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FISCAL AND MONETARY REGIMES: A STRATEGIC APPROACH

Abstract

This paper develops a full-fledged strategic analysis of Wallace's "game of chicken". A public sector facing legacy nominal liabilities is comprised of fiscal and monetary authorities that respectively set the primary surplus and the price level in a non-cooperative fashion. We find that the post 2008 feature of indefinitely postponed fiscal consolidation and rapid expansion of the Federal Reserve's balance sheet is consistent with a strategic setting in which neither authority can commit to a policy beyond its current mandate, and the fiscal authority has more bargaining power than the monetary one at each date.

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Fiscal and Monetary Regimes: A Strategic Approach*

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April 30, 2018

Abstract

This paper develops a full-fledged strategic analysis of Wallace's "game of chicken". A public sector facing legacy nominal liabilities is comprised of fiscal and monetary authorities that respectively set the primary surplus and the price level in a non-cooperative fashion. We find that the post 2008 feature of indefinitely postponed fiscal consolidation and rapid expansion of the Federal Reserve's balance sheet is consistent with a strategic setting in which neither authority can commit to a policy beyond its current mandate, and the fiscal authority has more bargaining power than the monetary one at each date.

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1 Introduction

Policy responses to the 2008 crisis have resulted in a massive increase in the liabilities of many fiscal and monetary authorities. In the face of stretched public finances, the question of the coordination of fiscal and monetary policies has come back to the foreground of policy discussions. At the origin of this longstanding question is the simple observation that the intertemporal budget constraint of the public sector imposes a restriction on the joint path of (nominal) public liabilities, (real) fiscal surpluses, and price levels. As Sargent (1986) simply puts it: *"Arithmetic makes the strategies of the monetary and fiscal authorities interdependent."*

Monetary economists often use a game-theoretic terminology to describe this interdependence. This can be traced back to Wallace's view of a "game of chicken" played among branches of government that control separate elements of the budget constraint.[1]

Yet despite these informal references to game theory, the interaction of fiscal and monetary authorities has not to our knowledge lent itself to a thorough strategic analysis. Broadly speaking, the macroeconomic literature has taken two routes to simplify away this interaction. A first route consists in summarizing the behavior of each authority with a policy rule. Monetary and fiscal rules must then be consistent with equilibrium conditions including the non-explosiveness of inflation and public debt (e.g., Leeper 1991). The second route posits that one authority has a free hand at policy making, and so the other authority has no option but accommodating it in order to ensure that the public sector is solvent. In the monetary regime, monetary policy by assumption has full backing by the fiscal authority. In the fiscal regime of Sargent and Wallace (1981), conversely, the monetary authority sets seignorage revenues (thereby giving up control of inflation) so that real government liabilities imposed by the fiscal authority and primary surpluses have the same present value. The goal of this paper is to develop a more general and agnostic strategic analysis of this interaction.

We model Wallace's "game of chicken" as...a game between a fiscal and a monetary authority. We posit that each authority controls a policy in-

[1] Examples abound. Recent ones include Svensson (2017): *"In spite of this interaction, normally monetary policy and fiscal policy are conducted separately, with each policy taking the conduct and effects of the other policy into account. This corresponds to a so-called Nash equilibrium in game theory, where each player chooses his or her instruments independently to achieve his or her goals, while taking into account the conduct of the policy by the other player."* See also Sims' 2013 presidential address: *"(...)With recent repeated Congressional games of chicken over the debt limit and inability to bargain to a resolution of long-term budget problems, the answer may now be in some doubt."*

strument. The fiscal authority sets the real fiscal surplus and the monetary one sets the price level at each date. Both authorities can also trade nominal intertemporal claims—government debt and remunerated reserves, respectively—with the private sector. Each authority incurs costs when its respective policy instrument strays away from a target. Both also incur costs from outright sovereign default.[2]

We do not offer particular micro-foundations for these costs. Our approach is instead one of "revealed preferences". We model each authority's payoff in a very flexible and general fashion. We also consider several strategic interactions between them (simultaneous game, bargaining) as well as various degrees of commitment and information structures. Our goal is to characterize the equilibria resulting from each set of assumptions on payoffs and strategic interactions, and from there to back out the set of assumptions that delivers the most plausible empirical implications. This way we seek insights into the question that Sargent and Wallace (1981) raise in conclusion of their unpleasant arithmetic: *"The question is, Which authority moves first, the monetary authority or the fiscal authority? In other words, Who imposes discipline on whom?"*

Section 2 starts with a simple static version of the game that generates two main insights.[3] It first highlights how the structure of default costs affects the nature of the game between authorities. This game is an actual "game of chicken" only when default costs are convex. In this case accommodation by one authority makes playing tough more appealing to the other. The fixed default cost that is commonly assumed in the literature on sovereign default introduces by contrast a coordination motive among authorities: Accommodation by one of them may make accommodation by the other relatively more attractive. As a result, pure-strategy equilibria with and without default coexist in the presence of fixed costs, whereas they do not when costs are convex. Second, and more technically, we find that simultaneous games have unappealing properties that limit their practical relevance. They either feature many equilibria and so lack predictive power or, in a version with uncertainty, counterfactually predict that the public sector frequently resorts to small defaults. We conclude that sequential games such as those implicit in the monetary and fiscal regimes described above

[2]Costs from outright default in practice include output losses due to financial-market exclusion, trade sanctions, banking crises and more generally financial instability, as well as private costs—electoral or more generally political costs for the fiscal authority and career concerns for central bankers.

[3]The static game is essentially akin to the multi-period game of Section 3 when both authorities can fully commit to a future action plan.

are more practically relevant theoretical tools. But then, selecting the most relevant game among these requires an answer to Sargent and Wallace's question about who moves first.

In order to tease out more testable implications from various assumptions regarding which authority moves first, Section 3 then studies a multi-period game. The version in which the fiscal and monetary authorities cannot commit beyond their current terms is the most plausible in our view. Comparing the situations in which the monetary authority moves first at each period ("monetary lead") with that of fiscal lead generates two main implications.

First, the fiscal authority can force the monetary one to depart from its target and inflate debt away even when the monetary authority moves first. To do so, it needs to roll over government debt until the outstanding amount is sufficiently large that the monetary authority cannot impose full fiscal accommodation, as the fiscal authority would prefer to default in this case. In other words, the fiscal authority gains bargaining power through endogenous fiscal irresponsibility even if the monetary one moves first by assumption. The monetary authority by contrast controls only whether the public sector owes money to itself or to the private sector via open-market operations, but cannot affect the schedule of repayments. Thus it cannot force the government to accommodate in a symmetric way under fiscal lead.

Second, we make the natural assumption that the public sector incurs immediate costs of default as soon as the path of public finances is not sustainable and features default at some future date. This implies that avoiding sovereign default seems less costly when the necessary fiscal and monetary adjustments are more remote. To overcome the resulting time-consistency issue, each authority can avail itself of a commitment device. The fiscal authority simply prepays some outstanding liabilities. The monetary authority expands its balance sheet by creating sufficiently large amounts of reserves that serve as a commitment to inflate beyond ex-post optimal levels in the future. This owes to the key but plausible assumption that reserves are the fundamental unit of value in the economy, and so outright default on reserves is infinitely costly and never occurs in equilibrium.

These predictions lead us to conclude that the version of the model in which the fiscal authority moves first is best suited to describe US public finances since 2008. First, this version is the only one in which the central bank significantly and immediately expands its balance sheet whereas the government postpones fiscal consolidation to the longer run.[4] Second, this

[4]Hall (2013) documents a shift away from stabilizing the debt to GDP ratio with fiscal policy that persists long after the post-2008 recession. Recent projections by the

version is the only one in which inflation is also postponed to the long-run. Overall, our strategic analysis thus suggests that the answer to Sargent and Wallace's question is: the fiscal authority moves first.

Related literature

As in Aguiar et al. (2013, 2015), we study how the public sector combines inflation, taxes, and outright default in order to cope with a liability shock, and we model the respective costs of these instruments in a reduced form. In Aguiar et al. (2013), the public sector is comprised of one single agent and the focus is on its optimal inflation credibility in the presence of self-justified solvency crises. Aguiar et al. (2015) formalize a monetary union as a public sector comprised of a monetary authority and atomistic fiscal authorities. Albeit in a simpler dynamic environment, we complement these papers with the study of the case in which both components of the public sector are comprised of strategic agents.

An older literature (Alesina 1987, Alesina and Tabellini 1987, Tabellini 1986, and more recently Dixit and Lambertini 2003) study like us Nash equilibria between multiple branches of government. We contribute to this literature in two ways. First, we include sovereign default as a feasible strategy profile. We view this as a pre-requisite to a full-fledged strategic analysis of Sargent and Wallace's unpleasant arithmetic: One needs to specify payoffs when the public sector defaults in order to derive fiscal or/and monetary accommodation along default-free equilibrium paths. More important, we show how the fiscal and monetary authorities can respectively use government debt and excess reserves as strategic devices to regulate their future bargaining powers in the absence of commitment.

Our point that balance-sheet expansion is a device to commit to future price levels that the monetary authority would find ex-post excessive but that are ex-ante desirable echoes similar rationales for quantitative easing in the face of a liquidity trap. In Battharai et al. (2015), the central bank commits to low future interest rates by owning long-term bonds that would depreciate under higher rates, thereby endangering its solvency. The depreciation of foreign reserves resulting from low domestic price levels acts as a similar threat to the solvency of the monetary authority in Jeanne and Svensson (2007). The mechanism that generates a commitment to high future price levels in our setup differs from these, however. By making its net wealth highly sensitive to the government's decision to (strategically)

Congressional Budget Office (CBO) tend to confirm this shift.

default, the central bank deliberately eliminates its future incentives to let the government default rather than inflate debt away since such a default would come with inflation anyway.

The restrictions that central-bank "solvency" imposes on feasible price-level paths rationalize unconventional measures in our setup. These restrictions are also central in the analyses of new-style central banking developed by Hall and Reis (2015) and Reis (2015, 2017). Our strategic approach delivers the novel insight that the swap of government debt with remunerated reserves is not neutral even when both types of claims are risk free and thus perfect substitutes *in equilibrium*. Such expansions of the central bank's balance sheet increase its exposure to the *out-of-equilibrium* threat of sovereign default. This reduces the future bargaining power of the monetary authority, thereby raising future price levels.

2 Wallace's game of chicken: Which game?

2.1 Setup

We consider a static game between two players, a fiscal authority F and a monetary authority M.

Actions. The fiscal authority F sets a real primary surplus $\tau \in \mathbb{R}$ and the monetary authority M a price level $p \in \mathbb{R}_+$. Players move simultaneously.

Payoffs. F and M face a maturing nominal liability $b \geq 0$. A strategy profile (τ, p) yields payoffs $U_X(\tau, p)$ to player $X \in \{F; M\}$ such that

$$U_F(\tau, p) = -f_F\left(\left(\frac{b}{p} - \tau\right)^+\right) - g_F\left(|\tau - \tau_F|\right), \tag{1}$$

$$U_M(\tau, p) = -f_M\left(\left(\frac{b}{p} - \tau\right)^+\right) - g_M\left(|p - p_M|\right), \tag{2}$$

where $(f_X, g_X)_{X \in \{F;M\}}$ are increasing, strictly convex and differentiable functions over $[0, +\infty)$, $\tau_F > 0$, $p_M > 0$, and $x^+ \equiv \mathbb{1}_{\{x \geq 0\}} x$. For conciseness, we restrict the analysis to the case in which g'_X spans $[0, +\infty)$ for $X \in \{F; M\}$.

In words, each authority incurs additively separable costs from i) setting its policy instrument away from a given target[5], and ii) from the (real)

[5]The analysis carries through if M has an inflation target.

loss resulting from sovereign default (equal to 0 if the liability is honored in full).[6] We could extend our approach to the case of non-separability in principle, presumably at the cost of additional analytical complexity. We leave this for future research mainly because we believe that the signs of the interaction terms would depend on the particular micro-foundations for these payoffs (e.g., nature of the cost of default). Note that these separable payoffs include the case in which F and M share the same objective

$$-f\left(\left(\frac{b}{p}-\tau\right)^{+}\right)-g_F\left(|\tau-\tau_F|\right)-g_M\left(|p-p_M|\right) \tag{3}$$

but do not cooperate to maximize it.[7]

These payoffs are such that the fiscal and monetary authorities have no reason to deviate from their respective targets other than avoiding to default on the maturing public liabilities b. This implies that there is no point for either authority to undershoot its target as it increases the loss given default. In game-theoretic language, τ_F dominates any strategy $\tau \leq \tau_F$ for F, and p_M dominates any strategy $p \leq p_M$ for M. This implies in particular that (τ_F, p_M) is the unique Nash equilibrium (and is an equilibrium in dominant strategies) when $b \leq \tau_F p_M$. The remainder of the paper focusses on the alternative situation of interest in which $b > \tau_F p_M$. In other words, we study a situation in which the policy (τ_F, p_M) that would be optimal in "normal times" must be revised in order to keep government debt sustainable following a significant shock on public liabilities, such as the one that resulted from the 2008 crisis.

In this case, if no player accommodates by overshooting its target, then there is outright default. The situation in which the fiscal authority stays on its primary-surplus target whereas the monetary authority fully accommodates to avoid default, i.e., inflates debt away, corresponds to what is commonly described as the fiscal regime in the monetary literature, whereas the symmetric situation corresponds to the monetary regime. In our strategic formulation, each regime corresponds to a particular strategy profile:

Definition. ***(Fiscal and monetary regimes)*** *The fiscal regime is the strategy profile* $(\tau_F, b/\tau_F)$ *and the monetary regime is the strategy profile* $(b/p_M, p_M)$.

[6] Results are qualitatively similar if the nominal loss given default matters.

[7] This is because adding to U_X terms that do not depend on X's instrument does not affect its optimal decision. This is only true in this simultaneous-move version of the game though.

We now solve for the Nash equilibria of this game, with a particular interest in identifying the circumstances under which either the fiscal regime or the monetary regime are Nash equilibria.

2.2 Equilibria

We first focus on equilibria in pure strategies. The functions $U_F(.,p)$ and $U_M(\tau,.)$ are strictly concave and bounded above and so admit each a unique maximizer which is the best response to the other authority's policy. Strict concavity of U_F and U_M also implies that these best responses are decreasing in the other player's action. In other words, M and F play a "game of chicken", whereby accommodation is a strategic substitute. More accommodation by one authority makes playing tough more appealing to the other.

Let us define

$$\tau_M = \sup_{\tau \geq \tau_F} \{g'_F(\tau - \tau_F) \leq f'_F(0)\}, \tag{4}$$

$$p_F = \sup_{p \geq p_M} \{p g'_M(p - p_M) \leq \tau_M f'_M(0)\}. \tag{5}$$

The following proposition characterizes the pure-strategy equilibria of the game:

Proposition 1. ***(Pure-strategy equilibria)*** *There exists $\tau_F p_M < b_- < b_+ < \tau_M p_F$ such that*

- *If $b \in (\tau_F p_M, b_-]$ then there exists a continuum of equilibria given by the strategy profiles $(\tau, b/\tau)$ where $\tau \in [\tau_F, b/p_M]$. Thus there is no default in equilibrium and both the monetary and fiscal regimes correspond to equilibria.*

- *If $b \in (b_-, b_+]$, there also exists a continuum of equilibria without default. Only one of the two regimes, either fiscal or monetary depending on parameter values, is an equilibrium outcome.*

- *If $b \in (b_+, \tau_M p_F]$, there also exists a continuum of equilibria without default that include neither the fiscal nor the monetary regime.*

- *If $b > \tau_M p_F$ there exists a unique equilibrium with default such that p, τ, and the loss given default all increase with respect to b.*

Proof. See the appendix. ■

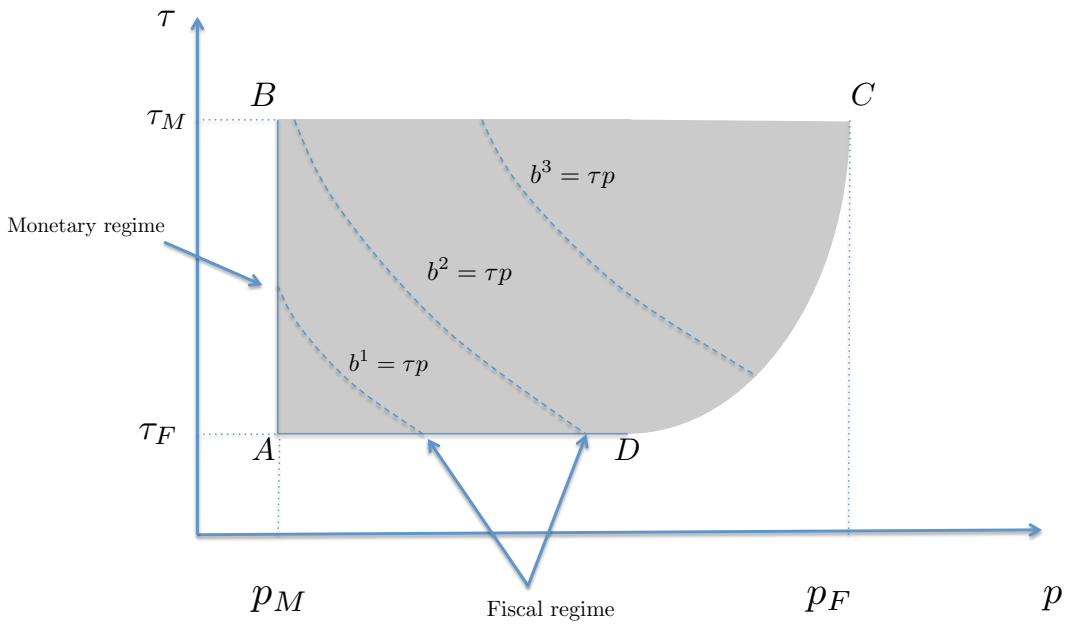


Figure 1. The dashed segments of hyperbolae represent the pure-strategy equilibria associated with three values b^1, b^2, b^3 such that $\tau_F p_M < b^1 < b_- < b^2 < b_+ < b^3 < \tau_M p_F$.

The equilibria are best described graphically in the plane (p, τ). Refer to Figure 1. If b is sufficiently large that the graph of the hyperbola $b = \tau p$ is to the northeast of C, then there is a unique equilibrium with default. Otherwise, the equilibria are default-free, described by the segment of the hyperbola $b = \tau p$ that is within the shaded area ABCD.[8] The intersection of this segment (if any) with AB is the monetary regime whereas that with AD (if any) corresponds to the fiscal regime. For sufficiently low levels of debt, both the monetary and fiscal regimes are equilibrium outcomes. This is illustrated by the case $b = b^1$ in Figure 1. As b increases, one of the two regimes ceases to be an equilibrium outcome. This is illustrated by the case $b = b^2$ in Figure 1 in which the monetary regime is not an equilibrium outcome.[9] Both authorities accommodate as b gets close to the value at which they default (as in the case $b = b^3$ in Figure 1).

Regarding the circumstances under which the fiscal and monetary regimes are Nash equilibria, Proposition 1 has two unsurprising but interesting implications. First, none of these regimes is an equilibrium outcome as soon

[8]This area collapses to A and so there is a unique equilibrium with default if $f'_F(0) = f'_M(0) = 0$.

[9]Either regime can survive in general in this area depending on which authority has the highest cost of default relative to deviation from target.

as public debt is sufficiently large other things being equal, as both parties accommodate in this case. Second, if the monetary or/and the fiscal regime are rationalizable outcomes, then unfortunately they coexist with a continuum of default-free equilibria in which both parties accommodate.

Mixed-strategy equilibria. Only pure-strategy equilibria are default-free when $b \leq \tau_M p_F$. There also exists for such values of b a large collection of mixed-strategy equilibria that all feature stochastic default.[10]

The next section reduces the number of equilibria by adding the ingredient that both authorities have imperfect control over their respective instruments.

2.3 Uncertainty

A simple way to reduce the number of equilibria is to introduce uncertainty in this game. This section does so by leaving the previous model unchanged, up to the additional assumption that F and M imperfectly control their respective policy variables. If F seeks to set the real surplus at τ, an actual surplus τe^{ϵ_F} is realized. Similarly, the realized price level is pe^{ϵ_M} if M sets a price level p. F and M set τ and p simultaneously. When doing so they share the prior belief that the random variable (ϵ_F, ϵ_M) admits a p.d.f. $1/\sigma^2 \varphi(\epsilon_F/\sigma, \epsilon_M/\sigma)$, where $\sigma > 0$ and φ has a bounded support[11] that includes a neighborhood of $(0,0)$.[12] F and M seek to maximize their respective expectations over the payoffs defined in (1) and (2).

Proposition 2. ***(Uncertainty yields equilibrium uniqueness)*** *There exists a unique equilibrium* $(\tau(\sigma), p(\sigma))$. *If* $b > \tau_M p_F$, *then* $\lim_{\sigma \to 0}(\tau(\sigma), p(\sigma))$ *is the strategy profile of the unique equilibrium obtained in the model without uncertainty. Otherwise,* $\lim_{\sigma \to 0}(\tau(\sigma), p(\sigma))$ *is the unique solution to*

$$b = \tau p, \tag{6}$$

$$p g'_M(p - p_M) f'_F(0) = \tau g'_F(\tau - \tau_F) f'_M(0). \tag{7}$$

In this case, the probability of default is strictly positive, increasing from 0 to 1 as b *spans* $(\tau_F p_M, \tau_M p_F]$. *Its limit for a given* $b \leq \tau_M p_F$ *as* $\sigma \to 0$ *is*

[10] It is easy to see for example that any pair of (default-free) pure-strategy equilibria (τ, p) and (τ', p') can be associated with an equilibrium with stochastic default in which F and M mix over the strategies associated with each equilibrium.

[11] This is only meant to abstract from convergence issues.

[12] A positive shock on output is likely both to generate a higher surplus than initially budgeted and to create inflationary pressure. A realistic assumption is thus that ϵ_M and ϵ_F be positively correlated, but this plays no role in our analysis.

$g'_F(\tau - \tau_F)/f'_F(0) = pg'_M(p - p_M)/\tau f'_M(0)$.

Proof. See the appendix. ■

On the one hand, this version of the model delivers a unique equilibrium in which both parties accommodate. On the other hand, accommodation always comes with a strictly positive probability of sovereign default. For a given b, the probability of default is in particular bounded away from 0 as uncertainty vanishes. The loss given default however becomes arbitrarily small for $b < \tau_M p_F$ in this limiting case. This prediction that the public sector uses small sovereign defaults as a routine tool whenever some accommodation is necessary is highly counterfactual. To be sure, the probability of default can be made small for b fixed by assuming that the marginal costs of default $f'_X(0)$ are sufficiently large other things being equal. Yet we find this description of the coordination of fiscal and monetary policies to be at odds with the commonly held view of a pecking order, whereby sovereign default is a last-resort strategy used only after other options have failed, at least in developed economies.

2.4 Non-convex cost of default

We aim at modelling payoffs in an agnostic and flexible fashion, and so we started above with a simple convex cost of default. The sovereign-default literature commonly assumes non-convexities, however, in the form of a fixed component to the cost of default. We introduce here such a fixed cost by assuming utilities:

$$U_F(\tau, p) = -\alpha_F \mathbb{1}_{\{b>\tau p\}} - \mid \tau - \tau_F \mid, \tag{8}$$

$$U_M(\tau, p) = -\alpha_M \mathbb{1}_{\{b>\tau p\}} - \mid p - p_M \mid, \tag{9}$$

where $\alpha_F, \alpha_M > 0$.[13] The assumption of a linear dependence in $\mid \tau - \tau_F \mid$ and $\mid p - p_M \mid$ is just a normalization given the fixed cost of default. We let

$$\beta_1 = \inf\{\tau_F(p_M + \alpha_M); (\tau_F + \alpha_F)p_M\}, \tag{10}$$

$$\beta_2 = \sup\{\tau_F(p_M + \alpha_M); (\tau_F + \alpha_F)p_M\}, \tag{11}$$

$$\beta_3 = (\tau_F + \alpha_F)(p_M + \alpha_M), \tag{12}$$

and characterize pure-strategy equilibria:

Proposition 3. ***(Pure-strategy equilibria with a fixed default cost)***

[13]The addition of a variable cost to the fixed one assumed here does not significantly affect the analysis.

- *If $b \in (\tau_F p_M, \beta_1]$ there exists a continuum of equilibria given by the strategy profiles $(\tau, b/\tau)$ where $\tau \in [\tau_F, b/p_M]$. Thus there is no default in equilibrium and both the monetary and fiscal regime correspond to equilibria.*

- *If $b \in (\beta_1, \beta_2]$, there also exists a continuum of equilibria without default. The fiscal regime is an equilibrium but not the monetary one if $\beta_1 = (\tau_F + \alpha_F)p_M$ whereas the opposite holds otherwise.*

- *If $b \in (\beta_2, \beta_3]$, there exists a continuum of equilibria without default that does not include the fiscal nor the monetary regime. There also exists an equilibrium in which F and M default and play their targets (τ_F, p_M).*

- *If $b > \beta_3$ then this latter equilibrium with default is the unique one.*

Proof. See the appendix. ■

As soon as b is sufficiently large that neither the fiscal nor the monetary regime are equilibrium outcomes ($b > \beta_2$), then the continuum of default-free equilibria coexist with the equilibrium with default. This is the salient and interesting difference with the case of a convex cost studied in Proposition 1, in which default-free equilibria do not coexist with the cum-default equilibrium in pure strategies. The intuition for this difference can be found in the strategic impact of the assumed fixed cost. With a fixed cost, the game is no longer a "game of chicken." Fiscal and monetary accommodations are no longer pure strategic substitutes as the fixed cost induces strategic complementarity. The cost of accommodation relative to the cost of default for one authority decreases as the other authority accommodates more because a smaller accommodating effort then suffices to avoid the fixed default cost. This creates a coordination motive between F and M that generates the additional multiplicity of equilibria—default-free versus cum-default.

One can show that as in Proposition 2, the introduction of small exogenous shocks to the policy variables reduces the number of equilibria. There remain however multiple equilibria under uncertainty in this case of a fixed cost. Because of the above-mentioned coordination problem, there still are two equilibria with varying probabilities of default in the limiting case of vanishing uncertainty. Also, the setup with uncertainty still has the unpalatable feature that default occurs even for small values of b.

A negative progress report. The analysis suggests that the simultaneous games considered thus far are tools of limited applicability to the study of fiscal and monetary coordination. Without uncertainty, the number of

equilibria is too large to generate useful predictions. The addition of uncertainty prunes equilibria, but small sovereign defaults arise as a routine tool in the remaining ones, contrary to the evidence that sovereign defaults typically are of significant magnitude. It is also at odds with the commonly held view that default is a "nuclear option" that is not seriously considered below very large debt levels. This motivates the restriction of the rest of the paper to games of sequential moves.

2.5 Sequential moves

One natural way to overcome the issues facing simultaneous games is to represent the interaction between F and M as a bargaining game with alternative offers. The seminal infinite-horizon setting of Rubinstein (1982) is not suited to this application in which players face fixed deadlines. We first study the simplest finite-horizon environment—ultimatum games—in the fixed-cost setup of Section 2.4. More precisely we assign payoffs (8) and (9) to F and M and consider two games. Under fiscal lead, F makes a take-it-or-leave-it offer to M, whereas the authorities swap roles under monetary lead.

Proposition 4. ***(Fiscal versus monetary lead)***

- *Under fiscal lead, there is a unique equilibrium:*

$$\begin{cases} (\tau_F, b/\tau_F) & \text{if } b \in (\tau_F p_M, \tau_F(p_M + \alpha_M)], \\ (b/(p_M + \alpha_M), p_M + \alpha_M) & \text{if } b \in (\tau_F(p_M + \alpha_M)), (\tau_F + \alpha_F)(p_M + \alpha_M)), \\ (\tau_F, p_M) & \text{if } b > (\tau_F + \alpha_F)(p_M + \alpha_M). \end{cases}$$

- *Under monetary lead, there is a unique equilibrium:*

$$\begin{cases} (b/p_M, p_M) & \text{if } b \in (\tau_F p_M, (\tau_F + \alpha_F)p_M], \\ (\tau_F + \alpha_F, b/(\tau_F + \alpha_F)) & \text{if } b \in ((\tau_F + \alpha_F)p_M, (\tau_F + \alpha_F)(p_M + \alpha_M)), \\ (\tau_F, p_M) & \text{if } b > (\tau_F + \alpha_F)(p_M + \alpha_M). \end{cases}$$

Proof. The leader just picks the simultaneous-game equilibrium that warrants him the highest payoff. Under fiscal lead, as b increases, the equilibrium is first the fiscal regime, F then partially accommodates once M accommodates so much that it is indifferent with default, and finally the public sector defaults. Monetary lead is the mirror image. ■

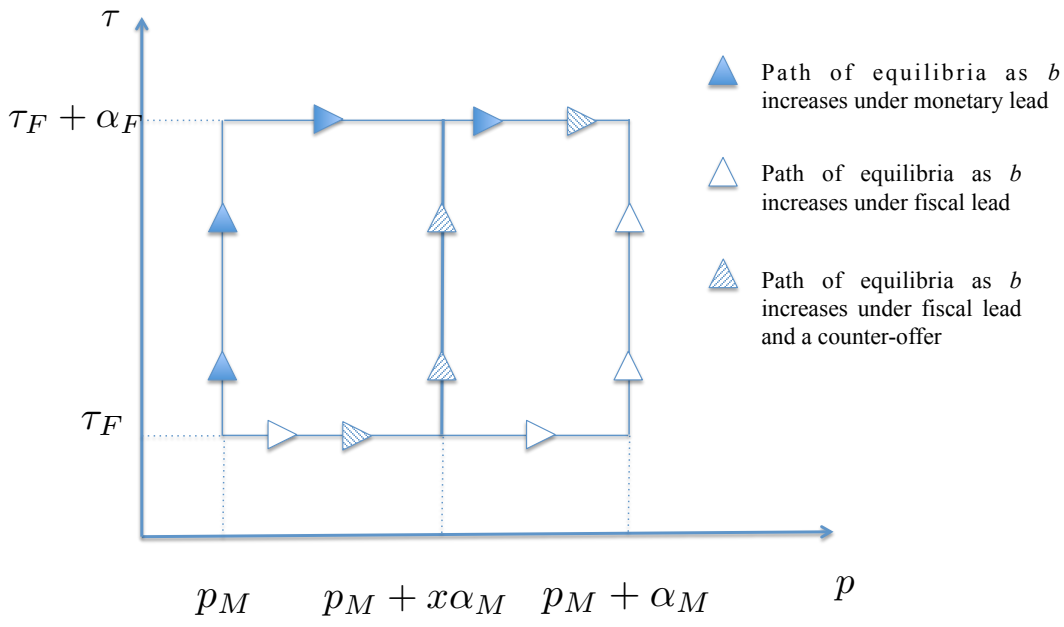


Figure 2. Equilibria under monetary and fiscal lead.

Figure 2 illustrates how the equilibrium evolves with b under monetary and fiscal lead as well as in the case of a counter-offer discussed below. It is transparent that the analysis of such ultimatum games is very similar in the case of a convex cost. The leader picks the most favorable equilibrium among the pure-strategy equilibria of the game with simultaneous move.

Counter-offers. Suppose to fix ideas that F moves first but that M can make a counter-offer which comes at a cost $x\alpha_M$, where $x \in (0,1)$.[14] A possible interpretation of this cost is that the monetary authority worries financial markets by dissenting with the fiscal one this way. It is straightforward to see that the equilibrium becomes:

$$\begin{cases} (\tau_F, b/\tau_F) & \text{if } b \in (\tau_F p_M, \tau_F(p_M + x\alpha_M)], \\ (b/(p_M + x\alpha_M), p_M + x\alpha_M) & \text{if } b \in (\tau_F(p_M + x\alpha_M)), (\tau_F + \alpha_F)(p_M + x\alpha_M)), \\ (\tau_F + \alpha_F, b/(\tau_F + \alpha_F)) & \text{if } b \in ((\tau_F + \alpha_F)(p_M + x\alpha_M)), (\tau_F + \alpha_F)(p_M + \alpha_M)), \\ (\tau_F, p_M) & \text{if } b > (\tau_F + \alpha_F)(p_M + \alpha_M). \end{cases}$$

Whereas the identity of the authority that picks the residual bill switches

[14] The case $x = 1$ is identical to that of fiscal lead whereas $x = 0$ corresponds to monetary lead.

once from M to F as b increases in the ultimatum game, there is one additional switch from F to M in the case of a counter-offer. Intuitively, introducing further rounds of offers would induce more frequent switches, and so the respective accommodating efforts of F and M would increase more smoothly with b as the number of rounds increases.

Who has the bargaining power? Bargaining games eliminate the issues of equilibrium indeterminacy or implausibility facing simultaneous games. But this comes at the cost of making non-obvious assumptions regarding the bargaining power of each authority. In other words, if one could derive the objectives (8) and (9) of F and M—or a more sophisticated version of them—out of a structural model of the economy, then one would still be left with little empirical guidance as to the exact structure of bargaining. One way to obtain some guidance is to tease out distinctive empirical implications from various assumptions about bargaining power. We now show that a dynamic version of this game with a richer debt structure delivers such implications.

3 Dynamic game of chicken: Public liabilities as commitment devices

This section studies dynamic versions of the simple games with sequential moves and fixed default costs studied in Section 2.5.[15]

3.1 Setup

Time is discrete and is indexed by $t \in \mathbb{N}$. At each date t, the fiscal authority F sets a real primary surplus $\tau_t \in \mathbb{R}$ and the monetary authority M a price level $p_t > 0$.[16]

Endogenous public liabilities. In addition to setting their respective policy instruments at each date, F and M can also promise nominal future payments to a private sector comprised of competitive risk-neutral investors

[15] Section 2 shows that the equilibria of the sequential games are qualitatively similar under convex and fixed costs.

[16] There are at least two simple cashless environments in which the monetary authority can set the price level at each date this way. In the cashless limit of the cash-in-advance model set up in Cochrane (2005), it does so by supplying the appropriate amount of intra-date money. In Hall and Reis (2017), it does so by promising an appropriate real repayment on one-period reserves.

who discount future consumption with a factor $\beta \in (0,1)$.[17] The fiscal authority F is free to trade any stream of fixed promised repayments with the private sector. All repayments promised by F at a given date are pari-passu. The monetary authority M can purchase outstanding government securities held by the private sector by issuing nominal claims with arbitrary repayment schedules. We deem these liabilities issued by the central bank "remunerated reserves". F and M make take-it-or-leave-it offers of prices and quantities to the private sector.[18]

Central bank reserves are the fundamental unit of value in every modern economy. A "default" on excess reserves is thus basically a currency reform. Such radical reforms clearly exceed the scope of central banks' mandates. We capture this in our setting by assuming that M finds any option preferable to a default on remunerated reserves.

Assumption 1. ***(Reserves are non-defaultable)*** *The monetary authority M incurs an arbitrarily large disutility from defaulting on remunerated reserves.*

We suppose that M receives an exogenous real income stream with date-1 present value $\epsilon > 0$. Central banks' income is in practice mainly comprised of seignorage—our environment is cashless, however—and of return from assets possibly purchased at distressed prices. For expositional simplicity, we study first the limiting case in which $\epsilon \downarrow 0$, and so this income has a negligible effect on the primary surplus if paid out as a dividend to the government. We address the case of a finite ϵ in Section 3.4.

Payoffs. A course of actions by F consists in real surpluses $(\tau_t)_{t\geq 0}$ and a set of promises to date-t' nominal repayments issued at date t, $(b_{t,t'})_{t'>t\geq -1}$. The actions of M are symmetrically comprised of price levels $(p_t)_{t\geq 0}$ and a set of date-t' nominal repayments promised at date t, $(r_{t,t'})_{t'>t\geq -1}$. Promises issued at date $t=-1$ represent exogenous liabilities inherited from an unmodelled past (the counterpart of b in the static model of Section 2).

Let Δ_t denote an indicator function equal to 1 if and only if these courses of actions imply default on government debt held by the private sector after

[17] We suppose that optimization by the private sector implies a transversality condition e.g., it is comprised of an infinitely lived representative agent.

[18] This rules out equilibrium multiplicity due to multiple self-justified prices in a Walrasian bond market, as in Aguiar et al. (2013).

date t.[19] The respective date-t payoffs of F and M are then

$$V_t^F = -\sum_{s\geq 0} \beta^s \mid \tau_{t+s} \mid -\beta(1+\beta)\alpha_F \Delta_t, \tag{13}$$

$$V_t^M = -\sum_{s\geq 0} \beta^s \mid p_{t+s} - p_M \mid -\beta(1+\beta)\alpha_M \Delta_t, \tag{14}$$

where $\alpha_F, \alpha_M, p_M > 0$.

Three remarks are in order. First, the target of the fiscal authority is normalized to zero in (13), and so it must accommodate to avoid default as soon as the public sector inherits some strictly positive liabilities at date 0 (as soon as $b_{-1,t} \geq 0$ with a strict inequality for some $t \geq 0$).[20] Second, the public sector incurs an immediate cost of default as soon as a path involves future default on government debt held by the private sector no matter how remote the date of this default. This reflects that the cost of default (financial instability, exclusion from capital markets,...) materializes as soon as the private sector understands that the public sector will break a promise at some future date.[21] Finally, the public sector discounts future adjustements with the same factor β as that of the private sector.

Initial public finances. We assume that at the outset, the public sector faces two identical exogenous government debt repayments b at dates 1 and 2. Formally, $b_{-1,1} = b_{-1,2} = b > 0$ and, for all $t \notin \{1;2\}$, $b_{-1,t} = 0$. There are no initial outstanding reserves ($r_{-1,t} = 0$ for all $t \geq 0$). This simple environment delivers all the important insights at a small analytical cost.

Unanticipated versus anticipated fiscal shock. We will study a version of the game in which the exogenous liabilities b due at dates 1 and 2 are *unanticipated*: F and M discover them at date 1. We will then study the situation in which these liabilities are *anticipated*: F and M discover them at the outset of date 0. The motivation is twofold. First, the unanticipated version is a useful building block to solve for the anticipated one. Second, each version of the game generates different insights that are clearer when presented separately.

[19] Formally, $\Delta_t = 1$ if and only if there exists a date $t' \geq t$ at which there is government debt held by the public and the public sector fails to fully honor it ($\sum_{i<t'} b_{i,t'} > \tau'_t p_{t'} + \sum_{j>t'} (q_{t',j} p_{t'}/p_j) b_{t',j}$, where $q_{t',j}$ is the (real) date-t' price asked for a date-j unit claim).

[20] Assuming a strictly positive surplus target clutters the exposition without adding insights.

[21] The factor $\beta(1+\beta)$ applied to the cost of default of F and M is just a convenient normalization.

Strategic interactions. Generically, an equilibrium is a set of actions by F and M such that each authority makes optimal decisions given correct expectations about the other authority's actions and all trades are fair to the private sector given correct expectations over future actions. We will compare the equilibria from games corresponding to different assumptions regarding order of moves and commitment.

Fiscal versus monetary lead. Under fiscal lead, at each date t, F sets τ_t and possibly issues bonds. Observing this, M sets p_t and possibly issues reserves. F and M swap these positions under monetary lead. We restrict the analysis to these situations in which one authority has the entire bargaining power in order to obtain clear insights into how bargaining power affects equilibrium outcomes.

Commitment power. Under full commitment, F and M can commit to any plan of future actions. Under limited commitment, they optimize at each date, and so the Nash equilibria resulting from their interactions are subgame perfect.

The following Section 3.2 first studies the situation of unanticipated fiscal shocks. Section 3.3 then tackles that in which the shocks are anticipated at date 0. Section 3.4 discusses the results.

3.2 Unanticipated fiscal shocks

In this Section 3.2, the public sector discovers the outstanding liabilities $b_{-1,1} = b_{-1,2} = b$ at the outset of date 1. This implies that they played their respective targets and did not issue claims at date 0. We first characterize the equilibrium when parties can commit to any future action plan, and then we do so when they cannot.

3.2.1 Full commitment

We characterize in turn equilibria under monetary and fiscal lead.

Proposition 5. ***(Unanticipated shocks, full commitment, M leads)***

1. *If $b \leq \beta\alpha_F p_M$ then M announces $p_t = p_M$ for all t, and the fiscal authority raises a stream of surpluses with date-1 present value $(1+\beta)b/p_M$.*

2. *If $\beta\alpha_F p_M < b \leq \beta\alpha_F(p_M + \beta\alpha_M)$, then M announces $p_1 = p_2 = b/\beta\alpha_F$, $p_t = p_M$ for $t > 2$, and F raises surpluses with date-1 present value $\beta(1+\beta)\alpha_F$.*

3. Otherwise F and M default and play their target at each date.

Proof. See the appendix. ■

Not surprisingly, the equilibrium is very similar to that in the static game described in Proposition 4. In case *1.*, M forces F to bear all the accommodation costs—this is the multiperiod equivalent of the monetary regime defined in Section 2. In case *2.*, F raises the maximum surplus $\beta(1+\beta)\alpha_F$—the one that makes it indifferent with default—and M picks up the residual accommodation by inflating away b at dates 1 and 2. Finally, there is a threshold for b beyond which the public sector defaults (case *3.*).

There is an asymmetry between F and M because the solvency constraint $b(1/p_1 + \beta/p_2) \leq \tau_1 + \beta\tau_2$ is linear in surpluses and strictly convex in price levels. Thus M has a unique optimal strategy that consists in setting the price level at the same level at dates 1 and 2 given that $b_{-1,1} = b_{-1,2} = b$.[22] On the other hand, F's strategy is indeterminate. Given its access to perfect bond markets and linear payoff, all that matters is the present value of the future surpluses that it commits to raise. Were its payoff (absent default) strictly concave in surpluses, F would obviously smooth the surpluses by permanently raising its per-period surplus target.

Proposition 6. ***(Unanticipated shocks, full commitment, F leads)***

1. *If $b \leq \beta\alpha_F(p_M + \beta\alpha_M)$ then F raises a stream of surpluses with date-1 present value $(1+\beta)b/(p_M + \beta\alpha_M)$, and M sets $p_1 = p_2 = p_M + \beta\alpha_M$ and $p_t = p_M$ for $t \notin \{1;2\}$.*

2. *Otherwise F and M default and play their target at each date.*

Proof. See the appendix. ■

The fiscal authority imposes maximum accommodation on the monetary authority and picks up any excess accommodation by raising surpluses (case *1.*), unless b is so large that the public sector defaults (case *2.*).

Note that, unlike under monetary lead, there is no pure fiscal regime whereby F sticks to its target for small levels of b. It is an uninteresting consequence of the normalization of the fiscal target to 0. F must obviously deviate from this target so that the public sector makes any nominal repayments at all. With a strictly positive surplus target, the pure fiscal regime would prevail for sufficiently low values of b.

[22] More generally, if $b_{-1,1} \neq b_{-1,2}$, then optimal price-level setting by M implies (ignoring the constraint $p_t \geq p_M$) $p_2/p_1 = \sqrt{(b_{-1,2}/b_{-1,1})}$.

3.2.2 Limited commitment

Again, we consider first monetary then fiscal lead. For conciseness, Proposition 7 below describes the equilibrium provided β is sufficiently large holding all parameters other than b fixed.[23]

Proposition 7. ***(Unanticipated shocks, limited commitment,*** M ***leads)***

1. *If* $b \leq \beta^2 \alpha_F p_M$*, then, as in the full-commitment case,* M *can implement the monetary regime* $p_t = p_M$ *for all* t*.*

2. *If* $\beta^2 \alpha_F p_M < b \leq \beta^2 \alpha_F (p_M + \beta \alpha_M)$*, then at date 1* F *sets* $\tau_1 = 0$ *and refinances the date-1 liability with debt due at date 2. Afterwards, it raises surpluses with date-2 present value* $\beta(1+\beta)\alpha_F$*, and so is indifferent between defaulting or not at date 2. Authority* M *sets* $p_1 = p_2 = b/(\beta^2 \alpha_F) > p_M$ *and* $p_t = p_M$ *otherwise.*

3. *Otherwise the public sector defaults.*

Proof. See the appendix. ■

Comparing cases *1.* in Propositions 5 and 7 shows that the range of values of b for which M can impose the monetary regime when moving first is strictly smaller absent commitment. The reason for this is as follows. Absent commitment, F's optimal strategy is to kick the can down the road by extending debt maturity so that all liabilities bunch at date 2. By making the date-2 liabilities sufficiently large, it forces M to accommodate at date 2 because F would credibly default at this date otherwise. Anticipating this, M starts accommodating at date 1 so as to optimally smooth price-level increases given the strict concavity of the solvency constraint. In other words, the model exhibits endogenous fiscal irresponsibility. All agents are far-sighted and share the same discount factor, and yet strategic concerns induce F to minimize date-1 fiscal consolidation by rolling over debt. This forces high future inflation, which the monetary authority mitigates by creating current inflation. As a result, M starts accommodating at date 1 whereas F does not until date 2 even though M has the entire bargaining power at each date. Finally, limited commitment also implies that default occurs at lower debt levels ($\beta^2 \alpha_F (p_M + \beta \alpha_M) < \beta \alpha_F (p_M + \beta \alpha_M)$) .

Proposition 8. ***(Unanticipated shocks, limited commitment,*** F ***leads)***

[23] The exact condition on β is stated in the proof of Proposition 7, which also explains how the results are modified for lower values of β. All insights carry over.

1. *If* $b \leq \underline{b} < \beta\alpha_F(p_M + \beta\alpha_M)$*, then* M *sets* $p_1 = p_M + \beta(1-\beta^2)\alpha_M < p_2 = p_M + \beta(1+\beta)\alpha_M$*, and* $p_t = p_M$ *for* $t > 2$*.* F *raises a surplus with date-1 present value* $b(1/p_1 + \beta/p_2)$*.*

2. *Otherwise the public sector defaults.*

Proof. See the appendix. ■

Proposition 8 highlights a fundamental asymmetry between M and F. Whereas, as shown in Proposition 7, F can extend the maturity of government debt so as to force monetary accommodation when M leads and lacks commitment, M cannot do so when F leads. The reason is that creating reserves in order to purchase government debt held by the public has no impact on the repayment schedule of the government. Hence M cannot kick the can down the road in order to force F to accommodate at lower debt levels than it would like to.

Absent commitment, F cannot help imposing maximum monetary accommodation at date 2. This is inefficient because it implies that p_1 must be set strictly inferior to p_2 so that M does not force default at date 1. This inefficient postponement of inflation implies that the default threshold $\underline{b}$ is strictly smaller than that under commitment.

Remark on the value of commitment. It is easy to see that the equilibria under full commitment described in Propositions 5 and 6 still obtain under the weaker assumption that only the leader (M in Proposition 5 and F in Proposition 6) can commit. This would however not be true if the duration of the inherited liabilities, measured for example by $b_{-1,2}/b_{-1,1}$, was sufficiently large. In this case, the follower could find a given date-2 deviation from target preferable to default at date 1, but no longer so at date 2 once it is no longer discounted by β. The simplifying assumption that $b_{-1,1} = b_{-1,2} = b$ enabled us to abstract from this time-inconsistency problem in this case of unanticipated shocks. The case of anticipated shocks studied in the following section is precisely meant to study this time inconsistency between dates 0 and 1.

3.3 Anticipated fiscal shocks

Consider now the situation in which the public sector learns at the outset of date 0 that it inherits $b_{-1,1} = b_{-1,2} = b$. The case in which parties can fully commit is a straightforward adaptation of that of full commitment and unanticipated shocks:

Proposition 9. ***(Anticipated shocks, full commitment)*** *Under full commitment, the respective equilibria under monetary and fiscal lead are respectively described by Proposition 5 and 6 up to the substitution of* $\{\alpha_M; \alpha_F\}$ *with* $\{\alpha_M/\beta; \alpha_F/\beta\}$.

Proof. See the appendix. ■

Intuitively, under full commitment, date-0 actions are immaterial. The schedule under which F collects surpluses is irrelevant, only their present value matters, and so assuming $\tau_0 = 0$ is without loss of generality. (This is only true under full commitment as we shall see below). Regarding M, there are no liabilities at date 0 and thus no gains from setting $p_0 > p_M$. The only difference with the unanticipated case stems from the fact that at date 0, F and M discount with β the date-1 present value of their respective costs of accommodations, whereas the cost of default along the equilibrium path is unchanged for them. Default is therefore relatively more costly than accommodation to the public sector at date 0 when the fiscal shock is more remote than at date 1. This induces time-inconsistency: F and M commit to accommodating higher levels of debt at date 0 than at date 1.

We are now equipped to solve for equilibria when the shock is anticipated and the public sector cannot commit beyond the current date. We view this case as the most empirically relevant one. First, an unexpected increase in *future* expenditures seems more realistic than a completely unforeseen large and *current* liquidity need. Second, a period in our model is a time interval over which the effects of a given fiscal or/and monetary policy fully materialize, and therefore corresponds reasonably well to a political or central-bank term beyond which it is difficult to credibly commit. We study in turn monetary and fiscal leads in this context.

Proposition 10. ***(Anticipated shocks, limited commitment,*** M ***leads)***

1. *If* $b \leq \beta^2\alpha_F(p_M + \beta\alpha_M)$ *then the equilibrium is as in the case of an unanticipated shock described in Proposition 7.*

2. *If* $\beta^2\alpha_F(p_M + \beta\alpha_M) < b \leq \alpha_F(p_M + \beta\alpha_M)$ *then* F *prepays at date 0 shares of* $b_{-1,1}$ *and* $b_{-1,2}$*, each with face value* $b - \beta^2\alpha_F(p_M + \beta\alpha_M)$*.* M *is inactive at date 0. The equilibrium at subsequent dates is described in Proposition 7 in the case* $b = \beta^2\alpha_F(p_M + \beta\alpha_M)$*.*

3. *If* $\alpha_F(p_M + \beta\alpha_M) < b \leq \alpha_F(p_M + \alpha_M)$ *then* M *issues reserves at date 0 in order to purchase date-*t *outstanding public debt for* $t \in \{1; 2\}$ *up to a face value equal to* $(b/\alpha_F - \beta\alpha_M)\epsilon/(1 + \beta)$ *at each date.* M *sets*

$p_1 = p_2 = b/\alpha_F$ *and* $p_t = p_M$ *for* $t \notin \{1;2\}$. *F raises surpluses*

$$\begin{cases} \tau_0 = \beta(1+\beta)(1-\beta^2)\alpha_F, \\ \tau_2 = \beta(1+\beta)\alpha_F, \\ \tau_t = 0 \text{ for } t \notin \{0;2\}. \end{cases} \tag{15}$$

4. Otherwise F and M default and play their target at each date.

Proof. See the appendix. ■

These results come from the time-inconsistency problem induced by the fact that default along the equilibrium path comes at the same current cost no matter how remote it is. This implies that the only difference between the unanticipated and anticipated cases is that the public sector is more willing to avert default at date 0 than at date 1. If b is sufficiently small that the public sector does not default in the unanticipated case (case *1.*), then the date-0 anticipation of the shock is immaterial. Beyond this threshold, F and M can use date-0 actions to commit to more accommodation than they would find ex-post optimal. F can commit by buying back some of the outstanding debt. M can commit to more inflation than it would find ex-post optimal with the creation of reserves backed by public debt which would threaten its solvency in case of sovereign default. We detail the mechanism through which such open-market operations serve as a commitment to future inflation in Section 3.4.

As in the static case (or in the case of full commitment), the leader M accommodates at date 0—that is, issues reserves to commit to future inflation—only if b is such that F has accommodated as much as possible and is indifferent with default (case *3.*). Before this, F is the only date-0 mover and bears the entire burden of accommodation by buying back debt.

Finally, Proposition 11 studies the case of fiscal lead. For brevity we state it assuming $\beta(1+\beta) \geq 1$—arguably the most plausible case, and discuss below how $\beta(1+\beta) < 1$ (slightly) affects the results.

Proposition 11. ***(Anticipated shocks, limited commitment, F leads)***

1. *If $b \leq \underline{b}$ then the equilibrium is as in the case of an unanticipated shock described in Proposition 8.*

2. *If $\underline{b} < b \leq \overline{b}$ then F is inactive at date 0 and subsequently sets surpluses with date-1 present value $\beta(1+\beta)\alpha_F$. At date 0, M issues reserves to purchase part of the publicly held debt due at date 1. It then sets $p_1 \in (p_M + \beta(1-\beta^2)\alpha_M, p_M + \beta(1+\beta)\alpha_M)$, $p_2 = p_M + \beta(1+\beta)\alpha_M$, and $p_t = p_M$ for $t \notin \{1;2\}$.*

3. If $\overline{b} < b \leq \alpha_F(p_M + \alpha_M)$, F prepays at date 0 a fraction $1 - \overline{b}/b$ of dates 1 and 2 liabilities, and then the equilibrium is as in 2. for $b = \overline{b}$.

4. Otherwise F and M default and play their target at each date.

Proof. See the appendix. ■

Again, if b is sufficiently small that the public sector would not default in the unanticipated case, then anticipation plays no role. Otherwise, the follower M is the only accommodating party at date 0 until its commitment to future inflation makes it indifferent with default. F then picks up the residual accommodation need up to the point at which the public sector finds default ex-ante optimal.

If, as assumed in Proposition 11, $\beta(1 + \beta) \geq 1$, then M reaches date-0 indifference with default before the value of p_1 that it commits to reaches p_2. For smaller values of β, an additional region of b arises between cases *2.* and *3.*, whereby M uses reserves to commit to higher values of both p_1 and p_2, with $p_1 = p_2$.

In sum, Propositions 10 and 11 show that debt prepayment and remunerated reserves are commitment devices that enable the public sector to accommodate a fiscal shock as long as it finds it optimal to do so under full commitment ($b \leq \alpha_F(p_M + \alpha_M)$ from Proposition 9) provided the shock is anticipated. Whether F and M can commit or not affects how they share the accommodation effort, however.

Summary of the main findings. This multiperiod analysis sheds light on two distinct implications from limited commitment. A first implication is purely due to the multiplicity of public-debt installments. A second implication stems instead from the time-inconsistency induced by a path-independent cost of default.

This latter implication is muted when the fiscal shock is unanticipated. In this case, under monetary lead, F can force M to accommodate more when commitment is more limited. Authority F does so by rolling over shorter-term government liabilities until they bunch with the longer-term ones, thereby creating a cumulated amount of public debt that F cannot be asked to mop up with fiscal surpluses only. Conversely (and unsurprisingly), lack of commitment hurts F when it leads, because it cannot help imposing that M inflate as much as possible at remote dates. Authority F therefore has no choice but inducing less inflation at shorter horizons. This failure to enable M to smooth debt monetization implies a higher ex-ante fiscal burden.

Assuming then that the fiscal shock is anticipated as of date 0 brings in the second commitment problem due to a path-independent cost of default.[24] Both F and M can in this case avail themselves of devices enabling them to accommodate more than they find ex-post optimal. The fiscal authority simply buys back some debt so that its long-term liabilities remain below the level at which its future behavior is ex-ante inefficient. The monetary authority swaps some public debt with remunerated reserves with the private sector as a credible commitment to ex-post excessive future inflation. As b increases, both authorities are first passive at date 0. The leading authority then remains passive but forces the follower to apply its respective commitment device at date 0, up to the point at which the follower is indifferent with default. Beyond this point the leader also commits to future accommodation.

So, who has the bargaining power since 2008? A natural empirical counterpart of the exogenous liabilities b that the public sector inherits in our model are the public liabilities that resulted from the bailout of many private agents during the 2008 crisis, and from the subsequent attempts at getting the economy out of a liquidity trap. The Federal Reserve largely contributed to the effort of the public sector with a massive creation of remunerated reserves. This can occur under two circumstances in our setup. Under monetary lead, this occurs when the fiscal authority has applied maximum fiscal discipline, to the point that it is indifferent with default (Proposition 10, case *3.*). Under fiscal lead, this may occur even before the fiscal authority attempts at any early fiscal consolidation (Proposition 11, case *2.*). We believe that the assumption of fiscal lead fits the response to the 2008 crisis much better than that of monetary lead for two reasons. First, the realized and projected primary federal deficits since 2008 are hardly consistent with maximum fiscal consolidation. In fact, comparing data prior to 2000 with post-2008 deficits, Hall (2013) argues that fiscal policy has seriously departed from the objective of stabilizing the debt to GDP ratio. To be sure, a standard fiscal-multiplier argument which is absent from our model can rationalize the presence of deficits during the post-2008 recession years—until 2011-12, say. It seems however more challenging to explain persistent realized and projected deficits since 2008 with such a fiscal-multiplier argument. Second, under monetary lead, M optimally sets a high price level as soon as of date 1 and then maintains the same level at date 2: Inflation

[24] In fact, this second time-consistency problem arises in the even simpler environment of a single future payment ($b_{-1,t} = 0$ for all $t \neq 1$ and $b_{-1,1} > 0$) as the multiplicity of installments is irrelevant for it.

occurs early at date 1 and the price level is then stable. On the other hand inflation arises in a more protracted fashion under fiscal lead, with the bulk of debt monetization taking place "in the long run" (at date 2). This is easier to reconcile with the absence of significant realized or expected inflation thus far since 2008.

3.4 Discussion

Inspecting the mechanism through which M purchases long-term public debt through open-market operations so as to commit to future inflation is instructive. Consider for example situations *2.* or *3.* in Proposition 11, in which M needs to commit to a date-1 price level p_1 strictly above the value $p_M + \beta(1-\beta^2)\alpha_M$ above which it would rather default ex post. Suppose M issues reserves with a face value $r < b$ due at date 1. In the (out-of-equilibrium) event that F fully defaults at date 1, M is forced to avoid default on reserves by setting the date-1 price level above the level p_1^d such that

$$p_1^d \epsilon = r. \tag{16}$$

This means that along the equilibrium path in which public debt is fully honored, F can extract a maximum price level p_1 such that

$$p_1 + \beta p_2 = \beta(1+\beta)\alpha_M + p_1^d + \beta p_M. \tag{17}$$

Condition (17) states that M is indifferent between defaulting or not along the default-free equilibrium path, understanding that the date-1 price will have to be at least p_1^d in case of sovereign default. Injecting the value of p_2 from Proposition 11 and that of p_1^d from (16) yields

$$p_1 = p_M + \beta(1-\beta^2)\alpha_M + \frac{r}{\epsilon}. \tag{18}$$

Expression (18) yields the amount of reserves r that M must issue at date 0 in order to credibly commit to $p_1 > p_M + \beta(1-\beta^2)\alpha_M$.

Relevance of QE. Reis (2017) notes that open-market operations such as the one that we consider are not immaterial if there is a probability of default on government debt *in equilibrium* whereas remunerated reserves are risk-free. Our strategic approach yields the stronger result that open-market operations, *by swapping two securities that are, along the equilibrium path, risk-free and thus perfect substitutes*, also has an impact on future price levels

because the *out-of-equilibrium* possibility of default affects future strategic interactions. This derivation yields four other interesting insights that we discuss in turn.

3.4.1 Finite ϵ and dividend policy

The central bank's dividend policy is immaterial in the limiting case of an arbitrarily small ϵ studied thus far. The analysis is straightforward when ϵ is finite. Expression (18) shows that the face value of reserves, r, which is necessary to induce a given date-1 price p_1 increases with ϵ. Thus r may exceed b for ϵ sufficiently large in principle, thereby violating our restriction to open-market operations in which public debt and reserves are swapped at fair value. The central bank can easily overcome this by paying out a dividend to the government at date 0, and retaining only a sufficiently small fraction of its income.[25] This retained fraction can in fact be arbitrarily small as we assumed: All that matters in our environment is that M's net wealth is strictly positive at date 1.

3.4.2 Helicopter money

We focus thus far on open-market operations: The creation of reserves only serves to buy outstanding public debt at the prevailing market price. This is a realistic but arbitrary restriction on M's action set. M could create reserves and directly pay out the proceeds to F at date 0 in order to commit to future inflation—a mechanism deemed "helicopter money". In this situation, reserves are backed only by M's income ϵ whether F defaults on its obligations or not, and so condition (16) implies that M commits to inflation r/ϵ by issuing reserves of size r.[26]

The reserves of M are completely unbacked by future taxes along the equilibrium path in the case of helicopter money whereas it is only an out-of-equilibrium threat in the case of open-market operations. Yet the same given quantity r of reserves yields the same commitment to future inflation under open-market-operations as with helicopter money. This implausible result obviously owes to the assumption of purely fixed costs of sovereign default, which implies in turn that F can credibly threaten with default on 100% of outstanding government debt. Suppose conversely that F cannot

[25] This may lead to a negative date-0 book value of the central bank's equity if this future income is not booked at fair value.

[26] Authority F may either rebate the payment from M to the private sector or buy back debt. Only the latter benefits M.

credibly threaten M with a default larger than a haircut $h \in (0, 1]$ on its liabilities. This does not affect the impact of reserves r on the date-1 price in the case of helicopter money since the reserves are fully unbacked anyway. On the other hand, with standard open-market, condition (16) becomes

$$p_1^d \epsilon + r(1 - h) = r, \tag{19}$$

and so the date-1 price level induced by reserves r is equal to $p_M + \beta(1 - \beta^2)\alpha_M + rh/\epsilon$ in the case of open-market operations, whereas it is still $p_M + \beta(1 - \beta^2)\alpha_M + r/\epsilon$ with helicopter money. The latter is above the former as soon as $h < 1$.

Notice finally that holding ϵ fixed, the amount of reserves r needed to reach a given commitment level becomes arbitrarily large as $h \to 0$.

3.4.3 Strong credibility and politically-insensitive assets

The contrapositive of the result that a highly leveraged central bank can credibly commit to high future inflation is that inflation targeting is more credible ex-ante if the central bank is sufficiently wealthy in all future states of nature, so that it is insensitive to the (out-of-equilibrium) threat of insolvency resulting from an (out-of-equilibrium) fiscal crisis. One way to achieve this is to augment the central bank's mandate with a covenant imposing that it is endowed with an amount of "politically-insensitive" assets that increases in the total liabilities of the public sector—not only the liabilities that are in the bank's balance sheet at a given date. Politically-insensitive assets include gold or foreign assets. In our elementary setup this means committing to an ϵ always sufficiently large that debt monetization is never credible.

3.4.4 What if abandoning the currency is also very costly to the fiscal authority?

Finally, we replace Assumption 1 with

Assumption 2. *F and M both incur an arbitrarily large disutility from a default on remunerated reserves.*

It seems reasonable to consider the possibility that the government at large incurs large political costs from a currency reform induced by a fiscal crisis. It is easy to see that the analysis is verbatim under fiscal lead: Propositions 6, 8, and 11 are unchanged. Basically the cost to F from default on

reserves are immaterial as F, when it leads, never needs to accommodate when reserves are at risk.

Conversely, Assumption 2 gives exorbitant power to M under monetary lead. Suppose that M wants to induce a payment θ from F at date t while staying on target p_M. Authority M can simply for example issue unbacked reserves due at date t, r_t, such that

$$p_M(\epsilon_t + \theta) = r_t, \tag{20}$$

where ϵ_t is the date-t market value of M's net wealth. This strong ability of the monetary authority to induce fiscal discipline seems highly unrealistic. This casts further doubt on the plausibility of the assumption that M has a very strong bargaining power (M leads).

4 Conclusion

This paper applies non-cooperative game theory to the study of the interaction between fiscal and monetary policy. We derive the implications from a wide array of assumptions regarding payoffs, timing, and commitment power. We conclude that the model in which the fiscal authority has significant bargaining power and the public sector cannot indefinitely commit is the one that delivers the most plausible predictions. We are admittedly only scratching the surface here. There are at least three interesting avenues for future research.

First, the obvious drawback from assuming reduced-form payoffs as we do is that it precludes substantial normative analysis. An interesting research route thus consists in taking a stand on the determinants of the policy targets and on the costs of sovereign default. This would connect equilibrium outcomes to the deep parameters of the economy, thereby opening up the opportunity to address a number of normative questions. One can in particular interpret the payoff of the central bank as resulting from a mandate that could be optimally designed. As discussed in Section 3.4, our approach suggests that the actions of the government as shareholder of the central bank have important strategic implications, and so the capitalization of the central bank should be regulated by the mandate ex-ante rather than being mostly left to ex-post negotiations, as is by and large currently the case. Such a micro-founded model could also inform the debate on the opportunity to put the central bank in charge of financial stability and banking supervision, as the bank in this case presumably puts more weight on sovereign default, and this deeply affects its whole equilibrium policies.

Second, we could extend the game to more players along the lines of Aguiar et al. (2015) or Sargent (1986). The "fiscal" authority could explicitly feature executive and legislative branches, and local tax authorities. The lack of cooperation and coordination across these branches could in particular be an endogenous source of commitment power vis-à-vis the monetary authority.

Third, uncertainty is an important feature of the long-term dynamics of public finances that we study, and so departing from perfect foresight is in order. One particularly interesting source of uncertainty regards the "types" of the politicians and central bankers that will be making future decisions—taking into account that politicians are elected and appoint central bankers. The theory of games under incomplete information could in addition be applied to the case in which decision makers privately observe their types and try to signal it to their counterparts and to the private sector.

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Appendix

Proof of Proposition 1

Step 1: Equilibria are either all default-free or all feature default. If a strategy profile (τ, p) constitutes an equilibrium then it must be that $b \geq \tau p$ otherwise at least one player could strictly benefit from getting closer to target. If $b = \tau p$ then it must be that

$$g'_F(\tau - \tau_F) \leq f'_F(0), \tag{21}$$

$$g'_M(p - p_M) \leq \frac{b}{p^2} f'_M(0). \tag{22}$$

If either inequality is not satisfied, the associated player could benefit from getting closer to target. Conversely if $b > \tau p$ then none of these inequalities holds, otherwise the associated player would benefit from accommodating more.

Suppose $b \leq \tau_M p_F$ and consider an equilibrium (τ, p). If $\tau > \tau_M$ then $p \leq p_F$ and $b/p^2 \geq \tau/p > \tau_M/p$, and so (21) is not satisfied whereas (22) is, a contradiction. Thus $\tau \leq \tau_M$ and any equilibrium must be such that $b = \tau p$. If $b > \tau_M p_F$, then an equilibrium cannot be such that $b = \tau p$ otherwise (21) and (22) would imply $b \leq \tau_M p_F$.

Step 2: Default-free equilibria. Suppose $b \leq \tau_M p_F$. Define

$$\tau(b) = \inf \left\{ \tau \in [\tau_F, \tau_M] \mid \frac{b}{\tau} g'_M \left(\frac{b}{\tau} - p_M \right) \leq \tau f'_M(0) \right\}, \tag{23}$$

$$b^1 = \sup\{b \geq \tau_F p_M \mid \tau(b) = \tau_F\}, \tag{24}$$

$$b^2 = \tau_M p_M, \tag{25}$$

$$b_- = \inf\{b^1; b^2\}, \tag{26}$$

$$b_+ = \sup\{b^1; b^2\}. \tag{27}$$

A profile (τ, p) is an equilibrium if and only if $\tau \geq \tau_F$, $p \geq p_M$, $\tau p = b$, and (τ, p) satisfies (21) and (22). The equilibria are therefore exactly $(\tau, b/\tau)$ for $\tau \in [\tau(b), \inf\{b/p_M; \tau_M\}]$, and so they can be described as in the proposition.

Step 3: Equilibrium with default. Suppose $b > \tau_M p_F$. For $d > 0$, the

conditions

$$g'_F(\tau - \tau_F) = f'_F(d), \tag{28}$$

$$g'_M(p - p_M) = \frac{b}{p^2} f'_M(d) \tag{29}$$

implicitly define $\tau(d)$ and $p(d, b)$ as strictly increasing functions. Furthermore, $b/p(d, b)$ is strictly increasing in b. Since $d(\tau, p, b) = b/p - \tau$ is strictly decreasing in p and τ,

$$d = \frac{b}{p(d, b)} - \tau(d) \tag{30}$$

admits a unique solution d for b fixed. This solution characterizes the unique equilibrium. Finally, the comparative statics with respect to b are a straightforward consequence from the fact $b/p(d, b)$ is strictly increasing in b holding d fixed.

Proof of Proposition 2

The payoffs $E[U_X(\tau, p)]$ for $X \in \{F; M\}$ are strictly concave, bounded above, and differentiable and so (τ, p) is a Nash equilibrium if and only if

$$\frac{\partial E[U_F(\tau, p)]}{\partial \tau} = \frac{\partial E[U_M(\tau, p)]}{\partial p} = 0. \tag{31}$$

A similar reasoning to that in Step 3 in the proof of Proposition 1 shows that this system has a unique solution (τ, p).

Consider now (τ, p) such that $\tau p < b$. Then

$$\lim_{\sigma \to 0} \frac{\partial E[U_F(\tau, p)]}{\partial \tau} = -g'_F(\tau - \tau_F) + f'_F\left(\frac{b}{p} - \tau\right), \tag{32}$$

$$\lim_{\sigma \to 0} \frac{\partial E[U_M(\tau, p)]}{\partial p} = -g'_M(p - p_M) + \frac{b}{p^2} f'_M\left(\frac{b}{p} - \tau\right). \tag{33}$$

If (τ, p) are such that $b = e^{\sigma k} \tau p$ for $k > 0$, then

$$\lim_{\sigma \to 0} \frac{\partial E[U_F(\tau, p)]}{\partial \tau} = -g'_F(\tau - \tau_F) + P(\epsilon_F + \epsilon_M < k) f'_F(0), \tag{34}$$

$$\lim_{\sigma \to 0} \frac{\partial E[U_M(\tau, p)]}{\partial p} = -g'_M(p - p_M) + P(\epsilon_F + \epsilon_M < k) \frac{b}{p^2} f'_M(0). \tag{35}$$

Continuity then implies that if $b > \tau_M p_F$, the unique equilibrium converges to that in the case without uncertainty when $\sigma \to 0$. Otherwise it converges to the unique solution to

$$b = \tau p, \tag{36}$$

$$p g'_M(p - p_M) f'_F(0) = \tau g'_F(\tau - \tau_F) f'_M(0). \tag{37}$$

The probability of default is strictly positive, increasing from 0 to 1 as b spans $(\tau_F p_M, \tau_M p_F]$.

Proof of Proposition 3

Given the fixed default cost, a strategy profile (τ, p) is an equilibrium if and only if either $\tau p = b$ or $(\tau, p) = (\tau_F, p_M)$. Default-free equilibria thus correspond to the portion of the hyperbolae $\tau p = b$ in the plane (τ, p) that is within the rectangle $[\tau_F, \tau_F + \alpha_F] \times [p_M, p_M + \alpha_M]$. Default is an equilibrium outcome if and only if $b \geq \tau_F(p_M + \alpha_M)$ and $b \geq (\tau_F + \alpha_F)p_M$, otherwise one authority always finds accommodation preferable to default.

Proof of Proposition 5

Suppose M plays $p_t = p_M$ for all $t \geq 1$. Default grants F a payoff $-\beta(1 + \beta)\alpha_F$. F may instead accommodate, which it can do in infinitely many ways as long as the stream of surpluses that it raises has date-1 present value $(1+\beta)b/p_M$. Therefore, as long as $b \leq \beta\alpha_F p_M$, M optimally commits to the profile $p_t = p_M$ for all $t \geq 1$ and F accommodates.

If $b > \beta\alpha_F p_M$, then M can avoid default with any strategy such that $p_t = p_M$ for $t \geq 3$ and $b/p_1 + \beta b/p_2 \leq \beta(1+\beta)\alpha_F$. Setting $p_1 = p_2 = b/(\beta\alpha_F)$ maximizes its payoff over these strategies, and so M plays this as long as $V_1^M = -(1 + \beta)(b/(\beta\alpha_F) - p_M) \geq -\beta(1 + \beta)\alpha_M$, or $b \leq \beta\alpha_F(p_M + \beta\alpha_M)$.

The public sector defaults otherwise.

Proof of Proposition 6

Suppose first that F not only raises surpluses and issues debt, but also sets p_1 and p_2 subject to $p_1 + \beta p_2 \leq (1 + \beta)(p_M + \beta\alpha_M)$. In this case, the best strategy that does not involve default consists in minimizing the real burden of debt $b/p_1 + \beta b/p_2$ by setting $p_1 = p_2 = p_M + \beta\alpha_M$, and in raising matching surpluses with date-1 present value $(1 + \beta)b/(p_M + \beta\alpha_M)$. This is preferable to default if and only if $b \leq \beta\alpha_F(p_M + \beta\alpha_M)$.

For such values of b, F can indeed induce M to set $p_1 = p_2 = p_M + \beta\alpha_M$ by announcing any strategy such that the public sector is exactly solvent with such prices p_1 and p_2 (and $p_t = p_M$ for $t \geq 3$). Authority F can for example set $\tau_1 = \tau_2 = b/(p_M + \beta\alpha_M)$, $b_{t,t'} = 0$ for all $t' > t \geq 1$. The monetary authority then finds it optimal to accommodate with $p_1 = p_2 = p_M + \beta\alpha_M$.

Proof of Proposition 7

We derive the equilibrium by backward induction.

Suppose that the public sector has to repay b' at date 2. From Proposition 4, M chooses $p_2 = p_M$ as long as F can bear the accommodation cost ($b' \leq \beta(1+\beta)\alpha_F p_M$). For $b' \in [\beta(1+\beta)\alpha_F p_M, \beta(1+\beta)\alpha_F(p_M+\beta(1+\beta)\alpha_M)]$, M forces F to accommodate as much as possible ($\tau_2 = \beta(1+\beta)\alpha_F$) and M deviates from target ($p_2 = b'/(\beta(1+\beta)\alpha_F)$). For larger values of b', the public sector defaults.

We now consider the problem from the date-1 perspective. Suppose M has played and p_1 is fixed. Authority F optimally seeks to maximize p_2 in order to minimize the real debt burden. From the date-2 analysis, p_2 is weakly increasing in b', and $b' = b + p_2(b/p_1 - \tau_1)/\beta$ is in turn increasing in p_2 and decreasing in τ_1. We now prove a series of auxiliary results.

Result 1. Authority M can set $p_1 = p_2 = p_M$ if and only if $b \leq \beta^2\alpha_F p_M$.
Proof. If $b \leq \beta^2\alpha_F p_M$, then $b'/p_2 \leq b/p_M + b/(\beta p_M) \leq \beta(1+\beta)\alpha_F$, and so from the date-2 analysis M can set $p_1 = p_2 = p_M$. Conversely if $b > \beta^2\alpha_F p_M$ then if M sets $p_1 = p_M$, F sets $\tau_1 = 0$ and rolls over date-1 debt so that $b' = b + p_2 b/(\beta p_M) > \beta(1+\beta)\alpha_F$, implying that M must accommodate at date 2. ■

Result 2. If $b > \beta^2\alpha_F p_M$, then $\tau_2 = \beta(1+\beta)\alpha_F$ in equilibrium.
Proof. If $\tau_2 < \beta(1+\beta)\alpha_F$, then it must be that $p_2 = p_M$ otherwise M would induce a higher τ_2 with a lower price level p_2. From Result 1, it must then be that $p_1 > p_M$. Furthermore, it must be that

$$b' + \tau_1 p_M/\beta \leq \beta(1+\beta)\alpha_F p_M, \tag{38}$$

otherwise F could force $p_2 > p_M$ at date 2 by reducing τ_1. If inequality (38) is strict then M can reduce p_1. If (38) binds, F can still strictly benefit by reducing p_1 and increasing p_2, a contradiction. ■

Result 3. If $b > \beta^2\alpha_F p_M$, then if an equilibrium is such that $\tau_1 > 0$ it is

also such that $p_2 = p_M + \beta(1+\beta)\alpha_M$.

Proof. From Result 2, $\tau_2 = \beta(1+\beta)\alpha_F$. So if $p_2 < p_M + \beta(1+\beta)\alpha_M$, F can reduce τ_1 to increase b' thereby forcing M to raise p_2 unless $\tau_1 = 0$. ■

Suppose $b > \beta^2\alpha_F p_M$. These results imply that if M seeks to implement an equilibrium such that $p_1 = p_2$ ("smoothing strategy") then it must be such that $\tau_1 = 0$, $\tau_2 = \beta(1+\beta)\alpha_F$, and so $p_1 = p_2 = b/(\beta^2\alpha_F)$. Such a smoothing strategy grants M a payoff (viewed from date 1)

$$(1+\beta)\left[\frac{b}{\beta^2\alpha_F} - p_M\right], \tag{39}$$

and is thus feasible as long as $b \leq \beta^2\alpha_F(p_M + \beta\alpha_M)$.

Alternatively, if M prefers to induce $\tau_1 > 0$ ("extracting strategy"), then it must sacrifice inflation smoothing by setting $p_1 < p_2 = \beta(1+\beta)\alpha_M$. For a given b the date-1 price level $p_1(b)$ solves in this case

$$\frac{b}{p_1(b)} + \frac{\beta b}{p_M + \beta(1+\beta)\alpha_M} = \beta(1+\beta)\alpha_F, \tag{40}$$

or $p_1(b) = p_M$ if (40) admits no solution larger than p_M. This extracting strategy can be sustained up to the point at which $p_1(b) = p_M + \beta(1-\beta^2)\alpha_M$ at which M is indifferent with default. This defines an upper bound $\underline{b}$ for the values of b such that the extracting strategy is admissible that solves

$$\underline{b} = \frac{\beta\alpha_F[p_M + \beta(1+\beta)\alpha_M][p_M + \beta(1-\beta^2)\alpha_M]}{p_M + \beta[1+\beta(1-\beta)]\alpha_M}. \tag{41}$$

The smoothing strategy is clearly dominant for b in a right neighborhood of $\beta^2\alpha_F p_M$ as it converges to the commitment solution whereas the extracting one does not. Furthermore, straightforward computations show that $\underline{b} \leq \beta^2\alpha_F(p_M + \beta\alpha_M)$ for β sufficiently close to 1. This implies that the smoothing strategy always dominates the extracting one in this case, and establishes the proposition.

For lower values of β, $\underline{b} > \beta^2\alpha_F(p_M + \beta\alpha_M)$. This implies that the equilibrium features an additional region $b \in (\beta^2\alpha_F(p_M + \beta\alpha_M), \underline{b})$ that stands between the smoothing-strategy region and the default region in which M implements the extracting strategy.

Proof of Proposition 8

Authority F ex-post optimally imposes full accommodation by M at date 2 unless the public sector defaults, and so $p_2 = p_M + \beta(1+\beta)\alpha_M$ in the absence of default. This implies that at date 1, F forces M to the maximum feasible accommodation given date-2 actions. The constraint $(p_1 - p_M) + \beta(p_2 - p_M) \leq \beta(1+\beta)\alpha_M$ implies $p_1 = p_M + \beta(1-\beta^2)\alpha_M$ in the absence of default. Authority F then raises a stream of surplus with date-1 present value $b(1/p_1 + \beta/p_2)$ as long as this accommodation cost is lower than $\beta(1+\beta)\alpha_F$, and otherwise the public sector defaults. The default cutoff is $\underline{b}$ defined in (41).

Proof of Proposition 9

Under commitment, there is nothing relevant that either authority can do at date 0 that it cannot do later on. Setting $p_0 > p_M$ is useless because there is no outstanding public debt at date 0, and the timing of surpluses is immaterial. Furthermore, the scaling of defaults costs by $1/\beta$ stems from the assumption that F and M incur these costs immediately given a non-sustainable path for public finances, as formalized in (13) and (14).

Proof of Proposition 10

Suppose first that $b \leq \beta\alpha_F^2(p_M + \beta\alpha_M)$. For such values of b, Proposition 7 shows that there is no default and characterizes the equilibrium in the case of an unanticipated fiscal shock and limited commitment. There is nothing that either party can do at date 0 that would improve its situation over this unanticipated case. First, as is shown below, issuing reserves can only lead M to set higher future price levels than in the absence of reserves. Second, pre-paying debt by raising $\tau_0 > 0$ would also be costly to F as it would enable M to reduce p_1 and p_2. The equilibrium is therefore as described in Proposition 7.

Consider then the case $b \in (\beta^2\alpha_F(p_M + \beta\alpha_M), \alpha_F(p_M + \beta\alpha_M)]$. For such values of b, Proposition 7 shows that there is default in the absence of commitment if the shocks are unanticipated, whereas Proposition 9 shows that the equilibrium under commitment does not feature default when shocks are anticipated as of date 0. In this case, M can let F bear the brunt of ex-ante accommodation by being inactive at date 0. The fiscal authority then optimally prepays the minimum amount of future debt at date 0 so as to elicit maximum accommodation by M at dates 1 and 2 ($p_1 = p_2 = p_M + \beta\alpha_M$). This implies reducing future outstanding liabilities at dates 1 and 2 from b to

$\beta^2\alpha_F(p_M+\beta\alpha_M)$. This requires raising $\tau_0 = \beta(1+\beta)[b/(p_M+\beta\alpha_M)-\beta^2\alpha_F]$. Given that F also raises $\tau_2 = \beta(1+\beta)\alpha_F$, it is willing to bear all the brunt of ex-ante accommodation this way as long as

$$\tau_0 + \beta^2\tau_2 \leq \beta(1+\beta)\alpha_F, \tag{42}$$

or

$$b \leq \alpha_F(p_M + \beta\alpha_M). \tag{43}$$

If $b \in (\alpha_F(p_M + \beta\alpha_M), \alpha_F(p_M + \alpha_M)]$, then again the public sector defaults in the absence of commitment if the fiscal shock is unanticipated (Proposition 7) whereas it prefers to make good under commitment if it anticipates the shock at date 0 (Proposition 9). The ex-ante accommodation effort of F described in the previous case is no longer sufficient, however, and so M must also contribute by committing to price levels p_1 and p_2 such that $\beta(1+\beta)\alpha_F \geq \beta(b/p_1 + \beta b/p_2)$. It does so at the lowest cost by committing to $p_1 = p_2 = b/\alpha_F$.

We now show that M can commit to such future price levels by issuing remunerated reserves. Suppose M issues reserves such that $r_{0,1} = r_{0,2} = r$. In case of sovereign default at date 1, M must set p_1^d at date 1 and p_2^d at date 2 such that

$$\frac{r}{p_1^d} + \frac{\beta r}{p_2^d} \leq \epsilon, \tag{44}$$

where ϵ is the date-1 present value of its (real) income. Authority M maximizes its payoff by doing so with $p_1^d = p_2^d = (1+\beta)r/\epsilon$. This implies that M is at date 1 indifferent between any prices p_1 and p_2 that avert default and defaulting if and only if

$$p_1 + \beta p_2 = \beta(1+\beta)\alpha_M + p_1^d + \beta p_2^d, \tag{45}$$

and so $p_1 = p_2 = b/\alpha_F$ is feasible for $r = \epsilon(b/\alpha_F - \beta\alpha_M)/(1+\beta)$.

Finally, if $b > \alpha_F(p_M + \alpha_M)$, then the public sector defaults as it does under full commitment.

Proof of Proposition 11

The proof mirrors that of Proposition 10 and we only sketch it.

First, if $b \leq \underline{b}$, where $\underline{b}$ is defined in (41), then F and M do not default when the fiscal shock is unanticipated. For the same reasons as when M leads, they both are inactive at date 0 and then play the equilibrium described in Proposition 8.

Second, there exists $\overline{b} > \underline{b}$ defined below such that for $b \in (\underline{b}, \overline{b}]$, F remains inactive at date 0 but induces M to commit to higher price levels than it finds ex-post optimal so that the public sector does not default. The difference with the situation in which M leads is that $p_1 < p_2$ ex-post, and so M only raises p_1 to a level $p_1(b)$ such that

$$\frac{b}{p_1(b)} + \frac{\beta b}{p_M + \beta(1+\beta)\alpha_M} = \beta(1+\beta)\alpha_F. \tag{46}$$

The threshold $\overline{b}$ is implicitly defined as

$$p_1(\overline{b}) - p_M + \beta^2(1+\beta)\alpha_M = (1+\beta)\alpha_M. \tag{47}$$

Note that $\beta(1+\beta) \geq 1$ ensures that $p_1(\overline{b}) < p_M + \beta(1+\beta)\alpha_M$ and so M never fully smoothes inflation. In order to commit to the price level $p_1(b)$, M issues a date-1 reserve $r = \epsilon[p_1(\overline{b}) - (p_M + \beta(1-\beta^2)\alpha_M)]$. This way it commits to a date-1 price level in case of default p_1^d such that $p_1^d \epsilon = r$. This means that along the equilibrium path in which public debt is fully honored, F can extract a maximum price level p_1 such that

$$p_1 + \beta p_2 = \beta(1+\beta)\alpha_M + p_1^d + \beta p_M, \tag{48}$$

or

$$p_1 = p_M + \beta(1-\beta^2)\alpha_M + \frac{r}{\epsilon} = p_1(\overline{b}). \tag{49}$$

For $b \in (\overline{b}, \alpha_F(p_M + \alpha_M)]$, F must also accommodate at date 0. It does so by prepaying a (nominal) amount $b - \overline{b}$ of public debt at each date 1,2 and then lets M accommodate as above for $b = \overline{b}$.

Otherwise the public sector defaults.