differential exposure to nationwide shocks, not from local, CZ-level shocks.

This paper makes four contributions. First, we show that under parallel-trends assumptions, if the treatment effect varies across CZs or over time, Bartik regressions may not identify well-defined averages of CZ-and-period-specific effects. Instead, they may identify weighted sums of those effects, with some negative weights. Due to the negative weights, Bartik regression coefficients may not satisfy the no-sign reversal property: they could be negative, even if the treatment effect is positive in every CZ and at every period. Importantly, those results are related to, but different from, those in Adão et al. (2019), Goldsmith-Pinkham et al. (2020), and Borusyak et al. (2022), and we will shortly discuss the connections and differences between our approaches. Our second contribution is to provide diagnostic tools researchers may use to assess if their Bartik regressions are robust to heterogeneous effects. Third, we propose two alternative correlated-random-coefficient estimators inspired from Chamberlain (1992), that are more robust to heterogeneous effects than Bartik regressions, though they still impose restrictions on effects' heterogeneity. Fourth, we use our results to revisit the canonical Bartik setting, and Autor et al. (2013), who use a Bartik instrument to measure the effect of exposure to Chinese competition on US employment. In both cases, we find that Bartik regressions do not estimate convex combinations of CZ-and-period-specific effects. In Autor et al. (2013), our alternative estimators are more negative than the Bartik estimator.

We now describe our results in more details. We follow Adão et al. (2019), and assume that the treatment evolution is generated by a model linear in the shocks, with sector-, commuting-zone-, and period-specific first-stage effects, while the outcome evolution is generated by a model linear in the treatment evolution, with commuting-zone- and period-specific second-stage effects. Then, we make two parallel-trends assumptions: in the absence of any industry shock, the treatment and the outcome would have followed the same evolution in every CZ. As we shall argue in Section 3, our parallel-trends assumption on the treatment should generally be fairly plausible. In Autor et al. (2013), we show that our parallel-trends assumption on the outcome can be tested, by regressing CZs employment evolution prior to the China shock on their Bartik instrument, a placebo test closely related to that conducted by Autor et al. (2013) in their Table 2. This test is not rejected, thus lending support to our parallel-trends assumption on the outcome in this application. In the canonical setting we are not able to test this assumption, but in Section 3 we argue again that it is fairly plausible.

Under those parallel-trends assumptions, we show that the first-stage (resp. reduced-form) Bartik regression identifies a weighted sum of industry- and commuting-zone-specific first-stage (resp. reduced-form) effects, where some weights *must* be negative. Accordingly, the first-stage and reduced-form Bartik regressions *never* estimate convex combinations of effects, a result that differs from that for two-way fixed effects regressions, that sometimes identify convex combinations of effects under parallel-trends assumptions (see de Chaisemartin & D'Haultfœuille 2020). Then, we show that the 2SLS Bartik regression identifies a weighted sum of commuting-zone specific second-stage effects, where some weights may be negative. Accordingly, under our parallel-trends assumptions, Bartik regressions may be misleading in the presence of heterogeneous first- or second-stage effects. The intuition for our results is that Bartik regressions compare the employment and wage evolutions of commuting zones that received high and low shocks. From that perspective, they are comparable to "fuzzy" differences-in-differences (DID) estimators, that also do not estimate convex combinations of effects under parallel-trends assumptions (see De Chaisemartin & D'Haultfœuille 2018).

The weights attached to the first-stage and reduced-form Bartik regressions can be estimated. The number of positive and negative weights attached to the 2SLS regression can also be estimated, under the assumption that the first-stage effects do not vary across industries and are all positive. Estimating the weights attached to the first-stage and reduced-form regressions, or counting the number of negative weights attached to the 2SLS regression, may help applied researchers assess the robustness of their Bartik regression to heterogeneous effects. One can perform that diagnosis under various restrictions on the first- and second-stage effects, to assess which type of heterogeneity is more problematic in a given application. For instance, when we revisit the canonical Bartik design, we find that without making any assumption, nearly half of the weights attached to the first-stage and reduced-form regressions are negative, and the negative weights sum to -1.22. On the other hand, assuming that first-stage effects do not vary across industries dramatically reduces the negative weights, which now sum to -0.04 only. Accordingly, it seems that Bartik regressions in this application may be robust to first- and second-stage effects that vary across commuting zones or over time, but may be much less robust to industry-specific first-stage effects. By contrast, when we revisit Autor et al. (2013), we find that Bartik regressions may not be robust to commuting-zone-specific first- and second-stage effects, while industry-specific first-stage effects seem less problematic. Those two examples show that our results can help researchers identify which restriction on treatment effect heterogeneity is key to lend a causal interpretation to their Bartik regression.

Finally, we propose two alternative estimators, that can be used when there are at least three periods in the data. The first one is robust to commuting-zone-specific first- and second-stage effects, but not to time-varying first- and second-stage effects or to sector-specific first-stage effects. The second one allows for time-varying effects, provided the first- and second-stage effects follow the same evolution over time in every location. Under our parallel-trends assumptions and the aforementioned assumptions on the first- and second-stage effects, we show that the mean of the treatment and outcome evolution conditional on the shocks are additively separable in a regression function with location-specific coefficients and a regression function with period-specific coefficients. Then, those conditional-mean-models are nested in the correlated-random-coefficient models studied in Chamberlain (1992), and the models' parameters can be estimated following similar steps as in that paper. Our first estimator can be written as a standard GMM estimator, so its asymptotic distribution follows from standard GMM asymptotic theory and one can use standard errors pre-canned in standard statistical softwares for inference. Our second estimator can be written as a GMM estimator, where the sample moment conditions depend on pre-estimated parameters, and where the number of pre-estimated parameters increases with the number of locations. Accordingly, its asymptotic distribution does not follow from standard GMM asymptotic theory, and studying it is left for future work. In Autor et al. (2013), our two estimators are substantially more negative than the Bartik 2SLS estimator, thus suggesting that the effect of Chinese import exposure on US manufacturing employment could have been more negative than that found by Autor et al. (2013). In the canonical setting, our two estimators are extremely close from the Bartik 2SLS estimator.

The paper is organized as follows. Section 2 discusses how our results relate to those in the recent Bartik literature. Section 3 introduces our notation and assumptions. Section 4 presents our identification results. Section 5 presents our alternative estimators. Section 6 presents our re-analysis of Autor et al. (2013). To preserve space, our re-analysis of the canonical Bartik setting is in Section A of the Web Appendix.

2 Connections Between our Results and the Pre-existing Bartik Literature

A recent literature has considerably improved our understanding of Bartik regressions. Goldsmith-Pinkham et al. (2020) show that they are equivalent to using local industry shares as instruments, and show that the Bartik 2SLS coefficient consistently estimates the treatment effect under the assumption that second-stage effects are homogeneous and that industry shares are exogenous, meaning uncorrelated with CZ's potential wage evolutions in the absence of any shock. Borusyak et al. (2022) instead assume that shocks are exogenous, and derive consistency results under that assumption. Finally, Adão et al. (2019) follow the exogenous-shocks approach first proposed by Borusyak et al. (2022), and show that using the robust standard errors of Bartik regression coefficients for inference leads to over-rejection because the instruments of locations with similar industry shares are correlated. They then propose new estimators of the standard errors.

While these papers sometimes assume homogeneous effects as a baseline case, they all have identification results allowing for heterogeneous effects. We now discuss the connections and the differences between those results and ours.

2.1 Our Results and Those in Borusyak et al. (2022) and Adão et al. (2019)

As we will explain in further details in Section 3.3.1, the key difference between our assumptions and those of Borusyak et al. (2022) and Adão et al. (2019) is that they do not make parallel-trends assumptions. Instead, they assume that the expectation of the shock is the same in every sector, the so-called randomly-assigned-shocks assumption. In the canonical Bartik design, this amounts to assuming that each industry has the same expected employment growth. Importantly, our parallel-trends assumptions are neither weaker or stronger than the randomly-assigned-shocks assumption: those two assumptions are non-nested. Under the randomly-assigned-shock assumption, Adão et al. (2019) show that the first-stage Bartik regression identifies a weighted average of industry-, commuting-zone-, and period-specific first-stage effects, with no negative weights. And Borusyak et al. (2022) and Adão et al. (2019) show that the 2SLS Bartik regression identifies a weighted average of commuting-zone- and period-specific second-stage effects, again without negative weights. Those results contrast with ours. Together, our and their results imply that whether Bartik regressions satisfy or not the no-sign reversal property depends on whether the randomly-assigned-shocks assumption or the parallel-trends assumption is more plausible. Therefore, a contribution of our paper may be to show that the robustness of Bartik regressions to heterogeneous effects in Borusyak et al. (2022) and Adão et al. (2019) critically relies on the randomly-assigned-shocks assumption.

Borusyak et al. (2022) have shown that a great appeal of the randomly-assigned-shocks assumption is that one can test it, by testing whether average shocks vary with sectors' characteristics. Unfortunately, in our two applications, we find that this test is rejected. In the canonical setting, some sectors are more likely to receive positive employment shocks than others. In Autor et al. (2013), some sectors are more likely to see an increase in their exposure to Chinese imports than other sectors. Borusyak et al. (2022) also test the randomly-assigned-shocks assumption in Autor et al. (2013), and they do not reject it. As explained in Section 6.2, the implication of the randomly-assigned-shocks assumption that we test is stronger than the one they test, so our test may be more powerful.

Overall, our approach may be appealing in applications where the randomly-assigned-shocks assumption is rejected, while our parallel-trends assumptions seem plausible and the tests we propose of those assumptions are not rejected. This is for instance the case in Autor et al. (2013).

Importantly, when they consider Bartik regressions with covariates, Borusyak et al. (2022) and Adão et al. (2019) allow sectors to have different expected shocks, provided those differences are fully accounted for by a linear model in sector-level covariates, see for instance Equation (24) in Adão et al. (2019). While this is an important relaxation of the randomly-assigned shocks assumption, one may still worry that there are characteristics determining sectors' shocks that are not controlled for in the regression.

2.2 Our Results and Those in Goldsmith-Pinkham et al. (2020)

Goldsmith-Pinkham et al. (2020) also show that with heterogeneous treatment effects, Bartik 2SLS regressions may identify weighted sums of effects with some negative weights, see their Equation (10). As we will explain in further details in Section 3.3.2, our assumptions on the second-stage Bartik regression are close to theirs, as they make an assumption that may be interpreted as a parallel-trends assumption on the outcome. On the other hand, our assumptions on the first-stage Bartik regression are very different. In fact, we show in Section 3.3.2 that our two sets of first-stage assumptions are "nearly mutually exclusive": under our first-stage assumptions, their first-stage assumptions can only hold under a condition that seems hard to rationalize, see Equation (3.4). Because our first-stage assumptions are different, the weights in our and in their decomposition of Bartik 2SLS regressions as weighted sums of location- and period-specific effects differ: their weights are functions of the so-called Rotemberg weights (see Rotemberg 1983), while our weights do not depend on said Rotemberg weights. Under our and their decompositions, the sets of locations $\times$ periods whose treatment effect is weighted negatively by the Bartik 2SLS regression differ. For instance, with two periods, if all industry shocks are positive and if all first-stage effects are positive, then the treatment effects of all locations with a Bartik shock below the average shock across locations are weighted negatively under our decomposition, but not under the decomposition in Goldsmith-Pinkham et al. (2020). See the first point of Theorem 3 for a closed-form expression of our weights, and see Equation (4.1) for a comparable expression of the weights in Equation (10) in Goldsmith-Pinkham et al. (2020). Importantly, Goldsmith-Pinkham et al. (2020) only use their assumptions on the first-stage Bartik regression to derive their Equation (10). Their other results assume homogeneous effects, and do not rely on their first-stage assumptions.

Another difference between our papers is that we show that the weights attached to the first-stage and reduced-form Bartik regressions can be estimated, and the sign of the weights attached to the 2SLS Bartik regression can be estimated under some assumptions. Thus, we provide researchers with diagnostic tools they may use to assess their regression's robustness to heterogeneous treatment effects. Goldsmith-Pinkham et al. (2020) do not propose such tools, again probably because heterogeneous treatment effects is not as central to their papers as it is to ours.

3 Set-up and Assumptions

We consider a data set with G locations, indexed by $g \in \{1, ..., G\}$. To make exposition as simple as possible, for now we assume the data has two time periods indexed by $t \in \{1, 2\}$, but we will relax that assumption later. Locations are typically geographical regions, for instance counties, states or commuting zones. Let $R_{g,t}$ denote the value of a generic variable R in location g and period t. Then, let $\Delta R_g = R_{g,2} - R_{g,1}$ denote the change of that variable from period 1 to 2 in location g. We are interested in how the evolution of a treatment variable ΔD_g affects the evolution of an outcome variable ΔY_g. For instance, the canonical setting first proposed by Bartik (1991) focused on how the evolution of (log) employment affects the evolution of (log) wages, and Autor et al. (2013) have studied how the evolution of exposure to Chinese competition affects the evolution of US local labor markets. We could regress ΔY_g on ΔD_g, but we worry that the treatment evolution may be endogenous. For example, in the canonical setting, the effect of employment growth on wage growth may be confounded by other labor market shocks, such as technological changes or inflation.

Instead, the following instrumental variable strategy has been proposed. Assume there are S sectors indexed by $s \in \{1, ..., S\}$. Sectors could for instance be industries. Let $R_{s,t}$ denote the value of a generic variable R in sector s and period t. Then, let $\Delta R_s = R_{s,2} - R_{s,1}$ denote the change of that variable from period 1 to 2 in sector s. For every $s \in \{1, ..., S\}$, let ΔZ_s denote a shock affecting sector s between periods 1 and 2. For example, in the canonical setting, ΔZ_s denotes the nationwide employment growth in sector s.

Definition 1 *Bartik Instrument: The Bartik instrument ΔZ_g is:*

$$\Delta Z_g = \sum_{s=1}^{S} Q_{s,g} \Delta Z_s.$$

For every g, $Q_{s,g}$ are positive weights summing to 1. Typically, $Q_{s,g}$ is a measure of the importance of sector s in location g at period 1. For instance, $Q_{s,g}$ could be the share that sector s accounts for in location g's employment at period 1. In the canonical setting, the Bartik instrument represents the employment evolution that location g would have experienced if all sectors in location g had experienced the same evolution as in the overall economy. Our results readily extend to applications where shares sum to less than 1, as is the case in Autor et al. (2013), see footnote 8 for further discussion.

Throughout the paper, we consider both the shares $Q_{s,g}$ and the shocks ΔZ_s as stochastic quantities. This approach is similar to that of Borusyak et al. (2022), and nests both that of Adão et al. (2019), who treat the shocks as stochastic and condition on the shares, and that of Goldsmith-Pinkham et al. (2020), who treat the shares as stochastic and condition on the shocks.

3.1 Assumptions on the first-stage Bartik regression

We start by introducing the notation and assumptions we use to analyze the first-stage Bartik regression. For any $(\delta_1, ..., \delta_S) \in \mathbb{R}^S$, let $\Delta D_g(\delta_1, ..., \delta_S)$ denote the potential treatment evolution that location g will experience if $(\Delta Z_1, ..., \Delta Z_S) = (\delta_1, ..., \delta_S)$. And let $\Delta D_g(\mathbf{0}) = \Delta D_g(0, ..., 0)$ denote the potential treatment evolution that location g will experience in the absence of any shocks. The actual treatment evolution is $\Delta D_g = \Delta D_g(\Delta Z_1, ..., \Delta Z_S)$.

We start by making the following assumption:

Assumption 1 *Linear First-Stage Equation: for all $g \in \{1, ..., G\}$, there are real numbers $(\beta_{s,g})_{s\in\{1,...,S\}}$ such that for any $(\delta_1, ..., \delta_S) \in \mathbb{R}^S$:*

$$\Delta D_g(\delta_1, ..., \delta_s) = \Delta D_g(\mathbf{0}) + \sum_{s=1}^{S} Q_{s,g}\delta_s\beta_{s,g}.$$

Assumption 1 requires that the effect of the shocks on the treatment evolution be linear. Similar assumptions are also made by Adão et al. (2019) (see their Equation (11)) and Goldsmith-Pinkham et al. (2020) (see their Equation (8), which we discuss in more details later). Increasing ΔZ_s by 1 unit, holding all other shocks constant, leads the treatment of location g to increase by $Q_{s,g}\beta_{s,g}$ units. Under Assumption 1,

$$\Delta D_g = \Delta D_g(\mathbf{0}) + \sum_{s=1}^{S} Q_{s,g}Z_s\beta_{s,g}. \tag{3.1}$$

Our key first-stage identifying assumption is the following one.

Assumption 2 *Exogenous Shares and Shocks and Common Trends for the Treatment:*

1. *For all $g \in \{1, ..., G\}$: $E(\Delta D_g(\mathbf{0})|\Delta Z_1, ..., \Delta Z_S, Q_{1,g}, ..., Q_{S,g}) = E(\Delta D_g(\mathbf{0}))$.*

2. *There is a real number μ^D such that $\forall g \in \{1, ..., G\}$: $E(\Delta D_g(\mathbf{0})) = \mu^D$.*

The first point of Assumption 2 requires that locations' potential treatment evolution without any shock be mean-independent of the sector-level shocks, and of locations' shares. The second point of Assumption 2 requires that the expectation of the potential treatment evolution in the absence of any shocks be the same for all locations. This second requirement is a parallel-trends assumption, similar to that made in Difference-in-Differences models (see Abadie 2005, de Chaisemartin & D'Haultfœuille 2020).

Assumption 2 is untestable, unless one observes the treatment change at a time period where sectors do not experience any shock (see Section 4.4). However, that condition is not always met: for instance, it is not met in our two empirical applications. Therefore, it is important to clarify the restrictions imposed by Assumption 2 in the context of a given application, to gauge its plausibility. We now do so in our two empirical applications.

In the canonical setting, ΔZ_s is the change in US employment in sector s, and ΔD_g is the change in commuting-zone (CZ) g's employment. $\Delta Z_1 = ... = \Delta Z_S = 0$ is a counterfactual where the US does not experience any employment growth in any sector. In that counterfactual, employment could be redistributed from less- to more-dynamic CZs, in which case $E(\Delta D_g(\mathbf{0}))$ would vary across CZs. However, for employment redistribution to lead to $\Delta Z_1 = ... = \Delta Z_S = 0$, workers moving across CZs should remain in the same sector. If some workers move from less- to more-dynamic CZs, they may be more likely to change sector and start working in a sector with a high employment share in their destination CZ. As sectoral shares are likely to differ across less- and more-dynamic CZs, this would lead to heterogeneous employment growth across sectors, thus violating $\Delta Z_1 = ... = \Delta Z_S = 0$. Therefore, Assumption 2 is violated if in the counterfactual where the US does not experience any employment growth in any sector, there is employment redistribution across CZs that does not lead to redistribution across sectors. That may not be a very plausible scenario.

In Autor et al. (2013), the treatment has a Bartik structure similar to that of the instrument. Let $\Delta M_{u,c,s}$ be the change in US imports from China in sector s, let $L_{u,s}$ be the US employment in sector s, let $L_{s,g}$ be CZ g's employment in sector s, and let L_g be CZ g's employment. Autor et al. (2013) define their treatment as $\Delta D_g = \sum_{s=1}^{S} Q_{s,g} \Delta D_s$, where $\Delta D_s = \Delta M_{u,c,s}/L_{u,s}$ is the per-worker change in US imports from China in sector s, and $Q_{s,g} = L_{s,g}/L_g$ is the employment share of sector s in CZ g (see their Equation (3)).[1] The shocks ΔZ_s are the per-worker change in imports from China in sector s, in some other high-income countries. Then, $\Delta Z_1 = ... = \Delta Z_S = 0$ is a counterfactual where other high-income countries do not experience a China shock. In that counterfactual, it is reasonable to assume that the US would also not experience a China shock. Then, one would have $\Delta D_s = 0$ for all s, thus implying that $\Delta D_g = 0$ for all g. Then, Assumption 2 mechanically holds.

Overall, Assumption 2 does impose some restrictions but still seems fairly plausible in the canonical setting, and almost mechanically holds in Autor et al. (2013).

[1] This definition of the treatment variable is micro-founded by a model where the change of location g's employment in sectors subject to Chinese competition is a linear function of ΔD_g, see Equation (2) in Autor et al. (2013).

3.2 Assumptions on the second-stage Bartik regression

We now introduce the notation and assumptions we use to analyze the second-stage Bartik regression. Let $\Delta Y_g(d_g)$ denote the potential outcome evolution that location g will experience if $\Delta D_g = d_g$.[2] $\Delta Y_g(0)$ is location g's potential outcome evolution without any treatment change.

Assumption 3 *Linear Second-Stage Equation: for all $g \in \{1, ..., G\}$, there is a real number α_g such that for any $d_g \in \mathbb{R}$:*

$$\Delta Y_g(d_g) = \Delta Y_g(0) + \alpha_g d_g.$$

Assumption 3 is analogous to Equation (7) in Goldsmith-Pinkham et al. (2020) without control variables, and to Equation (30) in Adão et al. (2019) allowing for location-specific second-stage effects. Combining Assumption 3 with Assumption 1, we have:

$$\begin{aligned}
\Delta Y_g(\Delta D_g(\delta_1, ..., \delta_S)) &= \Delta Y_g(\Delta D_g(\mathbf{0})) + \alpha_g(\Delta D_g(\delta_1, ..., \delta_S) - \Delta D_g(\mathbf{0})) \\
&= \Delta Y_g(\Delta D_g(\mathbf{0})) + \alpha_g \sum_{s=1}^{S} Q_{s,g}\delta_s\beta_{s,g} \\
&= \Delta Y_g(\Delta D_g(\mathbf{0})) + \sum_{s=1}^{S} Q_{s,g}\delta_s\gamma_{s,g},
\end{aligned}$$

where $\gamma_{s,g} = \alpha_g\beta_{s,g}$. Therefore, the potential outcome evolution is also linear in sector-level shocks.

Our key second-stage identifying assumption is the equivalent of Assumption 2 for $\Delta Y_g(\Delta D_g(\mathbf{0}))$, locations' potential outcome evolution without shocks.

Assumption 4 *Exogenous Shares and Shocks and Common Trends for the Outcome*

1. *For all $g \in \{1, ..., G\}$: $E(\Delta Y_g(\Delta D_g(\mathbf{0}))|\Delta Z_1, ..., \Delta Z_S, Q_{1,g}, ..., Q_{S,g}) = E(\Delta Y_g(\Delta D_g(\mathbf{0})))$.*
2. *There is a real number μ^Y such that $\forall g \in \{1, ..., G\}$: $E(\Delta Y_g(\Delta D_g(\mathbf{0}))) = \mu^Y$.*

The first point of Assumption 4 requires that locations' potential outcome evolution without any shock be mean-independent of the sector-level shocks, and of location' shares. The second point of Assumption 4 requires that the expectation of the potential outcome evolution in the absence of any shocks be the same for all locations. Assumption 4 is similar to Assumption A3 in De Chaisemartin (2013), and to Assumption A1 in Hudson et al. (2017).

Like Assumption 2, Assumption 4 is untestable, unless one observes the outcome change at a time period where sectors do not experience any shock (see Section 4.4).

[2] We implicitly make an exclusion restriction assumption: the shocks have no direct effect on the outcome evolution, they can only affect the outcome evolution through their effect on the treatment evolution, see Angrist & Imbens (1995).

This requirement is met in Autor et al. (2013): ΔY_g is the change in the manufacturing-employment share in CZ g, and that variable is observed before the China shock. The test is conclusive: changes in CZs manufacturing-employment shares before the China shock are uncorrelated with their Bartik instrument, a function of the shocks $(\Delta Z_1, ..., \Delta Z_S)$ and shares $(Q_{1,g}, ..., Q_{S,g})$. This lends credibility to Assumption 4 in this application.

We cannot implement the test in the canonical setting, but can still assess the plausibility of Assumption 4 on logical grounds. ΔY_g is the change in CZ g's average wage. Even in the absence of employment growth in any sector, sectors may experience non-zero and heterogeneous wage growths. For instance, some sectors may experience larger productivity growth than others. If, say, the labor market in each sector is competitive with homogenous workers, differential productivity growths across sectors leads to differential wage growth. In the short-run, workers may not be substitutable at all across sectors, say for instance because working in a given sector requires specific skills that take time to acquire. Then, differential wage growth is compatible with zero employment growth in all sectors, and CZ's wage growth would depend on their employment shares in sectors with a high/low wage growth, thus leading to a violation of Assumption 4. While it may be plausible in the short run, this scenario is implausible in the long run: then, differential wage growth across sectors should lead to differential employment growth. In the canonical setting, the shocks are sectoral employment growths over ten-years periods. Over this horizon, it is hard to envision how differential wage growths across sectors would not lead to differential employment growths, so Assumption 4 may be plausible.

In our analysis, we consider the first- and second-stage effects $\beta_{s,g}$ and α_g as deterministic. On the other hand, we consider $\left(\Delta Y_g(0), \Delta D_g(\mathbf{0}), (Q_{s,g})_{s\in\{1,...,S\}}\right)_{g\in\{1,...,G\}}$ and $(\Delta Z_s)_{s\in\{1,...,S\}}$ as random, and all the probabilistic statements above and below are with respect to the joint distribution of those random variables. Our last assumption is that the variables attached to different locations are independent conditional on the shocks.

Assumption 5 *Independent Locations:*
Conditional on $(\Delta Z_s)_{s\in\{1,...,S\}}$, *the vectors* $\left(\Delta Y_g(0), \Delta D_g(\mathbf{0}), (Q_{s,g})_{s\in\{1,...,S\}}\right)$ *are mutually independent across* g.

Goldsmith-Pinkham et al. (2020) make a similar assumption, see their Section I.A. Because they assume shocks are non-stochastic, they assume that locations' variables are independent without conditioning on shocks.

3.3 Comparing our identifying assumptions to those in pre-existing work

3.3.1 Comparing our assumptions to those in Adão et al. (2019) and Borusyak et al. (2022)

When studying the first-stage Bartik regression, Adão et al. (2019) make an assumption closely related to the first point of Assumption 2. Their Assumption 1 (ii) requires that for every s, ΔZ_s be mean independent of the vector $(\Delta D_1(\mathbf{0}), ..., \Delta D_G(\mathbf{0}))$. Assuming non-stochastic shares as they do, their mean-independence requirement is neither weaker nor stronger than that in the first point of our Assumption 2, and both assumptions are implied by the stronger condition $(\Delta Z_1, ..., \Delta Z_S) \perp\!\!\!\perp (\Delta D_1(\mathbf{0}), ..., \Delta D_G(\mathbf{0}))$. Similarly, when studying the second-stage Bartik regression, Adão et al. (2019) make an assumption closely related to the first point of Assumption 4 (see their Assumption 4 (ii)). Overall, both our and their approach require that shocks be unrelated to locations' potential treatment and outcome evolutions in the absence of any shock. Borusyak et al. (2022) also make a related assumption: shocks should be mean independent of locations' outcomes evolutions (see their Assumption 1).

On the other hand, Adão et al. (2019) and Borusyak et al. (2022) do not make the parallel-trends assumption in the second point of Assumption 2. Instead, they assume that the sector-level shocks are as-good-as randomly assigned, in the sense that they all have the same expectation, i.e. $E(\Delta Z_s) = \delta$, for some real number δ (see Assumption 1 (ii) in Adão et al. 2019 and Assumption 1 in Borusyak et al. 2022). This randomly-assigned-shocks assumption has first been proposed by Borusyak et al. (2022). Point 2 of Assumption 2 is neither weaker or stronger than the randomly-assigned-shocks assumption: those two assumptions are non-nested. As shown by Borusyak et al. (2022), a great appeal of the randomly-assigned-shocks assumption is that one can test it, by testing whether shocks are independent of industries' characteristics. Unfortunately, in our two applications, we find that this test is rejected. In the canonical setting, some sectors are more likely to receive positive employment shocks than others. In Autor et al. (2013), some sectors are more likely to see an increase in their exposure to Chinese imports than other sectors. Then, our approach may be appealing in applications where the randomly-assigned-shocks assumption is rejected, while our Assumptions 2 and 4 seem plausible. Our approach may be particularly appealing when the tests of Assumptions 2 and 4 we propose in Section 4.4 can be implemented and are conclusive.

3.3.2 Comparing our assumptions to those in Goldsmith-Pinkham et al. (2020)

Equation (8) and Assumption 3 are the two main assumptions on the first-stage Bartik regression in Goldsmith-Pinkham et al. (2020). Using our notation, and assuming the regression has no control variables, those require that for all (s,g),[3]

$$\Delta D_g = \mu^D + Q_{s,g}\Delta Z_s \beta_{s,g} + u_{s,g}, \tag{3.2}$$
$$\text{with } E(Q_{s,g}u_{s,g}) = 0. \tag{3.3}$$

Equation (3.2) is a linear first-stage equation similar to Equation (3.1), but where the first-stage effect of only one sector explicitly appears.

Under Assumptions 1 and 2, and momentarily assuming that shocks are non stochastic as in Goldsmith-Pinkham et al. (2020), it is difficult to rationalize Equations (3.2) and (3.3). Under Assumption 1, Equations (3.1) and (3.2) imply that

$$u_{s,g} = \Delta D_g(\mathbf{0}) - \mu^D + \sum_{s'\neq s} Q_{s',g}\Delta Z_{s'}\beta_{s',g}.$$

Then,

$$\begin{aligned} E(Q_{s,g}u_{s,g}) =& E(Q_{s,g}(\Delta D_g(\mathbf{0}) - \mu^D)) + \sum_{s'\neq s} E(Q_{s,g}Q_{s',g})\Delta Z_{s'}\beta_{s',g} \\ =& \sum_{s'\neq s} E(Q_{s,g}Q_{s',g})\Delta Z_{s'}\beta_{s',g}, \end{aligned}$$

where the second equality follows from Assumption 2. Therefore, Equations (3.2) and (3.3) can only hold if for all s,

$$\sum_{s'\neq s} E(Q_{s,g}Q_{s',g})\Delta Z_{s'}\beta_{s',g} = 0, \tag{3.4}$$

a condition that seems hard to rationalize.

Overall, the first-stage assumptions in Goldsmith-Pinkham et al. (2020) are hard to rationalize under our linear first-stage model and our parallel-trends assumption. Therefore, with a slight abuse of language, our two sets of assumptions may be considered as mutually exclusive: in applications where our first-stage assumptions are plausible, the first-stage assumptions in Goldsmith-Pinkham et al. (2020) may not be plausible. Conversely, in applications where the first-stage assumptions in Goldsmith-Pinkham et al. (2020) are plausible, our first-stage assumptions may not be plausible. Note that Equation (8) and Assumption 3 in Goldsmith-Pinkham et al. (2020) are only used to derive Proposition 4 and Equation (10) therein, which study the Bartik estimator under heterogeneous effects. All their other results assume homogeneous effects, and do not rest on Equation (8) and Assumption 3.

Perhaps surprisingly, while our first-stage assumptions are (nearly) incompatible, our second-stage assumptions are fairly close. Equation (7) and Assumption 2 are the two main assumptions on the second-stage Bartik regression in Goldsmith-Pinkham et al.

[3] Rather than Equation (3.3) below, Assumption 3 in Goldsmith-Pinkham et al. (2020) requires that $E(Q_{s,g}u_{s,g}\alpha_g) = 0$. Our discussion still applies if one replaces Equation (3.3) by $E(Q_{s,g}u_{s,g}\alpha_g) = 0$.

(2020). Without control variables, their Equation (7) is equivalent to our Assumption 3. Then, using our notation, their Assumption 2 requires that for all (s, g),

$$E(Q_{s,g}(\Delta Y_g(0) - \mu^Y)) = 0. \tag{3.5}$$

If instead of our Assumption 4, one were to make the same assumption but on $\Delta Y_g(0)$, then Equation (3.5) would automatically hold. Accordingly, Equation (3.5) may be interpreted as a parallel-trends assumption on the outcome without any treatment change, while Assumption 4 is a parallel-trends assumption on the outcome without shocks.

Assume that Assumption 2 holds with $\mu^D = 0$, as seems plausible both in the canonical setting and in Autor et al. (2013) as discussed above. Then, under Assumption 3 $\Delta Y_g(\Delta D_g(\mathbf{0})) = \Delta Y_g(0) + \alpha_g \Delta D_g(\mathbf{0})$, so parallel trends on $\Delta Y_g(0)$ implies parallel trends on $\Delta Y_g(\Delta D_g(\mathbf{0}))$: the two assumptions are closely related and Assumption 4 is slightly weaker. This means that in applications like the canonical setting or Autor et al. (2013), parallel trends on the outcome without any treatment change and Assumption 4 are essentially the same.

If Assumption 2 holds with $\mu^D \neq 0$, under Assumption 3 parallel trends on $\Delta Y_g(0)$ and Assumption 4 cannot jointly hold (unless $\alpha_g = \alpha$ for all g), so the two assumptions are substantively different. As explained above, an appealing feature of Assumption 4 is that it can be placebo-tested when one observes the outcome change at a time period where sectors do not experience any shock. Shocks in Bartik studies are sometimes caused by a policy change, and there may be time periods without any policy change, and therefore without any shock. Testing parallel trends on $\Delta Y_g(0)$ would require observing the outcome change at a time period where all locations do not experience any treatment change. But if Assumption 2 holds with $\mu^D \neq 0$, some locations will experience a treatment change even if there are no shocks, so one will not be able to placebo-test parallel trends on $\Delta Y_g(0)$.

Overall, Assumption 4 may be preferable to parallel trends on $\Delta Y_g(0)$: either the two assumptions are essentially equivalent, or they are not but then Assumption 4 is more likely to be placebo testable than parallel trends on $\Delta Y_g(0)$.

4 Identification Results for Bartik Regressions

4.1 First-Stage and Reduced-Form Bartik Regressions

Throughout the paper, we consider Bartik regressions that are not weighted, say, by locations' population. It is straightforward to extend all our results to weighted Bartik regressions. For instance, the decompositions in Theorems 1 to 3 still hold, except that the numerator and the denominator of the weights have to be multiplied by N_g, the weight assigned to location g, and $\Delta Z_.$ has to be redefined as $\sum_{g=1}^G \frac{N_g}{N} \Delta Z_g$, where $N = \sum_{g=1}^G N_g$.

We start by considering the first-stage Bartik regression.

Definition 2 *First-stage Bartik regression: Let $\hat{\beta}_C^D$ denote the coefficient of ΔZ_g in the first-stage regression of ΔD_g on ΔZ_g and a constant. Let $\beta_C^D = E[\hat{\beta}_C^D]$.*

Let $\Delta Z_. = \frac{1}{G}\sum_{g=1}^{G} \Delta Z_g$.

Theorem 1 *Suppose Assumptions 1, 2, and 5 hold.*

1. *Then,*

$$\beta_C^D = E\left(\sum_{g=1}^{G}\sum_{s=1}^{S} \frac{Q_{s,g}\Delta Z_s(\Delta Z_g - \Delta Z_.)}{\sum_{g=1}^{G}\sum_{s=1}^{S} Q_{s,g}\Delta Z_s(\Delta Z_g - \Delta Z_.)}\beta_{s,g}\right).$$

2. *If one further assumes $\beta_{s,g} = \beta_g$,*

$$\beta_C^D = E\left(\sum_{g=1}^{G} \frac{\Delta Z_g(\Delta Z_g - \Delta Z_.)}{\sum_{g=1}^{G} \Delta Z_g(\Delta Z_g - \Delta Z_.)}\beta_g\right).$$

The first point of Theorem 1 shows that β_C^D is equal to the expectation of a weighted sum of the first-stage effects $\beta_{s,g}$s, with weights summing to one. Accordingly, if all the first-stage effects $\beta_{s,g}$s are equal to a constant β, β_C^D identifies β under our assumptions. Note that the denominator of the weights is equal to $\sum_{g=1}^{G}(\Delta Z_g - \Delta Z_.)^2$ and is non-negative. Therefore, the weights are positive for all (s,g) such that ΔZ_s and $\Delta Z_g - \Delta Z_.$ are of the same sign, and negative otherwise. Interestingly, because $\Delta Z_g - \Delta Z_.$ must be strictly positive for some gs and strictly negative for other gs, there must be some $\beta_{s,g}$s that are weighted negatively, unless $Q_{s,g} = 0$ for all (s,g) such that $(\Delta Z_g - \Delta Z_.)\Delta Z_s < 0$, a condition that is very unlikely to hold. Therefore, under Assumptions 1, 2, and 5, β_C^D *never* estimates a convex combination of first-stage effects. This differs from two-way fixed effects regressions, which sometimes estimate a convex combination of effects under parallel-trends assumptions (see de Chaisemartin & D'Haultfœuille 2020). Because of the negative weights, β_C^D is not robust to heterogenous treatment effects. In particular, it does not satisfy the no-sign reversal property, which requires that if all first-stage effects are of the same sign, β_C^D should also be of that sign.

The second point of Theorem 1 shows that if we further assume $\beta_{s,g} = \beta_g$, β_C^D is equal to the expectation of a weighted sum of the β_gs, where some weights may still be negative. To fix ideas, assume $\Delta Z_. > 0$. Then, the β_gs of all locations that received a less than average positive shock ($0 < \Delta Z_g < \Delta Z_.$) enter with a negative weight in Point 2 of Theorem 1. If $\Delta Z_. = 0$ all the weights are positive, but $\Delta Z_. = 0$ is unlikely to hold in practice. If the instrument is redefined as $\Delta Z_g - \Delta Z_.$, all the weights are positive, but now our notation and assumptions need to hold for the recentered rather than original shocks. For instance, our notation for the potential treatment evolutions now implies that the treatment's evolution cannot depend on the average level of the shocks across sectors, a strong and implausible requirement.

Note that the weights in Theorem 1 can be estimated. Estimating the weights, and assessing if many are negative, can be used to assess the robustness of the first-stage Bartik regression to heterogeneous effects under parallel trends, as de Chaisemartin & D'Haultfœuille (2020) have proposed for two-way fixed effects regressions.

Here is some intuition on Theorem 1. If there are only two locations ($G = 2$),

$$\hat{\beta}_C^D = \frac{\Delta D_2 - \Delta D_1}{\Delta Z_2 - \Delta Z_1},$$

a Wald-DID estimator similar to that studied by De Chaisemartin & D'Haultfœuille (2018). Without loss of generality, let us assume that $\Delta Z_2 > \Delta Z_1$. Then,

$$\begin{aligned}
\beta_C^D =& E\left(\frac{E\left(\Delta D_2 | \Delta Z_1, ..., \Delta Z_S, Q_{1,2}, ..., Q_{S,2}\right) - E\left(\Delta D_1 | \Delta Z_1, ..., \Delta Z_S, Q_{1,1}, ..., Q_{S,1}\right)}{\Delta Z_2 - \Delta Z_1}\right) \\
=& E\left(\frac{E\left(\Delta D_2(\mathbf{0}) | \Delta Z_1, ..., \Delta Z_S, Q_{1,2}, ..., Q_{S,2}\right) + \sum_{s=1}^S Q_{s,2}\Delta Z_s \beta_{s,2}}{\Delta Z_2 - \Delta Z_1}\right) \\
-& E\left(\frac{E\left(\Delta D_1(\mathbf{0}) | \Delta Z_1, ..., \Delta Z_S, Q_{1,1}, ..., Q_{S,1}\right) + \sum_{s=1}^S Q_{s,1}\Delta Z_s \beta_{s,1}}{\Delta Z_2 - \Delta Z_1}\right) \\
=& E\left(\frac{\sum_{s=1}^S Q_{s,2}\Delta Z_s \beta_{s,2} - \sum_{s=1}^S Q_{s,1}\Delta Z_s \beta_{s,1}}{\Delta Z_2 - \Delta Z_1}\right),
\end{aligned}$$

where the first, second, and third equalities respectively follow from the law of iterated expectations and Assumption 5, Assumption 1, and Assumption 2. The previous display shows that in location two, the first-stage effects of negative shocks ($\Delta Z_s < 0$) are weighted negatively by β_C^D, while in location one, the first-stage effects of positive shocks are weighted negatively. If one assumes $\beta_{s,g} = \beta_g$, as in Point 2 of Theorem 1, the previous display rewrites as

$$\beta_C^D = E\left(\frac{\Delta Z_2 \beta_2 - \Delta Z_1 \beta_1}{\Delta Z_2 - \Delta Z_1}\right).$$

The Bartik first-stage regression compares the treatment evolution in locations receiving high and low Bartik shocks. But if the Bartik shock is positive in the low-shock location ($\Delta Z_1 > 0$), β_1, the first-stage effect in that location, is weighted negatively by β_C^D.

Our paper is not the first to note the analogy between Bartik regressions and DID estimators: this analogy is emphasized in Goldsmith-Pinkham et al. (2020). However, we believe that our paper is the first to note that owing to their similarity to "fuzzy" DID estimators, Bartik regressions are not robust to heterogeneous treatment effects.

In the previous literature on Bartik regressions, Adão et al. (2019) is the only paper that studies Bartik first-stage regressions with heterogeneous treatment effects (see their Proposition 1). Unlike our Theorem 1, they show that first-stage Bartik regressions identify a weighted average of first-stage effects under their randomly-assigned-shocks assumption. Accordingly, our two results show that whether first-stage Bartik regressions are robust to heterogeneous effects crucially depends on whether parallel trends or randomly-assigned-shocks is more plausible in the application under consideration.

Finally, we show a result very similar to Theorem 1 for reduced-form Bartik regressions.

Definition 3 *Reduced-form Bartik regression: Let $\hat{\beta}^Y_C$ denote the coefficient of ΔZ_g in the reduced-form regression of ΔY_g on ΔZ_g and a constant. Let $\beta^Y_C = E[\hat{\beta}^Y_C]$.*

Theorem 2 *Suppose Assumptions 1, 3, 4, and 5 hold.*

1. *Then,*

$$\beta^Y_C = E\left(\sum_{g=1}^{G}\sum_{s=1}^{S}\frac{Q_{s,g}\Delta Z_s(\Delta Z_g - \Delta Z_.)}{\sum_{g=1}^{G}\sum_{s=1}^{S}Q_{s,g}\Delta Z_s(\Delta Z_g - \Delta Z_.)}\gamma_{s,g}\right).$$

2. *If we further assume $\beta_{s,g} = \beta_g$ and let $\gamma_g = \alpha_g\beta_g$,*

$$\beta^Y_C = E\left(\sum_{g=1}^{G}\frac{\Delta Z_g(\Delta Z_g - \Delta Z_.)}{\sum_{g=1}^{G}\Delta Z_g(\Delta Z_g - \Delta Z_.)}\gamma_g\right).$$

Theorem 2 is equivalent to Theorem 1, replacing the treatment evolution by the outcome evolution. Accordingly, its interpretation is the same. Note that the weights in Theorem 2 are the same as in Theorem 1.

4.2 2SLS Bartik Regressions

In this section, we study 2SLS Bartik regressions.

Definition 4 *2SLS Bartik regression: Let $\hat{\beta}^{2SLS}_C = \hat{\beta}^Y_C/\hat{\beta}^D_C$ denote the coefficient of ΔD_g in the 2SLS regression of ΔY_g on ΔD_g and a constant, using ΔZ_g as the instrument for ΔD_g. Let $\beta^{2SLS}_C = \beta^Y_C/\beta^D_C$.*

Theorem 3 *Suppose Assumptions 1-5 hold.*

1. *Then,*

$$\beta^{2SLS}_C = E\left(\sum_{g=1}^{G}\frac{\sum_{s=1}^{S}\frac{Q_{s,g}\Delta Z_s(\Delta Z_g-\Delta Z_.)}{\sum_{g=1}^{G}\sum_{s=1}^{S}Q_{s,g}\Delta Z_s(\Delta Z_g-\Delta Z_.)}\beta_{s,g}}{E\left(\sum_{g=1}^{G}\sum_{s=1}^{S}\frac{Q_{s,g}\Delta Z_s(\Delta Z_g-\Delta Z_.)}{\sum_{g=1}^{G}\sum_{s=1}^{S}Q_{s,g}\Delta Z_s(\Delta Z_g-\Delta Z_.)}\beta_{s,g}\right)}\alpha_g\right).$$

2. *If $\Delta Z_s \geq 0$ for all s and $\beta^D_C \geq 0$, and one further assumes that $\beta_{s,g} \geq 0$ for all (s,g), α_g is weighted negatively in the decomposition in Point 1 if and only if $\Delta Z_g - \Delta Z_. < 0$.*

3. *If one further assumes $\beta_{s,g} = \beta_g$,*

$$\beta^{2SLS}_C = E\left(\sum_{g=1}^{G}\frac{\frac{\Delta Z_g(\Delta Z_g-\Delta Z_.)}{\sum_{g=1}^{G}\Delta Z_g(\Delta Z_g-\Delta Z_.)}\beta_g}{E\left(\sum_{g=1}^{G}\frac{\Delta Z_g(\Delta Z_g-\Delta Z_.)}{\sum_{g=1}^{G}\Delta Z_g(\Delta Z_g-\Delta Z_.)}\beta_g\right)}\alpha_g\right).$$

4. If one further assumes $\beta_{s,g}=\beta$,

$$\beta_C^{2SLS} = E\left(\sum_{g=1}^{G} \frac{\Delta Z_g(\Delta Z_g - \Delta Z_.)}{\sum_{g=1}^{G}\Delta Z_g(\Delta Z_g - \Delta Z_.)}\alpha_g\right).$$

The first point of Theorem 3 shows that β_C^{2SLS} is equal to the expectation of a weighted sum of the second-stage effects α_g, with weights that may not all be positive, thus implying that like β_C^D and β_C^Y, β_C^{2SLS} may not be robust to heterogeneous treatment effects under our assumptions.[4] Contrary to the weights in Theorems 1 and 2, the weights in the first point of Theorem 3 cannot be estimated, as they depend on the first stage effects $\beta_{s,g}$. One can still describe which α_gs get positively/negatively weighted. First, notice that the denominator of the weights is the first-stage estimand β_C^D, and to fix ideas let us assume that $\beta_C^D \geq 0$. Then, notice that

$$\sum_{g=1}^{G}\sum_{s=1}^{S} Q_{s,g}\Delta Z_s(\Delta Z_g - \Delta Z_.) = \sum_{g=1}^{G}(\Delta Z_g - \Delta Z_.)^2 \geq 0.$$

Then, locations whose α_g get weighted negatively are those for which $\Delta Z_g - \Delta Z_.$ is of a different sign than the effect of the shocks on their treatment evolution $\sum_{s=1}^{S} Q_{s,g}\Delta Z_s\beta_{s,g}$.

The second point of Theorem 3 shows that in applications where all the shocks ΔZ_s are positive and where the first-stage Bartik regression coefficient is positive, if one further assumes that the first-stage effects $\beta_{s,g}$ are all positive, an assumption similar to the monotonicity condition in Imbens & Angrist (1994), then α_g is weighted negatively by β_C^{2SLS} if and only if ΔZ_g is strictly lower than $\Delta Z_.$. This result has two important implications. First it implies that there must be gs whose second-stage effects are weighted negatively. Second, it implies that one can estimate the set of locations whose second-stage effects are weighted negatively. Then, one may compare the characteristics of those locations to the characteristics of locations whose second-stage effects are weighted positively, to assess if those two groups are likely to have different second-stage effects. Negative weights may not be problematic if those two groups are unlikely to have different second-stage effects, but they may be problematic otherwise.

The third and fourth points of Theorem 3 show that even under further restrictions on the heterogeneity of the first-stage effects, β_C^{2SLS} still identifies the expectation of a weighted sum of second-stage effects, with weights that may not all be positive. The third point of Theorem 3 also implies that if we assume that the first-stage effects are all positive and constant across sectors ($\beta_{s,g}=\beta_g \geq 0$), the sign of the weight assigned by β_C^{2SLS} to each second-stage effect is identified and equal again to the sign of $\Delta Z_g - \Delta Z_.$. Again, one can then compare positively and negatively weighted locations, to assess if their second-stage effects are likely to differ. The fourth point of Theorem 3 implies that if we assume that the first-stage effects are constant across sectors and locations ($\beta_{s,g}=\beta$), the weights attached to the second-stage effects can be estimated.

[4]The weights do not sum up to 1, but their expectations sum to 1.

In the previous Bartik literature, both Borusyak et al. (2022) and Goldsmith-Pinkham et al. (2020) study Bartik 2SLS regressions with heterogeneous treatment effects. Under their randomly-assigned-shock assumption, Borusyak et al. (2022) show that such regressions identify convex combination of location-specific second-stage effects (see their Proposition A1), provided first-stage effects are all positive. Very interestingly, their result does not rely on a linear second-stage effects assumption, thus showing that under their randomly-assigned-shocks assumption, Bartik 2SLS regressions are robust both to heterogeneous and non-linear effects. Again, our two results show that whether 2SLS Bartik regressions are robust to heterogeneous effects depends on whether parallel trends or randomly-assigned-shocks is more plausible in the application under consideration.

Under their assumptions, Goldsmith-Pinkham et al. (2020) also show that β_C^{2SLS} identifies a weighted sum of second-stage effects, potentially with some negative weights (see their Equation (10)). The weights in their and our decomposition differ, which is not surprising because the assumptions we make on the first-stage Bartik regression are very different. Expressed in our notation, the weight assigned to α_g in their decomposition is

$$\frac{\sum_{s=1}^{S}\left(\Delta Z_s\left(\sum_{g'=1}^{G}Q_{s,g'}(\Delta D_{g'}-\Delta D_.)\right)\left(Q_{s,g}-Q_{s,.}\right)^2\Delta Z_s\beta_{s,g}\right)}{\left(\sum_{s=1}^{S}\Delta Z_s\left(\sum_{g'=1}^{G}Q_{s,g'}(\Delta D_{g'}-\Delta D_.)\right)\right)\times\left(\sum_{g=1}^{G}\left(Q_{s,g}-Q_{s,.}\right)^2\Delta Z_s\beta_{s,g}\right)}$$
$$=\frac{\sum_{s=1}^{S}\left(\sum_{g'=1}^{G}Q_{s,g'}(\Delta D_{g'}-\Delta D_.)\right)\left(Q_{s,g}-Q_{s,.}\right)^2\Delta Z_s^2\beta_{s,g}}{\left(\sum_{g'=1}^{G}\Delta Z_{g'}(\Delta D_{g'}-\Delta D_.)\right)\times\left(\sum_{g=1}^{G}\left(Q_{s,g}-Q_{s,.}\right)^2\Delta Z_s\beta_{s,g}\right)}, \tag{4.1}$$

where

$$\frac{\Delta Z_s\left(\sum_{g'=1}^{G}Q_{s,g'}(\Delta D_{g'}-\Delta D_.)\right)}{\sum_{s=1}^{S}\Delta Z_s\left(\sum_{g'=1}^{G}Q_{s,g'}(\Delta D_{g'}-\Delta D_.)\right)}$$

is the so-called Rotemberg weight (see Rotemberg 1983).

Let us assume that $\sum_{g'=1}^{G}\Delta Z_{g'}(\Delta D_{g'}-\Delta D_.)>0$, meaning that the first-stage Bartik regression coefficient is positive. Notice that if $\sum_{s=1}^{S}Q_{s,g'}=1$, $\sum_{s=1}^{S}\sum_{g'=1}^{G}Q_{s,g'}(\Delta D_{g'}-\Delta D_.)=0$. Therefore, there are sectors such that $\sum_{g'=1}^{G}Q_{s,g'}(\Delta D_{g'}-\Delta D_.)>0$ (sectors whose shares are positively correlated with locations' treatment change), and sectors such that $\sum_{g'=1}^{G}Q_{s,g'}(\Delta D_{g'}-\Delta D_.)<0$ (sectors whose shares are negatively correlated with locations' treatment change). Then, in the decomposition of Goldsmith-Pinkham et al. (2020), locations whose second stage effects are weighted negatively are such that sectors for which $\left(Q_{s,g}-Q_{s,.}\right)^2\Delta Z_s^2\beta_{s,g}$ is the largest, meaning that g's share of sector s differs the most from s's average share across locations, tend to be sectors with a negative share correlation with locations' treatment change. Accordingly, locations whose second-stage effect is weighted negatively differ under their and our assumptions.

4.3 Extension to Applications with Multiple Periods

So far, we have assumed that the data only has two periods. All our previous results readily generalize to settings with more than two periods.

We begin by adapting our notation and assumptions to the case where there are multiple time periods indexed by $t \in \{1, ..., T\}$, $T \geq 3$. For all $t \geq 2$ and any location-level variable R_g, let $\Delta R_{g,t} = R_{g,t} - R_{g,t-1}$. Let $\Delta Z_{s,t}$ denote the shock affecting sector s between periods $t-1$ and t, and let $\mathbf{\Delta Z} = (\Delta Z_{s,t})_{(s,t)\in\{1,...,S\}\times\{2,...,T\}}$ denote a vector collecting all the shocks $\Delta Z_{s,t}$.

Definition 5 *Bartik Instrument: The Bartik instrument* $\Delta Z_{g,t}$ *is:*

$$\Delta Z_{g,t} = \sum_{s=1}^{S} Q_{s,g} \Delta Z_{s,t}.$$

With more than two periods, our definition of the Bartik instrument assumes that the shares $Q_{s,g}$ do not change over time. In practice, applied researchers sometimes use time-varying shares to define the Bartik instrument. Our results can readily be generalized to allow time-varying shares.

For any $(\delta_1, ..., \delta_S) \in \mathbb{R}^S$, let $\Delta D_{g,t}(\delta_1, ..., \delta_S)$ denote the potential treatment evolution that location g will experience from period $t-1$ to t if $(\Delta Z_{1,t}, ..., \Delta Z_{S,t}) = (\delta_1, ..., \delta_S)$. And let $\Delta D_{g,t}(\mathbf{0}) = \Delta D_{g,t}(0, ..., 0)$ denote the potential treatment evolution that location g will experience in the absence of any shocks. Finally, let $\Delta Y_{g,t}(d_{g,t})$ denote the potential outcome evolution that location g will experience from period $t-1$ to t if $\Delta D_{g,t} = d_{g,t}$.

The assumptions below generalize Assumptions 1-5 to instances with multiple periods.

Assumption 6 *Linear First-Stage Equation: for all* $g \in \{1, ..., G\}$, $t \in \{2, ..., T\}$, *there are real numbers* $(\beta_{s,g,t})_{s\in\{1,...,S\}}$ *such that for any* $(\delta_1, ..., \delta_S) \in \mathbb{R}^S$*:*

$$\Delta D_{g,t}(\delta_1, ..., \delta_s) = \Delta D_{g,t}(\mathbf{0}) + \sum_{s=1}^{S} Q_{s,g} \delta_s \beta_{s,g,t}.$$

Assumption 7 *Common Trends for the Treatment: for all* $t \in \{2, ..., T\}$, *there are real numbers* μ_t^D *such that* $\forall g \in \{1, ..., G\}$, $E(\Delta D_{g,t}(\mathbf{0})|\mathbf{\Delta Z}, (Q_{s,g})_{s\in\{1,...,S\}}) = \mu_t^D$.

Assumption 8 *Linear Second-Stage Equation: for all* $g \in \{1, ..., G\}$, $t \in \{2, ..., T\}$, *there is a real number* $\alpha_{g,t}$ *such that for any* $d_{g,t} \in \mathbb{R}$*:*

$$\Delta Y_{g,t}(d_{g,t}) = \Delta Y_{g,t}(0) + \alpha_{g,t} d_{g,t}.$$

Assumption 9 *Common Trends for the Outcome: for all* $t \in \{2, ..., T\}$, *there are real numbers* μ_t^Y *such that* $\forall g \in \{1, ..., G\}$, $E(\Delta Y_{g,t}(\Delta D_{g,t}(\mathbf{0}))|\mathbf{\Delta Z}, (Q_{s,g})_{s\in\{1,...,S\}}) = \mu_t^Y$.

Assumption 10 *Independent Locations:*
Conditional on $\mathbf{\Delta Z}$, *the vectors* $\left((\Delta Y_{g,t}(0), \Delta D_{g,t}(\mathbf{0}))_{t\in\{2,...,T\}}, (Q_{s,g})_{s\in\{1,...,S\}}\right)$ *are mutually independent across* g.

With several periods, the analog of the 2SLS regression with a constant is a 2SLS regression with period fixed effects.

Definition 6 *Let $\hat{\beta}^D_C$ denote the coefficient of $\Delta Z_{g,t}$ in the regression of $\Delta D_{g,t}$ on $\Delta Z_{g,t}$ and period fixed effects, and let $\beta^D_C = E[\hat{\beta}^D_C]$. Let $\hat{\beta}^Y_C$ denote the coefficient of $\Delta Z_{g,t}$ in the regression of $\Delta Y_{g,t}$ on $\Delta Z_{g,t}$ and period fixed effects, and let $\beta^Y_C = E[\hat{\beta}^Y_C]$. Let $\beta^{2SLS}_C = \beta^Y_C/\beta^D_C$.*

The following result extends Theorem 3 to applications with multiple periods. The corresponding extensions of Theorems 1 and 2 are in Appendix Section B.

Theorem 4 *Suppose Assumptions 6-10 hold.*

1. *Then,*

$$\beta^{2SLS}_C = E\left(\sum_{g=1}^{G}\sum_{t=2}^{T}\frac{\sum_{s=1}^{S}\frac{Q_{s,g}\Delta Z_{s,t}(\Delta Z_{g,t}-\Delta Z_{.,t})}{\sum_{g=1}^{G}\sum_{t=2}^{T}\sum_{s=1}^{S}Q_{s,g}\Delta Z_{s,t}(\Delta Z_{g,t}-\Delta Z_{.,t})}\beta_{s,g,t}}{E\left(\sum_{g=1}^{G}\sum_{t=2}^{T}\sum_{s=1}^{S}\frac{Q_{s,g}\Delta Z_{s,t}(\Delta Z_{g,t}-\Delta Z_{.,t})}{\sum_{g=1}^{G}\sum_{t=2}^{T}\sum_{s=1}^{S}Q_{s,g}\Delta Z_{s,t}(\Delta Z_{g,t}-\Delta Z_{.,t})}\beta_{s,g,t}\right)}\alpha_{g,t}\right).$$

2. *If one further assumes that $\beta_{s,g,t} = \beta_{g,t}$,*

$$\beta^{2SLS}_C = E\left(\sum_{g=1}^{G}\sum_{t=2}^{T}\frac{\frac{\Delta Z_{g,t}(\Delta Z_{g,t}-\Delta Z_{.,t})}{\sum_{g=1}^{G}\sum_{t=2}^{T}\Delta Z_{g,t}(\Delta Z_{g,t}-\Delta Z_{.,t})}\beta_{g,t}}{E\left(\sum_{g=1}^{G}\sum_{t=2}^{T}\frac{\Delta Z_{g,t}(\Delta Z_{g,t}-\Delta Z_{.,t})}{\sum_{g=1}^{G}\sum_{t=2}^{T}\Delta Z_{g,t}(\Delta Z_{g,t}-\Delta Z_{.,t})}\beta_{g,t}\right)}\alpha_{g,t}\right).$$

3. *If one further assumes that $\beta_{s,g,t} = \beta$,*

$$\beta^{2SLS}_C = E\left(\sum_{g=1}^{G}\sum_{t=2}^{T}\frac{\Delta Z_{g,t}(\Delta Z_{g,t}-\Delta Z_{.,t})}{\sum_{g=1}^{G}\sum_{t=2}^{T}\Delta Z_{g,t}(\Delta Z_{g,t}-\Delta Z_{.,t})}\alpha_{g,t}\right).$$

4. *If one further assumes that $\beta_{s,g,t} = \beta_g$ and $\alpha_{g,t} = \alpha_g$,*

$$\beta^{2SLS}_C = E\left(\sum_{g=1}^{G}\frac{\left(\sum_{t=2}^{T}\frac{\Delta Z_{g,t}(\Delta Z_{g,t}-\Delta Z_{.,t})}{\sum_{g=1}^{G}\sum_{t=2}^{T}\Delta Z_{g,t}(\Delta Z_{g,t}-\Delta Z_{.,t})}\right)\beta_g}{E\left(\sum_{g=1}^{G}\left(\sum_{t=2}^{T}\frac{\Delta Z_{g,t}(\Delta Z_{g,t}-\Delta Z_{.,t})}{\sum_{g=1}^{G}\sum_{t=2}^{T}\Delta Z_{g,t}(\Delta Z_{g,t}-\Delta Z_{.,t})}\right)\beta_g\right)}\alpha_g\right).$$

Points 1, 2, and 3 of Theorem 4 are generalizations of Points 1, 3, and 4 of Theorem 3. The important new implication that was not present in Theorem 3 is that Bartik regressions are also not robust to heterogeneous first- and second-stage effects over time. Point 4 is a new result that was not present in Theorem 3. There, it is assumed that first-stage effects do not vary across sectors and over time, and that second-stage effects do not vary over time ($\beta_{s,g,t} = \beta_g$ and $\alpha_{g,t} = \alpha_g$). It is useful to consider what Bartik regressions estimate under that assumption, because it underlies the first alternative estimator we propose in Section 5. Under that assumption, the second-stage Bartik regression estimates again a weighted sum of the location-specific second stage effects α_g, with weights that may be negative. This contrasts with our first alternative estimator: if $\beta_{s,g,t} = \beta_g$ and $\alpha_{g,t} = \alpha_g$, it estimates a weighted average of the second-stage effects α_g.

4.4 Testing the parallel-trends assumptions

When the data contains a time period where no shocks arise, our parallel-trends assumptions can be tested. Let us assume that $\Delta Z_{s,2} = 0$ for every $s \in \{1, ..., S\}$. In the canonical design, that means that the nationwide employment should remain stable from $t = 1$ to $t = 2$ in every sector. In Autor et al. (2013), that means that the exposure to Chinese imports should remain stable from $t = 1$ to $t = 2$ in every sector. Then, one can use the following regressions to test Assumptions 7 and 9.

Definition 7 *Placebo First-Stage Bartik regression: Let $\hat{\beta}^D_{pl}$ denote the coefficient of $\Delta Z_{g,3}$ in the regression of $\Delta D_{g,2}$ on $\Delta Z_{g,3}$ and a constant. Let $\beta^D_{pl} = E[\hat{\beta}^D_{pl}]$.*

Definition 8 *Placebo Reduced-Form Bartik regression: Let $\hat{\beta}^Y_{pl}$ denote the coefficient of $\Delta Z_{g,3}$ in the regression of $\Delta Y_{g,2}$ on $\Delta Z_{g,3}$ and a constant. Let $\beta^Y_{pl} = E[\hat{\beta}^Y_{pl}]$.*

β^D_{pl} is the coefficient of $\Delta Z_{g,3}$, the period-3 Bartik instrument, from a regression of $\Delta D_{g,2}$, the period-1-to-2 treatment evolution, on a constant and $\Delta Z_{g,3}$. If $\Delta Z_{s,2} = 0$ for every $s \in \{1, ..., S\}$, $\Delta D_{g,2} = \Delta D_{g,2}(\mathbf{0})$ for every g. Under Assumption 7, the expectation of $\Delta D_{g,2}(\mathbf{0})$ should be the same in every location, and should not correlate with locations' value of the Bartik instrument in the next period. Accordingly, $\beta^D_{\text{pl}} = 0$ so finding that $\hat{\beta}^D_{\text{pl}}$ is significantly different from 0 would imply that Assumption 7 is rejected. As per the same logic, finding that $\hat{\beta}^Y_{\text{pl}}$ is significantly different from 0 would imply that Assumption 9 is rejected. Theorem 5 below formalizes this argument.

Theorem 5 *Suppose Assumptions 6, 8, 10 hold, and $\Delta Z_{s,2} = 0$ for every $s \in \{1, ..., S\}$.*

1. *If Assumption 7 also holds, $\beta^D_{pl} = 0$.*

2. *If Assumption 9 also holds, $\beta^Y_{pl} = 0$.*

Theorem 5 shows that $\beta^D_{\text{pl}} = 0$ and $\beta^Y_{\text{pl}} = 0$ are testable implications of Assumptions 7 and 9. When $\Delta Z_{s,2} = 0$ for every $s \in \{1, ..., S\}$, testing whether $\hat{\beta}^D_{\text{pl}}$ (resp. $\hat{\beta}^Y_{\text{pl}}$) significantly differs from 0 is not the only way to test Assumption 7 (resp. 9). For instance, one could regress $\Delta D_{g,2}$ (resp. $\Delta Y_{g,2}$) on any of the $\Delta Z_{g,t}$ for $t \geq 3$. The reason we focus on the test presented above is that it is closely related to a placebo test commonly implemented by applied researchers, see for instance Autor et al. (2013). Theorem 5 shows that this placebo test is in fact a test of our parallel-trends assumptions.

5 Alternative estimators

In this section, we propose two alternative estimators. They are closely related to the correlated-random-coefficients estimator proposed by Chamberlain (1992). They can be used when the data has at least three periods, which is often the case in Bartik applications.[5]

[5] With two periods, the estimation method we propose cannot be used, but one may then be able to follow a similar estimation strategy as that proposed in Graham & Powell (2012).

5.1 Estimator robust to heterogeneous effects across locations

Our first alternative estimator relies on the following assumption:

Assumption 11 *Constant first-stage effects across sectors and over time, and constant second-stage effects over time: for all $g \in \{1, ..., G\}$, there are real numbers β_g and α_g such that $\beta_{s,g,t} = \beta_g$ for every $s \in \{1, ..., S\}$ and $t \in \{2, ..., T\}$ and $\alpha_{g,t} = \alpha_g$ for every $t \in \{2, ..., T\}$.*

Assumption 11 requires that first-stage effects be constant across sectors and over time, and that second-stage effects be constant over time, as in Point 4 of Theorem 4. Under Assumption 11, for every $g \in \{1, ..., G\}$, let $\gamma_g = \alpha_g \beta_g$ denote the reduced-form effect of the instrument on the outcome.

Let

$$\begin{aligned}\boldsymbol{\Delta Z}_g =&(\Delta Z_{g,2}, ..., \Delta Z_{g,T})' \\ \boldsymbol{\Delta D}_g =&(\Delta D_{g,2}, ..., \Delta D_{g,T})' \\ \boldsymbol{\Delta Y}_g =&(\Delta Y_{g,2}, ..., \Delta Y_{g,T})'\end{aligned}$$

be $(T-1) \times 1$ vectors stacking together the instruments, the evolutions of the treatment, and the evolutions of the outcome, respectively. Let

$$\begin{aligned}\boldsymbol{\mu}^D =&(\mu_2^D, ..., \mu_T^D)' \\ \boldsymbol{\mu}^Y =&(\mu_2^Y, ..., \mu_T^Y)'\end{aligned}$$

be $(T-1) \times 1$ vectors stacking together the common trends affecting the treatment and the outcome, defined in Assumptions 7 and 9. For every $g \in \{1, ..., G\}$, let

$$\boldsymbol{M}_g = \boldsymbol{I} - \frac{1}{\boldsymbol{\Delta Z}_g'\boldsymbol{\Delta Z}_g}\boldsymbol{\Delta Z}_g\boldsymbol{\Delta Z}_g',$$

where $\boldsymbol{I}$ denotes the $(T-1) \times (T-1)$ identity matrix.

Theorem 6 *Suppose that Assumptions 6-11 hold, $E\left(\frac{1}{G}\sum_{g=1}^G \boldsymbol{M}_g'\boldsymbol{M}_g\right)$ is invertible, and with probability 1 $\boldsymbol{\Delta Z}_g \neq 0$ for every $g \in \{1, ..., G\}$. Then:*

$$\boldsymbol{\mu}^D = E\left(\frac{1}{G}\sum_{g=1}^G \boldsymbol{M}_g'\boldsymbol{M}_g\right)^{-1} E\left(\frac{1}{G}\sum_{g=1}^G \boldsymbol{M}_g'\boldsymbol{M}_g\boldsymbol{\Delta D}_g\right) \tag{5.1}$$

$$\boldsymbol{\mu}^Y = E\left(\frac{1}{G}\sum_{g=1}^G \boldsymbol{M}_g'\boldsymbol{M}_g\right)^{-1} E\left(\frac{1}{G}\sum_{g=1}^G \boldsymbol{M}_g'\boldsymbol{M}_g\boldsymbol{\Delta Y}_g\right) \tag{5.2}$$

$$\frac{1}{G}\sum_{g=1}^G \beta_g = E\left(\frac{1}{G}\sum_{g=1}^G \frac{\boldsymbol{\Delta Z}_g'\left(\boldsymbol{\Delta D}_g - \boldsymbol{\mu}^D\right)}{\boldsymbol{\Delta Z}_g'\boldsymbol{\Delta Z}_g}\right) \tag{5.3}$$

$$\frac{1}{G}\sum_{g=1}^G \gamma_g = E\left(\frac{1}{G}\sum_{g=1}^G \frac{\boldsymbol{\Delta Z}_g'\left(\boldsymbol{\Delta Y}_g - \boldsymbol{\mu}^Y\right)}{\boldsymbol{\Delta Z}_g'\boldsymbol{\Delta Z}_g}\right). \tag{5.4}$$

Under Assumptions 6-11,

$$E(\boldsymbol{\Delta D}_g|\boldsymbol{\Delta Z},(Q_{s,g})_{s\in\{1,...,S\}}) = \boldsymbol{\mu}^D + \beta_g \boldsymbol{\Delta Z}_g, \tag{5.5}$$

an equation that is additively separable in the location-specific coefficient β_g and the common trends $\boldsymbol{\mu}^D$, and thus falls into the class of semi-parametric models studied in Chamberlain (1992). Then, identification follows from the same steps as in Chamberlain (1992). First, it is easy to check that $\boldsymbol{M}_g \boldsymbol{\Delta Z}_g = 0$. Accordingly, left-multiplying Equation (5.5) by $\boldsymbol{M}_g$, it follows that

$$E\left(\boldsymbol{M}_g \boldsymbol{\Delta D}_g|\boldsymbol{\Delta Z},(Q_{s,g})_{s\in\{1,...,S\}}\right) = \boldsymbol{M}_g \boldsymbol{\mu}^D, \tag{5.6}$$

so $\boldsymbol{\mu}^D$ is identified by a regression of $\boldsymbol{M}_g \boldsymbol{\Delta D}_g$ on $\boldsymbol{M}_g$, as in Equation (5.1). Similarly, Equation (5.3) follows after left-multiplying Equation (5.5) by $\boldsymbol{\Delta Z}_g'$, rearranging terms, using the law of iterated expectations, and averaging across g. Notice that

$$\frac{\boldsymbol{\Delta Z}_g'\left(\boldsymbol{\Delta D}_g - \boldsymbol{\mu}^D\right)}{\boldsymbol{\Delta Z}_g' \boldsymbol{\Delta Z}_g}$$

is the coefficient of $\Delta Z_{g,t}$ in the regression, within group g, of $\Delta D_{g,t} - \mu_t^D$ on $\Delta Z_{g,t}$ without a constant. Accordingly, once the common trends μ_t^D have been identified, Chamberlain's estimator of the average first-stage effect $\frac{1}{G}\sum_{g=1}^G \beta_g$ amounts to regressing $\Delta D_{g,t} - \mu_t^D$ on $\Delta Z_{g,t}$ without a constant in every group, and then averaging those coefficients across groups. To estimate the average reduced-form effect $\frac{1}{G}\sum_{g=1}^G \gamma_g$, the procedure is similar, replacing the treatment evolution by the outcome evolution.

Corollary 1 *Under the same assumptions as those in Theorem 6,*

$$\sum_{g=1}^G \frac{\beta_g}{\sum_{g=1}^G \beta_g}\alpha_g = \frac{E\left(\frac{1}{G}\sum_{g=1}^G \frac{\boldsymbol{\Delta Z}_g'\left(\boldsymbol{\Delta Y}_g - \boldsymbol{\mu}^Y\right)}{\boldsymbol{\Delta Z}_g' \boldsymbol{\Delta Z}_g}\right)}{E\left(\frac{1}{G}\sum_{g=1}^G \frac{\boldsymbol{\Delta Z}_g'(\boldsymbol{\Delta D}_g - \boldsymbol{\mu}^D)}{\boldsymbol{\Delta Z}_g' \boldsymbol{\Delta Z}_g}\right)}. \tag{5.7}$$

Corollary 1 directly follows from Equations (5.3) and (5.4) and the definition of γ_g. Under the assumption that all first-stage effects are positive, $\sum_{g=1}^G \frac{\beta_g}{\sum_{g=1}^G \beta_g}\alpha_g$ is a weighted average of location-specific second stage effects, that gives a higher weight to the second-stage effect of locations with a higher first-stage effect. Therefore, Corollary 1 implies that under Assumption 11 and if $\beta_g \geq 0$, our correlated-random-coefficient estimator estimates a weighted average of second-stage effects. Point 4 of Theorem 4 shows that under the same assumptions, the 2SLS Bartik regression may not estimate a weighted average of second-stage effects.

Operationally, estimators of $\boldsymbol{\mu}^D$, $\boldsymbol{\mu}^Y$, $\frac{1}{G}\sum_{g=1}^G \beta_g$, and $\frac{1}{G}\sum_{g=1}^G \gamma_g$ can be computed using the Generalized Method of Moments (GMM). Indeed, one has

$$
\begin{aligned}
E\left(\frac{1}{G}\sum_{g=1}^{G}\boldsymbol{M}_g(\boldsymbol{\Delta D}_g-\boldsymbol{\mu}^D)\right)&=0\\
E\left(\frac{1}{G}\sum_{g=1}^{G}\boldsymbol{M}_g(\boldsymbol{\Delta Y}_g-\boldsymbol{\mu}^Y)\right)&=0\\
E\left(\frac{1}{G}\sum_{g=1}^{G}\frac{\boldsymbol{\Delta Z}_g'\left(\boldsymbol{\Delta D}_g-\boldsymbol{\mu}^D\right)}{\boldsymbol{\Delta Z}_g'\boldsymbol{\Delta Z}_g}-\frac{1}{G}\sum_{g=1}^{G}\beta_g\right)&=0\\
E\left(\frac{1}{G}\sum_{g=1}^{G}\frac{\boldsymbol{\Delta Z}_g'\left(\boldsymbol{\Delta Y}_g-\boldsymbol{\mu}^Y\right)}{\boldsymbol{\Delta Z}_g'\boldsymbol{\Delta Z}_g}-\frac{1}{G}\sum_{g=1}^{G}\gamma_g\right)&=0,
\end{aligned}
\tag{5.8}
$$

a just-identified system with $2T$ moment conditions and $2T$ parameters.

All the moment conditions in (5.8) actually hold conditional on $(\Delta\boldsymbol{Z}_g)_{g\in\{1,...,G\}}$, thus implying that the parameters could be estimated using conditional rather than unconditional GMM. Applying results in Chamberlain (1992), one can derive the optimal estimator of $\boldsymbol{\mu}^D$ and $\boldsymbol{\mu}^Y$ attached to the first two equations in (5.8). An issue, however, is that Chamberlain's optimality results do not apply to the estimators of $\frac{1}{G}\sum_{g=1}^{G}\beta_g$ and $\frac{1}{G}\sum_{g=1}^{G}\gamma_g$, the building blocks of our target parameter. Moreover, the computation of the optimal estimator requires a non-parametric first-stage estimation. To our knowledge, no data-driven method has been proposed to choose the tuning parameters involved in this first stage. Accordingly, we prefer to stick with the unconditional GMM estimator above.

Under Assumption 10, conditional on $\mathbf{\Delta Z}$, the vectors $(\Delta Y_{g,t},\Delta D_{g,t},\Delta Z_{g,t})_{t\in\{2,...,T\}}$ are mutually independent across g. Then, to perform inference on $\sum_{g=1}^{G}\frac{\beta_g}{\sum_{g=1}^{G}\beta_g}\alpha_g$ conditional on the shocks, one may use the heteroskedasticity-robust standard errors attached to the GMM system in (5.8).[6] The entire time series of each location enters in the system's $2T$ moment conditions, so those heteroskedasticity-robust standard errors do not assume that the vectors $(\Delta Y_{g,t},\Delta D_{g,t},\Delta Z_{g,t})$ are independent across t: they only require that conditional on $\mathbf{\Delta Z}$, the vectors $(\Delta Y_{g,t},\Delta D_{g,t},\Delta Z_{g,t})_{t\in\{2,...,T\}}$ be independent across g. Accounting for the variance arising from the shocks would require extending the approach in Adão et al. (2019) to the estimators in Theorem 6, and without making the randomly-assigned shocks assumption. This important extension is left for future work. Similarly, in our applications, to draw inference on Bartik regression coefficients we use standard errors clustered at the location level: those do not account for the variance arising from the shocks, but they give valid estimators of the coefficients' standard errors conditional on the shocks under Assumption 10. The standard errors proposed by Adão et al. (2019) for Bartik regressions account for the shocks, but they are only valid under the random-shocks assumption.

[6]Theorem 6 is also valid conditional on the shocks.

5.2 Estimator robust to heterogeneous effects across locations and over time

While the estimator above is robust to heterogeneous first- and second-stage effects across locations, it is not robust to heterogeneous first- and second-stage effects over time, and to heterogeneous first-stage effects across sectors. Accordingly, it still rests on strong homogeneity assumptions. We now propose a second alternative estimator that relies on a weaker assumption.

Assumption 12 *Constant first-stage effects across sectors, and first- and second-stage effects additively separable in location and time: for all $g \in \{1, ..., G\}$ and $t \in \{2, ..., T\}$, there are real numbers β_g, α_g, λ_t^D and λ_t^Y such that $\beta_{s,g,t} = \beta_g + \lambda_t^D$ for every $s \in \{1, ..., S\}$, and $\alpha_{g,t} = \alpha_g + \lambda_t^Y$.*

λ_t^D and λ_t^Y respectively represent the change in the first- and second-stage effects from $t-1$ to t, which is assumed to be constant across locations. Accordingly, Assumption 12 maybe be interpreted as a parallel trends assumption on the first- and second-stage effects. Without loss of generality, we can assume that $\lambda_2^D = \lambda_2^Y = 0$. Then, let $\boldsymbol{\theta}^D = (\mu_2^D, \mu_3^D, \lambda_3^D, ..., \mu_T^D, \lambda_T^D)'$ and $\boldsymbol{\theta}^Y = (\mu_2^Y, \mu_3^Y, \lambda_3^Y, ..., \mu_T^Y, \lambda_T^Y)'$, let $\mathbf{0}_k$ denote a vector of k zeros, let

$$\mathbf{P}_g = \begin{pmatrix} 1, \mathbf{0}_{2T-4} \\ 0, 1, \Delta Z_{g,3}, \mathbf{0}_{2T-6} \\ \mathbf{0}_3, 1, \Delta Z_{g,4}, \mathbf{0}_{2T-8} \\ \vdots \\ \mathbf{0}_{2T-5}, 1, \Delta Z_{g,T} \end{pmatrix},$$

let $\Delta\tilde{Z}_{g,t} = \Delta Z_{g,t}(\beta_g + \lambda_t^D)$, let

$$\tilde{\boldsymbol{M}}_g = \boldsymbol{I} - \frac{1}{\boldsymbol{\Delta}\tilde{\boldsymbol{Z}}_g'\boldsymbol{\Delta}\tilde{\boldsymbol{Z}}_g}\boldsymbol{\Delta}\tilde{\boldsymbol{Z}}_g\boldsymbol{\Delta}\tilde{\boldsymbol{Z}}_g',$$

and let

$$\tilde{\mathbf{P}}_g = \begin{pmatrix} 1, \mathbf{0}_{2T-4} \\ 0, 1, \Delta\tilde{Z}_{g,3}, \mathbf{0}_{2T-6} \\ \mathbf{0}_3, 1, \Delta\tilde{Z}_{g,4}, \mathbf{0}_{2T-8} \\ \vdots \\ \mathbf{0}_{2T-5}, 1, \Delta\tilde{Z}_{g,T} \end{pmatrix}.$$

Theorem 7 *Suppose that Assumptions 6-10 and 12 hold, $E\left(\frac{1}{G}\sum_{g=1}^G \boldsymbol{P}_g'\boldsymbol{M}_g\boldsymbol{P}_g\right)$ and $E\left(\frac{1}{G}\sum_{g=1}^G \tilde{\boldsymbol{P}}_g'\tilde{\boldsymbol{M}}_g\tilde{\boldsymbol{P}}_g\right)$ are invertible, and with probability 1 $\boldsymbol{\Delta Z}_g \neq 0$ and $\boldsymbol{\Delta}\tilde{\boldsymbol{Z}}_g \neq 0$*

for every $g \in \{1, ..., G\}$*. Then:*

$$\boldsymbol{\theta}^D = E\left(\frac{1}{G}\sum_{g=1}^{G}\boldsymbol{P}_g'\boldsymbol{M}_g\boldsymbol{P}_g\right)^{-1} E\left(\frac{1}{G}\sum_{g=1}^{G}\boldsymbol{P}_g'\boldsymbol{M}_g\boldsymbol{\Delta D}_g\right) \tag{5.9}$$

$$\beta_g = E\left(\frac{\boldsymbol{\Delta Z}_g'\left(\boldsymbol{\Delta D}_g - \boldsymbol{P}_g\boldsymbol{\theta}^D\right)}{\boldsymbol{\Delta Z}_g'\boldsymbol{\Delta Z}_g}\right) \tag{5.10}$$

$$\boldsymbol{\theta}^Y = E\left(\frac{1}{G}\sum_{g=1}^{G}\tilde{\boldsymbol{P}}_g'\tilde{\boldsymbol{M}}_g\tilde{\boldsymbol{P}}_g\right)^{-1} E\left(\frac{1}{G}\sum_{g=1}^{G}\tilde{\boldsymbol{P}}_g'\tilde{\boldsymbol{M}}_g\boldsymbol{\Delta Y}_g\right) \tag{5.11}$$

$$\frac{1}{G}\sum_{g=1}^{G}\alpha_g = E\left(\frac{1}{G}\sum_{g=1}^{G}\frac{\boldsymbol{\Delta}\tilde{\boldsymbol{Z}}_g'\left(\boldsymbol{\Delta Y}_g - \tilde{\boldsymbol{P}}_g\boldsymbol{\theta}^Y\right)}{\boldsymbol{\Delta}\tilde{\boldsymbol{Z}}_g'\boldsymbol{\Delta}\tilde{\boldsymbol{Z}}_g}\right). \tag{5.12}$$

Under Assumptions 6-10 and 12,

$$E(\boldsymbol{\Delta D}_g|\boldsymbol{\Delta Z}, (Q_{s,g})_{s\in\{1,...,S\}}) = \boldsymbol{P}_g\boldsymbol{\theta}^D + \beta_g\boldsymbol{\Delta Z}_g, \tag{5.13}$$

an equation that is additively separable in the location-specific coefficient β_g and the common trends $\boldsymbol{\theta}^D$, like Equation (5.5), so identification of $\boldsymbol{\theta}^D$ follows from similar steps as the identification of $\boldsymbol{\mu}^D$ in Theorem 6. Once $\boldsymbol{\theta}^D$ is identified, Equation (5.10) directly follows from Equation (5.13), thus showing that β_g is identified for all g. Once β_g and $\boldsymbol{\theta}^D$ are identified, $\Delta\tilde{Z}_{g,t}$ is identified. Then, under Assumptions 6-10 and 12,

$$E(\Delta Y_{g,t}|\boldsymbol{\Delta Z}, (Q_{s,g})_{s\in\{1,...,S\}}) = \mu_t^Y + (\alpha_g + \lambda_t^Y)(\beta_g + \lambda_t^D)\Delta Z_{g,t} = \mu_t^Y + \lambda_t^Y\Delta\tilde{Z}_{g,t} + \alpha_g\Delta\tilde{Z}_{g,t}.$$

Therefore,

$$E(\boldsymbol{\Delta Y}_g|\boldsymbol{\Delta Z}, (Q_{s,g})_{s\in\{1,...,S\}}) = \tilde{\boldsymbol{P}}_g\boldsymbol{\theta}^Y + \alpha_g\boldsymbol{\Delta}\tilde{\boldsymbol{Z}}_g, \tag{5.14}$$

an equation similar to Equation (5.13), replacing $\boldsymbol{\Delta D}_g$ by $\boldsymbol{\Delta Y}_g$, and $\Delta Z_{g,t}$ by $\Delta\tilde{Z}_{g,t}$. Then, identification of $\frac{1}{G}\sum_{g=1}^{G}\alpha_g$ and $\boldsymbol{\theta}^Y$ follows from similar steps as above.

Note that once $\frac{1}{G}\sum_{g=1}^{G}\alpha_g$ and $\boldsymbol{\theta}^Y$ are identified,

$$\frac{1}{T-1}\sum_{t=2}^{T}\frac{1}{G}\sum_{g=1}^{G}(\alpha_g + \lambda_t^Y), \tag{5.15}$$

the average second-stage effect across all periods and locations, is also identified. This parameter is arguably more natural than the weighted average of second-stage effects in Corollary 1.

To estimate that parameter, one can proceed as follows:

1. Compute $\widehat{\boldsymbol{\theta}}^D = \left(\frac{1}{G}\sum_{g=1}^{G}\boldsymbol{P}_g'\boldsymbol{M}_g\boldsymbol{P}_g\right)^{-1}\left(\frac{1}{G}\sum_{g=1}^{G}\boldsymbol{P}_g'\boldsymbol{M}_g\boldsymbol{\Delta D}_g\right)$.
2. Compute $\widehat{\beta}_g = \frac{\boldsymbol{\Delta Z}_g'\left(\boldsymbol{\Delta D}_g - \boldsymbol{P}_g\widehat{\boldsymbol{\theta}}^D\right)}{\boldsymbol{\Delta Z}_g'\boldsymbol{\Delta Z}_g}$.

3. Compute $\Delta\widehat{\tilde{Z}}_{g,t} = \Delta Z_{g,t}(\widehat{\beta}_g + \widehat{\lambda}_t^D)$ and define $\widehat{\tilde{\boldsymbol{P}}}_g$ and $\widehat{\tilde{\boldsymbol{M}}}_g$ accordingly.

4. Compute $\widehat{\boldsymbol{\theta}}^Y = \left(\frac{1}{G}\sum_{g=1}^G \widehat{\tilde{\boldsymbol{P}}}_g' \widehat{\tilde{\boldsymbol{M}}}_g \widehat{\tilde{\boldsymbol{P}}}_g\right)^{-1} \left(\frac{1}{G}\sum_{g=1}^G \widehat{\tilde{\boldsymbol{P}}}_g' \widehat{\tilde{\boldsymbol{M}}}_g \boldsymbol{\Delta Y}_g\right)$.

5. Finally, compute $\widehat{\alpha}_g = \frac{\boldsymbol{\Delta}\widehat{\tilde{\boldsymbol{Z}}}_g'\left(\boldsymbol{\Delta Y}_g - \widehat{\tilde{\boldsymbol{P}}}_g \widehat{\boldsymbol{\theta}}^Y\right)}{\boldsymbol{\Delta}\widehat{\tilde{\boldsymbol{Z}}}_g' \boldsymbol{\Delta}\widehat{\tilde{\boldsymbol{Z}}}_g}$.

One can show that $\widehat{\boldsymbol{\theta}}^D$ is numerically equivalent to a standard GMM estimator. One can also show that $\widehat{\boldsymbol{\theta}}^Y$ and $\frac{1}{G}\sum_{g=1}^G \widehat{\alpha}_g$ are numerically equivalent to non-standard GMM estimators, with $G + T - 2$ preliminary estimated parameters to estimate the variables $\Delta\tilde{Z}_{g,t}$. Studying the large-sample properties of those estimators is left for future work.

6 Empirical Application: China Shock

In this section, we revisit Autor et al. (2013), who use the Bartik instrument to estimate the effects of exposure to Chinese imports on manufacturing employment in the US. We also revisit another empirical application, the canonical application in Bartik (1991), in Section A of our Web Appendix.

6.1 Data

We use the replication dataset of Autor et al. (2013) on the AEA website. In their main analysis, they use a CZ-level panel data set, with 722 CZs and 3 periods (1990, 2000, and 2007). The outcome variable $\Delta Y_{g,t}$ is the change in the manufacturing employment share of the working age population in CZ g between two consecutive periods. The treatment variable $\Delta D_{g,t}$ is the change in Chinese import exposure per worker in CZ g between two consecutive periods. The sectoral shocks $\Delta Z_{s,t}$ are the change in per-worker imports from China to other high-income countries in industry s, for 397 manufacturing industries. The share $Q_{s,g}$ in Autor et al. (2013) is the employment share of sector s in location g, and the shares are lagged by 1 period when constructing the Bartik instrument (see Equation (4) in Autor et al. 2013).[7] The shares do not sum up to 1 in each location, because there is employment in non-manufacturing industries as well (see Borusyak et al. 2022 for a discussion of this).[8] The replication dataset of Autor et al. (2013) does not contain

[7]Specifically, the exact definition of the treatment variable is $\Delta D_{g,t} = \sum_s P_{s,g,t}\Delta D_{s,t}$, where $P_{s,g,t} = L_{s,g,t}/L_{g,t}$ is the local employment share of sector s in cell (g,t), and $\Delta D_{s,t} = \Delta M_{u,c,s,t}/L_{u,s,t}$ is the per-worker change in US imports from China in sector s and period t. The Bartik instrument is defined as $\Delta Z_{g,t} = \sum_s Q_{s,g,t}\Delta Z_{s,t}$, where $Q_{s,g,t} = L_{s,g,t-1}/L_{g,t-1}$ is the lagged local employment share of sector s in location g, and $\Delta Z_{s,t} = \Delta M_{o,c,s,t}/L_{u,s,t-1}$ is the per-lagged-US worker change in other countries' imports from China in sector s and period t. See footnote 39 of Borusyak et al. (2022) for more discussion. We use the same variable definitions as Borusyak et al. (2022).

[8] Under their random-shock assumption, Borusyak et al. (2022) show that it is important to control for the sum of shares when it is not always equal to 1 (see their section 4.2). Intuitively, with shares summing

the shock and share variables. However, we obtained those variables from the replication dataset of Borusyak et al. (2022).

6.2 Testing the identifying assumptions

We start by assessing the plausibility of the random shock assumption, by testing whether shocks are independent of industries' characteristics. We use the five industry characteristics in Acemoglu et al. (2016) that are in the replication dataset of Borusyak et al. (2022). These characteristics are the share of production workers in each industry's employment in 1991, the ratio of the industry's capital to value-added in 1991, the industry's log real wages in 1991, the share of its investment devoted to computers in 1990, and the share of its high-tech equipment in total investment in 1990. These predetermined characteristics reflect the structure of employment and technology across industries. Table 1 shows regressions of sectoral shocks from 1990 to 2000 (in Column (1)) and from 2000 to 2007 (in Column (2)) on these characteristics. We follow Borusyak et al. (2022) and weight the regressions by the average industry exposure shares and cluster the standard errors at the level of three-digit SIC codes, but the results are very similar when the regressions are not weighted or when one uses robust standard errors.

The results show that large import shocks tend to appear in industries with low wages and more high-tech equipment investment, and we can reject the hypothesis that the import shocks are not correlated with any industry characteristic (p-value < 0.001 in Column (1), p-value $= 0.017$ in Column (2)). Therefore, there are industries with certain characteristics that make them more likely to receive a large import shock, and the random-shock assumption is rejected.h

Note that our test of the random shocks assumption is inspired from and closely related to that in Table 3 Panel A in Borusyak et al. (2022). The only difference is that they separately regress each industry characteristic on the shocks, while we regress the shocks on all the industry characteristics. Our results differ from those in Table 3 Panel A of Borusyak et al. (2022). In their univariate regressions, the authors find no significant correlation between characteristics and shocks. Reverting the dependent and the independent variables in their Table 3 Panel A would leave their t-stats unchanged, so the difference between our and their results comes from the fact they assess if each of the five industry-level characteristics can separately predict the average industry-level shocks, while we assess if the five industry-level characteristics can jointly predict the average industry-level shocks. Based on our results, one can reject the null that the mean

to less than one, the Bartik instrument is still well defined assuming there is a "missing sector" $s = 0$ with no sectoral shock: $\Delta Z_0 = 0$. But then, the random-shock assumption would require all the other sectors to have shocks with zero expectation. By controlling for the sum of shares, one can relax this restriction. In our setting, adding a missing sector with $\Delta Z_0 = 0$ does not impose additional restrictions on our parallel-trends assumptions, which can be extended to have an additional missing sector with no shock. Therefore, our results readily extend to settings where shares do not sum up to 1.

of the shocks conditional on those five characteristics is constant, thus implying that the random-shock assumption is violated. Based on their results, one cannot reject the null that the mean of the shocks conditional on each individual characteristic is constant. But it follows from the law of iterated expectations that the null in our test is stronger than the null in their test: denoting by $X_{1,s}, X_{2,s}, X_{3,s}, X_{4,s}, X_{5,s}$ the five characteristics in Acemoglu et al. (2016), $E(\Delta Z_s|X_{1,s}, X_{2,s}, X_{3,s}, X_{4,s}, X_{5,s}) = \mu \Rightarrow E(\Delta Z_s|X_{k,s}) = \mu$ for all $k \in \{1, ..., 5\}$. Because we test a stronger implication of the random-shock assumption, our test may be more powerful, which could explain why our test is rejected while theirs is not.

Table 1: Testing the Random Shock Assumption

	(1)	(2)
Variables	$\Delta Z_{s,t}$: 1990-2000	$\Delta Z_{s,t}$: 2000-2007
Production workers' share of employment$_{1991}$	0.178	1.528
	(5.179)	(24.436)
Ratio of capital to value-added$_{1991}$	0.209	8.389
	(0.822)	(3.000)
Log real wage (2007 USD)$_{1991}$	-8.946	-11.514
	(2.171)	(8.505)
Computer investment as share of total investment$_{1990}$	0.103	1.149
	(0.135)	(0.633)
High-tech equipment as share of total investment$_{1990}$	0.299	1.095
	(0.135)	(0.433)
Constant	29.414	25.161
	(9.787)	(40.623)
Observations	397	397
R-squared	0.082	0.067
F-test P-value	0.0000	0.0172

Notes: The table reports estimates of regressions of the industry-level change in per-worker imports from China to other high-income countries during 1990-2000 and 2000-2007 on a set of pre-determined industry characteristics. The dependent variable in Column (1) is the change in per-worker imports from China to other high-income countries from 1990 to 2000. The dependent variable in Column (2) is the change in per-worker imports from China to other high-income countries from 2000 to 2007. The industry characteristics are obtained from Acemoglu et al. (2016), and include an industry's share of production workers in employment in 1991, the ratio of its capital to value-added in 1991, its log real wages in 1991, the share of its investment devoted to computers in 1990, and the share of its high-tech equipment in total investment in 1990. The regressions are weighted by the average industry exposure shares. Standard errors clustered at the level of three-digit SIC codes are shown in parentheses. The F-test p-value is the p-value of the joint test that all the coefficients of the industry characteristics are equal to 0.

In this application, there are periods where all industries barely receive any shocks. As discussed in Autor et al. (2013), the growth in Chinese imports to the US was very small prior to 1990. Accordingly, 1990 and prior periods satisfy, or nearly satisfy, the requirement in Theorem 5 that $\Delta Z_{s,t}$ should be 0 for every industry. In fact, in their Table 2, Autor et al. (2013) implement a placebo test closely related to that we propose in Theorem 5. They average the 1990-to-2000 and 2000-to-2007 Bartik instrument of each CZ, and then estimate a 2SLS regression of 1970-to-1980 and 1980-to-1990 manufacturing employment growths on 1990-to-2000 and 2000-to-2007 Chinese import exposure growth, using the average Bartik as the instrument. Instead, Table 2 below follows Point 2 of Theorem 5 and presents placebo reduced-form regressions. In Column (1) (resp. (2)), we regress CZs 1970-to-1980 and 1980-to-1990 manufacturing employment growths on their 1990-to-2000 (resp. 2000-to-2007) Bartik instrument. We use standard errors clustered at the CZ level (between parenthesis). Unlike Autor et al. (2013), we do not weight the regression by CZ's population, to be consistent with Sections 3 to 5, but results remain similar with weighting. We also cluster our standard errors at the CZ rather than at the state level, but results are similar if we cluster at the state level. Finally, we do not average the 1990-to-2000 and 2000-to-2007 instruments, to be consistent with Theorem 5, but again results are similar if we instead use the average of the 1990-to-2000 and 2000-to-2007 instruments. Table 2 shows that the placebo reduced-form coefficients of the 1990-to-2000 and 2000-2007 Bartik instruments are small, and insignificant at the 5% level. Overall, it seems that CZs 1970-to-1980 and 1980-to-1990 manufacturing employment growths are uncorrelated with their 1990-to-2000 and 2000-to-2007 Bartik instruments, so Assumption 9, our parallel trends condition on the outcome evolution without shocks, seems plausible in this application. Unfortunately, we are unable to implement the placebo test for the first-stage regression, because trade data with China is unavailable before 1990, as explained by Autor et al. (2013).

Table 2: Testing the Common Trends Assumptions

	(1)	(2)
Dependent Variable	$\Delta Y_{g,t}, t \in \{1980, 1990\}$	$\Delta Y_{g,t}, t \in \{1980, 1990\}$
$\Delta Z_{g,2000}$	0.038	
	(0.077)	
$\Delta Z_{g,2007}$		0.053
		(0.031)
Observations	1,444	1,444

Notes: The table reports estimates of regressions using a US commuting-zone (CZ) level panel data set with five periods, 1970, 1980, 1990, 2000, and 2007. The dependent variable is the change of the manufacturing employment share in CZ g, from 1970 to 1980 and from 1980 to 1990. In Column (1) (resp. (2)), the independent variables are the 1990-to-2000 (resp. 2000-to-2007) Bartik instrument of each CZ, and an indicator equal to 1 if the independent variable is measured from 1980 to 1990. The construction of the Bartik instrument is detailed in the text. Standard errors clustered at the CZ level are shown in parentheses. All regressions are unweighted.

6.3 Results

Columns (1) to (3) of Table 3 below show the results of the Bartik first-stage, reduced-form, and 2SLS regressions. In Column (1), the first-stage coefficient is 0.867. In Column (2), the reduced form coefficient is -0.539. Finally, in Column (3), the 2SLS coefficient is -0.622. Robust standard errors clustered at the CZ level are shown between parentheses. All coefficients are statistically significant. The 2SLS coefficient slightly differs from that in Table 2 Column (3) in Autor et al. (2013), because our 2SLS regression is not weighted by CZ's population. This is just to be consistent with Sections 3 to 5, where we consider unweighted Bartik regressions.

Table 3: Bartik Regressions in Autor et al. (2013)

	Bartik FS	Bartik RF	Bartik 2SLS	Chamberlain 2SLS
	(1)	(2)	(3)	(4)
Dependent Variable	$\Delta D_{g,t}$	$\Delta Y_{g,t}$	$\Delta Y_{g,t}$	$\Delta Y_{g,t}$
Bartik Instrument $\Delta Z_{g,t}$	0.867	-0.539		
	(0.131)	(0.087)		
$\Delta D_{g,t}$			-0.622	-1.163
			(0.148)	(0.263)
Observations	1,444	1,444	1,444	722

Notes: Columns (1) to (3) report estimates of Bartik regressions with period fixed effects, using a US commuting-zone (CZ) level panel data set with $T = 3$ periods, 1990, 2000, and 2007. $\Delta Y_{g,t}$ is the change of the manufacturing employment share in CZ g, from 1990 to 2000 for $t = 2000$, and from 2000 to 2007 for $t = 2007$. $\Delta D_{g,t}$ is the change in exposure to Chinese imports in CZ g from 1990 to 2000 for $t = 2000$, and from 2000 to 2007 for $t = 2007$. $\Delta Z_{g,t}$ is the Bartik instrument, whose construction is detailed in the text. Columns (1), (2), and (3) respectively report estimates of the first-stage, reduced-form, and 2SLS Bartik regression coefficients. Standard errors clustered at the CZ level are shown in parentheses. All regressions are unweighted. Column (4) reports an alternative estimate of the second-stage effect, using the GMM Stata command, and with the system of moment conditions in Equation (5.8). The $2T = 6$ moment conditions have one observation per CZ, hence the number of observations. Heteroskedasticity-robust standard errors are shown in parentheses.

We now investigate if the first-stage, reduced-form, and 2SLS regressions in Table 3 respectively estimate convex combinations of first-stage, reduced-form, and second-stage effects. We follow Theorem B.1 in the Web Appendix, a straightforward generalization of Theorem 1 to applications with more than two periods, to estimate the weights attached to the first-stage regression in Table 3. Panel A of Table 4 reports summary statistics on those weights. Column (1) shows that without making any assumption on the first-stage effects, the first-stage regression estimates a weighted sum of 132,906 first-stage effects, where 70,422 effects receive a negative weight, and where negative weights sum to -0.095. Column (2) shows that under the assumption that first-stage effects do not vary across sectors ($\beta_{s,g,t} = \beta_{g,t}$), the first-stage regression estimates a weighted sum of 1,442 first-stage effects, where 854 effects receive a negative weight, and where negative weights sum to -0.084. Assuming that first-stage effects do not vary over time only slightly reduces the sum of negative weights, as shown in Column (3). Finally, Column (4) trivially shows that if the first-stage effect is fully homogeneous ($\beta_{s,g,t} = \beta$), the first-stage regression estimates β. Similarly, we can follow Theorem B.2 in the Web Appendix to estimate the weights attached to the reduced-form regression in Table 3. We do no report those weights, because they are identical to those in Panel A of Table 4, except that they are obtained under assumptions on the reduced-form effects $\gamma_{s,g,t}$.

Then, we follow Theorem 4 to estimate the weights attached to the 2SLS regression in Table 3. Panel B of Table 4 reports summary statistics on those weights. Column (1) shows that without making any assumption on the first-stage effects, we cannot estimate the number of negative weights or their sum. Column (2) shows that under the assumption that first-stage effects do not vary across sectors and are all positive ($\beta_{s,g,t} = \beta_{g,t} \geq 0$), the 2SLS regression estimates a weighted sum of 1,442 second-stage effects, where 854 effects receive a negative weight, even though we cannot estimate the sum of those negative weights. Similarly, Column (3) shows that under the assumption that first-stage effects do not vary across sectors and over time and are all positive ($\beta_{s,g,t} = \beta_g \geq 0$), and that second-stage effects do not vary over time ($\alpha_{g,t} = \alpha_g$), the 2SLS regression estimates a weighted sum of 722 second-stage effects, where 390 effects receive a negative weight. Finally, Column (4) shows that if the first-stage effect is fully homogeneous ($\beta_{s,g,t} = \beta$), the 2SLS regression estimates a weighted sum of 1,442 second-stage effects, where 854 effects receive a negative weight, and where negative weights sum to -0.084. Overall, Table 4 shows that under parallel-trends assumptions, the Bartik regressions in Table 3 do not estimate convex combinations of effects.

Table 4: Summary Statistics on the Weights Attached to Bartik Regressions in Table 3

Panel A: Weights Attached to the First-Stage Regression

Assumption on first-stage effects	None	$\beta_{s,g,t} = \beta_{g,t}$	$\beta_{s,g,t} = \beta_g$	$\beta_{s,g,t} = \beta$
	(1)	(2)	(3)	(4)
Number of negative weights	70,422	854	390	0
Number of positive weights	62,484	588	332	1
Sum of negative weights	-0.095	-0.084	-0.076	0

Panel B: Weights Attached to the 2SLS Regression

Assumption on first-stage effects	None	$\beta_{s,g,t} = \beta_{g,t} \geq 0$	$\beta_{s,g,t} = \beta_g \geq 0$	$\beta_{s,g,t} = \beta$
Assumption on second-stage effects	None	None	$\alpha_{g,t} = \alpha_g$	None
	(1)	(2)	(3)	(4)
Number of negative weights	?	854	390	854
Number of positive weights	?	588	332	588
Sum of negative weights	?	?	?	-0.084

Notes: Panel A of the table reports summary statistics on the weights attached to the first-stage regression in Table 3. The weights are estimated following Theorem B.1 in the Web Appendix. In Column (1), no assumption is made on first-stage effects. In Column (2), it is assumed that first-stage effects do not vary across sectors ($\beta_{s,g,t} = \beta_{g,t}$). In Column (3), it is assumed that first-stage effects do not vary across sectors and over time ($\beta_{s,g,t} = \beta_g$). Finally, in Column (4) it is assumed that the first-stage effects are fully homogeneous ($\beta_{s,g,t} = \beta$). Panel B of the table reports summary statistics on the weights attached to the 2SLS regression in Table 3. The weights are estimated following Theorem 4. In Column (1), no assumption is made on first- and second-stage effects. In Column (2), it is assumed that first-stage effects do not vary across sectors and are positive ($\beta_{s,g,t} = \beta_{g,t} \geq 0$). In Column (3), it is assumed that first-stage effects do not vary across sectors and over time and are positive ($\beta_{s,g,t} = \beta_g \geq 0$), and that second-stage effects do not vary over time ($\alpha_{g,t} = \alpha_g$). Finally, in Column (4) it is assumed that the first-stage effects are fully homogeneous ($\beta_{s,g,t} = \beta$). In Panel B, question marks indicate that the quantity under consideration cannot be estimated.

Finally, in Column (4) of Table 3, we follow Corollary 1 and present our first alternative estimator of the second stage effect. The system of moment conditions has one observation per CZ, hence the number of observations. Because the estimation only uses one observation per CZ, the heteroskedasticity-robust standard error shown between parenthesis below the estimate relies on the assumption that observations are independent across CZ, and is therefore comparable to the CZ-clustered standard errors in Columns (1) to (3). Our first alternative estimator is of the same sign as the 2SLS coefficient in Table 3, and is almost twice as large in absolute value.

Tables 3 and 4 suggest that even under Assumption 11, meaning that first- and second-stage effects only vary across CZs, the Bartik regressions in Table 3 may be biased. Column (2) of Table 4 shows that under that assumption, the sum of the negative weights attached to the first-stage and reduced-form Bartik regressions is still relatively large, around -0.076. Moreover, the weights attached to Bartik regressions are correlated with variables that may themselves be correlated with CZs' first- and second-stage effects. For instance, the correlation between the weights in Table 4 Panel A Column (3) and CZs' share of college-educated workers in 1990 is equal to -0.12 (p-value<0.01). In CZs with a higher proportion of college-educated workers, a rise in nationwide exposure to Chinese imports may lead to a lower rise in Chinese imports than the national average, because manufacturing jobs in those CZs may be more qualified, and less subject to Chinese competition. Accordingly, such CZs could have a lower first-stage effect. That would lead the Bartik first-stage regression to be upward biased for the average first-stage effect $\frac{1}{G}\sum_{g=1}^{G}\beta_g$. The Bartik second-stage regression could then be upward biased as well, assuming for instance a constant negative second-stage effect. Column (4) of Table 3 substantiates that concern: our first alternative estimator is substantially lower than the Bartik 2SLS regression coefficient, and unlike the Bartik regression, under Assumption 11 it estimates a convex combination of second-stage effects.

While it is more robust to heterogeneous treatment effects than the Bartik regression, our alternative estimator in Column (4) of Table 3 still assumes that the first- and second-stage effects are time invariant, and that the first-stage effects do not vary across sectors. Both assumptions are strong, in particular the first one: economic conditions in the US changed substantially over the period, and the effect of Chinese competition on US employment may have evolved over time. Our second alternative estimator allows for heterogeneous first- and second-stage effects over time, provided those follow the same evolution in all CZs. This estimator is equal to -0.867, in-between the 2SLS Bartik regression coefficient and our first alternative estimator. Accordingly, allowing for time-varying first- and second-stage effects, we still get a more negative estimate of the effect of Chinese imports on US employment than in Autor et al. (2013). Deriving the asymptotic distribution of this second alternative estimator is left for future work so we do not report a standard error for that estimate.

7 Conclusion

In this paper, we show that under parallel-trends assumptions, Bartik regressions may not be robust to heterogeneous treatment effects, across locations or over time. We provide diagnostic tools applied researchers may use to assess the robustness of their regressions. Finally, we propose a first alternative estimator that is robust to heterogeneous first- and second-stage effects across locations but not over time. We also propose a second alternative estimator allowing heterogeneous first- and second-stage effects over time, provided those effects follow the same evolution in every location. Studying the asymptotic distribution of that second alternative estimator is left for future work.

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8 Proofs

8.1 Theorem 1

Proof of Point 1

$$\hat{\beta}_C^D = \frac{\sum_{g=1}^G \Delta D_g \left(\Delta Z_g - \Delta Z_.\right)}{\sum_{g=1}^G \Delta Z_g \left(\Delta Z_g - \Delta Z_.\right)}.$$

Therefore,

$$\begin{aligned}
\beta_C^D &= E\left(\frac{\sum_{g=1}^G \Delta D_g \left(\Delta Z_g - \Delta Z_.\right)}{\sum_{g=1}^G \Delta Z_g \left(\Delta Z_g - \Delta Z_.\right)}\right) \\
&= E\left(\frac{\sum_{g=1}^G \left(\Delta Z_g - \Delta Z_.\right) E\left(\Delta D_g | \left(\Delta Z_s, (Q_{s,g'})_{g' \in \{1,...,G\}}\right)_{s\in\{1,...,S\}}\right)}{\sum_{g=1}^G \Delta Z_g \left(\Delta Z_g - \Delta Z_.\right)}\right). \qquad (8.1)
\end{aligned}$$

The second equality comes from the law of iterated expectations, and the fact ΔZ_g and $\Delta Z_.$ are functions of $\left(\Delta Z_s, (Q_{s,g'})_{g' \in \{1,...,G\}}\right)_{s\in\{1,...,S\}}$. Then,

$$\begin{aligned}
&\sum_{g=1}^G (\Delta Z_g - \Delta Z_.) E\left(\Delta D_g | \left(\Delta Z_s, (Q_{s,g'})_{g' \in \{1,...,G\}}\right)_{s\in\{1,...,S\}}\right) \\
=&\sum_{g=1}^G (\Delta Z_g - \Delta Z_.) E\left(\Delta D_g(\mathbf{0}) | \left(\Delta Z_s, (Q_{s,g'})_{g' \in \{1,...,G\}}\right)_{s\in\{1,...,S\}}\right) + \sum_{g=1}^G (\Delta Z_g - \Delta Z_.) \sum_{s=1}^S Q_{s,g}\Delta Z_s \beta_{s,g} \\
=&\sum_{g=1}^G (\Delta Z_g - \Delta Z_.) E\left(\Delta D_g(\mathbf{0}) | \left(\Delta Z_s, Q_{s,g}\right)_{s\in\{1,...,S\}}\right) + \sum_{g=1}^G (\Delta Z_g - \Delta Z_.) \sum_{s=1}^S Q_{s,g}\Delta Z_s \beta_{s,g} \\
=&\sum_{g=1}^G (\Delta Z_g - \Delta Z_.) \mu^D + \sum_{g=1}^G (\Delta Z_g - \Delta Z_.) \sum_{s=1}^S Q_{s,g}\Delta Z_s \beta_{s,g} \\
=&\sum_{g=1}^G (\Delta Z_g - \Delta Z_.) \sum_{s=1}^S Q_{s,g}\Delta Z_s \beta_{s,g}. \qquad (8.2)
\end{aligned}$$

The first equality follows from Assumption 1. The second equality follows from Assumption 5. The third equality follows from Assumption 2. The fourth equality follows after some algebra. Plugging (8.2) into (8.1),

$$\begin{aligned}
\beta_C^D &= E\left(\frac{\sum_{g=1}^G (\Delta Z_g - \Delta Z_.)\left(\sum_{s=1}^S Q_{s,g}\Delta Z_s \beta_{s,g}\right)}{\sum_{g=1}^G \Delta Z_g (\Delta Z_g - \Delta Z_.)}\right) \\
&= E\left(\frac{\sum_{g=1}^G (\Delta Z_g - \Delta Z_.)\left(\sum_{s=1}^S Q_{s,g}\Delta Z_s \beta_{s,g}\right)}{\sum_{g=1}^G \sum_{s=1}^S Q_{s,g}\Delta Z_s (\Delta Z_g - \Delta Z_.)}\right) \\
&= E\left(\sum_{g=1}^G \sum_{s=1}^S \frac{Q_{s,g}\Delta Z_s (\Delta Z_g - \Delta Z_.)}{\sum_{g=1}^G \sum_{s=1}^S Q_{s,g}\Delta Z_s (\Delta Z_g - \Delta Z_.)} \beta_{s,g}\right).
\end{aligned}$$

The second equality follows from Assumption 1.

Proof of Point 2

The result directly follows from plugging $\beta_{s,g} = \beta_g$ into Point 1.

8.2 Theorem 2

The proof is similar to that of Theorem 1, so it is omitted.

8.3 Theorem 3

Point 1 directly follows from Definition 4 and Theorems 1 and 2. Points 2, 3, and 4 directly follow from Point 1.

8.4 Theorem 4

The result directly follows from Definition 6 and Theorems B.1 and B.2, so it is omitted.

8.5 Theorem 5

Note that

$$\hat{\beta}^D_{pl} = \frac{\sum_{g=1}^G \Delta D_{g,2} \left(\Delta Z_{g,3} - \Delta Z_{.,3}\right)}{\sum_{g=1}^G \Delta Z_{g,3} \left(\Delta Z_{g,3} - \Delta Z_{.,3}\right)}.$$

Therefore,

$$\begin{aligned}
\beta^D_{pl} &= E\left(\frac{\sum_{g=1}^G \Delta D_{g,2} \left(\Delta Z_{g,3} - \Delta Z_{.,3}\right)}{\sum_{g=1}^G \Delta Z_{g,3} \left(\Delta Z_{g,3} - \Delta Z_{.,3}\right)}\right) \\
&= E\left(\frac{\sum_{g=1}^G E\left(\Delta D_{g,2} | \mathbf{\Delta Z}, (Q_{s,g'})_{g' \in \{1,\dots,G\}, s \in \{1,\dots,S\}}\right) \left(\Delta Z_{g,3} - \Delta Z_{.,3}\right)}{\sum_{g=1}^G \Delta Z_{g,3} \left(\Delta Z_{g,3} - \Delta Z_{.,3}\right)}\right) \\
&= E\left(\frac{\sum_{g=1}^G E\left(\Delta D_{g,2}(\mathbf{0}) | \mathbf{\Delta Z}, (Q_{s,g})_{s \in \{1,\dots,S\}}\right) \left(\Delta Z_{g,3} - \Delta Z_{.,3}\right)}{\sum_{g=1}^G \Delta Z_{g,3} \left(\Delta Z_{g,3} - \Delta Z_{.,3}\right)}\right) \\
&= E\left(\frac{\sum_{g=1}^G \mu^D_2 \left(\Delta Z_{g,3} - \Delta Z_{.,3}\right)}{\sum_{g=1}^G \Delta Z_{g,3} \left(\Delta Z_{g,3} - \Delta Z_{.,3}\right)}\right) \\
&= 0.
\end{aligned}$$

The second equality follows from the law of iterated expectation. The third equality follows from Assumptions 6 and 10, and the fact that $\Delta Z_{s,2} = 0$. The fourth equality follows from Assumption 7. The proof of $\beta^Y_{pl} = 0$ is similar, so it is omitted.

8.6 Theorem 6

Under Assumptions 6-11,

$$E(\boldsymbol{\Delta D}_g|\boldsymbol{\Delta Z},(Q_{s,g})_{s\in\{1,\ldots,S\}}) = \boldsymbol{\mu}^D + \beta_g \boldsymbol{\Delta Z}_g. \tag{8.3}$$

Because $\boldsymbol{M}_g\boldsymbol{\Delta Z}_g = 0$ and $\boldsymbol{M}_g$ is a function of $\boldsymbol{\Delta Z},(Q_{s,g})_{s\in\{1,\ldots,S\}}$, we left-multiply Equation (8.3) by $\boldsymbol{M}_g$, and it follows that

$$E\left(\boldsymbol{M}_g\boldsymbol{\Delta D}_g|\boldsymbol{\Delta Z},(Q_{s,g})_{s\in\{1,\ldots,S\}}\right) = \boldsymbol{M}_g\boldsymbol{\mu}^D.$$

Also note that

$$\begin{aligned}\boldsymbol{M}_g'\boldsymbol{M}_g &= \left(\boldsymbol{I} - \frac{1}{\boldsymbol{\Delta Z}_g'\boldsymbol{\Delta Z}_g}\boldsymbol{\Delta Z}_g\boldsymbol{\Delta Z}_g'\right)'\left(\boldsymbol{I} - \frac{1}{\boldsymbol{\Delta Z}_g'\boldsymbol{\Delta Z}_g}\boldsymbol{\Delta Z}_g\boldsymbol{\Delta Z}_g'\right)\\ &= \boldsymbol{I} - \frac{1}{\boldsymbol{\Delta Z}_g'\boldsymbol{\Delta Z}_g}\boldsymbol{\Delta Z}_g\boldsymbol{\Delta Z}_g' = \boldsymbol{M}_g,\end{aligned}$$

therefore, it follows that

$$E\left(\boldsymbol{M}_g'\boldsymbol{M}_g\boldsymbol{\Delta D}_g|\boldsymbol{\Delta Z},(Q_{s,g})_{s\in\{1,\ldots,S\}}\right) = \boldsymbol{M}_g'\boldsymbol{M}_g\boldsymbol{\mu}^D.$$

Therefore, by the law of iterated expectation:

$$E\left(\frac{1}{G}\sum_{g=1}^{G}\boldsymbol{M}_g'\boldsymbol{M}_g\boldsymbol{\Delta D}_g\right) = E\left(\frac{1}{G}\sum_{g=1}^{G}\boldsymbol{M}_g'\boldsymbol{M}_g\right)\boldsymbol{\mu}^D.$$

So Equation (5.1) holds.

Similarly, we left-multiply Equation (8.3) by $\boldsymbol{\Delta Z}_g'$, and it follows that

$$E(\boldsymbol{\Delta Z}_g'\boldsymbol{\Delta D}_g|\boldsymbol{\Delta Z},(Q_{s,g})_{s\in\{1,\ldots,S\}}) = \boldsymbol{\Delta Z}_g'\boldsymbol{\mu}^D + \beta_g\boldsymbol{\Delta Z}_g'\boldsymbol{\Delta Z}_g.$$

Therefore,

$$E\left(\frac{1}{\boldsymbol{\Delta Z}_g'\boldsymbol{\Delta Z}_g}\boldsymbol{\Delta Z}_g'\left(\boldsymbol{\Delta D}_g - \boldsymbol{\mu}^D\right)|\boldsymbol{\Delta Z},(Q_{s,g})_{s\in\{1,\ldots,S\}}\right) = \beta_g.$$

Then by the law of iterated expectation:

$$\frac{1}{G}\sum_{g=1}^{G}\beta_g = E\left(\frac{1}{G}\sum_{g=1}^{G}\frac{1}{\boldsymbol{\Delta Z}_g'\boldsymbol{\Delta Z}_g}\boldsymbol{\Delta Z}_g'\left(\boldsymbol{\Delta D}_g - \boldsymbol{\mu}^D\right)\right).$$

So Equation (5.3) holds. The proofs of Equations (5.2) and (5.4) are similar: one simply needs to replace $\boldsymbol{\Delta D}_g$, $\boldsymbol{\mu}^D$ and β_g by $\boldsymbol{\Delta Y}_g$, $\boldsymbol{\mu}^Y$ and γ_g.

8.7 Theorem 7

Under Assumptions 6-10 and 12, and because $\boldsymbol{P}_g$ is a function of $\boldsymbol{\Delta Z}, (Q_{s,g})_{s\in\{1,\dots,S\}}$,

$$E(\boldsymbol{\Delta D}_g|\boldsymbol{\Delta Z}, (Q_{s,g})_{s\in\{1,\dots,S\}}) = \boldsymbol{P}_g\boldsymbol{\theta}^D + \beta_g\boldsymbol{\Delta Z}_g, \tag{8.4}$$

Because $\boldsymbol{M}_g\boldsymbol{\Delta Z}_g = 0$ and $\boldsymbol{M}_g$ is a function of $\boldsymbol{\Delta Z}, (Q_{s,g})_{s\in\{1,\dots,S\}}$, we left-multiply Equation (8.3) by $\boldsymbol{M}_g$, and it follows that

$$E(\boldsymbol{M}_g\boldsymbol{\Delta D}_g|\boldsymbol{\Delta Z}, (Q_{s,g})_{s\in\{1,\dots,S\}}) = \boldsymbol{M}_g\boldsymbol{P}_g\boldsymbol{\theta}^D,$$

which in turn implies

$$E(\boldsymbol{P}_g'\boldsymbol{M}_g\boldsymbol{\Delta D}_g|\boldsymbol{\Delta Z}, (Q_{s,g})_{s\in\{1,\dots,S\}}) = \boldsymbol{P}_g'\boldsymbol{M}_g\boldsymbol{P}_g\boldsymbol{\theta}^D.$$

Therefore, by the law of iterated expectation:

$$E\left(\frac{1}{G}\sum_{g=1}^{G}\boldsymbol{P}_g'\boldsymbol{M}_g\boldsymbol{\Delta D}_g\right) = E\left(\frac{1}{G}\sum_{g=1}^{G}\boldsymbol{P}_g'\boldsymbol{M}_g\boldsymbol{P}_g\right)\boldsymbol{\theta}^D.$$

So Equation (5.9) holds.

Similarly, we left-multiply Equation (8.4) by $\boldsymbol{\Delta Z}_g'$, and it follows that

$$E(\boldsymbol{\Delta Z}_g'\boldsymbol{\Delta D}_g|\boldsymbol{\Delta Z}, (Q_{s,g})_{s\in\{1,\dots,S\}}) = \boldsymbol{\Delta Z}_g'\boldsymbol{P}_g\boldsymbol{\theta}^D + \beta_g\boldsymbol{\Delta Z}_g'\boldsymbol{\Delta Z}_g.$$

Equation (5.10) follows from rearranging and the law of iterated expectations.

Similarly, under Assumptions 6-10 and 12, and because $\tilde{\boldsymbol{P}}_g$ is a function of $\boldsymbol{\Delta Z}, (Q_{s,g})_{s\in\{1,\dots,S\}}$,

$$E(\boldsymbol{\Delta Y}_g|\boldsymbol{\Delta Z}, (Q_{s,g})_{s\in\{1,\dots,S\}}) = \tilde{\boldsymbol{P}}_g\boldsymbol{\theta}^Y + \alpha_g\boldsymbol{\Delta}\tilde{\boldsymbol{Z}}_g. \tag{8.5}$$

Then, the proofs of Equations (5.11) and (5.12) are similar to those of Equations (5.9) and (5.10).

Web Appendix: not for publication

A Empirical Application: Canonical Bartik design

In this section, we revisit the canonical application in Bartik (1991), where the Bartik instrument is used to estimate the inverse elasticity of labor supply.

A.1 Data

Our data construction closely follows Goldsmith-Pinkham et al. (2020). We construct a decennial continental US commuting-zone (CZ) level panel data set, from 1990 to 2010, with CZ wages and employment levels. For 1990 and 2000, we use the 5% IPUMS sample of the U.S. Census. For 2010, we pool the 2009-2011 ACSs (Ruggles et al. 2019). Sectors are IND1990 industries. We follow Autor & Dorn (2013) to reallocate Public Use Micro Areas level observations of Census data to the CZ level. We also follow Autor et al. (2013) to aggregate the Census industry code ind1990 to a balanced panel of industries for the 1990 and 2000 Censuses and the 2009-2011 ACS, with new industry code ind1990dd.[A.1] In our final dataset, we have 3 periods, 722 CZs and 212 industries.

The outcome variable is $\Delta Y_{g,t} = \Delta \log w_{g,t}$, the change in log wages in CZ g from $t-10$ to t, for $t \in \{2000, 2010\}$. The treatment variable is $\Delta D_{g,t} = \Delta \log E_{g,t}$, the change in log employment in CZ g from $t-10$ to t. We use people aged 18 and older who are employed and report usually working at least 30 hours per week in the previous year to generate employment and average wages. We define $Q_{s,g}$ as the employment share of industry s in CZ g in 1990, and then construct the Bartik instrument using 1990-2000 and 2000-2010 leave-one-out sectoral employment growth rates. Specifically, to construct the nationwide employment growth rate of industry s for CZ g, we use the change in log employment in industry s over all CZs excluding CZ g, following Adão et al. (2019).[A.2]

A.2 Testing the identifying assumptions

We start by assessing the plausibility of the random shock assumption in Adão et al. (2019) and Borusyak et al. (2022), by testing whether shocks are independent of industries' characteristics. Table A.1 shows regressions of the nationwide sector-level shocks, namely the employment growth of each industry from 1990 to 2000 (in Column (1)) and from 2000

[A.1] Crosswalk files are available online at https://www.ddorn.net/data.htm. The original crosswalk file for industry code only creates a balanced panel of industries up to the 2006-2008 ACSs. We extend the crosswalk approach to one additional industry (shoe repair shops, crosswalked into miscellaneous personal services) to create a balanced panel of industries up to the 2009-2011 ACSs.

[A.2] As discussed in Adão et al. (2019) and Goldsmith-Pinkham et al. (2020), we use the leave-one-out definition to construct the national growth rates, in order to avoid the finite sample bias that comes from using own-observation information. In practice, because we have 722 locations, whether one uses leave-one-out or not to estimate the national growth rates barely changes the results.

to 2010 (in Column (2)), on a set of pre-determined industry characteristics measured in 1990.[A.3] These industry characteristics are the log of average wages, the proportion of male workers, the proportion of white workers, the average age of workers, and the proportion of workers with some college education. The results show that large employment shocks tend to appear in industries with low average wages and more educated workers, and in both cases we can reject the hypothesis that the employment shocks are not correlated with any industry characteristic (p-value < 0.001). Therefore, there are industries with certain characteristics that make them more likely to receive a large employment shock, and the random-shock assumption is rejected.

Table A.1: Testing the Random Shock Assumption

	(1)	(2)
Variables	$\Delta \log E$: 1990-2000	$\Delta \log E$: 2000-2010
$\log w_{1990}$	-0.372	-0.375
	(0.093)	(0.081)
Male_{1990}	0.212	0.337
	(0.200)	(0.146)
White_{1990}	-0.738	-0.182
	(0.547)	(0.472)
Age_{1990}	-0.027	0.009
	(0.015)	(0.011)
$\text{Some College}_{1990}$	1.095	1.122
	(0.229)	(0.199)
Constant	4.772	2.760
	(1.345)	(0.964)
Observations	212	212
R-squared	0.161	0.152
F-test P-value	0.0000	0.0000

Notes: The table reports estimates of regressions of the industry-level employment growth during 1990-2000 and 2000-2010 on a set of pre-determined industry characteristics measured in 1990. The dependent variable in Column (1) is the change in log nationwide employment in the industry from 1990 to 2000. The dependent variable in Column (2) is the change in log nationwide employment in the industry from 2000 to 2010. $\log w_{1990}$ denotes log average wages in the industry in 1990. Male_{1990} denotes the proportion of male workers in the industry in 1990. White_{1990} denotes the proportion of white workers in the industry in 1990. Age_{1990} denotes the average age of workers in the industry in 1990. $\text{Some College}_{1990}$ denotes the proportion of workers with at least some college education in the industry in 1990. Robust standard errors in parentheses. The F-test p-value is the p-value of the joint test that all the coefficients of the industry characteristics are equal to 0.

Unfortunately, in this application we cannot implement the tests of our parallel-trends

[A.3] When constructing the industry characteristics, we only use the workers from continental U.S. CZs, in order to be consistent with our main sample.

assumptions in Theorem 5, as there are no consecutive time periods where the nationwide employment remains stable in every industry.

A.3 Results

Columns (1) to (3) of Table A.2 below show the results of the first-stage, reduced-form, and 2SLS Bartik regressions. In Column (1), the first-stage coefficient is 0.818. In Column (2), the reduced form coefficient is 0.390. Finally, in Column (3), the 2SLS coefficient is 0.477. If interpreted causally, this 2SLS coefficient means that a 1% increase in employment leads to a 0.477% increase in wages. Robust standard errors clustered at the CZ level are shown between parentheses. All coefficients are statistically significant.

Table A.2: Bartik Regressions in the Canonical Setting

	Bartik FS	Bartik RF	Bartik 2SLS	Chamberlain 2SLS
	(1)	(2)	(3)	(4)
Dependent Variable	$\Delta D_{g,t}$	$\Delta Y_{g,t}$	$\Delta Y_{g,t}$	$\Delta Y_{g,t}$
Bartik Instrument $\Delta Z_{g,t}$	0.818	0.390		
	(0.055)	(0.031)		
$\Delta D_{g,t}$			0.477	0.483
			(0.039)	(0.061)
Observations	1,444	1,444	1,444	722

Notes: Columns (1) to (3) report estimates of Bartik regressions with period fixed effects, using a decennial US commuting-zone (CZ) level panel data set from 1990 to 2010. $\Delta Y_{g,t}$ is the change in log wages in CZ g from $t-10$ to t, for $t \in \{2000, 2010\}$. $\Delta D_{g,t}$ is the change in log employment in CZ g from $t-10$ to t. $\Delta Z_{g,t}$ is the Bartik instrument, whose construction is detailed in the text. Columns (1), (2), and (3) respectively report estimates of the first-stage, reduced-form, and 2SLS Bartik regression coefficients. Standard errors clustered at the CZ level are shown in parentheses. Column (4) reports an alternative estimate of the second-stage effect, using the GMM Stata command, and with the system of moment conditions in Equation (5.8). The $2T = 6$ moment conditions have one observation per CZ, hence the number of observations. Heteroskedasticity-robust standard errors are shown in parentheses.

We now investigate if the first-stage, reduced-form, and 2SLS regressions in Table A.2 respectively estimate convex combinations of first-stage, reduced-form, and second-stage effects. We follow Theorem B.1 in the Web Appendix, a straightforward generalization of Theorem 1 to applications with more than two periods, to estimate the weights attached to the first-stage regression in Table A.2. Panel A of Table A.3 reports summary statistics on those weights. Column (1) shows that without making any assumption on the first-stage effects, the first-stage regression estimates a weighted sum of 273,187 first-stage effects, where 134,493 effects receive a negative weight, and where negative weights sum to -1.249. Column (2) shows that under the assumption that first-stage effects do not vary across sectors ($\beta_{s,g,t} = \beta_{g,t}$), the first-stage regression estimates a weighted sum of 1,444 first-stage effects, where 446 effects still receive a negative weight, and where negative weights sum to -0.040. Assuming that first-stage effects do not vary over time reduces the number and the sum of negative weights even further, as shown in Column (3). Finally, Column (4) trivially shows that if the first-stage effect is fully homogeneous ($\beta_{s,g,t} = \beta$), the first-stage regression estimates β. Similarly, we can follow Theorem B.2 in the Web Appendix to estimate the weights attached to the reduced-form regression in Table A.2. We do no report those weights, because they are identical to those in Panel A of Table A.3, except that they are obtained under assumptions on the reduced-form effects $\gamma_{s,g,t}$.

Then, we follow Theorem 4 to estimate the weights attached to the 2SLS regression in Table A.2. Panel B of Table A.3 reports summary statistics on those weights. Column (1) shows that without making any assumption on the first-stage effects, we cannot estimate the number of negative weights or their sum. On the other hand, Column (2) shows that under the assumption that first-stage effects do not vary across sectors and are all positive ($\beta_{s,g,t} = \beta_{g,t} \geq 0$), the 2SLS regression estimates a weighted sum of 1,444 second-stage effects, where 446 effects receive a negative weight, even though we cannot estimate the sum of those negative weights. Similarly, Column (3) shows that under the assumption that first-stage effects do not vary across sectors and over time and are all positive ($\beta_{s,g,t} = \beta_g \geq 0$), and that second-stage effects do not vary over time ($\alpha_{g,t} = \alpha_g$), the 2SLS regression estimates a weighted sum of 722 second-stage effects, where 75 effects receive a negative weight. Finally, Column (4) shows that if the first-stage effect is fully homogeneous ($\beta_{s,g,t} = \beta$), the 2SLS regression estimates a weighted sum of 1,444 second-stage effects, where 446 effects receive a negative weight, and where negative weights sum to -0.04. Overall, Table A.3 shows that under parallel-trends assumptions, the Bartik regressions in Table A.2 do not estimate convex combinations of effects.

Table A.3: Summary Statistics on the Weights Attached to Bartik Regressions in Table A.2

Panel A: Weights Attached to the First-Stage Regression

Assumption on first-stage effects	None	$\beta_{s,g,t} = \beta_{g,t}$	$\beta_{s,g,t} = \beta_g$	$\beta_{s,g,t} = \beta$
	(1)	(2)	(3)	(4)
Number of negative weights	134,493	446	75	0
Number of positive weights	138,694	998	647	1
Sum of negative weights	-1.249	-0.040	-0.010	0

Panel B: Weights Attached to the 2SLS Regression

Assumption on first-stage effects	None	$\beta_{s,g,t} = \beta_{g,t} \geq 0$	$\beta_{s,g,t} = \beta_g \geq 0$	$\beta_{s,g,t} = \beta$
Assumption on second-stage effects	None	None	$\alpha_{g,t} = \alpha_g$	None
	(1)	(2)	(3)	(4)
Number of negative weights	?	446	75	446
Number of positive weights	?	998	647	998
Sum of negative weights	?	?	?	-0.040

Notes: Panel A of the table reports summary statistics on the weights attached to the first-stage regression in Table A.2. The weights are estimated following Theorem B.1 in the Web Appendix. In Column (1), no assumption is made on the first-stage effects. In Column (2), it is assumed that first-stage effects do not vary across sectors ($\beta_{s,g,t} = \beta_{g,t}$). In Column (3), it is assumed that first-stage effects do not vary across sectors and over time ($\beta_{s,g,t} = \beta_g$). Finally, in Column (4) it is assumed that the first-stage effects are fully homogeneous ($\beta_{s,g,t} = \beta$). Panel B of the table reports summary statistics on the weights attached to the 2SLS regression in Table A.2. The weights are estimated following Theorem 4. In Column (1), no assumption is made on first- and second-stage effects. In Column (2), it is assumed that first-stage effects do not vary across sectors and are positive ($\beta_{s,g,t} = \beta_{g,t} \geq 0$). In Column (3), it is assumed that first-stage effects do not vary across sectors and over time and are positive ($\beta_{s,g,t} = \beta_g \geq 0$), and that second-stage effects do not vary over time ($\alpha_{g,t} = \alpha_g$). Finally, in Column (4) it is assumed that the first-stage effects are fully homogeneous ($\beta_{s,g,t} = \beta$). In Panel B, question marks indicate that the quantity under consideration cannot be estimated.

Finally, in Column (4) of Table A.2, we follow Corollary 1 and present our first alternative estimator of the second stage effect. The system of moment conditions has one observation per CZ, hence the number of observations. Because the estimation only uses one observation per CZ, the heteroskedasticity-robust standard error shown between parenthesis below the estimate relies on the assumption that observations are independent across CZ, and is therefore comparable to the CZ-clustered standard errors in Columns (1) to (3). Our first alternative estimator is very close to the 2SLS coefficient in Table 3. We compute our second alternative estimator, which is robust to heterogeneous effects across location and over time, provided all locations experience the same evolution of their first- and second-stage effects over time. We find that it is equal to 0.464, so it is extremely close to the Bartik 2SLS regression coefficient and to our first alternative estimator.

Overall, Tables A.2 and A.3 suggest that under our parallel-trends assumptions, the Bartik regressions in Table A.2 are robust to heterogeneous effects across CZs. Under the assumption that first- and second-stage effects only vary across CZs, Column (3) of Table A.3 shows that the sum of the negative weights attached to the first-stage and reduced-form Bartik regressions is quite small, around -0.010. The implicit weights attached to Bartik regressions could still be problematic, as the weights are correlated with variables that may themselves be correlated with CZs' second-stage effects. For instance, the correlation between the weights in Table A.3 Panel A Column (3) and CZs' unemployment rate in 1990 is equal to -0.07 (p-value=0.08). Commuting zones with a higher unemployment rate may have a lower second-stage effect, as their labor market is less tight. That would lead, say, the Bartik reduced-form regression to overestimate the average reduced-form effect $\frac{1}{G}\sum_{g=1}^{G}\gamma_g$, by putting less weight on those CZs. Column (4) of Table A.2 appeases that concern: under the assumption that first- and second-stage effects only vary across CZs, the correlated random coefficient estimators of the average first-stage, reduced-form, and second-stage effects are remarkably close to the Bartik regression coefficients.

Table A.3 may also suggest that the Bartik regressions in Table A.2 may be reasonably robust to heterogeneous effects over time: even if one allows first- and second-stage effects to vary across CZs and over time, Column (2) of Table A.3 shows that the sum of the negative weights attached to the first-stage and reduced-form Bartik regressions is still quite small, around -0.040.

On the other hand, Table A.3 strongly suggests that the Bartik regressions in Table A.2 are not robust to heterogeneous first-stage effects across sectors. If first-stage effects vary across sectors, locations, and time, Column (1) of Table A.3 shows that the sum of the negative weights attached to the first-stage and reduced-form Bartik regressions becomes very large. It seems that whether Bartik regressions can or cannot receive a causal interpretation in this application crucially depends on whether it is plausible to assume homogeneous first-stage effects across sectors.

B Identification Results with Multiple Periods

In this section, we use the same notation, definitions, and assumptions as in Section 4.3 of the paper. We first consider the Bartik first-stage regression. Let $\Delta Z_{.,t} = \frac{1}{G}\sum_{g=1}^{G}\Delta Z_{g,t}$.

Theorem B.1 *Suppose Assumptions 6, 7, 10 hold.*

1. *Then,*

$$\beta_C^D = E\left(\sum_{g=1}^{G}\sum_{t=2}^{T}\sum_{s=1}^{S}\frac{Q_{s,g}\Delta Z_{s,t}(\Delta Z_{g,t}-\Delta Z_{.,t})}{\sum_{g=1}^{G}\sum_{t=2}^{T}\sum_{s=1}^{S}Q_{s,g}\Delta Z_{s,t}(\Delta Z_{g,t}-\Delta Z_{.,t})}\beta_{s,g,t}\right).$$

2. *If one further assumes that* $\beta_{s,g,t} = \beta_{g,t}$*,*

$$\beta_C^D = E\left(\sum_{g=1}^{G}\sum_{t=2}^{T}\frac{\Delta Z_{g,t}(\Delta Z_{g,t} - \Delta Z_{.,t})}{\sum_{g=1}^{G}\sum_{t=2}^{T}\Delta Z_{g,t}(\Delta Z_{g,t} - \Delta Z_{.,t})}\beta_{g,t}\right).$$

3. *If one further assumes that* $\beta_{s,g,t} = \beta_g$*,*

$$\beta_C^D = E\left(\sum_{g=1}^{G}\left(\sum_{t=2}^{T}\frac{\Delta Z_{g,t}(\Delta Z_{g,t} - \Delta Z_{.,t})}{\sum_{g=1}^{G}\sum_{t=2}^{T}\Delta Z_{g,t}(\Delta Z_{g,t} - \Delta Z_{.,t})}\right)\beta_g\right).$$

Then we consider the reduced-form Bartik regression. Let $\gamma_{s,g,t} = \alpha_{g,t}\beta_{s,g,t}$.

Theorem B.2 *Suppose Assumptions 6, 8, 9, 10 hold.*

1. *Then,*

$$\beta_C^Y = E\left(\sum_{g=1}^{G}\sum_{t=2}^{T}\sum_{s=1}^{S}\frac{Q_{s,g}\Delta Z_{s,t}(\Delta Z_{g,t} - \Delta Z_{.,t})}{\sum_{g=1}^{G}\sum_{t=2}^{T}\sum_{s=1}^{S}Q_{s,g}\Delta Z_{s,t}(\Delta Z_{g,t} - \Delta Z_{.,t})}\gamma_{s,g,t}\right).$$

2. *If one further assumes that* $\beta_{s,g,t} = \beta_{g,t}$*, and let* $\gamma_{g,t} = \alpha_{g,t}\beta_{g,t}$*,*

$$\beta_C^Y = E\left(\sum_{g=1}^{G}\sum_{t=2}^{T}\frac{\Delta Z_{g,t}(\Delta Z_{g,t} - \Delta Z_{.,t})}{\sum_{g=1}^{G}\sum_{t=2}^{T}\Delta Z_{g,t}(\Delta Z_{g,t} - \Delta Z_{.,t})}\gamma_{g,t}\right).$$

3. *If one further assumes that* $\beta_{s,g,t} = \beta_g$ *and* $\alpha_{g,t} = \alpha_g$*, and let* $\gamma_g = \alpha_g\beta_g$*,*

$$\beta_C^Y = E\left(\sum_{g=1}^{G}\left(\sum_{t=2}^{T}\frac{\Delta Z_{g,t}(\Delta Z_{g,t} - \Delta Z_{.,t})}{\sum_{g=1}^{G}\sum_{t=2}^{T}\Delta Z_{g,t}(\Delta Z_{g,t} - \Delta Z_{.,t})}\right)\gamma_g\right).$$

C Proofs of results in Web Appendix

C.1 Theorem B.1

Note that by the Frisch-Waugh theorem,

$$\hat{\beta}_C^D = \frac{\sum_{g=1}^G \sum_{t=2}^T \Delta D_{g,t} \left(\Delta Z_{g,t} - \Delta Z_{.,t}\right)}{\sum_{g=1}^G \sum_{t=2}^T \Delta Z_{g,t} \left(\Delta Z_{g,t} - \Delta Z_{.,t}\right)},$$

where $\Delta Z_{g,t} - \Delta Z_{.,t}$ is the residual from a regression of $\Delta Z_{g,t}$ on period fixed effects. Therefore,

$$\begin{aligned} \beta_C^D &= E\left(\frac{\sum_{g=1}^G \sum_{t=2}^T \Delta D_{g,t} \left(\Delta Z_{g,t} - \Delta Z_{.,t}\right)}{\sum_{g=1}^G \sum_{t=2}^T \Delta Z_{g,t} \left(\Delta Z_{g,t} - \Delta Z_{.,t}\right)}\right) \\ &= E\left(\frac{\sum_{g=1}^G \sum_{t=2}^T \left(\Delta Z_{g,t} - \Delta Z_{.,t}\right) E\left(\Delta D_{g,t} | \mathbf{\Delta Z}, \left(Q_{s,g'}\right)_{g' \in \{1,...,G\}, s \in \{1,...,S\}}\right)}{\sum_{g=1}^G \sum_{t=2}^T \Delta Z_{g,t} \left(\Delta Z_{g,t} - \Delta Z_{.,t}\right)}\right). \end{aligned} \quad \text{(C.1)}$$

The second equality comes from the law of iterated expectations, and the fact that $\Delta Z_{g,t}$ and $\Delta Z_{g,.}$ are functions of $\left(\mathbf{\Delta Z}, \left(Q_{s,g'}\right)_{g' \in \{1,...,G\}, s \in \{1,...,S\}}\right)$. Then,

$$\begin{aligned} &\sum_{g=1}^G \sum_{t=2}^T (\Delta Z_{g,t} - \Delta Z_{.,t}) E\left(\Delta D_{g,t} | \mathbf{\Delta Z}, \left(Q_{s,g'}\right)_{g' \in \{1,...,G\}, s \in \{1,...,S\}}\right) \\ =&\sum_{g=1}^G \sum_{t=2}^T (\Delta Z_{g,t} - \Delta Z_{.,t}) E\left(\Delta D_{g,t}(\mathbf{0}) | \mathbf{\Delta Z}, \left(Q_{s,g'}\right)_{g' \in \{1,...,G\}, s \in \{1,...,S\}}\right) \\ +&\sum_{g=1}^G \sum_{t=2}^T (\Delta Z_{g,t} - \Delta Z_{.,t}) E\left(\sum_{s=1}^S Q_{s,g} \Delta Z_{s,t} \beta_{s,g,t} | \mathbf{\Delta Z}, \left(Q_{s,g'}\right)_{g' \in \{1,...,G\}, s \in \{1,...,S\}}\right) \\ =&\sum_{g=1}^G \sum_{t=2}^T (\Delta Z_{g,t} - \Delta Z_{.,t}) \left(E\left(\Delta D_{g,t}(\mathbf{0}) | \mathbf{\Delta Z}, \left(Q_{s,g}\right)_{s \in \{1,...,S\}}\right)\right) \\ +&\sum_{g=1}^G \sum_{t=2}^T (\Delta Z_{g,t} - \Delta Z_{.,t}) \sum_{s=1}^S Q_{s,g} \Delta Z_{s,t} \beta_{s,g,t} \\ =&\sum_{g=1}^G \sum_{t=2}^T (\Delta Z_{g,t} - \Delta Z_{.,t}) \mu_t^D + \sum_{g=1}^G \sum_{t=2}^T (\Delta Z_{g,t} - \Delta Z_{.,t}) \sum_{s=1}^S Q_{s,g} \Delta Z_{s,t} \beta_{s,g,t} \\ =&\sum_{g=1}^G \sum_{t=2}^T (\Delta Z_{g,t} - \Delta Z_{.,t}) \sum_{s=1}^S Q_{s,g} \Delta Z_{s,t} \beta_{s,g,t}. \end{aligned} \quad \text{(C.2)}$$

The first equality follows from Assumption 6. The second equality follows from Assumption 10. The third equality follows from Assumption 7. The fourth equality is by algebra.

Plugging (C.2) into (C.1),

$$
\begin{aligned}
\beta_C^D &= E\left(\frac{\sum_{g=1}^{G}\sum_{t=2}^{T}(\Delta Z_{g,t}-\Delta Z_{.,t})\left(\sum_{s=1}^{S}Q_{s,g}\Delta Z_{s,t}\beta_{s,g,t}\right)}{\sum_{g=1}^{G}\sum_{t=2}^{T}\Delta Z_{g,t}(\Delta Z_{g,t}-\Delta Z_{.,t})}\right)\\
&= E\left(\frac{\sum_{g=1}^{G}\sum_{t=2}^{T}(\Delta Z_{g,t}-\Delta Z_{.,t})\left(\sum_{s=1}^{S}Q_{s,g}\Delta Z_{s,t}\beta_{s,g,t}\right)}{\sum_{g=1}^{G}\sum_{t=2}^{T}\sum_{s=1}^{S}Q_{s,g}\Delta Z_{s,t}(\Delta Z_{g,t}-\Delta Z_{.,t})}\right)\\
&= E\left(\sum_{g=1}^{G}\sum_{t=2}^{T}\sum_{s=1}^{S}\frac{Q_{s,g}\Delta Z_{s,t}(\Delta Z_{g,t}-\Delta Z_{.,t})}{\sum_{g=1}^{G}\sum_{t=2}^{T}\sum_{s=1}^{S}Q_{s,g}\Delta Z_{s,t}(\Delta Z_{g,t}-\Delta Z_{.,t})}\beta_{s,g,t}\right).
\end{aligned}
$$

Note that when $\beta_{s,g,t} = \beta_{g,t}$:

$$
\begin{aligned}
\beta_C^D &= E\left(\sum_{g=1}^{G}\sum_{t=2}^{T}\sum_{s=1}^{S}\frac{Q_{s,g}\Delta Z_{s,t}(\Delta Z_{g,t}-\Delta Z_{.,t})}{\sum_{g=1}^{G}\sum_{t=2}^{T}\sum_{s=1}^{S}Q_{s,g}\Delta Z_{s,t}(\Delta Z_{g,t}-\Delta Z_{.,t})}\beta_{s,g,t}\right)\\
&= E\left(\frac{\sum_{g=1}^{G}\sum_{t=2}^{T}(\Delta Z_{g,t}-\Delta Z_{.,t})\beta_{g,t}\sum_{s=1}^{S}Q_{s,g}\Delta Z_{s,t}}{\sum_{g=1}^{G}\sum_{t=2}^{T}\Delta Z_{g,t}(\Delta Z_{g,t}-\Delta Z_{.,t})}\right)\\
&= E\left(\frac{\sum_{g=1}^{G}\sum_{t=2}^{T}(\Delta Z_{g,t}-\Delta Z_{.,t})\beta_{g,t}\Delta Z_{g,t}}{\sum_{g=1}^{G}\sum_{t=2}^{T}\Delta Z_{g,t}(\Delta Z_{g,t}-\Delta Z_{.,t})}\right)\\
&= E\left(\sum_{g=1}^{G}\sum_{t=2}^{T}\frac{\Delta Z_{g,t}(\Delta Z_{g,t}-\Delta Z_{.,t})}{\sum_{g=1}^{G}\sum_{t=2}^{T}\Delta Z_{g,t}(\Delta Z_{g,t}-\Delta Z_{.,t})}\beta_{g,t}\right).
\end{aligned}
$$

C.2 Theorem B.2

The proof is similar to that of Theorem B.1, so it is omitted.