The Signaling Effect of Raising Inflation

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Abstract

This paper argues that central bankers should raise inflation when anticipating liquidity traps to signal their credibility to forward guidance policies. As stable inflation in normal times either stems from central banker's credibility, e.g. through reputation, or from his aversion to inflation, the private sector is unable to infer the central banker's type from observing stable inflation, jeopardizing the efficiency of forward guidance policy. We show that this signaling motive can justify level of inflation well above 2% but also that the low inflation volatility during the Great Moderation was insufficient to ensure fully efficient forward guidance when needed.

Keywords: Forward Guidance, Inflation, Signaling.

JEL Classification: E31, E52, E65.

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1 Introduction

Optimal monetary policy often requires to manipulate private agents expectations, potentially leading to time-inconsistent policies, as the central banker may find desirable to renege on past promises. As a solution, Rogoff (1985) advocated for delegating monetary policy to an independent and inflation averse central banker (e.g. with a unique objective of price stability) so as to ensure a credible stabilization of inflation. This partially explains why many central banks have adopted a clear price stability mandate.

However, such a delegation to an inflation-averse central banker falls short of optimally stabilizing the economy in the presence of a liquidity trap, that results from the zero lower bound (ZLB, thereafter) on nominal interest rates.[1] In this case, the central banker can still stimulate the economy by promising to generate an inflationary boom after the deflationary shock disappears - the so-called forward guidance (see Eggertsson and Woodford, 2003; Werning, 2012). Yet, such an inflationary boom is unlikely to be engineered by an inflation-averse central banker, thus jeopardizing the efficiency of forward guidance.[2] And so, we need to find conditions under which a central banker is also credible to deviate from inflation stabilization after liquidity traps and to convince private agents of such a credibility once the liquidity trap occurs.

In this paper, we provide a theory of why forward guidance may be more or less efficient depending on the central banker's objectives and his past behavior in terms of responses to shocks and the inflation rate. To this purpose, we study optimal monetary policy when private agents are uncertain about the central banker's preferences that determine his credibility to implement an inflationary boom after a liquidity trap. More specifically, we link central banker's preference to the sustainability of the Ramsey outcome through the threat of reversion to the discretionary outcome. We determine whether standard monetary policy instruments can be used as a signal to ensure credibility and, hence, the efficiency of forward guidance. Our main result is that raising inflation before a liquidity trap occurs can achieve such a signaling so that private agents anticipate a forward guidance policy once the economy actually falls into a liquidity trap. Turning to historical data, we show that the low inflation volatility during the Great Moderation was insufficient to generate such signaling and to ensure the efficiency of forward guidance once in a liquidity trap as observed during the Great Recession.

But why signal through higher inflation? Recall that a forward guidance policy is least likely to be followed through by inflation-averse central bankers. And so, a credible

[1]The lower bound on nominal interest rates may be slightly below zero as we have already observed slightly negative rates in Switzerland or in the euro area.

[2]This echoes the so-called "forward guidance puzzle" documented by Del Negro et al. (2013) and McKay et al. (2015) among others.

central banker has every incentive to be distinguished from such conservative central bankers. And because conservative central bankers are inclined to stabilize inflation in normal times, increasing inflation is the only available tool so as to ensure credibility before a liquidity trap occurs.

To formalize that argument, we consider a new Keynesian model facing a period of negative natural interest rate that pushes the economy into a liquidity trap. Section 2 presents the model under perfect information and proves that an intermediate but positive weight on the output gap stabilization objective in central banker's preferences is necessary to sustain the Ramsey allocation when introducing reputation concerns. However, such a central banker will conduct a policy that is similar to the one conducted by the conservative central banker in normal times. We then introduce asymmetric information on the weight of the output gap stabilization objective and limit the central banker's types to two: either he maximizes social welfare (the benevolent central banker) or he only takes inflation into account in his objective function (the conservative central banker) and therefore is not able to sustain forward guidance in the benchmark model.

Our main insight (Section 3) is that inflation before the liquidity trap occurs can be used as a signaling device for the benevolent central banker. First, signaling is not possible during a liquidity trap as the policy instrument is constraint by the zero lower bound. Thus, if signaling is optimal, it must take place prior to the liquidity trap. Second, as inflation is the only variable entering into the conservative central banker's objective function, it is also the only means to distinguish between the benevolent central banker and the conservative one as the latter will not be inclined to mimic the former. The benevolent central banker then optimally sets inflation at the rate at which the conservative central banker would be indifferent between mimicking the benevolent central banker and adopting a conservative monetary policy. This inflation rate increases with the probability, the intensity and the length of liquidity traps.

Furthermore, signaling is possible only if the benevolent central banker weights output gap stabilization in his objective and has some inflation bias. Even though this inflation bias does not affect the equilibrium outcome, it makes inflation relatively less costly for him than for the conservative central banker.[3] This also means that forward guidance can be credible only when the central banker has an objective of output gap stabilization and not only an objective of inflation stabilization.

Turning to quantification (Section 4), the required inflation level is significantly higher than 0. When calibrating our model, we show that this signaling motive can generate a level of inflation of around 2%. In addition, we observe that even a small increase

[3] In other words, the inflation bias in benevolent central bankers' preferences plays the role of a Spence-Mirrlees condition in the signaling problem.

in the probability or the expected length of the liquidity trap can increase the level of signaling inflation to above 3%. Note that, in our framework, inflation does not reduce the probability of a liquidity trap and does not help to reduce the consequences of this liquidity trap, so that these values only reflect our signaling motive. In particular, this explains why we find different values compared with Coibion et al. (2012) who consider the buffer motive of raising inflation.

But how costly is signaling? To investigate further this issue, we consider three possible extensions of our benchmark model that may affect either the benevolent or the conservative central banker's incentives: the presence of cost-push shocks, repeated liquidity traps and the possibility of being mistakenly considered as an inflation-biased discretionary central banker when raising inflation.

First, cost-push shocks may help the benevolent central banker to produce inflation, and so, separating without cost from being conservative, before the liquidity trap (Section 5.1).[4] Indeed, this type of shocks leads to different responses under perfect information depending on the weight of the output gap stabilization objective of the central banker. To assess their effects, we perform two quantitative experiments suggesting that cost-push shocks were not sufficiently large to prevent the conservative central banker from imitating the benevolent one. First, cost-push shock needs to be 16 times larger than standard estimated sizes of cost-push shocks to trigger separating. Second, even if we attribute all inflation volatility to cost-push shocks, inflation volatility during the Great Moderation was not sufficient to trigger such a separating. Therefore, it is unlikely that cost-push shocks during the Great Moderation[5] were sufficient *per se* to signal the central banker type before the 2008 crisis.

Second, the repetition of liquidity traps (Section 5.2) may also give incentives for the conservative central banker to sustain forward guidance and, thus, to limit his willingness to imitate the benevolent one. As a result, the benevolent central banker does not need anymore to signal his type prior to liquidity traps and so, to raise inflation. We show that raising inflation is still needed when traps are insufficiently frequent or, more importantly, when the central banker anticipates a low probability to experience a second liquidity trap during his term.

Third, increasing inflation may lead the benevolent to be considered wrongly as an inflation-biased central banker, thus increasing the cost of signaling. We extend (Section 5.3) our signaling problem to such a case and we show that our main result is robust: benevolent central bankers need to raise inflation before a trap occurs, even if this implies

[4]When occurring after the liquidity trap, such shocks reinforce the desirability for the benevolent central banker to sustain the Ramsey allocation compared to the conservative one.

[5]The Great Moderation was characterized by low volatility of inflation and weak output gap/inflation trade-off (Justiniano et al., 2013).

that they are pooled with the inflation-biased central banker as long as private agents do not sufficiently believe that the central banker in office is the benevolent one when observing zero inflation.

Finally, these three elements lead to the following testable implications: the efficiency of forward guidance (i) increases with the occurrence of cost-push shocks prior to liquidity traps, (ii) increases with the frequency of repeated liquidity traps or with the length of the term of the central banker, (iii) but decreases with the past reputation of the central banker to be inflation-biased.

We get our main results analytically by focusing on the best sustainable allocation from a timeless perspective (that is when the game starts at $-\infty$) and by assuming that separating emerges only if the central banker ensures that the other central banker has no incentive to imitate him at any date. These two assumptions make the problem tractable and allow us to transform the infinite-horizon game into a simplified three-period game. We then assess the robustness of our results by relaxing these assumptions. To this purpose, we study in Section 6.1 the optimal path of inflation when the policymakers take their decision at a finite time horizon and when signaling can be achieved in one period for all. In this case, the optimal level of inflation set by the benevolent central banker to signal his type is initially high and then converges to zero slowly. In the benchmark calibration, the level of inflation starts at 8% and remains higher than 1.5% during first four years. Finally, Section 6.2 discusses how to extend our results to the possibility of contracting with the central banker, where we provide mild conditions under which contracting cannot avoid the necessity of signaling.

In the end, this paper provides a new motive for raising inflation, not to manage the probability of falling in a liquidity trap but to signal the central banker's credibility once the liquidity trap occurs, so as to ensure the efficiency of forward guidance. In particular, we show that this motive can result in levels of inflation above 2%. Furthermore, this paper links weak evidence of forward guidance efficiency with the private uncertainty about the central banker's future action. Eventually, we show that the very stable business cycle during the Great Moderation era was unlikely to give the possibility for private agents to verify the central banker's willingness to tolerate an inflationary boom after the 2008 crisis.

Related literature. Raising inflation has been suggested as a possible policy to avoid liquidity traps (see Blanchard et al., 2010, among others). By widening the size of shocks required to hit the zero lower bound on nominal interest rates, inflation can act as a buffer. This proposal has also been defended by Ball (2014) and the Fed's president Rosengren (April 16, 2015), who argue that the cost of liquidity traps is large, policies used to escape the trap are inefficient and the probability of a liquidity trap has increased. Aruoba and Schorfheide (2013) show that recovery from the Great Recession in the US would have been a year shorter if the Fed had targeted 4% inflation instead of the observed 2.5%. Yet, from a normative standpoint, the optimal size of this buffer has been found to be quantitatively small as liquidity traps are nevertheless rare events (see Coibion et al., 2012).

An alternative way to enforce promises at the ZLB is Quantitative Easing as studied by Bhattarai et al. (2015). By making interest rate hikes costly to the central bank, large purchases of sufficiently long term assets make future deviations from low interest rates less likely. Our paper's main difference is that we rely only on reputation concerns to sustain forward guidance policies, while they consider *ex post* real costs of reneging on past promises, that, in the end, hinge on the central bank's ability to make losses on its balance sheet (see Sims, 2013, among others).

Bassetto (2015) analyses communication on forward guidance policies. Costly signaling as raising inflation is consistent with his insight that pure communication on credibility is cheap talk. Yet, in our case, communicating about preferences (the inflation bias), as it is connected with the credibility, would not affect the set of achievable outcomes for central bankers: announcing inflation bias, and thus credibility, is also cheap talk in our case.

Our paper is also connected to the literature on imperfect credibility at the ZLB as in Bodenstein et al. (2012). The main difference is that imperfect credibility in our framework is an equilibrium outcome that can be altered by a monetary authority's actions. In our model, credibility results from reputation concerns as in Barro and Gordon (1983) or Alesina (1987). In addition, Andrade et al. (2015) provide evidence on heterogeneous understanding of forward guidance policies and link this to imperfect credibility of the central bank.

Our paper is closely related to the literature on learning on central bankers' types as in Backus and Driffill (1985), Cukierman and Meltzer (1986) or Barro (1986) (see also Sleet and Yeltekin (2007) for a recent contribution). Our paper's main difference is that we introduce the possibility of liquidity traps and we allow the credible central banker to optimally react to mimicking strategies by a discretionary central banker. More generally, our paper enters in the literature on reputation and asymmetric information as pioneered by Kreps and Wilson (1982) and Milgrom and Roberts (1982).

Finally, our results contrast with recent results by Nakata and Schmidt (2015) who argue that delegating monetary policy to a conservative central banker remains optimal even in presence of liquidity traps. In our paper, we allow for reputation concerns to sustain forward guidance policy while these authors only consider discretionary policies.

They thus do not account for a possible link between central banker's preference and credibility.

2 Model under perfect information

In this section, we introduce the standard new-Keynesian model in the presence of a zero lower bound as in Eggertsson and Woodford (2003) and Werning (2012). Our main assumptions concern the process of the natural rate of interest, that may push the economy at the ZLB to allow for tractability. We relax most of these assumptions once we quantify the signaling mechanism. Then, we determine in this environment the behavior of central bankers of different types under full information. This allows us to identify whether they have similar incentives, when we will consider asymmetric information on their types in the next sections.

2.1 Model

Time is discrete and denoted by $t \in \{-\infty, ...\infty\}$. We consider an economy summarized by two equations:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t, \tag{NKPC}$$

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - r_t^n\right). \tag{EE}$$

where π_t denotes (log of) date-t inflation rate, y_t the output gap, i_t the nominal interest rate and r_t^n the natural rate of interest. Scalars σ, β and κ are positive constants. Finally, E_t denotes the time-t expectations operator.

The New Keynesian Phillips curve (NKPC) describes the dynamics of prices. The Euler equation (EE) derives from the consumer's inter temporal consumption choice.

Finally, the Zero Lower Bound on nominal interest rate imposes:

$$i_t \geq -\bar{i}, \text{ with } \bar{i} \geq 0. \tag{ZLB}$$

Assumption 1 (Process)**.** *The natural rate of interest (r_t^n) is* 0 *with probability* $1-\gamma$ *and* $-r$ *otherwise. Once the natural rate of interest equals* $-r$*, it returns to 0 next period and for any subsequent period. We assume that* $-r$ *is sufficiently low.*

This assumption is convenient for deriving analytical results but involves no loss of generality. We depart from this assumption in the numerical application, where we consider a liquidity trap lasting more than one period. We also discuss the role of possible future liquidity traps in Section 6.2.

Welfare We suppose that the date-s economy's welfare is given by the opposite of the following quadratic loss function:[6]

$$L_s = \frac{1}{2} E_s \sum_{t=s}^{\infty} \beta^{t-s} \left[\pi_t^2 + \lambda^* \left(y_t - y^* \right)^2 \right],$$

where the scalar $y^* \geq 0$ denotes the desired level of the output gap, also referred to as the inflation bias, and the scalar $\lambda^* \geq 0$ denotes the weight of output fluctuations in the welfare function.

For deriving analytical results, we restrict the policy parameter, λ^* as follows:

Assumption 2 (Output gap weight)**.** *The weight on the output gap in the welfare-relevant loss function is sufficiently high, i.e.* $\sigma\kappa < \lambda^*$*.*

When this condition is satisfied, the optimal monetary policy under commitment leads to a forward guidance type of policy similar to Eggertsson and Woodford (2003) characterized by a positive inflation rate and output gap after the liquidity trap. More about this assumption can be found in Werning (2012). In the numerical applications, we relax this assumption.

Central bankers We consider a continuum of central bankers indexed by λ, so that the λ-central banker's preference is:

$$L_s^{\lambda} = \frac{1}{2} E_s \sum_{t=s}^{\infty} \beta^{t-s} \left[\pi_t^2 + \lambda \left(y_t - y^* \right)^2 \right]$$

When the preference parameter, λ, is equal to zero we call the central banker the conservative central banker. When this parameter is equal to λ^*, we call him the benevolent central banker as he maximizes the welfare function.

2.2 Optimal policies under full information

Let us first describe the optimal monetary policy and the resulting Ramsey allocation in this economy. Then we compute optimal policies of a λ-central banker under discretion. Finally, we derive the policy implemented by central bankers in the presence of reputation concerns. In all this subsection, we suppose that the central banker's type, indexed by λ, is common knowledge.

The Ramsey allocation. The Ramsey allocation maximizes date-s welfare with respect to the entire sequence of nominal interest rate $\{i_t\}_{t \geq s}$ under the constraint (ZLB),

[6] A micro-foundation of this loss can be found in Woodford (2003).

given that the allocation $\{\pi_t, y_t\}_{t\geq s}$ solves equations (NKPC) and (EE). Date s is supposed to be in the distant past ($s \to -\infty$) to allow for a stationary equilibrium before the liquidity trap. We call the resulting outcome the Ramsey allocation.

Problem 1 (Ramsey program)**.**

$$\min_{i_t, t\geq s} L_s$$

under the constraints (NKPC), (EE) and (ZLB).

The Lagrangian of this problem can be written as:

$$E_s \sum_{t=s}^{\infty} \beta^{t-s} \left[\frac{1}{2}(\pi_t^2 + \lambda^* \left(\hat{y}_t - y^*\right)^2) + \mu_t(\pi_t - \beta E_t \pi_{t+1} - \kappa \hat{y}_t) \right.$$
$$\left. + \nu_t(y_t - E_t y_{t+1} + \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - r_t^n\right)) + \zeta_t(i_t + \bar{i}) \right],$$

where μ_t, ν_t and ζ_t are the Lagrange multipliers associated to the constraints (NKPC), (EE) and (ZLB). This latter constraint being an inequality, the Lagrange multiplier ζ_t is strictly negative if the constraint is slack, zero otherwise. First order conditions lead to:

$$\pi_t + \mu_t - \mu_{t-1} - \frac{\nu_{t-1}}{\beta\sigma} = 0, \tag{1}$$

$$\lambda^*(y_t - y^*) - \kappa\mu_t + \nu_t - \frac{1}{\beta}\nu_{t-1} = 0, \tag{2}$$

$$\frac{1}{\sigma}\nu_t + \zeta_t = 0. \tag{3}$$

The Ramsey allocation is the solution of the system defined by first-order conditions and equations (NKPC), (EE) and (ZLB).

Prior to the trap, as the date s is supposed to be in the distant past,[7] inflation is fully stabilized ($\pi_t = 0$). However, the expectation of a liquidity trap triggers a positive output gap, $-\frac{\beta\gamma}{\kappa}\pi_l$, where π_l is the rate of inflation at the ZLB. After the trap, the Ramsey allocation can be implemented through forward Guidance policy associated with positive inflation, π_{l+1}, and output gap, y_{l+1}, at the end of the trap. Afterward, both inflation and output gap converge to 0.

λ-central banker under discretion Under full information and under discretion, the λ-central banker solves at each period the following optimization program:

[7]This assumption is sometimes referred to as optimal policy in a timeless perspective, see Woodford (2003)

Problem 2 (Under discretion)**.**

$$\forall t, \min_{i_t} \left[\pi_t^2 + \lambda \left(y_t - y^*\right)^2\right],$$

under the constraints (NKPC), (EE) and (ZLB).

The Lagrangian writes:

$$1/2 \left[\pi_t^2 + \lambda \left(y_t - y^*\right)^2\right] + ...$$
$$.... + \mu_t \left(\pi_t - \beta E_t \pi_{t+1} - \kappa y_t\right) + \nu_t \left(y_t - E_t y_{t+1} + \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - r_t^n\right)\right) + \zeta_t (i_t + \bar{i}),$$

and the resulting first order conditions lead to:

$$\kappa \pi_t + \lambda \left(y_t - y^*\right) = \sigma \zeta_t,$$

where ζ_t stands for the Lagrange multiplier associated with the constraint (ZLB). As a result, inflation and the output gap follow:

In any period after the liquidity trap:

$$\pi_t = \lambda \frac{\kappa y^*}{\kappa^2 + \lambda(1-\beta)} \text{ and } y_t = \lambda \frac{(1-\beta) y^*}{\kappa^2 + \lambda(1-\beta)},$$

During the liquidity trap:

$$\pi_t = \kappa \left[\lambda y^* \frac{1 + \frac{\kappa}{\sigma}}{\kappa^2 + \lambda(1-\beta)} - \frac{1}{\sigma}\left(\bar{i} - r\right)\right] \text{ and } y_t = \lambda y^* \frac{(1-\beta) + \frac{\kappa}{\sigma}}{\kappa^2 + \lambda(1-\beta)} - \frac{1}{\sigma}\left(\bar{i} - r\right),$$

Before the liquidity trap:

$$\pi_t = \lambda \frac{\beta \gamma \pi_l + \kappa y^*}{\kappa^2 + (1 - \beta(1-\gamma))\lambda} \text{ and } y_t = \frac{(1 - \beta(1-\gamma))\lambda y^* - \kappa \beta \gamma \pi_l}{\kappa^2 + (1 - \beta(1-\gamma))\lambda}.$$

In particular, note that inflation before the trap equals 0 whatever the probability of ZLB (γ) if and only if λ equals zero.[8]

Sustainable policies. We now investigate whether a λ-type central banker can sustain the Ramsey allocation under a trigger strategy in the spirit of Chari and Kehoe (1990) and as in Kurozumi (2008) and Loisel (2008). We focus on revert-to-discretion strategies

[8]For a given level of γ, there also exists a specific value of y^* such that $\pi_t = 0$ as $y_t = y^*$, but we discard such solution as we concentrate on cases where the discretionary central banker stabilizes inflation for all values of γ.

so that the the resulting sustainability condition at date t is:

$$\sum_{k=t}^{\infty} \beta^{t-k} \left(\pi_k^2 + \lambda(y_k - y^*)^2\right) \le U_t^d + W^D, \tag{4}$$

where U^d is the current gain from the best deviation given private agents' expectations consistent with the Ramsey allocation and W^D is the present value of future periods' losses with discretion. The set of these conditions at all dates is sufficient to ensure sustainability.Using the results from previous paragraphs, this condition can be rewritten when the ZLB is not binding as follows:

$$\sum_{k=t}^{\infty} \beta^{t-k} \left(\pi_k^2 + \lambda(y_t - y^*)^2\right) \le \left(\frac{(\kappa y^* + \beta E_t \pi_{t+1})^2}{\kappa^2 + \lambda}\right) \lambda + \frac{\beta}{1-\beta} \kappa^2 \lambda (y^*)^2 \frac{\lambda + \kappa^2}{\left(\kappa^2 + \lambda(1-\beta)\right)^2}. \tag{5}$$

In particular, in the case of $\lambda = 0$, condition (4) boils down to:

$$\sum_{t}^{\infty} \beta^{k-t} \left(\pi_t^2\right) \le 0, \text{ for every period t.} \tag{6}$$

As a result, when $\lambda = 0$, $\pi_t = 0$ for any period after the trap. In particular, this holds whatever the value of the discount factor β.

This leads to the following proposition:

Proposition 1 (Full information). *When $\gamma > 0$, the Ramsey allocation is sustainable by a λ-central banker, only if λ, the weight on the output gap stabilization objective, is intermediate, i.e. $\lambda \in [\underline{\lambda}_D, \overline{\lambda}_D]$ where $0 < \underline{\lambda}_D \le \overline{\lambda}_D < \infty$. In this case, inflation is fully stabilized before the trap.*

In addition, there exists at least one central banker who triggers no inflation before the trap when λ is lower than $\underline{\lambda}_D$, that is the conservative central banker ($\lambda = 0$).

Proof. See Appendix B.1 □

Proposition 1 provides several insights. First of all, inflation stabilization before a trap occurs can be achieved both by credible - with intermediate λ - and conservative - $\lambda = 0$ - central bankers. Second, only the first category of central bankers is able to sustain the Ramsey allocation and especially forward guidance policy. For low weight on the output gap stabilization objective, the central banker is not able to sustain this allocation. In particular, the conservative central banker will never follow through any kind of forward guidance policies after the liquidity trap as he can achieve full inflation stabilization afterward.

The presence of the zero lower bound is key to obtain this disconnection between inflation stabilization before the trap and ability to sustain forward guidance. In the absence of ZLB ($\gamma = 0$), the Ramsey allocation coincides with the preferred outcome of the conservative central banker ($\lambda = 0$). Yet, such coincidence that motivated Rogoff (1985)'s delegation result, does not arise anymore when liquidity trap requires forward guidance policy.

In the end, this means that, even under full information, the central banker in charge should not only have an inflation stabilization objective but also the objective to stabilize output.

Remark. Note that the existence of positive values of λ that would ensure sustainability may be obtained as in standard folk theorems. In contrast, this is not the case when $\lambda = 0$, as there exists no future costs for such a central banker who will always fully stabilize inflation.

In the end, under perfect information, positive inflation bias ($\lambda > 0$) is required to ensure both optimal policies before and after the trap. In the rest of the paper, we make the following assumption:

Assumption 3. *The benevolent central banker is able to achieve the Ramsey allocation, i.e.* $\lambda^* \in [\underline{\lambda}_D, \overline{\lambda}_D]$.

This assumption discards situations in which the social welfare does not allow reputation concerns to sustain the Ramsey allocation.

3 Model under imperfect information

We now turn to the equilibrium under imperfect information on central bankers' types. To this purpose, we first define such an equilibrium before characterizing it. We then obtain that a central banker that can sustain a forward guidance policy may find optimal to run positive inflation before the liquidity trap occurs.

In this section and the next one (Section 4), we focus on the case where the central banker's types are either benevolent ($\lambda = \lambda^*$) or conservative ($\lambda = 0$). We consider this case as our benchmark case, as we have shown in proposition 1 that these two types stabilize inflation prior to a liquidity trap but the former can credibly use forward guidance policy while the latter cannot.

3.1 Beliefs and equilibrium definition.

We denote agents beliefs about the central banker's type in period t by $q_t \in [0, 1]$, with the following meaning: when $q_t = 1$, agents are sure that the central banker is benevolent, while, when $q_t = 0$, agents are certain that the central banker is conservative.

In particular, q denotes the agents' prior beliefs.

Definition 1. *An equilibrium is an allocation $\{\pi_t, y_t\}_t$, a sequence of policy decisions $\{i_t\}_t$ a sequence of beliefs $\{q_t\}_t$ such that, for each period t:*

- *Given the sequences of beliefs and policy decisions, π_t and y_t solve (NKPC) and (EE), where expectations are consistent with beliefs and policy.*
- *Given sequences of beliefs and allocations, both central bankers optimize their objective function under the constraint (ZLB).*
- *Private agents update beliefs on the central banker's type following the Bayes' law whenever possible.*

Such a definition of equilibrium allows for trigger strategies and sustainability based on reputation but also asymmetric information on the central banker's preference. In the following, we suppose that economic agents always coordinate on the best sustainable equilibrium.

3.2 Central banker's actions and payoffs.

After the trap. To start with, let us note that once the trap ends, the conservative central banker has no incentive to imitate the benevolent central banker anymore as he can fully stabilize inflation. This implies that our problem will behave in the long run as in the full information case. This helps us to simplify the dynamic component of our signaling game.

Separating before and during the trap. Let us first describe the benevolent central banker's policy when separating between types occurs. Such a separating emerges if and only if the benevolent central banker's policy implements a policy that prevents the conservative central banker from imitating his policy prior to the liquidity trap. When policies are chosen at date $t < t_l$,[9] such a constraint can be written as follows:

$$\sum_{\tau=t}^{\infty} (\beta(1-\gamma))^{\tau-t} \left[\frac{1}{2}\pi_\tau^2 + \beta\gamma L_{t_l}^0(S,1)\right] \geq \sum_{\tau=t}^{\infty} (\beta(1-\gamma))^{\tau-t} \beta\gamma L_{t_l}^0(S,0), \qquad \text{(REV)}$$

where we denote by $L_{t_l}^0(S,0)$ the loss of the conservative central banker during and after the trap when he follows his own policy and by $L_{t_l}^0(S,1)$ when he replicates the benevolent central banker's policy before and during the trap.

[9] In timeless perspective, $t = -\infty$.

In general, it is sufficient to satisfy this constraint *only in the first period* of the economy, as agents have then learnt the type of the central banker for any subsequent periods. However, in what follows, we will assume that this constraint has to be satisfied for all periods date t strictly lower than the first period of the liquidity trap, denoted by t_l. Such an assumption is only made for tractability as it allows to have a constant rate of inflation prior to the trap. From an economic perspective, imposing such a constraint may correspond to some form of inattention by private agents, which forces the central banker to send signals over multiple periods. In Section 6.1, we relax this assumption and illustrate that signaling inflation is slightly decreasing over time (at a rate $\beta(1-\gamma)$) and is initially *higher* than the constant rate of signaling inflation that we obtain in this section.

The resulting problem for the benevolent central banker in a separating equilibrium is:

Problem 3 (Benevolent central banker's program under separating equilibrium)**.** Given the following sequence of beliefs: $q_t = 1$ for all periods,

$$\min_{\{i_t\}_t} L_{-\infty},$$

under the incentive compatibility constraint (REV) satisfied for all $t < t_l$ and constraints (NKPC), (EE) and (ZLB) at each date.

The following proposition describes the solution of the benevolent central banker's problem under separating equilibrium.

Proposition 2. *The solution of Problem 3 takes the following form:*

(i) *for any period t before the trap, inflation is constant ($\pi_t = \pi$) and solves:*

$$\pi^2 \geq \beta\gamma\left[L^0_{t_l}(S,0) - L^0_{t_l}(S,1)\right]$$

(ii) *The net present loss during and after the trap $L^\lambda_{t_l}(S,1)$ is higher than the one obtained under perfect information.*

Furthermore, under Assumption 3, this solution is sustainable by the benevolent central banker.

Proof. See Appendix B.2. □

First, the optimal policy in a separating equilibrium is to set a positive and constant level of inflation prior to the liquidity trap. This positive level ensures truth-telling of the conservative central banker. Second, the benevolent central banker also implements a less powerful forward guidance policy to deter the conservative central banker from imitating him. Finally, while this optimal policy differs from the Ramsey allocation in perfect information, it remains sustainable. The benevolent central banker does not lower inflation before the trap to ensure the incentive compatibility constraint and after the trap the incentive to deviate decreases as he implements a weaker forward guidance.

Pooling before and during the trap. We now turn to the description of the benevolent central banker's policy when both central bankers are pooled. It is worth noting that because the problem before the liquidity trap is stationary, the sequence of beliefs in pooling verifies: $q_t = q$ for periods t before and during the trap, and $q_t = 1$ for periods after the trap.

Problem 4 (Benevolent central banker's program under pooling.)**.** Given the sequence of beliefs $q_t = q < 1$ for $t \leq t_l$ and $q_t = 1$ after the trap,

$$\min_{i_t, t} L_{-\infty},$$

under the constraints (NKPC), (EE) and (ZLB).

Let us now describe how private beliefs q affect central banks' outcomes once the economy is in a liquidity trap. Real functions $L_{t_l}(P, q)$ and $L^0_{t_l}(P, q)$ denote the losses of the benevolent and the conservative central bankers once the liquidity trap hits the economy given private beliefs on the central banker's type, q.

Proposition 3. *The solution to Problem 4 is to implement zero inflation prior to the liquidity trap, that is* $\forall t < t_l$, $\pi_t = 0$. *Furthermore, central bankers' losses satisfy:*

(i) *The welfare loss during the liquidity trap* $L_{t_l}(P, q)$ *is decreasing in* q. *In addition,* $L_{t_l}(P, 1) \leq L_{t_l}(S, 1)$, *where* $L_{t_l}(S, 1)$ *denotes the loss in the signaling case.*

(ii) *The conservative central banker's loss* $L^0_{t_l}(P, q)$ *is decreasing in* q.

Proof. See Appendix B.3. □

The more agents are convinced that the central banker is benevolent (q approaching 1), the lower the losses of both central bankers due to the liquidity trap. Indeed, larger beliefs mean that agents are more convinced that the central banker can commit to forward guidance policy which is beneficial for both types. Even though conservative central banker will, *ex post*, deviate from forward guidance promise, he benefits from the private beliefs in forward guidance's future easing monetary policy. In the end, both central bankers have an incentive to raise beliefs q.

Note that Proposition 3 remains silent about the sustainability by the benevolent central banker of the solution to Problem 4. When beliefs are such that $q < 1$, this may require the central banker to engage into more monetary stimulus after the end of the trap, so as to compensate for lack of credibility.[10] In turn, this can make the solution to Problem 4 not sustainable. To keep tractability, we make the following assumption in the remaining of the section:

Assumption 4. *The solution to Problem 4 is sustainable by the benevolent central banker.*

Such an assumption works against us as the sustainability of the solution to Problem 4 actually increases the value of being pooled for the benevolent central banker and so reduces the need for signaling. Yet, it does not affect the incentive constraint of the conservative central banker, that determines the level of signaling inflation, as it will become clear in the next paragraph.

Summary. We have successively examined separating and pooling cases. Separating will be eventually chosen if separating dominates pooling from the benevolent central banker point of view. Thus, separating emerges if and only if the following two incentive constraints hold:

$$\pi^2 \geq \beta\gamma \left[L^0_{t_l}(S,0) - L^0_{t_l}(S,1)\right] \tag{7}$$

$$\beta\gamma \left(L_{t_l}(P,q) - L_{t_l}(S,1)\right) \geq \pi^2 + \lambda^*[(y-y^*)^2 - (y_p - y^*)^2] \tag{8}$$

with π and y are inflation and the output gap chosen by the benevolent central banker before the trap in case of separating equilibrium. In the pooling equilibrium, inflation is zero but the output gap y_p is positive to compensate deflationary expectations due to future zero lower bound.

Equation (7) is the incentive constraint for the conservative central banker not to mimic the benevolent central banker's policy and who suffers from loss before the trap. This incentive constraint pins down the *price of credibility* as it determines the minimum level of inflation that is required to separate from the conservative central banker. Note that this equation does not preclude signaling through deflation as the conservative central banker's preferences are symmetric.

[10]See Andrade et al. (2015) for a similar insight on the hump-shaped forward guidance with respect to private beliefs on credibility.

Equation (8) captures the incentive constraint for the benevolent central banker not to be pooled with the conservative central banker and who then enjoys the full gains from forward guidance, i.e. $\beta\gamma\left(L_{t_l}(P,q) - L_{t_l}(S,1)\right)$. This equation then corresponds to the benevolent central banker's *willingness to pay* the cost of credibility, that is signaling through inflation. In particular, as the benevolent central banker has a preference for positive output gaps ($y^* > 0$), he will always prefer signaling himself through inflation rather than through deflation.

In the end, the inflation rate before the trap should be sufficiently low not to deter too much the benevolent central banker's outcome, but it should be sufficiently high to cause losses to the conservative central banker to convince him not to mimic the benevolent central banker's policy.

3.3 Equilibrium description.

The following theorem describes the equilibrium outcome of the signaling game played by the two central bankers. In particular, it identifies when agents are able to elicit information on central bankers' type, both before and after the trap.

Theorem 4. *When the inflation bias y^* is sufficiently low, there exists $\bar{q} < 1$ such that the equilibrium is as follows:*

(i) *For any period t after the trap, the equilibrium is separating and agents are certain about the central banker's type: $q_t \in \{0, 1\}$.*

(ii) *For any period t before and during the trap:*

- *If $q \leq \bar{q}$, the equilibrium is separating ($q_t \in \{0,1\}$) and the inflation rate set by the benevolent central banker is positive before the trap (strictly positive when $q > 0$).*
- *Otherwise, the equilibrium is pooling ($q_t = q$) and the inflation rate equals 0 for both central banker types before the trap.*

Proof. See Appendix. □

When prior beliefs are too biased towards the conservative central banker ($q \leq \bar{q}$), the cost of imperfect credibility on the effectiveness of the forward guidance is so great that the benevolent central banker increases inflation before the trap as a costly signaling device. This results in a separating equilibrium.

When prior beliefs are biased towards the benevolent central banker, such costly signaling is not desirable any more, as the resulting gains in the trap are lower. This results into a pooling equilibrium. At the limit, when economic agents fully believe that the central banker is benevolent, there is no reason to avoid pooling as pooling does not limit the efficiency of the forward guidance while the incentive compatibility constraint (REV) imposes a suboptimal constraint to the optimization program. That explains why the threshold, $\bar{q}$ is strictly lower than one.

In either case, agents observe perfectly the type of central banker after the trap, except in the degenerate case where $q = 0$, i.e. when the prior belief that the central banker is benevolent equals 0.

The existence of separating equilibria. The next proposition states necessary conditions to obtain separating equilibrium and determines how the set of such separating equilibria evolves with respect to the inflation bias.

Proposition 5. *$\bar{q} > 0$ only if $\lambda^* > 0$. In addition, $\bar{q}$ is strictly increasing in y^*, when $\lambda^* > 0$ and increasing with λ^*.*

Proof. See Appendix B.5. □

Proposition 5 clarifies the condition under which the benevolent central banker signals his type (separating equilibrium). The mechanism is as follows. By raising inflation before the trap the benevolent central banker incurs a loss due to the quadratic term in inflation in its loss similar to the conservative type; but he also benefits from lower output loss because the output gap is closer to the efficient level, y^*. This latter gain is specific to the benevolent central banker, depends on the weight on output gap stabilization in the welfare function and explains why he enjoys fewer costs compared to the conservative type when raising inflation.

Inflation, beliefs and probability of ZLB. To minimize the required inflation rate, equation (7) holds with equality. This leads to the following corollary:

Corollary 6. *Before the trap, the inflation rate in the separating equilibrium, π, weakly increases with:*

(i) *The probability of a liquidity trap γ and there is no inflation when $\gamma = 0$. When $q \leq \bar{q}$, π strictly increases with γ.*

(ii) *The intensity of the liquidity trap r. When $q \leq \bar{q}$, π strictly increases with r.*

Proof. See Appendix. □

More likely and more intense liquidity traps imply a higher inflation target, as the gains for the conservative central banker type from mimicking the benevolent central banker are higher and the loss for this latter type from pooling is higher.

Do more likely liquidity traps imply more desirable signaling? How does the threshold $\bar{q}$ evolve with respect to the probability of liquidity traps? This is determined by incentive constraints (8) and (7) and, more specifically, how the term:

$$[(y - y^*)^2 - (y_p - y^*)^2] = (y - y_p)(y + y_p - 2y^*)$$

evolve with respect to γ.

Increasing inflation involves a direct welfare loss but also affects welfare through the higher level of the output gap. On the other hand, in a separating equilibrium, the benevolent central bank is better able to smooth inflation and the output gap during the liquidity trap. In turn, lower expected deflationary pressures reduce monetary accommodation before the trap (that allows to maintain inflation at 0) and, hence, the level of output is lower. In the end, in a separating equilibrium, the sign of output in comparison with a pooling one is ambiguous.

We however report the variation of the threshold $\bar{q}$ with respect to the probability γ, when this probability is sufficiently small:

Proposition 7. *For sufficiently small probability of ZLB γ, $\bar{q}$ is increasing in γ.*

Proof. See Appendix B.7. □

As a result of this proposition, the level of inflation can even be a discontinuous function of the probability of ZLB. Let q be above but arbitrarily close to $\bar{q}(\gamma)$. The inflation rate is then zero before the trap. A small increase in the probability of hitting the ZLB of $d\gamma$ leads q to be below $\bar{q}(\gamma + d\gamma)$ and so to a strictly positive inflation rate.

3.4 Time-varying probabilities of ZLB.

Suppose that before a trap occurs, there are two states associated with two different probabilities of ZLB. For simplicity, we assume that the natural rate of interest is the same in these two states. The corresponding transition matrix is:

$$\begin{pmatrix} \Pi_1 & 1 - \Pi_1 - \gamma_1 & \gamma_1 \\ 1 - \Pi_2 - \gamma_2 & \pi_2 & \gamma_2 \\ 0 & 0 & 1 \end{pmatrix}$$

with $\gamma_1 > \gamma_2$, that is the ZLB is more likely in state 1 than in state 2.

Proposition 8. *A necessary condition for the equilibrium to be separating is that level of inflation in state i implemented by the benevolent central banker satisfies the following* *condition:*

$$\pi_i^2 = \beta\gamma_i(L^0_{t_l}(S,0) - L^0_{t_l}(S,1)). \tag{9}$$

Proof. See Appendix B.8 □

Proposition 8 implies that the higher the probability of hitting the ZLB, the higher the level of signaling inflation. Furthermore, the signaling level of inflation only depends on the current probability to reach the zero lower bound and, in particular, the signaling level of inflation (π_1 and π_2) is independent of transition probabilities from state to state outside the liquidity trap (Π_1 and Π_2).

To gain intuition from this result, let us recall that, because there is always a possibility to stay in the same state before reaching the liquidity trap, equation (9) needs to be satisfied in both states. Conversely, if equation (9) holds for both states, any linear combinations of those inequalities are also satisfied ensuring separating equilibrium whatever the states history.

Remark. Note that under the conditions of Proposition 8, the equilibrium is fully separating, i.e. separating in all states before the trap and whatever the history of state realization.

4 How large is signaling inflation?

In this section, we quantitatively assess how great the level of inflation to signal the central banker's type to private agents should be.

Definition	Parameters	Value
	Structural parameters	
Discount factor	β	0.99
Risk aversion	σ	1
NKPC slope	κ	0.02
Interest rate steady state	$\bar{i}$	0.01
	Liquidity trap parameters	
Probability of ZLB	γ	0.007
Size of the shock	r	0.015
Duration of the trap	N_{trap}	12
	Social welfare parameters	
Weight of the output gap	λ^*	0.025
Inflation bias	y^*	0.0625

Table 1 – Baseline calibration

In Table 1, we report our baseline calibration. Most of structural parameters are standard and calibrated following Woodford (2003) or Eggertsson and Woodford (2003). We calibrate the frequency of the liquidity trap to a very low level of 0.007% per quarter as in Coibion et al. (2012). The size and the duration of the trap are consistent with the estimates of Barsky et al. (2014) for the post world war II US economy. The trap is hence rare but when occurring severe and relatively long. Finally, concerning the social welfare parameters, there is no clear consensus in the literature about the weight of the output gap, so we choose to calibrate this parameter to an intermediate value of 0.025. Finally, we calibrate the inflation bias, y^* to 0.0625 consistently with Woodford (2003). We provide for many alternative calibrations below.

Optimal signaling inflation. Table 2 reports the level of optimal signaling inflation and the threshold for prior beliefs above which pooling emerges, $\bar{q}$.

In our baseline specification the inflation level is slightly lower than 2 while the threshold belief, $\bar{q}$ is around 0.3. This suggests that the inflation level is significantly positive even if below the often chosen inflation target of 2%. However, signaling is only required if prior beliefs are very biased toward the conservative central banker. In Appendix A, Figure 3 displays inflation, the output gap and the nominal interest rate when the economy is hit by a liquidity trap during 12 periods for different prior beliefs in the baseline calibration.

Probability of the ZLB is a key determinant of the inflation level. For instance, a probability of 5% per quarter leads to more than 4% inflation. Equation (7) proves that the signaling level of inflation is proportional to the root square of the probability of ZLB:

$$\pi \geq \sqrt{\beta\gamma\left[L^0_{t_l}(S,0) - L^0_{t_l}(S,1)\right]},$$

and, so, the marginal effect of a change in probability when the probability of hitting the ZLB is close to 0 is large.

Besides, the threshold $\bar{q}$ below which separating equilibria arises is very sensitive to both the output gap weight in the central banker's objective function λ^* and the inflation bias y^*. In particular, if one of these two preference parameters are sufficiently large, then separating equilibrium emerges whatever the beliefs. Conversely, the signaling level of inflation decreases only modestly with this parameter. This is consistent with the fact that these parameters determine how much the benevolent central banker's preferences are different from the conservative one while they do not enter into the latter one's preferences.

Risk aversion is also crucial as it directly affects the effectiveness of forward guidance. The higher the risk aversion, the lower the inflation, as the gain from forward guidance

Cases	Inflation level, π	threshold beliefs, $\bar{q}$
baseline	1.5	0.30
Trap spell		
Shorter ($N_{TRAP} = 8$)	0.5	0.10
Longer ($N_{TRAP} = 16$)	3.4	0.14
Frequency of the ZLB		
Less frequent ($\gamma = 0.005$)	1.3	0.31
Slightly more frequent ($\gamma = 0.02$)	2.5	0.30
More frequent ($\gamma = 0.05$)	4.0	0.32
Severity of the ZLB		
Less severe ($r = 0.04$ annualized)	1.0	0.33
More severe ($r = 0.08$ annualized)	2.0	0.29
Risk aversion		
Low risk aversion ($\sigma = 1.5$)	0.9	0.21
High risk aversion ($\sigma = 1/2$)	3.7	0.16
output gap weight		
Small weight ($\lambda^* = 0.0025$)	1.5	0.06
Large weight ($\lambda^* = 0.05$)	1.5	1
Inflation bias		
Close to zero ($y^* = 0.00625$)	1.5	0.27
Large bias ($y^* = 0.625$)	1.5	1

Table 2 – Optimal signaling inflation

is lower. Inflation goes from 0.9% to 3.7% for sigma between 1.5 to 0.5. Given the uncertainty and the importance of this parameter, the choice of a standard logarithmic utility function ($\sigma = 1$) appears a neutral stance.

Signaling inflation and inflation target. We have assumed that the optimal steady state level of inflation is zero in the absence of inflation bias. However, there may be reasons to target a positive level of inflation in the welfare function. The European Central Bank, for instance, cites three reasons to target a positive rather than a zero level of inflation: limit the probability of a liquidity trap, take into account possible positive bias in the measurement of price level changes and a sufficient buffer to avoid deflation in one country in the euro area due to inflation differentials. These last two reasons are beyond the scope of this paper. We hence wonder whether our results would be affected by such a positive price stability definition. To answer this question, we modify the preferences of central bankers by adding a positive inflation target, $\bar{\pi}$ as follows:[11]

[11]We do not consider cases in which one of the central bankers targets a positive level and not the other as this would generate de facto a strong incentives not to imitate the other central banker.

$$\tilde{L}_t^{\lambda} = \frac{1}{2} E_s \sum_{t=s}^{\infty} \beta^{t-s} \left[(\pi_t - \bar{\pi})^2 + \lambda \left(y_t - y^* \right)^2 \right]$$

We, then, can rewrite the incentive constraints with a positive inflation target and compute the threshold $\bar{q}$ and the level of inflation, π. Assuming that the inflation target $\bar{\pi}$ is 2% in annual term, we find the same level of threshold, $\bar{q} = 0.3$ and a level of inflation needed to signal the benevolent central banker type around 3.4% in our benchmark calibration. Thus, our main result does not change much when including a positive inflation target.

It is worth noticing that in this exercise we do not take into account the impact of the inflation target on the probability of the liquidity trap as the shock is sufficiently large to always generate a zero lower bound once it hits the economy. However, the probability of ZLB we consider is consistent with historical data for the US economy that has experienced positive average inflation.

Discounted Euler equation. The level of signaling inflation depends on the gains of forward guidance at the zero lower bound. Yet, as argued by Del Negro et al. (2013) forward guidance may be less effective compared to what is predicted by the model.[12] In the following, we integrate a discounted Euler equation as introduced by McKay et al. (2015) to account for this lower efficiency:[13]

$$y_t = \alpha E_t y_{t+1} - \frac{\zeta}{\sigma}(i_t - E_t \pi_{t+1} - r_t^n), \tag{10}$$

where both α and ζ parameters are below 1 and hence reduce both the severity of a natural rate of interest shock and forward guidance effectiveness.

For comparability reasons, we calibrate all parameters as in the baseline calibration of McKay et al. (2015). The risk aversion is set to $\sigma = 2$, duration of the trap is set to 20, the size of the shock is -2% in annual term. The discounting parameters are calibrated as in the benchmark case of McKay et al. (2015): $\alpha = 0.97$ and $\zeta = 0.75$.

In this case, for a probability of ZLB as in Coibion et al. (2012): inflation is 0.5. This result is not due to limited effectiveness of forward guidance but rather due to the small recession and hence the small loss generated by the shock they consider, as well as the

[12] Note that an intermediate level of q can also explain lack of effect of forward guidance in our model. See Andrade et al. (2015) for the connection between private beliefs and the so-called forward guidance puzzle.

[13] They actually show that incomplete markets and heterogeneous agents can reduce the efficiency of forward guidance and they then show that the discounted Euler equation is a shortcut to reproduce their findings in a representative agents formulation.

limited impact due to the discounted Euler equation and the high risk aversion. Indeed, in the discretionary case output falls by 5% and inflation by less than 4%.

We thus can consider this calibration as a very conservative one that gives a lower bound for the signaling inflation level. Even if this level is rather small, it does not mean that it is always negligible. When the probability of ZLB is high (5% per quarter) and the severity of the crisis is more in line with estimated values (Barsky et al., 2014), signaling inflation can become significantly non zero (4% in this particular case). This suggests that even if the exact level of inflation needed to ensure credibility may change from one specification to the other, the level of inflation can easily be greater than two when the liquidity trap becomes more likely and/or more severe.

5 Other sources of signaling and reputations.

In this section, we consider three extensions of our benchmark model that may affect either benevolent or the conservative central banker's incentives and hence the need for signaling. We first introduce cost-push shocks both before and after the liquidity trap. Our main insight is that cost-push shocks are usually too small to lead to separating *per se*. We then study repeated liquidity trap by introducing a probability of hitting the zero lower bound after the first liquidity trap. We show that if the probability of future liquidity trap is sufficiently high, separating is achieved by the means of an infinitely small costly signal. Finally, we introduce a third central banker type to allow the possibility of being considered wrongly as an inflation-biased discretionary central banker when raising inflation. We prove that our main result goes through as long as private agents beliefs are sufficiently biased toward the conservative central banker's type in case of pooling.

5.1 The role of cost-push shocks.

In this subsection, we investigate to what extent the presence of cost-push shocks, i.e. shocks leading to an output gap/inflation trade-off, can help the benevolent central banker to signal his type. Our first insight is that cost-push shocks may reduce the need of signaling if they occur before the trap, but such a signaling is still required when cost-push shocks remain sufficiently small. We then show that cost-push shocks are unlikely to be sufficiently large to replace a costly signal as described in this paper through two quantitative experiments. Finally, we discuss the role of cost-push shocks after the trap and we highlight that they give more punishment ability to the private sector and so they provide relatively more credibility to the benevolent central banker - or more generally to central bankers having a positive weight on the output gap stabilization objective in their preferences - than to the conservative central banker.

We consider cost-push shocks u_t affecting the (NKPC) as follows:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + u_t.$$

Before the trap. As the monetary policy response to cost-push shocks cannot ensure perfect inflation and output gap stabilization, the corresponding optimal response also requires credibility. In the end, the optimal response to such shocks, if sufficiently large, may potentially trigger separation so that signaling through higher inflation compared with perfect information is no longer necessary.

This leads to the following testable implication of our model:

Claim 1. *The more the economy experiences cost-push shocks before a liquidity trap, the less the benevolent central banker has to costly signal his type and, thus, the more efficient is forward guidance.*

Eventually, the question whether cost-push shocks can be a relevant signaling device for the benevolent central banker is a quantitative issue.

Experiment 1. We now prove that standard estimated shocks are not sufficient *per se* to signal central banker's type. To do this, we calibrate the size of the cost-push shocks along the line of Lubik and Schorfheide (2004) who estimate a model very similar to the one considered in this paper and Justiniano et al. (2010) who estimate a more realistic DSGE model; all other parameters are calibrated as in our benchmark model. In both cases, we simulate the impact of a one standard deviation cost-push shock to inflation and compute the loss incurred by the conservative central banker (basically, the discounted sum of squared inflation) to imitate the Ramsey allocation.

The general specification for the cost-push shocks is an ARMA(1,1) process defined as follows:

$$u_t - \rho_p u_{t-1} = \sigma_p(\epsilon_t - \theta_p \epsilon_{t-1}),$$

where parameters ρ_p, σ_p and θ_p are calibrated following estimates by Lubik and Schorfheide (2004) and Justiniano et al. (2010). We report both calibrations in Table 3. In this table we also compute the discounted inflation loss due to a one-standard deviation cost-push shock in the Ramsey allocation, which also coincides with the loss of the conservative if he imitates the benevolent central banker before a liquidity trap. These losses have to be compared with the gain for the conservative central banker to be pooled with the benevolent central banker due to expectations of future forward guidance: 820.10^{-6}.[14] Therefore, a one-standard deviation cost-push shock is far from being sufficient to trigger separating. To trigger such a separating, the cost-push shock has to be 16 times larger than the estimated standard deviation at least. Eventually, cost-push shocks are unlikely to be quantitatively sufficient to generate separating and raising inflation seems to remain optimal even if cost-push shocks hit the economy.

	Lubik and Schorfheide (2004)	Justiniano et al. (2010)
	Calibration	
ρ_p	0.85	0.94
θ_p	0	0.77
σ_p	0.32	0.56
	Loss generated by a 1-standard deviation cost-push shock	
	17.10^{-6}	$7.9.10^{-6}$
	Size of the cost-push shock needed to get separating (in std)	
	16	24

Table 3 – Size of cost-push shocks and signaling

Experiment 2. We now run a second experiment attributing any inflation departure from its average to cost-push shocks. While this historical decomposition may appear far-fetched, it provides an upper bound on non-costly signaling due to cost-push shocks. In addition, let us assume that private agents infer the type of the central banker from the past. They can identify a conservative from a benevolent central banker by computing the weighted sum of past inflation prior to the liquidity trap. Figure 1 plots these weighted sums expressed as the constant inflation level that would lead to the same loss for a conservative central banker during the Greenspan's term (in blue) and during the Bernanke's term prior to 2008Q4 (in red) and compare them to the signaling inflation rate computed in Section 4. These constant inflation levels are computed as follows:

$$\bar{\pi} = 400\sqrt{(1-\beta(1-\gamma))\sum_k [\beta(1-\gamma)]^k \pi_k^2},$$

where π_k denotes the observed quaterly change in Consumer Price Index for the US and $\bar{\pi}$ is the level of inflation that would lead to the same level of loss in annualized terms. The vertical line refers to the change of the chairman of the Fed in February 2006.

Compared to the signaling inflation rate from our benchmark scenario, observed volatility of inflation was not sufficient to generate a clear signal in favor of the benevolent

[14]This loss is 640.10^{-6} in a time-0 perspective instead of a timeless perspective.

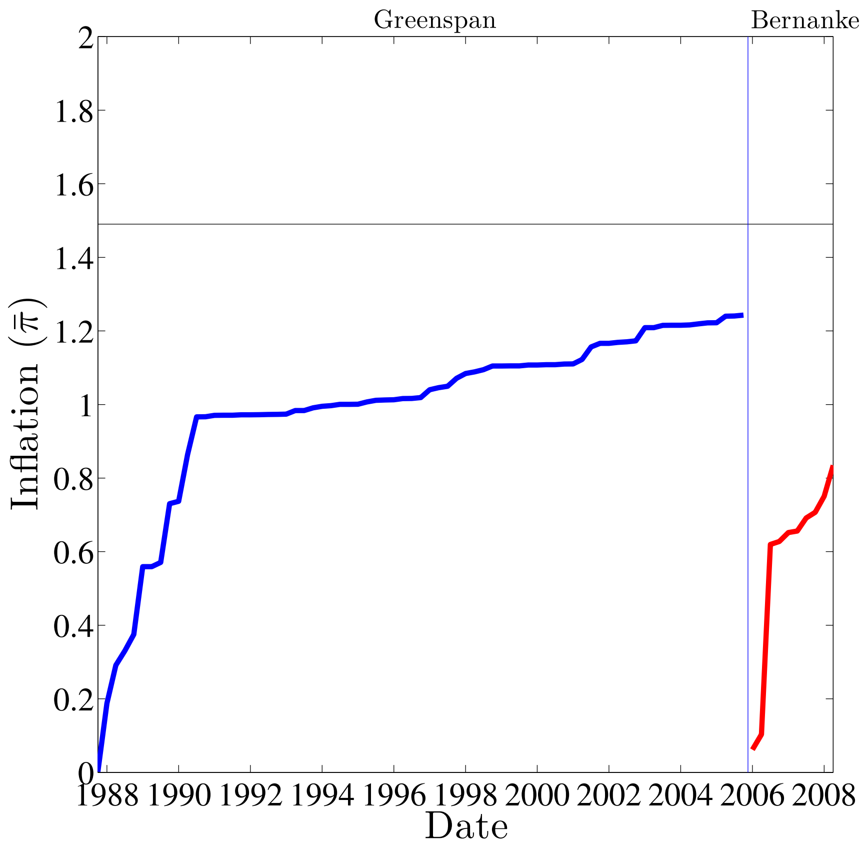

Figure 1 – Observed cumulated losses of a conservative central banker from past inflation fluctuations.

central banker during the Greenspan and Bernanke tenures. Looking backward, private agents were not able to disentangle between conservative or benevolent central bankers since a conservative central banker had still sufficient incentives to imitate the benevolent one.

This implies that our first experiment's finding does not only result from the low level of output/inflation trade-off during the great moderation but also from the low level of inflation volatility itself.

In the end, both experiments prove that because the Great Moderation was a period of low inflation, low inflation volatility but also of limited inflation/output trade-off (see Justiniano et al., 2013), additional signaling through higher inflation would have been required to provide central bankers with full credibility to implement forward guidance when the economy fell into the liquidity trap during the Great Recession.

After the trap. Cost-push shocks occurring after the trap lead to very different responses depending on the central banker's weight on inflation. If the central banker is conservative, he will perfectly stabilize inflation, no matter the size of the cost-push shock. Conversely, when the weight on the output gap in the objective function is positive, the central banker would like to smooth the effect of the shock over multiple periods. Yet, such an optimal response requires credibility as it is time-inconsistent: the central banker has the temptation to re-stabilize inflation once the cost-push shock is over - this is known as the *stabilization bias*. As a result, private agents can punish the central banker by not believing in his promise to smooth inflation over multiple periods.

This possibility for the private sector to punish the benevolent central banker results in stronger reputation concerns (i.e. higher punishment capacity) and so, relatively more credibility. In contrast, the conservative central banker does not care about managing his credibility. Thus, cost-push shocks after the trap mirror the inflation bias in our benchmark analysis.

5.2 Repeated liquidity traps.

Our benchmark model rules out the possibility of future subsequent liquidity traps. Such an assumption makes the signaling problem tractable as central bankers are ultimately separated. Yet, traps can still occur with some probabilities in the future. We show in this subsection that repeated traps may reduce the need of signaling. Yet, when conservative central bankers can be short-term, our results are unchanged compared to our benchmark model in the presence of repeated traps.

Let us suppose that the probability of a new liquidity trap after the end of the first liquidity trap is γ'. We do not necessarily assume that the natural rate of interest shock is i.i.d. (which would correspond to $\gamma = \gamma'$) to allow for distinguishing between a high likelihood of a liquidity trap in the short run and the probability of such traps in the long run.

Sufficiently large discount factors. One may easily obtain some form of a Folk theorem: when the probability of future liquidity traps, γ', is strictly positive and economic agents are sufficiently patient, i.e. β sufficiently large, the conservative and the benevolent central bankers can both sustain their optimal policy under commitment under perfect information. This is how the result by Nakata (2014) can be interpreted.

If both types can sustain their first best policy, that is the one obtain under commitment, there are no more incentives for any central bankers to imitate any other types. In this case, any infinitely small signal will be sufficient to signal. For example, as the two policymakers share different objectives, the benevolent central banker should choose a level of inflation, $\epsilon > 0$, where ϵ is a small positive scalar. There is thus no need for a (significant) costly signal.

This result does not contradict Corollary 6 and proposition 8 in which we prove that signaling inflation increases with the probability of the liquidity trap. Indeed, in these results we study a variation of the probability of hitting for the first time the liquidity trap, γ, while in this section we investigate the role of future liquidity trap in the ability of the conservative central banker to sustain forward guidance through a punishment on future forward guidance policies.

On the other side of the spectrum, when the probability of subsequent liquidity traps is 0, we are back to our benchmark case, where the conservative central banker does not have any incentives to sustain forward guidance. Thus, for intermediary values, one can expect that the conservative central banker may sustain some, if not entirely, the inflationary boom so as to better smooth inflation. This leads to the following testable implication:

Claim 2. *The lower the probability of future liquidity traps, the more willing the benevolent central banker will be to signal his type and the greater the signaling inflation will be and, conversely, in the absence of signaling, the less efficient is forward guidance.*

Finite terms. An alternative reason why repeated traps may not lead the conservative central banker to be able to sustain forward guidance is finite horizons for the central banker (e.g. because of finite terms). The game would then also be between long-term central bankers (that maximize infinite-horizon objective functions) against short-term ones (that maximize finite-horizon objective functions, in the spirit of the literature on reputation effects in games where long-run players face sequence of short-run opponents (see the seminal contribution of Kreps and Wilson, 1982; Milgrom and Roberts, 1982).

In the end, the benevolent central banker has to signal himself from being a short-term conservative central banker and not only from being a long-term central banker.

The only elements that we need to show for ensuring that we can extend our reasoning to this more complex game is that the short-term conservative central banker implements 0 inflation after the trap and that the incentive constraint corresponding to this central banker is not too different compared with our original game.

The first point is ensured by the standard backward induction argument: as the short-term central banker is going to implement 0 inflation in the last period of his term, the same central banker is always going to implement a 0 inflation rate after the trap.

For the second point, we may even showthat the no-imitation constraint is unmodified in this game. Indeed, in such a situation the constraint writes:

$$\left[1 + \beta(1-\gamma) + ... + \beta^T(1-\gamma)^T\right]\pi^2 \leq \cdots \tag{11}$$

$$\cdots \beta\gamma\left[1 + \beta(1-\gamma) + ... + \beta^T(1-\gamma)^T\right]\left[L^0_{t_l}(S,0) - L^0_{t_l}(S,1)\right], \tag{12}$$

where $L^0_{t_l}(S,0)$ and $L^0_{t_l}(S,1)$ have to be adjusted for the case of finite horizon. Such a constraint can, in turn, be rewritten as:

$$\pi^2 \leq \beta\gamma \left[L^0_{t_l}(S,0) - L^0_{t_l}(S,1)\right], \tag{13}$$

which is the same constraint as in the case of the infinite horizon term.

In the end, signaling does not deal only with signaling from long-term conservative central bankers but also from short-term ones that may find profitable to be taken as benevolent in the short-run. This highlights another dimension along which central bankers' preferences may differ from the society's welfare.

5.3 Inflation-biased central banker.

Raising inflation to ensure credibility may also be costly as private agents may mistakenly believe that the central banker acts under discretion with an inflation bias. In this subsection, we investigate the incentives of benevolent central banker to raise inflation when such an inflation can also be taken as a sign of lack of credibility.

To this purpose, we assume that central bankers may also differ with respect to their discount factor. To simplify our problem, we assume that the discount factor can take two values: a high one as in the benchmark model, β and a low one equals to 0 for simplicity. We call this latter type, the inflation-biased central banker. This latter type cannot sustain the Ramsey equilibrium whatever the weight on output gap stabilization, λ.

We show that the benevolent central banker can be optimally pooled with the inflation-biased central banker. In addition, the former's incentive to be pooled with the inflation-biased central banker *ex ante* can be strengthened by his ability to dis-inflate the economy once the liquidity trap ends.

The model. On top of the two types that we already considered, we now consider that central bankers' preferences can also be $\beta = 0$ and $\lambda \in]0,\infty)$.[15] We thus now denote by a bi-dimensional vector the central banker's type, (β, λ). The prior beliefs that the central banker is benevolent, (β, λ^*), is denoted by q, the one that he is conservative, $(\beta, 0)$, by q_{cons} and there is a probability distribution function g so that the prior belief that central banker is of type $(0, \lambda)$ is $g(\lambda)$. Prior beliefs have to sum to one so, $q + q_{cons} + \int_0^\infty g(\lambda)d\lambda = 1$.

Note that the central banker with a positive λ but with a zero discount factor ($\beta = 0$) cannot act strategically and so, it would be easy for the benevolent central banker to always separate himself from the inflation-biased central banker, just by creating noise in

[15]We exclude $\lambda = 0$ because it would be redundant with the conservative central banker.

the *ex ante* rate of inflation. Yet, such strategy is not robust to positive discount factors for the inflation-biased central banker and, in order not to lose generality, we impose that the benevolent central banker adopts a constant rate of inflation before a trap occurs.

5.3.1 Incentive constraints.

Positive inflation prior to the liquidity trap. As long as the benevolent central banker implements positive inflation, π, prior to the liquidity trap, there always exists an inflation-biased central banker leading to the same level of inflation. The benevolent central banker is hence pooled with a discretionary central banker $(0, \tilde{\lambda})$:

$$\tilde{\lambda} = \kappa \frac{y^* - \pi\kappa}{(\pi(1 - \beta(1 - \gamma)) - \beta\gamma\pi_l)}$$

where π_l is the level of inflation during the trap.

As a result, before the trap, beliefs over central bankers with no discount factor go to zero except the one corresponding with $\lambda = \tilde{\lambda}$. The equilibrium is never fully separating but only partially. However, the exact reallocation of beliefs is indeterminate and many equilibria are compatible with our Bayesian Perfect Equilibrium definition. We thus parametrize beliefs once such a partial pooling emerges. We denote by q' and $1 - q'$ the beliefs associated with the benevolent and the $(0, \tilde{\lambda})$-type central bankers.[16]

After the trap, by choosing a different level of inflation, the benevolent central banker can distinguish himself from the $(0, \tilde{\lambda})$ central banker so that the equilibrium becomes fully separating as soon as the trap is over.

Zero inflation case. When the benevolent central banker sets a zero inflation rate prior to the trap, economic agents hence form (indeterminate) beliefs over only two central banker's types: either the benevolent type, q'', or the conservative's one, $1 - q''$. Indeed, we exclude that an inflation-biased central banker attributes zero weight on the output gap stabilization objective.

Incentive constraint of the benevolent central banker. We denote by $L_{t_l}(0, q'')$ the present discounted loss of the benevolent central banker at the in the zero inflation case associated with beliefs over the benevolent type, q'' and by $L_{t_l}(\pi, q')$ his loss if the benevolent central banker is pooled with the $(0, \tilde{\lambda})$ central banker and when beliefs over the benevolent type is q'. The benevolent central banker prefers setting a positive inflation

[16] Because of the Bayes' law, beliefs satisfy: $1 - q \geq 1 - q' \geq g(\tilde{\lambda})$ and $q' \geq q$.

level prior to the liquidity trap if the gain in terms of forward guidance, $L_{t_l}(0, q'') - L_{t_l}(\pi, q')$, compensates the loss due to positive inflation prior to the trap, hence:

$$\beta\gamma \left(L_{t_l}(0, q'') - L_{t_l}(\pi, q')\right) \geq \pi^2 + \lambda^*[(y(0, q'') - y^*)^2 - (y(\pi, q') - y^*)^2], \qquad (14)$$

where $y(0, q'')$ and $y(\pi, q')$ are the output gap if the benevolent central banker is pooled with the conservative and the inflation-biased central bankers respectively. A rise in inflation, π trigger two effects. First, it increases the loss before the liquidity trap through higher inflation. Second, it affects expectations at the liquidity trap through a change in $\tilde{\lambda}$. This second effect can potentially reduce or enhance the severity of the deflation during the trap.

Incentive constraint of the conservative central banker. The conservative central banker imitates the benevolent central banker if the gains from forward guidance dominates the costs of inflation prior to the liquidity trap. Therefore, the relevant incentive constraint is:

$$\pi^2 \geq \beta\gamma \left(L^0_{t_l}(\pi, q') - L^0_{t_l}(0, q')\right). \qquad (15)$$

Contrary to Section 3, the loss of the conservative if mimicking the benevolent central banker, $L^0_{t_l}(\pi, q')$, depends on the inflation level π chosen by the benevolent central banker. Indeed, the third central banker type characterized by $(0, \tilde{\lambda})$ depends on the inflation level π and he matters for the efficiency of the forward guidance. For instance, if π is low, the benevolent central banker will be pooled with a low $\tilde{\lambda}$ type producing low level of inflation after the liquidity trap making the liquidity trap more costly, on the contrary high level of π and hence high level of $\tilde{\lambda}$ will reduce the cost of the trap. In other words, $L^0_{t_l}(\pi, q')$ is decreasing with π for a given q'. Finally, if economic agents do not consider inflation-biased central banker in their beliefs, $q' = 1$, then inequalities (15) and (7) are identical.

5.3.2 Equilibrium description

Proposition 9. *The equilibrium is as follows*

(i) *For any period t after the trap, the equilibrium is separating and agents are certain about the central banker's type.*

(ii) *For any period t before and during the trap:*

- *If $q'' \leq \tilde{q}$, the equilibrium is partially separating: the benevolent central banker is pooled with the $(0, \tilde{\lambda})$ central banker but separated from the conservative one. The inflation rate set by the benevolent central banker is positive before the trap.*
- *Otherwise, the benevolent central banker is pooled with the conservative central banker and the inflation rate equals 0 before the trap.*

The introduction of central bankers with zero discount factor does not qualitatively modify results from Theorem 4. The benevolent central banker desires not to be mistakenly considered as conservative if prior beliefs are strongly biased toward this former type (in case of pooling). In this case, once again inflation acts as a costly signal for the benevolent central banker.

Our findings lead to the following testable implication:

Claim 3. *The more the central banker has the past reputation to be inflation-biased (the larger is q'), the lower the gains to be separated from the conservative ones and so, the lower the efficiency of forward guidance.*

Connection with Coibion et al. (2012). In some equilibria, learning due to signaling leads to a degenerate distribution peaked on the inflation-biased central banker, i.e. $q' = 0$. In this case, signaling is impotent to reveal the credibility to agents that attribute inflation only to inflation bias. In the end, *ex ante*, such a situation corresponds to the optimal inflation target when the central banker acts under discretion as in Coibion et al. (2012). They show that the buffer stock effect of higher inflation rate already explain an inflation target of at least 1.5% in their benchmark case.

Inflation-biased central bankers with non-zero discount factor. A positive discount factor of inflation-biased central bankers might reduce the ex ante incentive of the benevolent central banker to be pooled with them. Indeed the ex-post cost of disinflation policy is higher because inflation-biased central bankers will choose to strategically mimic the benevolent central banker's policy after the trap. Finally, the disinflation will require higher real rates leading to larger fall of the output gap. However, in the long run, as long as β is sufficiently low, separating will emerge.[17]

[17] Otherwise, it means that the discount factor is sufficiently large to make the Ramsey allocation sustainable and inflation-biased central banker does not differ from the benevolent central banker anymore.

6 Extensions and robustness

In this section, we extend and discuss our model along two dimensions to check for robustness. First, we determine the optimal policy of the benevolent central banker when he starts being in office at a finite date. This allows us to study the optimal time-varying signaling inflation in a more realistic setup. Second, we discuss possible contracting with the central banker to avoid costly signaling.

6.1 Time-varying signaling inflation.

In this subsection, we extend our results to the case where the game begins at a finite date and where the central banker is not forced to signal at each date before the trap. Indeed, in Section 3, we assume that under separating, the benevolent central banker makes sure the incentive constraint (REV) is satisfied at any date prior to the liquidity trap. This allows to have a static game prior to the liquidity trap and hence a constant level of signaling inflation. To study the optimal dynamics of inflation leading to separation, we relax the following assumptions: first, the signaling game starts at date 0 instead of date $s \to -\infty$; second, the incentive constraint (REV) needs to be satisfied only at date 0. Our main finding is that signaling occurs with a large peak of inflation at date 0 and persistent inflation afterward (more persistent than the date 0 optimal solution under perfect information).

The resulting problem for the benevolent central banker that separates from the conservative one is:

Problem 5. Given the sequence of beliefs: $q_t = 1$ for all periods,

$$\min_{\{i_t\}_{t\geq 0}} L_0,$$

under the time-0 incentive compatibility constraint (REV) and constraints (NKPC), (EE) and (ZLB) at each date $t \geq 0$.

The first order conditions until a trap occurs are the following:

$$\pi_t + \mu_t - \mu_{t-1} - \pi_t \left(\beta(1-\gamma)\right)^t \eta = 0, \tag{16}$$

$$\lambda^*(y_t - y^*) - \kappa\mu_t = 0. \tag{17}$$

where η is the Lagrange multiplier associated with (REV) satisfied at $t = 0$ and the initial Lagrange multiplier $\mu_{-1} = 0$. Inspecting these equations suggests that the signaling motive (which affects these equations through the term $\pi_t \left(\beta(1-\gamma)\right)^t \eta$) has a decreasing effect that converges to 0 when time t goes to infinity.

Figure 2 plots the dynamics of inflation under separation (thin black line). We contrast its evolution with the benchmark case in which inflation is constant prior to the liquidity trap (dashed line) and with the date-0 optimal monetary policy in full information (red thick line).

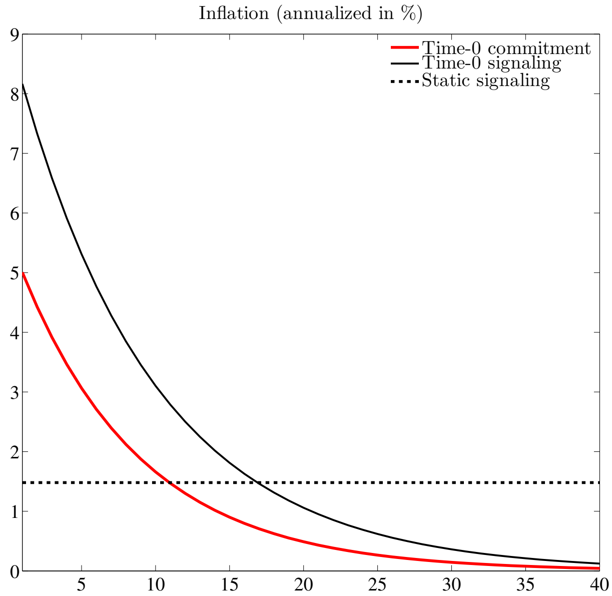

Figure 2 – Inflation dynamics under separation

Our main observation is that signaling occurs with a peak of inflation at the initial date of the policy decision (date 0) and then leads to a persistent rate of inflation. More specifically, with our benchmark calibration, the initial level of signaling inflation is 8% (in annual term) while it is only 5% under perfect information. Inflation decreases afterward but remains above 1.5% - the baseline level of signaling inflation - for 4 years.

In the end, signaling through a inflation peak is optimal from a date-0 point of view. The main reason is that this allows inflation to converge to 0 in the long run as in the perfect information case, thus reducing the welfare cost of signaling. At the same time, inflation should initially be higher to ensure a sufficiently large cost of imitation for the conservative central banker, thus explaining the large rate of inflation at the initial peak.

Importantly, the signaling strategy through peak relies on the assumption that one period of revelation is sufficient to ensure that types will be perfectly identified in *any* future period and, in particular, in the very long run.

6.2 Contracting with the central banker

The main problem associated with the implementation of the Ramsey allocation is the alignment of the central banker's preferences with welfare. And so, rather than relying on inefficient policies such as signaling through inflation, one can argue that private agents can also contract on the central banker's compensation to provide credibility. Such a performance contract has been shown to implement the second best policy in the absence of liquidity traps (see Walsh, 1995; Svensson, 1997).

To start with, note that if the state-space of contracts is sufficiently rich and if agents can commit to contracts, then the Ramsey allocation can be implemented whatever the preferences of the central banker. In particular, the society can write a contract with the central banker to reveal his type and that can also implement forward guidance once the trap is over. For example, a contingent transfer that gives an infinite amount of money to the central banker as long as he follows the Ramsey policy can implement the Ramsey allocation whatever the initial preference of the central banker. However, as forward guidance requires history dependence, this contract is also history dependent.

If we restrict the set of contracts to more plausible history independent contracts, then a transfer that would give the proper inflation bias to the conservative central banker can restore his credibility. However this type of transfer does not work in two important cases.

First, if the conservative central banker's discount rate is sufficiently low, giving an inflation bias to him will not increase his credibility but rather translates into higher inflation both before and after the trap. Thus, this transfer does not implement the Ramsey allocation and may be inefficient.

Second, if the responsiveness of the central banker to transfers is private information, the exact level of transfer needed to restore credibility is unknown. More precisely, let suppose that the central banker's preference is of the form:

$$\Gamma \pi_t^2,$$

where Γ is private information (it does not appear in any of the policy implemented by the conservative central banker). As a result, when augmented with a transfer that creates an inflation bias, these preferences are:

$$\Gamma \pi_t^2 + \lambda^*(y - y^*)^2.$$

Prior to the liquidity trap and under timeless perspective, observing the nominal interest rate will not be sufficient to pin down the exact weight on output λ^*/Γ and hence credibility is not guaranteed.

7 Conclusion.

This paper argues that raising inflation is a way to solve an asymmetric information problem concerning the central banker's motive to stabilize inflation: a central banker that stabilizes inflation because of his credibility has an incentive to distinguish himself from a conservative central banker acting under discretion. Indeed, this latter type does not follow through forward guidance policy once the shocks leading to the liquidity trap is over and hence being pooled with this central banker type reduces the efficiency of forward guidance policy. Our main result is that signaling can be achieved solely by raising inflation. Quantitatively, we show that the benevolent central banker should optimally raise the inflation rate above standard inflation targets followed by central banks around the world especially if the probability, the length or the severity of the liquidity trap is high.

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A The effect of belief on credibility - For Online Publication.

Figure 3 plots inflation, the output gap and the nominal interest rate when a 12-period liquidity trap hits the economy in period 1. Thin, thick, dashed and thick with crosses denote the dynamics of the economy under the benevolent central banker if the prior beliefs on the credibility of the central banker, q, are respectively 1, 0.5, 0.05 and 0.

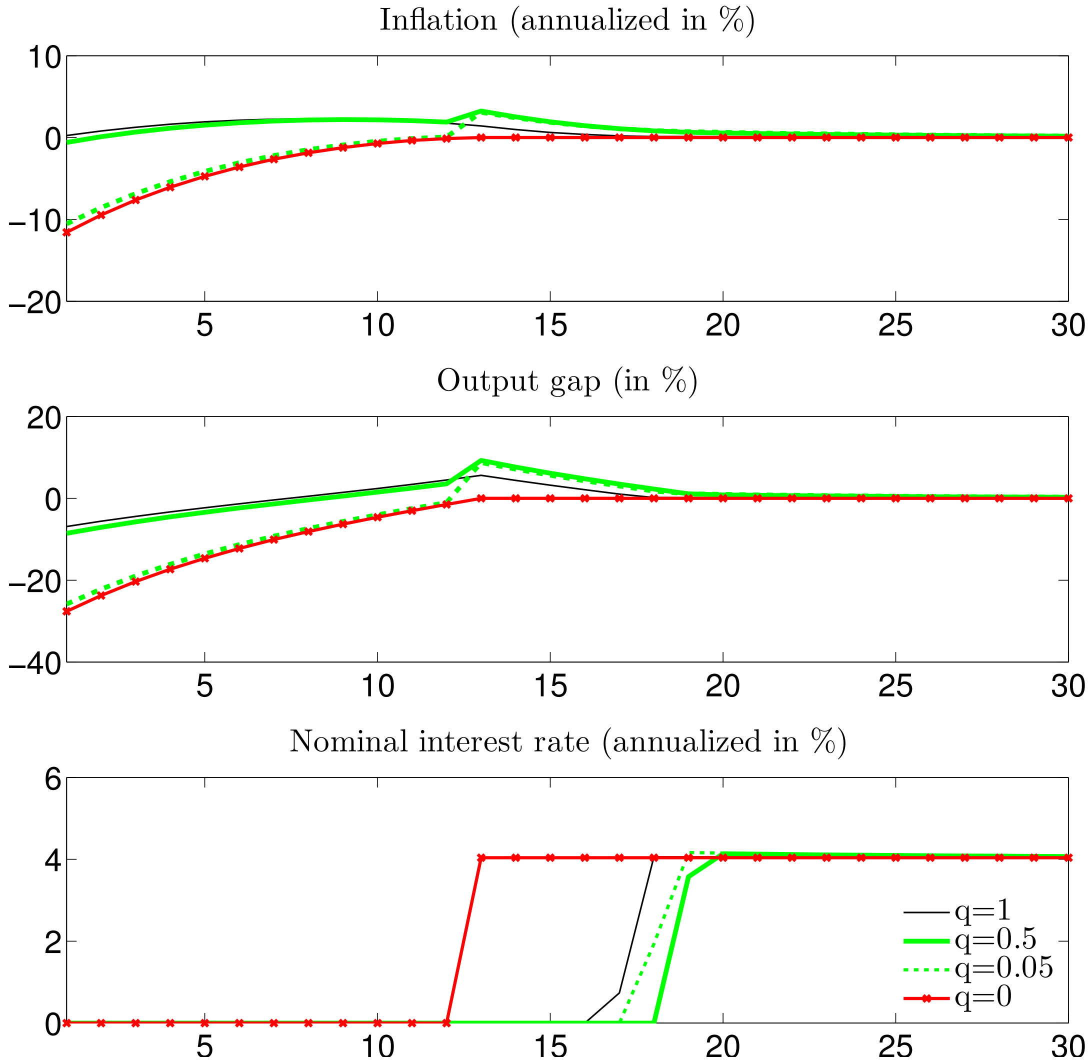

Figure 3 – Liquidity trap and prior beliefs

When $q = 1$, the dynamics of the economy is not affected by imperfect information and the efficiency of the forward guidance is maximal. Inflation is close to 0 and the boom needed after the liquidity trap is relatively small. The forward guidance lasts 5 periods. In the contrary, when private agents believe that the central banker is the conservative one, the forward guidance is useless and the dynamics is the discretionary one. The recession is deep and long lasting. For intermediate cases, the forward guidance is longer than in the credibility case ($q = 1$) leading to a large boom after the trap ended. The efficiency of the forward guidance is quite effective except when beliefs are very biased toward the conservative central banker. At the limit, when q is very close to 0, the cost of the forward guidance after the trap dominates the gain during the trap and the duration of the forward guidance diminishes down to 0.

B Proofs - For Online Publication

B.1 Proof of Proposition 1.

Given the Ramsey allocation, we can observe that inequality (5) is neither satisfied for $\lambda = 0$ nor for $\lambda = \infty$. In the former case positive inflation after the liquidity trap is impossible to sustain. In the latter case, asymptotic zero inflation and zero output gap after the liquidity trap leads to an infinite loss on the left-hand-side of inequality 5, while the best deviation leads to finite loss (right hand side).

As a result, the set of λ for which inequality (5) holds is bounded. In addition, as inequality (5) is a weak inequality, this set is closed and so it is a compact, if non-empty, or it is the empty set. In the former case, there exists a lower bound of this set that we denote by $\underline{\lambda}_D$. Then, for any $\lambda < \underline{\lambda}_D$, inequality (5) is not satisfied. Symmetrically, we can define $\overline{\lambda}_D$ as the supremum of λ values satisfying inequality (5).

B.2 Proof of Proposition 2.

Solution to problem 5. The first order conditions until a trap occurs are:

$$\pi_t + \mu_t - \mu_{t-1} - \pi_t \sum_{\tau=s}^{t} \beta^{t-\tau}(1-\gamma)^{t-\tau}\eta_\tau = 0, \tag{18}$$

$$\lambda^*(y_t - y^*) - \kappa\mu_t = 0. \tag{19}$$

where η_τ is the Lagrange multiplier associated with the revelation constraint (REV) at date τ. We crucially assume that the optimization takes place at $t = -\infty$. Assuming that Lagrange multipliers μ_t and η_t converge prior to the liquidity trap toward μ and η respectively, the stationary solution must solve:

$$\pi\left(1 - \frac{1}{1-\beta(1-\gamma)}\eta\right) = 0, \tag{20}$$

$$\lambda^*(y - y^*) - \kappa\mu = 0. \tag{21}$$

As a result, $(1-\beta(1-\gamma)) = \eta$. This also implies that π_t before the trap is constant and determined by (REV), that can be rewritten as follows:

$$\pi^2 \geq \beta\gamma\left[L^0_{t_l}(S,0) - L^0_{t_l}(S,1)\right]$$

where the index t_l means that the loss is computed during the trap.

During the trap the system of first order conditions is:

$$\pi_t + \mu_t - \mu_{t-1} - \frac{\nu_{t-1}}{\beta\sigma} - \pi_t \sum_{\tau=-\infty}^{t_l} \beta^{t-\tau}(1-\gamma)^{t_l-\tau}\gamma\eta_\tau = 0, \tag{22}$$

$$\lambda^*(y_t - y^*) - \kappa\mu_t + \nu_t - \frac{1}{\beta}\nu_{t-1} = 0, \tag{23}$$

$$\frac{1}{\sigma}\nu_t + \zeta_t = 0. \tag{24}$$

Substituting η by its value yields:

$$\pi_t\left(1-\beta^{t-t_l-1}\gamma\right)+\mu_t-\mu_{t-1}-\frac{\nu_{t-1}}{\beta\sigma}=0, \tag{25}$$

$$\lambda^*(y_t-y^*)-\kappa\mu_t+\nu_t-\frac{1}{\beta}\nu_{t-1}=0, \tag{26}$$

$$\frac{1}{\sigma}\nu_t+\zeta_t=0. \tag{27}$$

After the trap, the system of first order conditions is as under perfect information:

$$\pi_t+\mu_t-\mu_{t-1}-\frac{\nu_{t-1}}{\beta\sigma}=0, \tag{28}$$

$$\lambda^*(y_t-y^*)-\kappa\mu_t+\nu_t-\frac{1}{\beta}\nu_{t-1}=0, \tag{29}$$

$$\frac{1}{\sigma}\nu_t+\zeta_t=0. \tag{30}$$

Sustainability. After the liquidity trap, the central banker type is unrevealed. Therefore, Proposition 1 applies and according to assumption 3, the benevolent central banker will not choose to deviate. In numerical applications, we check that this assumption is well satisfied.

If the inflation level set by the benevolent central banker is below the steady state level of inflation under discretion, then the benevolent central banker would like to raise the inflation rate further. Compared to the sustainability constraint for the benevolent central banker in full information, the sustainability constraint, equation (4) is slacker. The deviation is more costly as inflation expectations are higher due to the positive inflation level before the trap and the loss under the Perfect Bayesian equilibrium is lower has a higher inflation level leads to lower loss. This would lead to an increase in the deviation loss.

If the signaling inflation level is above the steady state level of inflation under discretion (for instance, y^* is close to 0), the benevolent central banker would like to lower the level of inflation but it is impossible as at the equilibrium, constraint (REV) is binding and hence a lower inflation would lead to pooling.

B.3 Proof of Proposition 3.

We first derive economic dynamics under pooling to prove that inflation is negative at the zero lower bound and positive just after it. Then, we apply the envelope condition to prove Proposition 3.

Dynamics. For any period before the trap, the ZLB has not hit, and so, $\zeta_t=\nu_t=0$. The set of equations is then:

$$\pi_t+\frac{\lambda^*}{\kappa}(y_t-y_{t-1})=0, \tag{31}$$

associated with (NKPC) and (EE). In a timeless perspective, this implies that $\pi_t=0$ and $y_t=-\frac{\kappa}{\beta\gamma}\pi_l$. The resulting value for μ_t is $\mu_t=-\frac{\lambda^*}{\kappa}\left(\frac{\kappa}{\beta\gamma}\pi_l+y^*\right)$. We denote by $\tilde{\mu}_t$ the modified Lagrange multiplier $\tilde{\mu}_t=\tilde{mu}_t+\frac{\lambda^*}{\kappa}y^*$.

Lemma 10. *If $\sigma\kappa/\lambda<1$, for all $t>l$, π_t and y_t are strictly positive and π_l and y_l are strictly negative, for any q.*

Proof. We first show that after the trap and after leaving the ZLB, π_t and y_t have the same sign and converge to 0. Indeed, they solve, jointly with $\tilde{\mu}_t$ the following problem:

$$\pi_t = \beta\pi_{t+1} + \kappa y_t \tag{32}$$
$$\lambda^* y_t - \kappa\tilde{\mu}_t = 0 \tag{33}$$
$$\pi_t + \tilde{\mu}_t - \tilde{\mu}_{t-1} = 0 \tag{34}$$

This allows to solve for π_t and y_t given an initial value for $\tilde{\mu}_{t-1}$. $\tilde{\mu}_t$ follows a second order linear difference equation:

$$\beta\tilde{\mu}_{t+1} - (\beta + \frac{\kappa^2}{\lambda} + 1)\tilde{\mu}_t + \tilde{\mu}_{t-1} = 0 \tag{35}$$

whose (bounded) solution is:

$$\tilde{\mu}_t = \Gamma_3\tilde{\mu}_{t-1} \tag{36}$$

where $\Gamma_3 = [\frac{1}{2} + \frac{\kappa^2}{2\beta\lambda} + \frac{1}{2\beta}] - \sqrt{[\frac{1}{2} + \frac{\kappa^2}{2\beta\lambda} + \frac{1}{2\beta}]^2 - 1} \in (0,1)$. As a consequence, $\tilde{\mu}_t$ monotonically converges to 0, which implies that π_t and y_t converge to 0 as well. In addition, either $\tilde{\mu}_t$ is decreasing and positive, and so are π_t and y_t, or $\tilde{\mu}_t$ is increasing and negative, and so are π_t and y_t.

After the trap not at the ZLB, first period:

$$\pi_t = \beta\pi_{t+1} + \kappa y_t \tag{37}$$
$$\lambda^* y_t - \kappa\tilde{\mu}_t - \frac{\sigma}{\beta} q\zeta_{t-1} = 0 \tag{38}$$
$$\pi_t + \tilde{\mu}_t - q\tilde{\mu}_{t-1} - q\frac{\zeta_{t-1}}{\beta} = 0 \tag{39}$$

We use the solution of the first equations to substitute π_{t+1} by something proportional to $\tilde{\mu}_t$, and we find π_t, y_t and $\tilde{\mu}_t$ as functions of ζ_{t-1} and $\tilde{\mu}_{t-1}$.

This is the same equations than before except that there is now a constant due to the past Lagrange multiplier associated with past ZLB. $\tilde{\mu}_t$ follows a second order linear difference equation:

$$\beta\tilde{\mu}_{t+1} - (\beta + \frac{\kappa^2}{\lambda} + 1)\tilde{\mu}_t + q\tilde{\mu}_{t-1} = -\frac{q}{\beta}(\frac{\sigma\kappa}{\lambda} - 1)\zeta_{t-1} \tag{40}$$

Thus, the solution should be as follows:

$$\tilde{\mu}_t = q\left[\Gamma_2\zeta_{t-1} + \Gamma_3\tilde{\mu}_{t-1}\right], \tag{41}$$

where $\Gamma_2 = \frac{\beta + \frac{\kappa^2}{\lambda} + 1}{\beta}(\frac{\sigma\kappa}{\lambda} - 1)$. When, $\sigma\kappa/\lambda < 1$ (see Werning (2012)), $\Gamma_2 < 1$.

In period l, the set of equations is:

$$\pi_l = \beta q\pi_{l+1} + \kappa y_l, \tag{42}$$
$$\lambda^* y_l - \kappa\tilde{\mu}_l - \sigma\zeta_l = 0, \tag{43}$$
$$\pi_l + \tilde{\mu}_l = 0. \tag{44}$$

Equations (34) and (36) prove that inflation is always positive after the trap (for $t > l + 2$) if only if μ_{l+1} is positive. Then, equation (41) shows that it depends on the value of $\tilde{\mu}_l$. We now prove that $\tilde{\mu}_l$ is positive, i.e. π_l is negative.

We combine equation (NKPC) at the ZLB period and equations giving π_{l+1} and y_l with respect to π_l and ζ_l and we obtain:[18]

$$\left[1+\frac{\kappa^2}{\lambda}+\beta q^2(1-\Gamma_3)\right]\pi_l=\left[\frac{\sigma\kappa}{\lambda}+\frac{\beta q^2}{\beta}-q\Gamma_2\right]\zeta_l \tag{45}$$

Assuming that $\frac{\sigma\kappa}{\lambda}<1$, and remarking that ζ_l is the Lagrange multiplier associated with (ZLB) and hence is negative, we thus obtain that $\pi_l<0$. Equation (43) shows that y_l is also negative in such a case.

□

Losses. The envelope condition of the two loss functions with respect to q leads to:

$$\frac{\partial L_{t_l}(P,q)}{\partial q}=-\beta\mu_l\pi_{l+1}-\nu_l\left(y_{l+1}+\frac{\pi_{l+1}}{\sigma}\right),$$

where $l=t_l$ is the period at which the economy is facing the shock. $\mu_l=-\pi_l>0$, $\nu_l=-\sigma\zeta_l>0$. Variables taken at time $l+1$ correspond to the situation if the central banker is benevolent. Thus, $\pi_{l+1}>0$ as well as $y_{l+1}>0$. Finally, these derivatives are negative, which yields our result. We can observe that the loss of the conservative central banker is proportional to the square of π_l, thus, inflation is increasing in q during the liquidity trap. The same computation proves that $L^0_{t_l}(P,q)$ is decreasing in q.

B.4 Proof of Theorem 4.

The benevolent central banker prefers the separating equilibrium over the pooling equilibrium if:

$$\frac{\pi^2+\lambda^*(y-y^*)^2+\beta\gamma L_{t_l}(S,1)}{1-\beta(1-\gamma)}\leq\frac{\pi_p^2+\lambda^*(y_p-y^*)^2+\beta\gamma L_{t_l}(P,q)}{1-\beta(1-\gamma)} \tag{46}$$

where π and y (resp. π_p and y_p) are the output gap and inflation in the separating (pooling resp.) equilibrium before the liquidity trap. One can observe that $\pi_p=0$ because it corresponds to the best policy of the conservative central banker. The (NKPC) leads to $y=\frac{1-\beta(1-\gamma)}{\kappa}\pi+y_p(q)$ and $y_p=-\frac{\beta\gamma}{\kappa}\pi_l$. This inequality can be rewritten as:

$$\beta\gamma\left(L_{t_l}(P,q)-L_{t_l}(S,1)\right)\geq\pi^2+\lambda^*[(y-y^*)^2-(y_p-y^*)^2] \tag{47}$$

$$\tag{48}$$

The conservative central banker prefers imitating the benevolent central banker if equation (REV) is violated thus the two relevant incentive constraints are:

[18]We here neglect the impact of the deviation of the Lagrange multiplier before the liquidity trap due to the expectations of liquidity trap, μ_{l-1}. However, its contribution is negligible as long as probability of the liquidity trap is sufficiently small. In the numerical applications, we however formally take this past multiplier into account.

$$\beta\gamma\left(L_{t_l}(P,q) - L_{t_l}(S,1)\right) \geq \pi^2 + \lambda^*[(y-y^*)^2 - (y_p - y^*)^2] \tag{49}$$

$$\pi^2 \geq \beta\gamma\left[L^0_{t_l}(S,0) - L^0_{t_l}(S,1)\right] \tag{50}$$

No liquidity trap. When $\gamma = 0$, the two conditions (49) and (50) are equalities when both types follow their optimal policies, thus, they are indifferent between pooling and separating equilibria. Thus, the equilibrium is independent of the prior, q.

Liquidity trap. Ex ante output gains due to the anticipation of ZLB dominate the steady state output inefficiency:

$$\gamma\left(L_{t_l}(P,q) - L_{t_l}(S,1)\right) \geq (1-\gamma)\left[(1 + \frac{\lambda^*}{\kappa}(1-\beta(1-\gamma))\pi^2 + 2\frac{\lambda^*}{\kappa}(1-\beta(1-\gamma))\pi(y_p - y^*)\right] \tag{51}$$

$$\pi^2 \geq \beta\gamma\left[L^0_{t_l}(S,0) - L^0_{t_l}(S,1)\right] \tag{52}$$

Note that, if y^* sufficiently low, as y_p decreases with q, $\pi^2 + \lambda^*[(y-y^*)^2 - (y_p - y^*)^2]$ increases with q. This allows to conclude as follows. Either $L_{t_l}(0)$ is such that (51) holds and, then, there exists $\tilde{q}$ such that, for $q \leq \bar{q}$, (51) holds or $\bar{q} = 0$. Finally, note that $\bar{q}$ is strictly below 1 as $L_{t_l}(S,1) > L_{t_l}(P,1)$.

B.5 Proof of Proposition 5.

Suppose that $\lambda^* = 0$. Combining (8) and (7) yields

$$\left(L_{t_l}(P,q) - L_{t_l}(S,1)\right) \geq \left[L^0_{t_l}(S,0) - L^0_{t_l}(S,1)\right],$$

which implies that $\bar{q} = 0$ and there exists no separating equilibrium.

Conversely, suppose that $\lambda^* > 0$. Combining (8) and (7) yields:

$$\beta\gamma\left(L^0_{t_l}(S,1) - L^0_{t_l}(S,0) + L_{t_l}(P,0) - L_{t_l}(S,1)\right) \geq \lambda^*[(y-y^*)^2 - (y_p - y^*)^2] \tag{53}$$

When y^* goes to infinity, this inequality becomes: $\beta\gamma\left(L_{t_l}(P,0) - L_{t_l}(S,1)\right) \geq 0$, which is always strictly satisfied. Finally, note that we also have $\beta\gamma\left(L_{t_l}(P,0) - L_{t_l}(P,1)\right) \geq 0$ and, as $L_{t_l}(P,q)$ is decreasing in q, this implies that $\bar{q}$ is strictly positive, when y^* is sufficiently large.

The second part of the proposition results from the fact that the derivative of the right hand term of (8) with respect to y^* is negative.

B.6 Proof of Corollary 6.

When $q \leq \bar{q}$, the equilibrium is separating, inflation under the benevolent central banker is strictly positive (except if $q = 0$) and given by the IC of the conservative central banker (7). Hence, inflation is increasing with q and reaches its maximum for $q = \bar{q}$.

π is determined as follows:

$$\pi^2 \geq \beta\gamma\left[L^0_{t_l}(S,0) - L^0_{t_l}(S,1)\right].$$

Thus, π is increasing in γ. As $L^0_{t_l}(S,0) - L^0_{t_l}(S,1)$ increases with r, π increases with r as well.

B.7 Proof of Proposition 7.

We want to determine the derivative of

$$[(y-y^*)^2-(y_p-y^*)^2]=(y-y_p)(y+y_p-2y^*).$$

Using (NKPC), we obtain:

$$\begin{aligned} y-y_p(q) &= \frac{1-\beta(1-\gamma)}{\kappa}\pi-\frac{\beta\gamma}{\kappa}\left(\pi_l(1)-\pi_l(q)\right), \\ y+y_p(q)-2y^* &= \frac{1-\beta(1-\gamma)}{\kappa}\pi-\frac{\beta\gamma}{\kappa}\left(\pi_l(1)+\pi_l(q)\right)-2y^*. \end{aligned}$$

When γ is close to 0, the two expressions lead to:

$$\begin{aligned} y-y_p(q) &= \frac{1-\beta}{\kappa}A\gamma^{1/2}+o(\gamma^{1/2}), \\ y+y_p(q)-2y^* &= -2y^*+A^2\gamma+o(\gamma), \end{aligned}$$

with $A=\left(\beta\left[L^0_{t_l}(S,0)-L^0_{t_l}(S,1)\right]\right)^{1/2}$. Then:

$$[(y-y^*)^2-(y_p-y^*)^2]=-2Ay^*\gamma^{1/2}+A^2\gamma+o(\gamma).$$

B.8 Proof of Proposition 8.

The incentive constraint for the benevolent central banker writes:

$$\begin{aligned} &\frac{\pi_1^2+\lambda^*(y_1-y^*)^2+\beta\gamma_1L_{t_l}(S,1)}{1-\beta\Pi_1}+\beta(1-\gamma_1-\Pi_1)\frac{\pi_2^2+\lambda^*(y_2-y^*)^2+\beta\gamma_2L_{t_l}(S,1)}{1-\beta\Pi_2}\geq\cdots \\ &\cdots\frac{\lambda^*(y_p-y^*)^2+\beta\gamma_1L_{t_l}(P,q)}{1-\beta\Pi_1}+\beta(1-\gamma_1-\Pi_1)\frac{\lambda^*(y_p-y^*)^2+\beta\gamma_2L_{t_l}(P,q)}{1-\beta\Pi_2} \end{aligned}$$

The incentive constraint for the conservative central banker writes:

$$\frac{\pi_1^2+\gamma_1L^0_{t_l}(S,1)}{1-\beta\Pi_1}+\beta(1-\gamma_1-\Pi_1)\frac{\pi_2^2+\gamma_2L^0_{t_l}(S,1)}{1-\beta\Pi_2}\geq\frac{\beta\gamma_1}{1-\beta\Pi_1}L^0_{t_l}(S,0)+\beta(1-\gamma_1-\Pi_1)\frac{\beta\gamma_2}{1-\beta\Pi_2}L^0_{t_l}(S,0)$$

Similarly:

$$\frac{\pi_2^2+\gamma_2L^0_{t_l}(S,1)}{1-\beta\Pi_2}+\beta(1-\gamma_2-\Pi_2)\frac{\pi_1^2+\gamma_1L^0_{t_l}(S,1)}{1-\beta\Pi_1}\geq\frac{\beta\gamma_2}{1-\beta\Pi_2}L^0_{t_l}(S,0)+\beta(1-\gamma_2-\Pi_2)\frac{\beta\gamma_1}{1-\beta\Pi_1}L^0_{t_l}(S,0)$$

These two inequalities hold with equality at the optimum. Multiplying the second one by $\beta(1-\gamma_1-\Pi_1)$ and subtracting it from the first one, we can then simplify the expression that we find, and divide it by $(1-\beta^2(1-\gamma_2-\Pi_2)(1-\gamma_1-\Pi_1))$. We then obtain:

$$\pi_1^2=\beta\gamma_1(L^0_{t_l}(S,0)-L^0_{t_l}(S,1))\text{ and }\pi_2^2=\beta\gamma_2(L^0_{t_l}(S,0)-L^0_{t_l}(S,1)),$$

which yields the result.