“Stock Market Volatility and Learning” by K. Adams, A. Marcet & J.P. Nicolini

Philippe Weil, Sciences Po, OFCE, Paris, France

Conference on Monetary Policy, Asset Prices and Learning ECB
November 6-7, 2006

Introduction

- Ambitious project

Introduction

- Ambitious project
- A *parti pris* of simplicity.

- Ambitious project
- A *parti pris* of simplicity.
- Nothing exotic or non-standard in the model.

Introduction

- Ambitious project
- A *parti pris* of simplicity.
- Nothing exotic or non-standard in the model.
- Avoid freeing up too many parameters.

- Ambitious project
- A *parti pris* of simplicity.
- Nothing exotic or non-standard in the model.
- Avoid freeing up too many parameters.
- An determined attempt to take learning models into the mainstream of macro.

General comments

Objectives of the paper

- Show that during the (slow) process of convergence to rational expectations, an OLS learning model generates data that closely match the behavior of actual stock markets.

Objectives of the paper

- Show that during the (slow) process of convergence to rational expectations, an OLS learning model generates data that closely match the behavior of actual stock markets.
- Mechanism: beliefs affect prices and prices affect beliefs $\Longrightarrow$ possibility of **momentum** effect

Objectives of the paper

- Show that during the (slow) process of convergence to rational expectations, an OLS learning model generates data that closely match the behavior of actual stock markets.
- Mechanism: beliefs affect prices and prices affect beliefs $\Longrightarrow$ possibility of **momentum** effect
- Key result: this effect is strong during the transition to R.E., yet weak close to it.

Caveats

- The name of the game remains the somewhat mechanical characterization of the behavior of a stochastic non-linear difference equation.

Caveats

- The name of the game remains the somewhat mechanical characterization of the behavior of a stochastic non-linear difference equation.
- OLS learning rule is intuitive, has desirable properties, yet remains arbitrary. At the very least, the exogeneity of the gain sequence is problematic.

Gripes

- Hard to disentangle which results are general, and which are specific to the OLS learning rule

Gripes

- Hard to disentangle which results are general, and which are specific to the OLS learning rule
  - ▷ What is the respective contribution to the results of the paper of *each* of the essential ingredients (pricing equation, OLS learning rule)?

Gripes

- Hard to disentangle which results are general, and which are specific to the OLS learning rule
  - What is the respective contribution to the results of the paper of *each* of the essential ingredients (pricing equation, OLS learning rule)?
  - What is the implication of the auxiliary assumption that agents have RE about dividends, and learning is only about prices?

Gripes

- Hard to disentangle which results are general, and which are specific to the OLS learning rule
  - ▷ What is the respective contribution to the results of the paper of *each* of the essential ingredients (pricing equation, OLS learning rule)?
  - ▷ What is the implication of the auxiliary assumption that agents have RE about dividends, and learning is only about prices?
- No attention paid to the fact that, in this model, the *fundamental* value of the asset can be perfectly computed by all investors

Under the hood

Simplifications

- No dividend uncertainty: everyone knows $D_t = 1$ for all t ($\Longrightarrow$ focus on learning about *prices*).

Simplifications

- No dividend uncertainty: everyone knows $D_t = 1$ for all t ($\Longrightarrow$ focus on learning about *prices*).
- Continuous time ($\Longrightarrow$no exotic dynamics due to discrete time)

Simplifications

- No dividend uncertainty: everyone knows $D_t = 1$ for all t ($\Longrightarrow$ focus on learning about *prices*).
- Continuous time ($\Longrightarrow$no exotic dynamics due to discrete time)
- Timing: observe $p(t)$, compute $p^E(t+dt)$, observe $p(t+dt)$ etc.

Simplifications

- No dividend uncertainty: everyone knows $D_t = 1$ for all t ($\Longrightarrow$ focus on learning about *prices*).
- Continuous time ($\Longrightarrow$no exotic dynamics due to discrete time)
- Timing: observe $p(t)$, compute $p^E(t+dt)$, observe $p(t+dt)$ etc.
- Interest rate: $r = \delta$

Simplifications

- No dividend uncertainty: everyone knows $D_t = 1$ for all t ($\Longrightarrow$ focus on learning about *prices*).
- Continuous time ($\Longrightarrow$no exotic dynamics due to discrete time)
- Timing: observe $p(t)$, compute $p^E(t+dt)$, observe $p(t+dt)$ etc.
- Interest rate: $r = \delta$
- No rational bubbles (e.g., Ramsey model)

Under risk neutrality

$$\boxed{\frac{1+\dot{p}^E}{p} = r}$$

Rational expectations

$$\frac{1 + \dot{p}^E}{p} = r$$

Rational expectations

$$\frac{1 + \dot{p}^E}{p} = r$$

- Perfect foresight on prices: $\dot{p}^F = \dot{p}$.

Rational expectations

$$\frac{1 + \dot{p}^E}{p} = r$$

- Perfect foresight on prices: $\dot{p}^F = \dot{p}$.
- If there are no bubbles, the RE price is $p = 1/r$.

Rational expectations

$$\frac{1+\dot{p}^E}{p} = r$$

- Perfect foresight on prices: $\dot{p}^F = \dot{p}$.
- If there are no bubbles, the RE price is $p = 1/r$.
- Investors rationally expect

$$\dot{p}^E/p = 0$$

$$\frac{1+\dot{p}^E}{p} = r$$

$$\frac{1+\dot{p}^E}{p} = r$$

- Assume that people irrationally expect

$$\dot{p}^E / p = \beta, \qquad \beta < r.$$

Irrational expectations

$$\frac{1+\dot{p}^E}{p} = r$$

- Assume that people irrationally expect

$$\dot{p}^E/p = \beta, \qquad \beta < r.$$

- Then $p = 1/(r-\beta)$.

Irrational expectations

$$\frac{1+\dot{p}^E}{p} = r$$

- Assume that people irrationally expect

$$\dot{p}^E / p = \beta, \qquad \beta < r.$$

- Then $p = 1/(r-\beta)$.
- Actual price growth is

$$\dot{p}/p = p\dot{\beta} = \dot{\beta}/(r-\beta).$$

$$\frac{1+\dot{p}^E}{p} = r$$

- Assume that people irrationally expect

$$\dot{p}^E/p = \beta, \qquad \beta < r.$$

- Then $p = 1/(r-\beta)$.
- Actual price growth is

$$\dot{p}/p = p\dot{\beta} = \dot{\beta}/(r-\beta).$$

- Hence both the *level* and *rate of change* of price-growth expectations determine actual growth. **General result**

Two important results

$$\frac{\dot{p}}{p} = p\dot{\beta} = \frac{1}{r-\beta}\dot{\beta}$$

- Price growth generated by the learning model exceeds the fundamental growth rate ($\dot{p}/p > 0$) if growth expectations are increasigly bullish ($\dot{\beta} > 0$).

Two important results

$$\frac{\dot{p}}{p} = p\dot{\beta} = \frac{1}{r-\beta}\dot{\beta}$$

- Price growth generated by the learning model exceeds the fundamental growth rate ($\dot{p}/p > 0$) if growth expectations are increasigly bullish ($\dot{\beta} > 0$).
- If the current price is high (β close to r), then even a small degree of bearishness ($\dot{\beta} < 0$) can cause a huge price collapse. When the current price is low, $\dot{\beta}$ does not have a large influence on prices.

Two important results

$$\frac{\dot{p}}{p} = p\dot{\beta} = \frac{1}{r-\beta}\dot{\beta}$$

- Price growth generated by the learning model exceeds the fundamental growth rate ($\dot{p}/p > 0$) if growth expectations are increasigly bullish ($\dot{\beta} > 0$).
- If the current price is high (β close to r), then even a small degree of bearishness ($\dot{\beta} < 0$) can cause a huge price collapse. When the current price is low, $\dot{\beta}$ does not have a large influence on prices.

But these are simply **results 1 and 4** in the paper!

Two important results

- They do not depend on the OLS learning rule, on the randomness of dividends or on discrete time.

Two important results

- They do not depend on the OLS learning rule, on the randomness of dividends or on discrete time.
- They can be for sure be extended to a stochastic discount factor.

Two important results

- They do not depend on the OLS learning rule, on the randomness of dividends or on discrete time.
- They can be for sure be extended to a stochastic discount factor.
- They are **general.**

Learning rule

- Assume β is updated according to

$$\boxed{\dot{\beta} = \theta(\frac{\dot{p}}{p} - \beta)} \qquad \theta > 0.$$

Learning rule

- Assume β is updated according to

$$\boxed{\dot{\beta} = \theta(\frac{\dot{p}}{p} - \beta)} \qquad \theta > 0.$$

- Now remember that from the pricing condition

$$\frac{\dot{p}}{p} = \frac{1}{r - \beta}\dot{\beta}.$$

Learning rule

- Assume β is updated according to

$$\boxed{\dot{\beta} = \theta(\frac{\dot{p}}{p} - \beta)} \qquad \theta > 0.$$

- Now remember that from the pricing condition

$$\frac{\dot{p}}{p} = \frac{1}{r - \beta}\dot{\beta}.$$

- Hence

$$\dot{\beta} = -\theta' \beta, \qquad \theta' = \frac{\theta}{1 - \frac{\theta}{r - \beta}}$$

More results

$$\dot{\beta} = -\theta' \beta, \qquad \theta' = \frac{\theta}{1 - \frac{\theta}{r - \beta}}$$

More results

$$\dot{\beta} = -\theta' \beta, \qquad \theta' = \frac{\theta}{1 - \frac{\theta}{r-\beta}}$$

- In the *long run*, the gain θ is small, so $\theta' \approx \theta > 0$: convergence to RE ($\beta \to 0$).

More results

$$\dot{\beta} = -\theta' \beta, \qquad \theta' = \frac{\theta}{1 - \frac{\theta}{r-\beta}}$$

- In the *long run*, the gain θ is small, so $\theta' \approx \theta > 0$: convergence to RE ($\beta \to 0$).
- But in the *short run*, the gain θ is large so that we can have $\theta' < 0$ if
$$r > \beta > r - \theta.$$

$$\dot{\beta} = -\theta' \beta, \qquad \theta' = \frac{\theta}{1 - \frac{\theta}{r-\beta}}$$

- In the *long run*, the gain θ is small, so $\theta' \approx \theta > 0$: convergence to RE ($\beta \to 0$).
- But in the *short run*, the gain θ is large so that we can have $\theta' < 0$ if
$$r > \beta > r - \theta.$$
- If this inequality is verified, anything that throws β off its (zero) SS RE value is reinforced in the short run by self-referential learning: **momentum, bubbles.**

More results

- The paper assumes $\beta_0 = 0$ and relies on small dividend shocks to move β away from zero right away (big initial gain).

More results

- The paper assumes $\beta_0 = 0$ and relies on small dividend shocks to move β away from zero right away (big initial gain).
- The alternative (followed here) is to eliminate shocks and allow $\beta_0 \neq 0$.

Moral

- What does the stripped-down model teaches us about the paper?

- What does the stripped-down model teaches us about the paper?
  - ▷ Results 1 and 4 are very robust.

Moral

- What does the stripped-down model teaches us about the paper?
  - ▷ Results 1 and 4 are very robust.
  - ▷ Other results (momentum, bubbles) intimately linked to shape of learning rule, i.e. more fragile.

Moral

- What does the stripped-down model teaches us about the paper?
  - ▷ Results 1 and 4 are very robust.
  - ▷ Other results (momentum, bubbles) intimately linked to shape of learning rule, i.e. more fragile.
- What the stripped-down model teaches us more generally about learning and stock prices?

Moral

- What does the stripped-down model teaches us about the paper?
  - ▷ Results 1 and 4 are very robust.
  - ▷ Other results (momentum, bubbles) intimately linked to shape of learning rule, i.e. more fragile.
- What the stripped-down model teaches us more generally about learning and stock prices?
  - ▷ Short- and long-run stability properties of learning rules may differ a lot.

Moral

- What does the stripped-down model teaches us about the paper?
  - ▷ Results 1 and 4 are very robust.
  - ▷ Other results (momentum, bubbles) intimately linked to shape of learning rule, i.e. more fragile.
- What the stripped-down model teaches us more generally about learning and stock prices?
  - ▷ Short- and long-run stability properties of learning rules may differ a lot.
  - ▷ Crucial role of decreasing gain.

Moral

- What does the stripped-down model teaches us about the paper?
  - ▷ Results 1 and 4 are very robust.
  - ▷ Other results (momentum, bubbles) intimately linked to shape of learning rule, i.e. more fragile.
- What the stripped-down model teaches us more generally about learning and stock prices?
  - ▷ Short- and long-run stability properties of learning rules may differ a lot.
  - ▷ Crucial role of decreasing gain.
  - ▷ The shape of the learning rule is key to understanding how small dividend shocks interact, on impact, with learning to produce big movements in prices.

Fundamentals

Crashes

- The model predicts that stock market crashes are possible.

- The model predicts that stock market crashes are possible.
- Problem: the price cannot fall below its fundamental value $1/r$:

- The model predicts that stock market crashes are possible.
- Problem: the price cannot fall below its fundamental value $1/r$:
  - ▷ Proof: stock = ownership of dividends + right to resell. Value of the latter non-negative under free disposal.

Crashes

- The model predicts that stock market crashes are possible.
- Problem: the price cannot fall below its fundamental value $1/r$:
  - ▷ Proof: stock = ownership of dividends + right to resell. Value of the latter non-negative under free disposal.
  - ▷ Hence the fundamental (**which the paper assumes can be calculated**) is a lower bound on prices.

Crashes

- The model predicts that stock market crashes are possible.
- Problem: the price cannot fall below its fundamental value $1/r$:
  - ▷ Proof: stock = ownership of dividends + right to resell. Value of the latter non-negative under free disposal.
  - ▷ Hence the fundamental (**which the paper assumes can be calculated**) is a lower bound on prices.
  - ▷ The argument goes through for any holding period T (e.g., in an OLG model). We must have

$$p_t \geq \int_t^{t+T} \mathrm{e}^{-r(s-t)} D_s \, ds.$$

Crashes

- The model predicts that stock market crashes are possible.
- Problem: the price cannot fall below its fundamental value $1/r$:
  - ▷ Proof: stock = ownership of dividends + right to resell. Value of the latter non-negative under free disposal.
  - ▷ Hence the fundamental (**which the paper assumes can be calculated**) is a lower bound on prices.
  - ▷ The argument goes through for any holding period T (e.g., in an OLG model). We must have

$$p_t \geq \int_t^{t+T} e^{-r(s-t)} D_s \, ds.$$

- Moral: to understand price crashes, one needs a model in which the fundamental i) must be learned and ii) can crash.

Bubbles

- Uneasy feeling about bubble-like phenomena in the model.

- Uneasy feeling about bubble-like phenomena in the model.
- The Timmerman and Cogley-Sargent *projection facility* ($\dot{\beta} = 0$ when $\beta > r$) that is used to keep $\beta < r$ and prices positive is ad hoc:

- Uneasy feeling about bubble-like phenomena in the model.
- The Timmerman and Cogley-Sargent *projection facility* ($\dot{\beta} = 0$ when $\beta > r$) that is used to keep $\beta < r$ and prices positive is ad hoc:
- Justification: eliminates "crazy" actions and phenomena.

- Uneasy feeling about bubble-like phenomena in the model.
- The Timmerman and Cogley-Sargent *projection facility* ($\dot{\beta} = 0$ when $\beta > r$) that is used to keep $\beta < r$ and prices positive is ad hoc:
- Justification: eliminates "crazy" actions and phenomena.
  - ▷ Rationality is creeping in through the backdoor.

Bubbles

- Uneasy feeling about bubble-like phenomena in the model.
- The Timmerman and Cogley-Sargent *projection facility* ($\dot{\beta} = 0$ when $\beta > r$) that is used to keep $\beta < r$ and prices positive is ad hoc:
- Justification: eliminates "crazy" actions and phenomena.
  - ▷ Rationality is creeping in through the backdoor.
  - ▷ Dangerous argument: why don't agents compare price growth (which is hard to predict) to dividend growth (which is easy to predict)? Why don't they know it is "crazy" that prices move more than dividends?

Bubbles

- Uneasy feeling about bubble-like phenomena in the model.
- The Timmerman and Cogley-Sargent *projection facility* ($\dot{\beta} = 0$ when $\beta > r$) that is used to keep $\beta < r$ and prices positive is ad hoc:
- Justification: eliminates “crazy” actions and phenomena.
  - ▷ Rationality is creeping in through the backdoor.
  - ▷ Dangerous argument: why don’t agents compare price growth (which is hard to predict) to dividend growth (which is easy to predict)? Why don’t they know it is “crazy” that prices move more than dividends?
- Moral: use economic arguments instead (TVC condition probably rules out equilibria with $\dot{p}^E / p = \beta > r$).

Conclusion

- A thought-provoking paper that tries to play the asset pricing game without affording itself extra degrees of freedom.

Conclusion

- A thought-provoking paper that tries to play the asset pricing game without affording itself extra degrees of freedom.
- It should be possible to go further by disentangling what is general from what is specific to the learning rule and what comes from auxiliary modeling assumptions.

Conclusion

- A thought-provoking paper that tries to play the asset pricing game without affording itself extra degrees of freedom.
- It should be possible to go further by disentangling what is general from what is specific to the learning rule and what comes from auxiliary modeling assumptions.
- Generalize H-J bounds to learning economies?