Love Thy Children

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LOVE THY CHILDREN
Reflections on the Barro Debt Neutrality Theorem*

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This paper proves that Barro's (1974) debt neutrality proposition, whose relevance hinges, in an economy with bequest.motives, on bequests being operative in the economy without public debt, is not applicable to a wide class of overlapping generation economies – those with a 'weak' bequest motive. In particular, it is shown that the bequest motive is always too weak, and public debt therefore non-neutral, when non-physical assets have a well-defined function (reducing oversavings) in the corresponding economy without bequest motive, i.e., when the non-altruistic economy is dynamically inefficient.

1. Introduction

This paper investigates the theoretical relevance of the Barro (1974) debt neutrality proposition.

In his celebrated 1974 article, Barro had shown that, provided that agents loved their descendants and actually left them positive bequests, finite horizons were no impediment to the operation of a debt neutrality theorem. If finitely-lived consumers had already chosen to leave positive bequests to heirs in the absence of government debt, the introduction of public debt would not affect the agents' optimal consumption plans as it would not create new opportunities to transfer resources from children to parents. The size of the national debt would thus not matter in equilibrium.

The applicability of this neo-Ricardian argument clearly hinges, in an economy with a bequest motive, on whether bequests are indeed operative in the economy without government debt. Although this is an important issue, it has not been resolved by previous authors. Barro (1974, p. 1106) had some feeling for the factors likely to generate operative bequests, but he did not provide any formal analysis. Drazen (1978) followed Barro's lead, but his paper did not yield any explicit condition under which bequests may be

operative. While both Buiter (1979) and Carmichael (1982) described sta-
tionary equilibria with operative intergenerational transfers, neither derived
any existence condition.

To derive such a condition, it is necessary to realize that bequests are, within
the two-period overlapping generation framework used by Barro (1974), an
intergenerational transfer *from the old to the young*. Now we know, in particu-
lar since Gale (1973) and from the work of Wallace (1980) and Tirole (1985),
that there exists a deep connection between the efficiency properties of
overlapping generation economies without bequest motives and the direction
of Pareto-improving intergenerational transfers.[1] Under reasonable assump-
tions, the dynamic inefficiency that may occur in Diamond (1965) economies
may be removed by the introduction of intergenerational schemes, such as fiat
money, bubbles, social security and government debt, which transfer goods
from the young to the old. Hence, on theoretical grounds, there exists a
relationship between the efficiency characteristics of the economy without
bequest motive, and the possibility of bequests being operative in the economy
with public debt.

We indeed prove that a necessary and sufficient condition for bequests to be
operative, either in an exchange economy or, under some assumptions, in a
stationary production economy, is that parents love their children 'enough'.
More precisely, we show that the intercohort discount factor applied by
parents to their heirs' utility must not be smaller than a threshold level which
depends on the discrepancy between the steady state interest rate of the
economy without bequest motive and the economy's growth rate – i.e., on the
'size' of the (in)efficiency of the economy without bequest motive. In particu-
lar, we prove that bequests cannot be operative at all dates in the no-debt
economy when the economy without bequest motive is dynamically inefficient.
As a consequence, the neo-Ricardian debt neutrality proposition is inapplica-
ble to a wide class of overlapping generation economies.

The next section of this paper derives the condition for operative bequests
in a simple exchange economy with a bequest motive. This condition is
extended, in section 3, to a production economy. Section 4 introduces uncer-
tainty. The conclusion summarizes the paper.

2. Bequests: Exchange economy

We consider a simple two-period overlapping generations exchange econ-
omy with a bequest motive: parents love their children. We ask the following
question. Under which condition will parents choose to leave positive bequests
to their descendants? Answering this question obviously amounts to determin-

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[1]As Cass, Okuno and Zilcha (1980) have shown, this connection may disappear when within-
cohort heterogeneity is introduced. This paper will follow Barro's (1974) article by assuming that
all consumers belonging to the same generation are identical.

ing the applicability of Barro's (1974) debt neutrality theorem to an economy with a bequest motive.[2]

2.1. The basic framework

We adopt Barro's (1974) formalization of the bequest motive: parents care about their children's *utility*, and have the same preference ordering over goods as their heirs. We assume that utility is separable, both intertemporally and intergenerationally.[3] Agents live for two periods, with lifespans overlapping so that the children's first period of life corresponds to the parent's second period. All consumers are identical, except for their age. Output is non-produced and non-storable, and agents receive endowments $e_1 > 0$ when young and $e_2 \geq 0$ when old of this consumption good. A young agent can buy or sell claims on next period's endowment at the real interest rate R. The structure of the model implies, however, that the market for consumption loans must be inactive in equilibrium, since intragenerational trade is precluded by the assumption that all agents of the same age are identical, and intergenerational loans are impossible when agents live only for two periods. Therefore, we will not introduce consumption loans explicitly below but will instead implicitly define the real interest rate as the marginal rate of substitution between present and future consumption. As we are interested in the equilibrium without government debt or other non-physical asset, the only possibility effectively offered to an agent to smooth out his consumption profile is to give a (pleasurable) bequest to each of his/her $(1 + n)$ children, $n \geq 0$. We assume that fertility is exogenous (n is a given parameter), that birth occurs by parthenogenesis (there is no marriage), and that all children are equally treated by their parent [each parent leaves the same bequest to each of its $(1 + n)$ heirs].[4]

[2] In the sole presence of a bequest motive, Barro's (1974) debt neutrality theorem is applicable if and only if bequests are operative at all dates in the absence of government debt. As mentioned by Barro (1974) himself and emphasized by Carmichael (1982), however, the debt neutrality proposition may also hold in the presence of operative *gifts* from children to parents. We purposefully exclude gift motives (children loving their parents) from our analysis, as the specification of preferences, the characterization of equilibrium, and the possible absence of an intertemporal government budget constraint in the presence of a gift motive are issues outside the scope of this paper [see Carmichael (1982), Buiter and Carmichael (1984), Burbridge (1983,1984), Abel (1985) and especially Kimball (1986) on this point].

[3] The assumption of intertemporal separability could easily be relaxed, in this one-good economy, without affecting our results. Intergenerational linear separability, however, is crucial to our analysis. Abel (1986) has studied the implications of relaxing this assumption by introducing 'concave altruism'.

[4] For a relaxation of the first assumption, see Becker and Barro (1986). For the implications of a model with bequest motives and marriage, refer to Bernheim and Bagwell (1985). As for environments in which the assumption of equal treatment might be violated (e.g., primogeniture economies) and in which the neo-Ricardian debt neutrality proposition obviously does not hold, they are not the focus of this article.

A consumer representative of generation t thus chooses c_{1t}, c_{2t}, b_{t+1} to maximize

$$U_t = u(c_{1t}) + \beta u(c_{2t}) + \gamma U^*_{t+1}, \tag{1}$$

subject to

$$c_{1t} = e_1 + b_t, \tag{2}$$

$$c_{2t} + (1+n)b_{t+1} = e_2, \tag{3}$$

$$c_{1t}, c_{2t}, b_{t+1} \geq 0, \tag{4}$$

$$e_1, e_2, b_t \text{ given,}$$

where

$u(\cdot)$ = utility derived from goods, $u' > 0$, $u'' < 0$, $u'(0) = \infty$, $u'(\infty) = 0$,

U^*_{t+1} = maximum utility attainable by a generation $t+1$ agent, given the bequest received from his parent,

c_{1t} = first-period consumption of an agent born at t,

c_{2t} = second-period consumption of an agent born at t,

b_{t+1} = bequest left by an agent born at t to each of its $(1+n)$ generation $t+1$ children,

β = intertemporal discount factor $(0 < \beta \leq 1)$,

γ = intercohort discount factor $(0 < \gamma \leq 1)$.[5]

The first-order conditions for an optimum are given by (1), (2), (3), (4) and

$$-\beta(1+n)u'(c_{2t}) + \gamma u'(c_{1t+1}) \leq 0$$

$$= 0 \quad \text{if} \quad b_{t+1} > 0. \tag{5}$$

Formula (5) simply states that, if one chooses to leave a positive bequest, the utility value, at time t, of bequeathing one unit of good to each of $(1+n)$ children must be equal to the pleasure derived from seeing these children consume that bequest in the first period of their life. Notice that, because of the assumption that the consumption good is not storable, and because the

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[5] The assumption $\gamma \leq 1$ imposes *intercohort* utility discounting. To see this, rewrite $\gamma = \beta\gamma'(1+n)$, to reflect the fact that γ incorporates both a pure time preference element β (my children enjoy their utils tomorrow) and a pure *interpersonal* discount factor γ' (one util accruing to one of my children is worth γ' utils to me), and the assumption that each parent cares equally about its $(1+n)$ children. γ is thus the overall contribution to a parent's utility of an extra util accruing tomorrow to each of his/her $(1+n)$ children. This specification is equivalent to Burbridge's (1983,1984). In the presence of a preference for future cohorts' utils $(\gamma > 1)$, the existence of an optimal consumption program is problematic.

consumption loan market is inactive in equilibrium, we need not invoke the envelope theorem to compute dU_{t+1}^*/db_{t+1}.

From these first-order conditions, we find that the optimal bequest left by an agent of generation t must satisfy

$$u'[e_1 + b_{t+1}] - \beta(1+n)\gamma^{-1}u'[e_2 - (1+n)b_{t+1}] \le 0 \qquad (6)$$

$$= 0 \quad \text{if} \quad b_{t+1} > 0.$$

Define the implicit real interest rate in the absence of bequests as

$$1 + \overline{R} = u'(e_1)/[\beta u'(e_2)].$$

We then immediately have:

Proposition 1.   Bequests are operative $(b > 0)$ in the exchange economy if and only if

$$\gamma > (1+n)/(1+\overline{R}),$$

i.e., if and only if the bequest motive is strong enough. When this condition is satisfied, equilibrium bequests are time-independent and uniquely defined by the equation

$$u'[e_1 + b] = \beta(1+n)\gamma^{-1}u'[e_2 - (1+n)b].$$

Proof.   Obvious from the properties of $u(\cdot)$, (6) and the definition of $\overline{R}$.

The condition of Proposition 1 relates all the parameters characterizing our exchange economy. It provides an intuitive result, namely that bequests motives must not be too weak if bequests are to be operative. But, more interestingly, we have the following:

Corollary.   If the economy without bequest motive is dynamically inefficient $(\overline{R} < n)$, then bequests cannot be operative in the economy with a bequest motive.

Proof.   When $\overline{R} < n$, satisfaction of the condition of Proposition 1 requires $\gamma > 1$, which is impossible.

This corollary confirms the intuitive reasoning of the introduction, in which we had established the connection between the efficiency characteristics of the

economy without bequest motive and the possibility that bequests may be operative when parents love their children. Heuristically, when the economy without bequest motive is dynamically inefficient, it takes intergenerational transfers from young to old (fiat money, social security) to achieve a Pareto-superior allocation. In that context, bequests, which transfer goods in precisely the opposite direction, would result in a Pareto-inferior allocation – which is the reason why they cannot be chosen positive by altruistic agents who, through a bequest motive, act as social, or rather dynastic, planners. In fact, it is straightforward to show that when $\bar{R} < n$, parents would indeed like to choose negative bequests ($b < 0$), so that children would indeed make *gifts* to their parents. This outcome is however ruled out in this economy in which children do not love their parents and in which gifts cannot, as a consequence, be voluntary.

Proposition 1 and its corollary thus show that dynamic efficiency of the economy without bequest motive is a necessary condition for the neo-Ricardian debt neutrality theorem to be applicable to an economy with a bequest motive. Moreover we see, from (5), that the (implicit) real interest rate when bequests are operative is simply

$$u'(c_1)/[\beta u'(c_2)] = (1+n)/\gamma \geq 1+n.$$

Hence although the presence of operative bequests lowers the real interest rate relative to the no bequest economy [as $(1+n)/\gamma < 1 + \bar{R}$], it never does so to such an extent that the economy with a bequest motive becomes inefficient.

In the following section, we examine whether it is possible to generalize these results to a neo-classical production economy.

3. Bequests: Production economy

We now modify the framework of the previous section, and assume that output, instead of being non-storable manna from heaven, is produced through a neo-classical production function, which we write, in intensive form, $f(k)$, where k is capital per young agent. We assume that the production function exhibits constant returns to scale, and that $f' > 0$, $f'(0) = \infty$, and $f'' < 0$. There is only one good in the economy, which can either be consumed or used, in combination with labor, as an input into the production process.

Competitive profit maximization by firms leads to the following conditions:

$$r_t = f'(k_t) \equiv r(k_t), \tag{7}$$

$$w_t = f(k_t) - k_t f'(k_t) \equiv w(k_t), \tag{8}$$

where r_t and w_t denote the interest and wage rate at t.

As in Diamond's (1965) model, we assume that the young inelastically supply one unit of labor when young, and retire when old. An agent born at t can buy x_t units of capital when young. A consumption loan market is not introduced explicitly, as it will again be inactive in equilibrium. To be consistent with the previous section, we assume that children receive their bequest at birth, i.e., that bequests are transmitted to children at the *beginning* of the parent's retirement.[6]

Under this specification, the budget constraints facing consumers, and the first-order conditions for an *interior optimum with operative bequests* are, using the envelope theorem to compute dU_{t+1}^*/db_{t+1},

$$c_{1t} + x_t = w_t + b_t, \tag{9}$$

$$c_{2t} + (1+n)b_{t+1} = (1+r_{t+1})x_t, \tag{10}$$

$$c_{1t}, c_{2t}, x_t, b_{t+1} \geq 0, \tag{11}$$

$$u'[c_{1t}] = \beta(1+r_{t+1})u'[c_{2t}], \tag{12}$$

$$\beta(1+n)u'[c_{2t}] = \gamma u'[c_{1t+1}]. \tag{13}$$

Eq. (13) is a static relation, implicitly giving, for any given amount $C_{t+1} \equiv c_{1t+1} + c_{2t}/(1+n)$ of per capita aggregate consumption, the optimal intergenerational allocation at $t+1$:

$$c_{2t} = \Phi(C_{t+1}), \qquad c_{1t+1} = C_{t+1} - (1+n)^{-1}\Phi(C_{t+1}),$$

with

$$\Phi(0) = 0 \text{ and } 0 < \Phi' < (1+n).$$

Using this definition of $\Phi(\cdot)$, it is straightforward to show that (12), in combination with (13), is simply the first-order condition of the infinite-horizon dynastic intertemporal allocation problem:

$$\max \sum_{t=0}^{\infty} \gamma^t v(C_t),$$

subject to

$$C_t + x_t = (1+r_t)x_{t-1}/(1+n) + w_t,$$

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[6] The presence of a physical store of value makes it possible to envisage an alternative specification in which bequests are transmitted by the old at death, and hence received by children at the beginning of their retirement. It would lead to results identical with ours.

where
$$v(C) \equiv u'[\Phi(C)].^7$$

This decomposition of the dynastic optimization program into separate intra- and intergenerational problems is, of course, the decentralized analogue of Samuelson's (1968) two-part golden rule.

3.1. Steady-state equilibrium

Substituting the equilibrium condition $x_t = (1+n)k_{t+1}$, (7) and (8) into (9) to (13), we find that a steady-state perfect foresight equilibrium must satisfy

$$c_1 = w(k) - (1+n)k + b, \tag{14}$$

$$c_2 = [1 + r(k)](1+n)k - (1+n)b, \tag{15}$$

$$u'(c_1) = \beta[1 + r(k)]u'(c_2), \tag{16}$$

$$1 + r(k) = (1+n)/\gamma \equiv 1 + r^*, \tag{17}$$

$$c_1, c_2 \geq 0, \tag{18}$$

$$b \geq 0. \tag{19}$$

Eq. (17) states that, in steady state, the equilibrium interest rate must be equal to r^*, which is simply the modified golden rule interest rate. Note that, as in Samuelson (1968), the steady-state interest rate does *not* depend on β, but only on the intergenerational discount factor γ. Let $k^* \equiv f'^{-1}(r^*)$ denote the *modified golden rule* capital stock. Eqs. (14), (15), and (16) uniquely determine c_1^*, c_2^*, and b^* as functions of k^*.

To determine the conditions under which this solution satisfies the non-negativity constraint on bequests, notice first that, from the properties of the $u(\cdot)$,

$$b^* \gtreqless 0 \quad \text{as} \quad u'[w(k^*) - (1+n)k^*]$$
$$\gtreqless \beta(1+r^*)u'[(1+r^*)(1+n)k^*]. \tag{20}$$

Now let $S[w, r]$ be the solution to $u'[w - S] = \beta(1+r)u'[(1+r)S]$, i.e., the savings function of an economy without bequest motive. We can then rewrite (20) as

$$b^* \gtreqless 0 \quad \text{as} \quad S[w(k^*), r(k^*)] \lesseqgtr (1+n)k^*. \tag{21}$$

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[7] The $v(\ )$ function has all the properties of the $u(\ )$ function: $v' > 0$, $v'(0) = \infty$, $v'(\infty) = 0$, $v'' < 0$. If $u(\cdot)$ is isoelastic, then $v(C) = z(C)$, where z is a positive constant depending on β, γ, n and the elasticity of marginal utility. Thus, with isoelastic tastes, the same utility function can be used to evaluate aggregate and individual consumption (up to the constant z). This is obviously not true in general.

To replicate the results of the exchange economy, we need to make the following assumptions:

Assumption 1. *The equation $g(k) \equiv S[w(k), r(k)] - (1 + n)k = 0$ has a unique positive solution, denoted $\bar{k}$.*

Assumption 2. *The function $g(k)$ has the following property:*

$$g(k) > 0 \quad \text{for all} \quad k \in \,]0, \bar{k}[,$$

$$= 0 \quad \text{for} \quad k = \bar{k},$$

$$< 0 \quad \text{for all} \quad k > \bar{k}.$$

Assumption 1 guarantees the unicity of the steady-state capital stock, defined as $\bar{k}$, of the economy *without bequest motive* studied by Diamond (1965). Assumption 2 can easily be shown to be equivalent to the assumption that this steady-state capital stock is stable and that convergence to $\bar{k}$ is non-oscillatory; Assumption 2 also ensures that positive steady-state bequests be associated with an *increase* in capital accumulation (relative to the Diamond economy without bequest motive), or, equivalently, with a decline in the real interest rate. Since the same decline occurs in the exchange economy, Assumption 2 is needed to make the production economy analogous to the exchange economy. Both assumptions are used, in the guise of existence, uniqueness and stability assumptions, by Diamond (1965).

We can now extend our results to the case of a production economy in:

Proposition 2. *Under Assumptions 1 and 2, bequests are operative ($b^* > 0$) in the stationary production economy if and only if*

$$\gamma > (1 + n)/(1 + \bar{r}),$$

where $\bar{r} \equiv f'(\bar{k})$ is the steady-state interest rate of the Diamond (1965) economy without bequest motive.

Proof. Follows immediately from (21) and Assumptions 1 and 2.

A heuristic interpretation of this proposition runs as follows. Assumptions 2 imposes that intergenerational transfers from old to young (positive bequests) effectively increase, in long-run equilibrium, capital accumulation and decrease the steady-state interest rate – which is an arguably reasonable restriction to impose on tastes and technology.[8] Now we know, from (17), that the

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[8] Were we to abandon Assumption 2, our results would be reversed and our model would yield counterintuitive conclusions, as would Diamond's (1965).

Table 1[a]

	n'			
h	1%	3%	5%	7%
0.25	0.49	0.59	0.66	0.70
0.50	0.73	0.98	1.18	1.32
0.75	0.87	1.25	1.60	1.86

[a] This table gives the minimum discount factor $(1 + n)/(1 + \bar{r})$ consistent with bequests being positive in steady state. It is assumed that tastes are logarithmic, so that $h = \beta/(1 + \beta)$ is the propensity to save out of first-period income in the absence of bequests. The production function is taken as being Cobb–Douglas, with a share of capital equal to 25%. n' is the annual growth rate of the economy, so that $1 + n = (1 + n')^{35}$ given a period (half-life) of 35 years. An entry greater than one in the table indicates that the associated Diamond economy is dynamically inefficient.

long-run interest rate is equal to r^*, the modified golden rule interest rate. If intergenerational transfers are operative from old to young, it must be the case that they increase the steady-state capital stock relative to the Diamond (1965) economy. A necessary condition for bequests to be operative in the long run is thus that $k^* > \bar{k}$, or $1 + r^* = (1 + n)/\gamma < 1 + \bar{r}$. Proposition 2 shows that this condition is also sufficient. In table 1, we compute the minimum intercohort discount factor, $(1 + n)/(1 + \bar{r})$, consistent with bequests being operative in steady state for the case of logarithmic utility and production functions. This table suggests that operative bequests require, for plausible parameter values, extremely low intercohort discount rates.[9]

As in the case of the exchange economy, dynamic inefficiency of the economy without bequest motive is sufficient to rule out operative bequests in the economy with a bequest motive; similarly, dynamic efficiency of the Diamond economy is a necessary, but not sufficient condition for bequests to be operative when parents love their children.

3.2. Transition path

Substituting (7) and (8) into (12) and (13), and using the definition of $v(\cdot)$, we find that, *if bequests are operative at all dates*, the equilibrium dynamics of aggregate consumption and of the capital/labor ratio are given by

$$v'(C_{t+1}) = \left\{ [1 + r^*]/[1 + r(k_{t+1})] \right\} v'(C_t), \tag{22}$$

$$C_t = k_t + f(k_t) - (1 + n)k_{t+1}, \tag{23}$$

[9] Some care must be exercised in inferring from this table the minimum required *interpersonal* discount factor (γ'), as the *intercohort* discount factor (γ) potentially depends both on n and β (see footnote 5).

a pair of difference equations admitting a unique steady state (k^*, C^*), with $C^* = f(k^*) - nk^*$. The resulting bequest is then given, from (10), by

$$b_t = [1 + r(k_t)] k_t - \Phi(C_t)/(1 + n). \tag{24}$$

The initial capital stock k_0 found by the original generation in the Garden of Eden will, however, in general differ from its long-run value k^*. Unlike in the exchange economy in which there is no state variable, we must worry, in the production economy, about the equilibrium dynamics of this economy. It can be shown that, *if bequests are operative at all dates*, then the modified golden rule capital stock is globally saddlepoint stable with k_t and C_t increasing (decreasing) towards their long-run levels if $k_0 < (>)k^*$, and that this saddlepath trajectory is the unique equilibrium (as the dynastic equilibrium replicates the standard infinite horizon, representative agent equilibrium). The difficulty, however, resides in determining necessary and sufficient conditions under which bequests will be operative at all dates given an initial capital stock k_0 differing from k^*.

As it is unfortunately very hard to analytically derive these conditions, we adopted a numerical approach. For a given intergenerational discount factor γ, we searched for the minimum capital stock $k_{\min}$ such that, along the saddlepath leading to the modified golden rule k^*, bequests be operative if $k_t \geq k_{\min}$. We specified the production function as being Cobb–Douglas, with a 25% share of capital in output, and we took $u(\cdot)$, and thus $v(\cdot)$, as being isoelastic, with an Arrow–Pratt curvature σ. Setting β, n, σ, and varying γ, we computed b_t from (24) and established the following:

Numerical summary

(1) *If $\gamma < (1 + n)/(1 + \bar{r})$, bequests cannot be operative at all dates whatever the initial capital stock.*

(2) *If $\gamma = (1 + n)/(1 + \bar{r})$, bequests are operative at all dates if $k_0 > k^* = \bar{k}$. They cannot be operative at all dates if $k_0 < k^*$. They are equal to zero at all dates if $k_0 = k^*$.*

(3) *If $\gamma > (1 + n)/(1 + \bar{r})$, bequests are operative at all dates if and only if $k_0 \geq k_{\min}$, where $k_{\min} < k^*$ is a decreasing function of γ satisfying $k_{\min} \uparrow k^*$ as $\gamma \downarrow (1 + n)/(1 + \bar{r})$. Bequests are an increasing function of the capital stock.*

These simulation results, which hold for all choices of β, n and σ, are very intuitive. If the condition of Proposition 2 is violated (case 1 of the numerical summary), bequests cannot be operative at all dates, for, if they were, the capital stock would converge, from what was said above, to k^* and bequests to $b^* < 0$! Even if steady-state bequests are positive (case 3 of the summary),

bequests cannot be operative at all dates on the transition path if agents are 'poor' initially, in the sense that the initial capital stock is too small (relative to k_{min}). The rationale of this result is that, with a small initial capital stock, an optimal dynastic consumption profile would involve, if bequests were not constrained to be non-negative, redistributing goods away from the children towards their parents, i.e., letting bequests be negative as long as $k_t < k_{min}$, and positive thereafter. As should be expected, the agents' perception of 'poverty', k_{min}, depends on how they value their children's utility, i.e., on γ: the less the parents value their heirs' utility (the smaller γ), the larger the minimum capital stock required for bequests to be positive (the larger k_{min}).

What happens when $k_0 < k_{min}$ or when $\gamma \leq (1 + n)/(1 + \bar{r})$ remains an open question. A natural conjecture is that as long as k_t is below k_{min} bequests are at a corner at zero, and capital accumulation proceeds as in a Diamond (1965) economy. When $k_t \geq k_{min}$, bequests are non-negative and at an interior, and capital accumulates as in (22) and (23). If this conjecture is verified, the capital stock of an economy with bequest motive converges, whatever k_0, to the modified golden rule k^* if $\gamma > (1 + n)/(1 + \bar{r})$, and to the Diamond (1965) steady state $\bar{k}$ if $\gamma < (1 + n)/(1 + \bar{r})$ – with bequests at a zero corner in both cases as long as $k_t < k_{min}$.

4. Bequests and uncertainty

In the previous section, we have derived a condition for operative bequests within a framework in which, because of the absence of uncertainty, the safe interest rate was always equal to the marginal productivity of capital. These two magnitudes will however in general differ when uncertainty is introduced, so that it is both conceptually and practically important to derive conditions for operative bequests within a stochastic environment, and to determine which rate of return, on safe or risky assets, matters for operative bequests.

In order to keep the answer as simple as possible, we revert to the exchange economy with no storage studied in section 2, but now assume that the second period endowment e_2 is stochastic, with $\pi_j = \text{Prob}\{e_2 = e_{2j}\}$, $j = 1, \ldots, J < \infty$ and $\sum_j \pi_j = 1$. The first-period endowment remains non-random. Agents are assumed to be von Neumann–Morgenstern expected utility maximizers, so that, letting b_{t+1j} denote the bequest an agent born at t plans to leave to his/her children at $t + 1$ if state j is realized, we have the following first-order conditions:

$$\beta(1 + n)u'\left[e_{2j} - (1 + n)b_{t+1j}\right] = \gamma u'\left[e_1 + b_{t+1j}\right], \qquad j = 1, \ldots, J.$$

$$(25)$$

The interpretation of (25) is the same as in the deterministic economy: agents choose state-contingent bequests such that, in every state, the marginal

cost of bequeathing one unit of good to each of the $(1 + n)$ heirs be equal to the marginal altruistic benefit derived from observing these descendants consume this bequest in their young age. It is clear that the solutions to (25) are time-independent, with $b_{t+1j} = b_j$ for all t, $j = 1, \ldots, J$. Moreover, it is easily checked that b_j is an increasing function of e_{2j}, so that the old leave larger bequests to their heirs in 'good' states of nature. More importantly, we find that

$$b_j > 0 \Leftrightarrow \gamma > (1 + n)/(1 + \hat{R}_j), \tag{26}$$

where $1 + \hat{R}_j \equiv u'(e_1)/[\beta u'(e_{2j})]$ is the marginal rate of substitution between first- and second-period consumption in state j when there are no bequests. (26) is analogous to the condition of Proposition 1; it tells us that for bequests to the operative in every state of nature, the bequest motive must be so strong that even in 'catastrophic' states of nature (in which e_{2j} is at its minimum) agents choose to transfer goods to their descendants.

(26) is obviously a very strong condition. In particular, it suffices that there be one state of nature such that the second-period endowment be zero with some positive probability to rule out that bequests be operative in every state [as we have assumed $u'(0) = \infty$].

An *implication* of (26) is that a necessary, but not sufficient, condition for bequests to be operative in every state is that

$$\gamma > (1 + n)/(1 + \hat{R}), \tag{27}$$

where $1 + \hat{R} \equiv \{\sum_j \pi_j (1 + \hat{R}_j)^{-1}\}^{-1}$ is the implicit *safe* rate of interest prevailing in the no-bequest intergenerational autarkic allocation. This is an interesting result, which establishes that, although it is the safe rate of interest and not the average risky rate which matters, the condition for operative bequests in a stochastic economy is more restrictive than the condition which prevails in a deterministic economy, even when the interest rate is suitably redefined. The fact that (27) is only necessary reflects the selection by agents of state-contingent bequests.

What remains true in the stochastic economy is that dynamic inefficiency of the economy without bequest motive is sufficient to ensure that bequests cannot be operative in all states of nature. This results from the fact that when $\hat{R} < n$, i.e., when the safe interest rate is below the growth rate, the economy without bequest motive is dynamically inefficient, in the (constrained optimality) sense that the *ex ante* welfare of each generation could be improved by intergenerational transfers $T > 0$ from young to old. Such transfers clearly benefit the original old; as for later generations, the effect of this social

security program on their expected utility is measured by

$$\mathrm{d}\mathrm{E}\,U(T)/\mathrm{d}T = -u'[e_1 - T] + \beta(1+n)\sum_j \pi_j u'\big[e_{2j} + (1+n)T\big],$$

(28)

which proves that the economy without bequest motive is inefficient whenever

$$\mathrm{d}\mathrm{E}\,U(0)/\mathrm{d}T = -u'[e_1] + \beta(1+n)\sum_j \pi_j u'[e_{2j}] > 0,$$ (29)

i.e., when $\hat{R} < n$. But if $\hat{R} < n$, (27) is violated as $\gamma \leq 1$, so that bequests cannot be operative in all states if the economy without bequest motive is dynamically inefficient.

We conjecture that these results generalize to more complex stochastic environments, and that as long as bequests are state-contingent the condition for operative bequests involves a state-by-state comparison, and not simply the type of average comparison between γ, $\hat{R}$, and n contained in (27). As a consequence of the very restrictive nature of this state-by-state comparison, it is very unlikely that bequests may be operative in every state of nature in a stochastic economy.

5. Conclusion

This paper has proved that Barro's (1974) debt neutrality proposition, whose relevance hinges, in an economy with bequest motives, on bequests being operative in the absence of public debt, is not applicable to a wide class of overlapping generation economies – those with a weak bequest motive. In particular, we have shown that the bequest motive is always too weak, and public debt therefore non-neutral, when non-physical assets have a well-defined function (reducing oversavings) in the corresponding economy without bequest motive, i.e., when the non-altruistic economy is dynamically inefficient.

This result was reached without adding any of the usual ingredients delivering non-neutrality (e.g., heterogeneous tastes, distortionary taxes) to Barro's (1974) model, and within a variety of environments (exchange and production economies without uncertainty, random exchange economy).

An important caveat to be kept in mind in interpreting the wider implications of this conclusion is that a modification of consumers' tastes to include a gift motive (concern for the welfare of ascendants) would, of course, modify our results. In particular, situations which lead to inoperative bequests may, but in general need not, be conducive to operative gifts and to the neutrality of

intertemporal lump-sum tax reallocations – a fact which would restrict, but not eliminate (relative to the range circumscribed in this article) the domain of inapplicability of Barro's (1974) debt neutrality theorem.

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