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PARTIAL LANGUAGE COMPETENCE

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Partial Language Competence*

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Abstract

This paper proposes an equilibrium concept, Language-Based Expectation Equilibrium, which accounts for partial language understanding in sender-receiver cheap talk games. Each player is endowed with a privately known language competence which represents all the messages that he understands. For the messages he does not understand, he has correct but only coarse expectations about the equilibrium strategies of the other player. In general, a language-based expectation equilibrium outcome differs from Nash and communication equilibrium outcomes, but is always a Bayesian solution. Partial language competence of the sender rationalizes information transmission and lies in pure persuasion problems, and facilitates information transmission from a moderately biased sender.

KEYWORDS: Analogy-based expectations; Bayesian solution; bounded rationality, cheap talk; language; pure persuasion; strategic information transmission.

JEL CLASSIFICATION: C72; D82

1 Introduction

In this article we propose a novel framework to capture partial language competence in sender-receiver cheap talk games. The sender is privately informed about a payoff-relevant state (his *payoff type*), and the receiver chooses an action that affects the utility of both players. Before the receiver chooses his action, the sender can strategically transmit information to the receiver by directly sending a cheap talk message to the receiver.

We depart from standard cheap talk (e.g., Crawford and Sobel, 1982) by incorporating language competence into the model. Each player is privately informed about his language

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competence (his *language type*), which represents all the messages that he understands. Language competence determines how a player perceives the other player's strategy. If the sender does not understand some messages, he perceives only coarsely (but correctly on average) the response of the receiver to such messages. Similarly, if the receiver does not understand some messages, he perceives only coarsely the sender's strategy over such messages; hence, after these messages, the receiver updates only coarsely (but in a Bayesian way) his information about the sender's payoff type.

Language competence constrains neither the set of messages the sender is able to send to the receiver nor the set of messages the receiver is able to receive and distinguish. That is, partial language understanding does not restrict the set of possible strategies of the players. Instead, partial language competence is modeled as bounded cognitive rationality: it determines players' ability to predict the meaning and impact of the messages. Precisely, we incorporate players' language competence into the solution concept by adopting the notion of analogy-based expectation equilibrium developed by Jehiel (2005), Jehiel and Koessler (2008), Ettinger and Jehiel (2010). According to this concept, every player bundles the nodes at which his opponent acts into analogy classes, and best-responds to the average play in theses classes. In our communication game, the analogy classes are determined by the language competences of the sender and the receiver: the sender puts in the same class the receiver's reactions to all the messages that he does not understand, and the receiver puts in the same class all nodes at which the sender uses messages that the receiver does not understand.[1]

As a concrete example, imagine that the sender is a French tourist going to a restaurant in China. He would like to order food, say rice or noodles, but the menu is only written in Chinese and does not contain any picture of the meals. The tourist can pick and show any item from the menu to the waiter, but he cannot figure out exactly what he will get. If three quarters of the dishes actually served in the restaurant are rice dishes and one quarter of them are noodles, the fact that the tourist partially but correctly perceives the response of the Chinese waiter means that, in a language-based expectation equilibrium, by picking any item from the menu he expects to get rice with probability three quarters and noodles with probability one quarter. Now imagine that the tourist tries to ask for food in French. If the Chinese waiter does not speak French, he might be able to observe a phonetic difference between a tourist asking for rice and a tourist asking for noodles, but he would not understand the correlation between what the tourist wants to eat and what the tourist asks for. The waiter may still decide to serve noodles because he knows that, on average, most French-speaking customers wish to order noodles.

[1]We can view each player's perception of the other player's strategy as resulting from a learning process: senders and receivers from a large population are randomly matched and play a best response to the feedback they receive about the past behavior in the population, with the property that feedbacks of a player are aggregated whenever a message outside his language competence has been used (see, e.g., Ettinger and Jehiel, 2010 or Esponda and Pouzo, 2016 for more details).

We describe how we incorporate language competence in sender-receiver cheap talk games in Section 2. We define a language-based expectation equilibrium (LBEE) for such games in Section 3. When players are fully competent, that is, if they understand all messages available to them, then the definition of a LBEE coincides with the standard Nash equilibrium. In Section 4, we study general properties of equilibrium outcomes arising under partial language competence. As in standard cheap talk games, there always exists an equilibrium with no information transmission, that is, the babbling (non-revealing) outcome is always a LBEE outcome. Hence, the set of LBEE outcomes is always non-empty. The set of LBEE outcomes coincides with the set of babbling outcomes in the extreme case in which the receiver has no language competence, i.e., when he understands no message at all.

We then ask what is the set of all equilibrium outcomes that could arise due to partial language competence. The answer to this question turns out to be very simple: a LBEE outcome is always a Bayesian solution (Forges, 1993, 2006) of the basic game without communication. A Bayesian solution of the basic game corresponds to an outcome that can be achieved with some information system consistent with the prior probability distribution over the payoff-relevant states. It can be viewed in a canonical way: it is the set of outcomes obtained when an omniscient mediator (knowing the payoff-relevant state) makes recommendations of actions to the receiver, and the receiver rationally follows these recommandations.[2] The best Bayesian solution for the sender at the ex-ante stage, before he learns the state, corresponds to the solution of the Bayesian persuasion problem (Kamenica and Gentzkow, 2011).

For some language competences, the set of LBEE outcomes is equal to the set of Bayesian solutions. In particular, the two sets coincide if the sender has no language competence, the receiver is fully competent, and the message space is large enough. Since the set of Bayesian solutions may strictly include the set of communication equilibrium outcomes (Myerson, 1982; Forges, 1986), a LBEE outcome might not be implementable with any (even mediated) communication extension of the game with fully competent players. A LBEE might also Pareto dominate all communication equilibria (see Example 4 and Section 5.2). While partial language competence can improve communication, it can also be a barrier to communication, whether the sender or the receiver is partially competent. As in Blume and Board (2013), efficiency in common interest games might not be achieved when language competence is privately known, even when players' language competence would be sufficient to achieve efficiency if language competence were commonly known (see Examples 5 and 6).

In Section 5 we apply our approach to pure persuasion problems (i.e., communication games in which the sender's preference is state-independent) and to the uniform-quadratic model of

[2]Since in our model the informed player's type exhausts payoff-relevant information, the set of Bayesian solutions also coincides with the set of Bayes correlated equilibrium outcomes (Bergemann and Morris, 2013, 2016).

Crawford and Sobel (1982). In these applications, we consider a receiver whose optimal action simply corresponds to his expected value of the sender's payoff type. We also consider a language system involving a fully competent receiver and a sender whose language competence is perfectly correlated with his payoff type and increasing with this type: a sender with a high payoff type understands more messages than a sender with a lower payoff type. This setting could correspond, for example, to a job interview situation in which the candidate (sender) is characterized by his technical ability (payoff type) for a certain task and his understanding of the technical jargon (message) depends on his ability. To convince the recruiter (receiver) to give him a job, he must decide whether he speaks only about what he correctly understands, or whether he tries to mislead the recruiter by using a technical jargon beyond his own competence. This setting could also correspond to a situation in which a seller (of food, financial or information technology services) decides how to describe the characteristics of his product in order to convince a consumer to buy.

In the pure persuasion context, the sender only cares about the receiver's estimate of the state and wants to maximize it. Under standard cheap talk (i.e., cheap talk involving two fully competent players), communication cannot be influential. With the language system described above, we show that there exists a threshold equilibrium in which (i) sender types higher than the threshold reveal the threshold (which is the highest message that they commonly understand); (ii) sender types below the threshold lie about their type by sending messages they do not understand, with the hope that they will be identified by the receiver as higher types than they truly are. We show that this outcome constitutes a LBEE for every threshold that corresponds to a payoff type of the sender. When the sender's utility can only take two values (for example, the sender wants to convince the receiver to choose an alternative action against the status-quo), one of these threshold equilibria implements the best Bayesian solution for the sender (i.e., the one that maximizes the probability that the alternative action is chosen).

In the uniform-quadratic model of Crawford and Sobel (1982), the sender wants the receiver's estimate to match the state plus a bias. We show that there exists a unique threshold equilibrium which is such that (i) sender types below the threshold reveal their type truthfully; (ii) sender types above the threshold pool. This equilibrium Pareto dominates and is more informative than every Nash equilibrium of the standard cheap talk game.

Related Literature The most closely related papers in the literature are Blume and Board (2013) and Giovannoni and Xiong (2019). As in our framework, they model language competence by considering a privately known language type for every player. However, in these articles, a language type of the sender characterizes the set of messages that he is able to send, and a language type of the receiver characterizes the set of messages he is able to distinguish. Blume and Board (2013) and Giovannoni and Xiong (2019) use the standard Nash equilibrium as a solution concept, and language competence restricts the set of possible strategies of the players by directly modifying the extensive form of the cheap talk game (the set of available messages for the sender, and the information sets for the receiver). In their framework, the tourist of the previous example would not be able to pick one item from the menu because these items (messages) do not belong to his language type. Hence, our approach does not usually lead to the same equilibrium outcomes as their approach. This difference in modeling language competence also makes a significant difference in our applications to pure persuasion and cheap talk with a biased sender. With their approach, if a higher payoff type of the sender understands messages which are not understood by lower types, then the games in these applications are equivalent to disclosure games with hard information (as in Grossman, 1981; Milgrom, 1981, Seidmann and Winter, 1997) and complete information disclosure would always be an equilibrium.

Blume and Board (2013) focus on common interest games. They show that influential communication is possible with private language types. However, they show that efficiency may not be attained, even if it would be attained if players' language competences were commonly known. Then, they consider a specific class of common interest sender-receiver games. When only the sender has partial language competence, they show that in the best equilibrium the sender uses all his messages, that messages also transmit information about language types, and that there exists a language type and a message such that this message does not induce the same belief for the receiver as the one that the receiver would have if he knew the sender's language type. When only the receiver has partial language competence, they show that in the best equilibrium the receiver's response to a message depends on his language type, and is therefore stochastic from the point of view of the sender. The role of higher-order uncertainty about language types is investigated further in Blume (2018).

Giovannoni and Xiong (2019) show that standard Nash equilibria can be replicated in a game with language barriers (in the sense of Blume and Board, 2013) whenever messages can be sent multiple times. They also show that there always are language barriers that achieve a welfare which is at least as good as (and sometimes stricter better than) the one achievable with mediated or noisy communication.

The role of language structures has also been studied in the economics literature without strategic considerations. For example, Rubinstein (1996) proposes to explain some common structures of natural language using binary relations, and Cremer, Garicano, and Prat (2007) characterize the optimal choice of technical language in organizations when there are restrictions on the number of messages that agents can use.

2 Model

Basic Game. There are two players: the sender (S) and the receiver (R). The set of *payoff types* of the sender is T. Let $p^0 \in \Delta(T)$ be the prior probability distribution over payoff types. The set of actions of the receiver is A. Unless specified otherwise, the sets T and A are assumed to be finite. A mixed action of the receiver is denoted by $y \in \Delta(A)$. The utility of the sender (receiver, respectively) is given by $u(a;t)$ ($v(a;t)$, respectively) when the payoff type of the sender is $t \in T$ and the action of the receiver is $a \in A$. With some abuse of notation, when the mixed action of the receiver is $y \in \Delta(A)$, the utilities are extended as follows:

$$u(y;t) = \sum_{a \in A} y(a)u(a;t) \text{ and } v(y;t) = \sum_{a \in A} y(a)v(a;t).$$

For any $p \in \Delta(T)$ we also denote

$$u(y;p) := \sum_{t \in T} p(t)u(y;t) \text{ and } v(y;p) := \sum_{t \in T} p(t)v(y;t).$$

Let $A(p) := \arg\max_{a \in A} v(a;p)$ be the set of optimal actions of the receiver when his belief is $p \in \Delta(T)$, and let $Y(p) := \Delta(A(p))$ be the set of optimal mixed actions of the receiver when his belief is $p \in \Delta(T)$. This basic game (without communication) is denoted by $G = (A, T, p^0, u, v)$.

Messages and Languages. The set of all possible messages is M. A *language type* of player i, $i = S, R$, is a subset $\lambda_i \subseteq M$ of messages, that is interpreted as the set of messages that player i understands when of type λ_i. The set of all possible language types of player i is a non-empty set $\Lambda_i \subseteq 2^M$ (we allow $\emptyset \in \Lambda_i$). Let $\Lambda := \Lambda_S \times \Lambda_R$ be the set of profiles of language types. We assume that sender's and receiver's language types are independent, and that the language type of the receiver is independent of the payoff type (so that he learns no payoff-relevant information from his language type). However, we allow the language type of the sender to be correlated with his payoff type. Let $\pi_R \in \Delta(\Lambda_R)$ be the prior probability distribution over the language types of the receiver and let $\pi_S : T \to \Delta(\Lambda_S)$ be the conditional probability distribution over the language types of the sender. For every payoff type $t \in T$ and language type of the sender $\lambda_S \in \Lambda_S$, $\pi_S(\lambda_S \mid t)$ denotes the probability that the sender's language type is λ_S when his payoff type is t.

We say that player i is *fully competent* if $\lambda_i = M$, *incompetent* if $\lambda_i = \emptyset$, and *partially competent* in all other cases. It is commonly known that he is fully competent (incompetent, resp.) if $\Lambda_i = \{M\}$ ($\Lambda_i = \{\emptyset\}$, resp.). $L = (M, \Lambda_S, \Lambda_R, \pi_S, \pi_R)$ is called a *language system*. We call it "standard" when $\Lambda_S = \Lambda_R = \{M\}$.

Cheap Talk Extension. We are interested by the following extended cheap talk game (G, L):

1. Nature selects the sender's payoff type, $t \in T$, according to the probability distribution $p^0 \in \Delta(T)$, and the language types $(\lambda_S, \lambda_T) \in \Lambda_S \times \Lambda_R$, according to the probability distributions $(\pi_S(\cdot \mid t), \pi_R) \in \Delta(\Lambda_S) \times \Delta(\Lambda_R)$;

2. The sender is privately informed about $t \in T$ and $\lambda_S \in \Lambda_S$, and the receiver is privately informed about $\lambda_R \in \Lambda_R$;

3. The sender sends a message $m \in M$ to the receiver;

4. The receiver observes the message m and chooses an action $a \in A$.

A strategy for the sender is a mapping $\sigma_S : T \times \Lambda_S \to \Delta(M)$. A strategy for the receiver is a mapping $\sigma_R : M \times \Lambda_R \to \Delta(A)$. An outcome (distribution of actions induced by players' strategies in each state) of the cheap talk game is denoted by $\mu : T \to \Delta(A)$. That is, for every $a \in A$ and $t \in T$:

$$\mu(a \mid t) = \sum_{(\lambda_S, \lambda_R) \in \Lambda} \pi_S(\lambda_S \mid t) \pi_R(\lambda_R) \sum_{m \in M} \sigma_S(m \mid t, \lambda_S) \sigma_R(a \mid m, \lambda_R).$$

3 Language-Based Expectation Equilibrium

Given a strategy σ_S for the sender, denote by

$$\Pr_{\sigma_S}(m) := \sum_{t \in T} p^0(t) \sum_{\lambda_S \in \Lambda_S} \pi_S(\lambda_S \mid t) \sigma_S(m \mid t, \lambda_S),$$

the total probability that the sender sends message $m \in M$, or simply $\Pr(m)$ when there is no ambiguity about the strategy of the sender.

Sender's Coarse Perception. The next definition describes how an actual strategy profile (σ_S, σ_R) translates into a *strategy of the receiver perceived by the sender* depending on the sender's language type. For each message that the sender understands, he correctly perceives the receiver's response. For messages that the sender does not understand (if such messages are sent with strictly positive probability according to σ_S), he perceives the actual average reaction of the receiver to all such messages. In the terminology of Jehiel (2005), this perceived strategy corresponds to the analogy-based expectation of a sender who bundles in the same analogy class the receiver' decisions nodes that follow a message that the sender does not understand.

Definition 1 Given (σ_S, σ_R), a *strategy of the receiver perceived by the sender* with language type $\lambda_S \in \Lambda_S$ is a strategy $\tilde{\sigma}_R^{\lambda_S} : M \times \Lambda_R \to \Delta(A)$ such that for every $a \in A$ and $m \in M$:

$$\tilde{\sigma}_R^{\lambda_S}(a \mid m, \lambda_R) = \begin{cases} \sigma_R(a \mid m, \lambda_R) & \text{if } m \in \lambda_S \\ \sum_{m' \notin \lambda_S} \Pr_{\sigma_S}(m' \mid m' \notin \lambda_S)\sigma_R(a \mid m', \lambda_R) & \text{if } m \notin \lambda_S \text{ and } \Pr_{\sigma_S}(m' \notin \lambda_S) > 0, \end{cases}$$

where for $m' \notin \lambda_S$,

$$\Pr_{\sigma_S}(m' \mid m' \notin \lambda_S) = \frac{\Pr_{\sigma_S}(m')}{\Pr_{\sigma_S}(m' \notin \lambda_S)} = \frac{\sum_{t \in T} p^0(t) \sum_{\lambda'_S \in \Lambda_S} \pi_S(\lambda'_S \mid t)\sigma_S(m' \mid t, \lambda'_S)}{\sum_{m'' \notin \lambda_S} \sum_{t \in T} p^0(t) \sum_{\lambda'_S \in \Lambda_S} \pi_S(\lambda'_S \mid t)\sigma_S(m'' \mid t, \lambda'_S)}.$$

Example 1 Let $T = \{t_1, t_2, t_3\}$, with a uniform prior probability distribution. Let $M = \{m_1, m_2, m_3\}$, $\Lambda_S = \{\{m_1\}\}$ and $\Lambda_R = \{M\}$. The strategy of the receiver perceived by the sender is given by

$$\tilde{\sigma}_R^{\lambda_S}(m_1) = \sigma_R(m_1)$$

$$\tilde{\sigma}_R^{\lambda_S}(m_2) = \tilde{\sigma}_R^{\lambda_S}(m_3) = \Pr_{\sigma_S}(m_2 \mid \{m_2, m_3\})\sigma_R(m_2) + \Pr_{\sigma_S}(m_3 \mid \{m_2, m_3\})\sigma_R(m_3).$$

For example, if the receiver's action matches the message (i.e., $\sigma_R(m_k) = a_k$) and the equilibrium probabilities of messages m_2 and m_3 are the same and strictly positive (i.e., $\Pr_{\sigma_S}(m_2) = \Pr_{\sigma_S}(m_3) > 0$), then the sender believes that the receiver uniformly randomizes over $\{a_2, a_3\}$ when the sender sends a message that he does not understand:

$$\tilde{\sigma}_R^{\lambda_S}(a_2 \mid m) = \tilde{\sigma}_R^{\lambda_S}(a_3 \mid m) = \frac{1}{2}, \text{ for } m \in \{m_2, m_3\}.$$

◇

Receiver's Coarse Perception. The next definition describes how an actual strategy profile (σ_S, σ_R) translates into a *strategy of the sender perceived by the receiver* depending on the receiver's language type. For each message that the receiver understands, he correctly perceives the correlation between the sender's information and the message received. For messages that the receiver does not understand (if such messages are sent with strictly positive probability according to σ_S), he perceives correctly the correlation between the sender's information and the fact that he gets a message that he does not understand. However, he perceives the same correlation between the sender's information and each message that he does not understand. To obtain this perceived strategy as an analogy-based expectation of a receiver with language type λ_R, one can consider that the sender's strategy is built in two stages. First, the sender either picks a message $m \in \lambda_R$ or decides not to send a message in λ_R; second, if he decided not to send a message in λ_R, he picks a message $m \notin \lambda_R$. The receiver then bundles in the same analogy class the sender's decisions nodes of the second stage.

Definition 2 Given (σ_S, σ_R), a *strategy of the sender perceived by the receiver* with language type $\lambda_R \in \Lambda_R$ is a strategy $\tilde{\sigma}_S^{\lambda_R} : T \times \Lambda_S \to \Delta(M)$ such that for every $m \in M$ we have:

$$\tilde{\sigma}_S^{\lambda_R}(m \mid t, \lambda_S) = \begin{cases} \sigma_S(m \mid t, \lambda_S) & \text{if } m \in \lambda_R \\ \Pr_{\sigma_S}(m \mid m \notin \lambda_R) \sum_{m' \notin \lambda_R} \sigma_S(m' \mid t, \lambda_S) & \text{if } m \notin \lambda_R \text{ and } \Pr_{\sigma_S}(m' \notin \lambda_R) > 0. \end{cases} \tag{1}$$

Example 2 Let $T = \{t_1, t_2, t_3\}$, with a uniform prior probability distribution. Let $M = \{m_1, m_2, m_3\}$, $\Lambda_S = \{M\}$ and $\Lambda_R = \{\{m_1\}\}$. Consider the following fully separating strategy for the sender: $\sigma_S(t_k) = m_k$. Then, the unique strategy of the sender perceived by the receiver is given by

$$\tilde{\sigma}_S^{\lambda_R}(t_1) = \sigma_S(t_1) = m_1$$

$$\tilde{\sigma}_S^{\lambda_R}(m_2 \mid t_2) = \Pr_{\sigma_S}(m_2 \mid \{m_2, m_3\}) \sum_{m \in \{m_2, m_3\}} \sigma_S(m \mid t_2)$$

$$= \frac{1}{2} = \tilde{\sigma}_S^{\lambda_R}(m_3 \mid t_2) = \tilde{\sigma}_S^{\lambda_R}(m_2 \mid t_3) = \tilde{\sigma}_S^{\lambda_R}(m_3 \mid t_3).$$

◇

The next lemma shows that the receiver's belief about the payoff type facing two different messages that do not belong to his language type is the same. Hence, even though he could potentially choose different actions after these two messages, he will only do so when indifferent between several actions at the corresponding belief.

Lemma 1 *For every* $m \in M$ *such that* $\Pr_{\sigma_S}(m) > 0$ *we have*

$$\Pr_{\tilde{\sigma}_S^{\lambda_R}}(t \mid m) = \begin{cases} \Pr_{\sigma_S}(t \mid m) & \textit{if } m \in \lambda_R, \\ \Pr_{\sigma_S}(t \mid m \notin \lambda_R) & \textit{if } m \notin \lambda_R. \end{cases}$$

Proof. For $m \notin \lambda_R$ we have

$$\Pr_{\tilde{\sigma}_S^{\lambda_R}}(t \mid m) = \frac{p^0(t) \Pr_{\tilde{\sigma}_S^{\lambda_R}}(m \mid t)}{\Pr_{\tilde{\sigma}_S^{\lambda_R}}(m)},$$

with $\Pr_{\tilde{\sigma}_S^{\lambda_R}}(m) = \Pr_{\sigma_S}(m)$ and, using (1),

$$\Pr_{\tilde{\sigma}_S^{\lambda_R}}(m \mid t) = \Pr_{\sigma_S}(m \mid m \notin \lambda_R) \sum_{m' \notin \lambda_R} \Pr_{\sigma_S}(m' \mid t),$$

so

$$\Pr_{\tilde{\sigma}_S^{\lambda_R}}(t \mid m) = \frac{p^0(t) \sum_{m' \notin \lambda_R} \Pr_{\sigma_S}(m' \mid t)}{\sum_{m' \notin \lambda_R} \Pr_{\sigma_S}(m')} = \Pr_{\sigma_S}(t \mid m \notin \lambda_R).$$

For $m \in \lambda_R$, Bayes rule applies as in the benchmark case since the receiver perceives the exact strategy of the sender: $\Pr_{\tilde{\sigma}_S^{\lambda_R}}(t \mid m) = \Pr_{\sigma_S}(t \mid m)$. ■

With the sender's and receiver's coarse perceptions in hands, we are now ready to define our equilibrium concept.

Definition 3 A strategy profile (σ_S, σ_R) is a *language-based expectation equilibrium* (LBEE) of the cheap talk extension of G with language system L if there exists perceived strategies $(\tilde{\sigma}_R^{\lambda_S}, \tilde{\sigma}_S^{\lambda_R})$, $(\lambda_S, \lambda_T) \in \Lambda_S \times \Lambda_R$, such that the following conditions are satisfied:

For the sender: for every $t \in T$, $\lambda_S \in \Lambda_S$ and $m^* \in M$ satisfying $\sigma_S(m^* \mid t, \lambda_S) > 0$:

$$m^* \in \arg\max_{m \in M} \sum_{\lambda_R \in \Lambda_R} \pi_R(\lambda_R) \sum_{a \in A} \tilde{\sigma}_R^{\lambda_S}(a \mid m, \lambda_R) u(a; t).$$

For the receiver: for every $m \in M$ satisfying $\Pr_{\sigma_S}(m) > 0$, every $\lambda_R \in \Lambda_R$ and every a^* satisfying $\sigma_R(a^* \mid m, \lambda_R) > 0$:

$$a^* \in \arg\max_{a \in A} \sum_{t \in T} \Pr_{\tilde{\sigma}_S^{\lambda_R}}(t \mid m) v(a; t).$$

Notice that when the language system is standard ($\Lambda_S = \Lambda_R = \{M\}$), the definition above coincides with the definition of a Nash equilibrium of the cheap talk game.

Example 3 Let $T = \{t_1, t_2, t_3\}$, with a uniform prior probability distribution. Consider the following language system: $M = \{m_1, m_2, m_3\}$, $\Lambda_S = \{\{m_1\}, M\}$ and $\Lambda_R = \{M\}$. Consider the following payoff matrix:

	a_1	a_2	a_3
t_1	$1, 1$	$0, 0$	$0, 0$
t_2	$1, 0$	$3, 1$	$-3, 0$
t_3	$1, 0$	$-3, 0$	$3, 1$

Let $\sigma_R(m_k) = a_k$, $k = 1, 2, 3$, be the strategy of the receiver. There is a LBEE in which the sender fully reveals when his language type is $\lambda_S = M$ (i.e., $\sigma_S(t_k, \lambda_S) = m_k$, $k = 1, 2, 3$)

and always sends the message he understands when his language type is $\lambda_S = \{m_1\}$ (i.e., $\sigma_S(t_k, \lambda_S) = m_1$, $k = 1, 2, 3$). However, if we replace the sender's payoffs -3 by 0, then this strategy profile is not a LBEE anymore: when $t \in \{t_2, t_3\}$, the sender of language type $\lambda_S = \{m_1\}$ deviates by sending a message that he does not understand. ◇

4 Relation to other solution concepts

4.1 LBEE and Nash Equilibrium

We first mention that a Nash Equilibrium is not necessarily a LBEE. In the previous example for instance, full revelation of information can occur in a Nash equilibrium but not in a LBEE with the language system proposed. The following example shows that the reverse can also be true, namely that we can obtain full revelation as a LBEE while there is no fully informative Nash equilibrium.

Example 4 (Pareto improving revelation of information by an incompetent sender) Consider the following basic game with uniform priors:

	a_1	a_2
t_1	$1, 1$	$0, 0$
t_2	$1, -2$	$0, 0$

Clearly, full revelation cannot be sustained as a Nash equilibrium outcome because type t_2 would have an incentive to mimic type t_1. Consider now that the sender is incompetent and the receiver is fully competent, i.e., $\Lambda_S = \{\emptyset\}$ and $\Lambda_R = \{\{m_1, m_2\}\}$. The strategy profile $\sigma_S(t_1) = m_1$, $\sigma_S(t_2) = m_2$, $\sigma_R(m_1) = a_1$, $\sigma_R(m_2) = a_2$ is a LBEE. The sender perceives the same receiver's reaction following m_1 and m_2, and therefore has no incentive to deviate from full revelation. ◇

In the previous example, partial language competence of the sender makes every player better off. We now provide two examples illustrating the more negative result that, even in common interest games, the efficient Nash equilibrium outcome may not be achieved as a LBEE outcome. As in Blume and Board (2013), these examples demonstrate that efficiency might not be reached when language competence is privately known even if language competences are sufficient to do so (that is, efficiency would be reached for every language competence considered in these examples provided that the language competence were commonly known). The first example involves a fully competent receiver, and the second example a fully competent sender.

Example 5 (Inefficiency due to private information about the sender's competence) Consider the following basic game with uniform priors and common interest:

	a_1	a_2	a_3
t_1	$0, 0$	$-2, -2$	$-2, -2$
t_2	$0, 0$	$1, 1$	$-2, -2$
t_3	$0, 0$	$-2, -2$	$1, 1$

Consider the following language system: $M = \{m_1, m_2, m_3\}$, the sender's language types are $\lambda_S^1 = \{m_1\}$, $\lambda_S^2 = \{m_2\}$, $\lambda_S^3 = \{m_3\}$ and are uniformly distributed, and $\Lambda_R = \{M\}$. Therefore, the sender has nine equally likely types in $T \times \Lambda_S$, and the receiver has a single type. The efficient (first best) outcome is $\mu(t_k) = a_k$, $k = 1, 2, 3$. This outcome can be implemented only if the sender uses a separating strategy and the receiver chooses a different action for each message. Consider such a strategy profile. If the receiver chooses a_1 after m_1 then the sender with language type λ_S^1 and payoff type t_2 or t_3 would like to deviate and send message m_1 since his payoff is 0 by sending m_1 but his perceived payoff is $\frac{1}{2}(1) + \frac{1}{2}(-2) < 0$ if he sends m_2 or m_3. Similarly, if the receiver chooses a_1 after m_2 then the sender with language type λ_S^2 and payoff type t_2 or t_3 would like to deviate, and if the receiver chooses a_1 after m_3 then the sender with language type λ_S^3 and payoff type t_2 or t_3 would like to deviate.

However, observe that if the sender's language type is commonly known, then there is an efficient LBEE. For example, if $\Lambda_S = \{\{m_1\}\}$, then $\sigma_S(t_1) = m_2$, $\sigma_S(t_2) = m_1$, $\sigma_S(t_3) = m_3$ and $\sigma_R(m_1) = a_2$, $\sigma_R(m_2) = a_1$, $\sigma_R(m_3) = a_3$ is an efficient LBEE. A symmetric argument applies when $\Lambda_S = \{\{m_2\}\}$ or $\Lambda_S = \{\{m_3\}\}$. $\diamond$

Example 6 (Inefficiency due to private information about the receiver's competence) Consider the following basic game with uniform priors and common interest:

	a_1	a_2	a_3
t_1	$1, 1$	$-3, -3$	$0, 0$
t_2	$-3, -3$	$1, 1$	$0, 0$

The optimal action of the receiver as a function of his belief p on t_1 is given by

$$A(p) = \begin{cases} a_2 & \text{if } p < 1/4; \\ a_3 & \text{if } 1/4 < p < 3/4; \\ a_1 & \text{if } p > 3/4. \end{cases}$$

Consider the following language system: $M = \{m_1, m_2, m_3\}$, the receiver's language types are $\lambda_R^1 = \{m_1\}$, $\lambda_R^2 = \{m_2\}$, $\lambda_R^3 = \{m_3\}$ and are uniformly distributed, and $\Lambda_S = \{M\}$. The efficient (first best) outcome is $\mu(t_k) = a_k$, $k = 1, 2$. For such an outcome to be implemented by players' strategies, these strategies must satisfy

$$\text{supp}[\sigma_S(t_1)] \cap \text{supp}[\sigma_S(t_2)] = \emptyset,$$

$$\sigma_R(m) = \begin{cases} a_1 & \text{if } m \in \text{supp}[\sigma_S(t_1)]; \\ a_2 & \text{if } m \in \text{supp}[\sigma_S(t_2)]. \end{cases}$$

Consider without loss of generality the following strategy for the sender (the reasoning is similar for other sender's strategies satisfying $\text{supp}[\sigma_S(t_1)] \cap \text{supp}[\sigma_S(t_2)] = \emptyset$):

$$\sigma_S(m_1 \mid t_1) = 1, \ \sigma_S(m_2 \mid t_2) = 1 - \sigma_S(m_3 \mid t_2) = \alpha.$$

The induced beliefs for the receiver types λ_R^2 and λ_R^3 are given by

$$\Pr_{\tilde{\sigma}_S^{\lambda_R^2}}(t_1 \mid m_1) = \Pr_{\tilde{\sigma}_S^{\lambda_R^2}}(t_1 \mid m_3) = \Pr(t_1 \mid \{m_1, m_3\}) = \frac{1}{2 - \alpha},$$

$$\Pr_{\tilde{\sigma}_S^{\lambda_R^3}}(t_1 \mid m_1) = \Pr_{\tilde{\sigma}_S^{\lambda_R^3}}(t_1 \mid m_2) = \Pr(t_1 \mid \{m_1, m_2\}) = \frac{1}{1 + \alpha}.$$

Hence, their incentive constraints imply $\frac{1}{2-\alpha} \geq \frac{3}{4}$ and $\frac{1}{1+\alpha} \geq \frac{3}{4}$, i.e., $\alpha \geq \frac{2}{3}$ and $\alpha \leq \frac{1}{3}$, a contradiction. Therefore, there is no LBEE implementing the efficient outcome.

However, as in Example 5, if language types were commonly known, then there would be an efficient LBEE. For example, if $\Lambda_R = \{\{m_1\}\}$, then $\sigma_S(t_1) = m_1$, $\sigma_S(t_2) = m_2$ and $\sigma_R(m_1) = a_1$, $\sigma_R(m_2) = a_2$, is an efficient LBEE. A symmetric argument applies when $\Lambda_R = \{\{m_2\}\}$ or $\Lambda_R = \{\{m_3\}\}$. ◇

The next proposition establishes that, for every language system L, the set of LBEE outcomes of (G, L) is non-empty because it always includes the set of babbling outcomes.

Proposition 1 *For every language system L the set of babbling outcomes $\{\mu : T \to \Delta(A) : \mu(t) \in Y(p^0)$ for every $t \in T\}$ is included in the set of LBEE outcomes of (G, L). If $\Lambda_R = \{\emptyset\}$ (i.e., it is commonly known that the receiver is incompetent), then the two sets coincide.*

Proof. To show that a babbling outcome is always a LBEE, it suffices to consider a constant strategy for the receiver, who plays an action in $Y(p^0)$ for every message, and a constant strategy for the sender, who always send the same message. If $\Lambda_R = \{\emptyset\}$, then from Lemma 1 the belief of the receiver after every message sent with positive probability is equal to the prior p^0 regardless of the strategy of the sender. Hence, in all equilibria the receiver plays a mixed action in $Y(p^0)$ after every message sent with positive probability. Since $Y(p^0)$ is convex, the distribution of actions $\mu(t)$ belongs to $Y(p^0)$ for every t. ■

In Section 5, we will focus on communication games with the following language system: $M = T \subseteq [0,1]$, the receiver is fully competent, and $\lambda(t) = \{s \in T : s \leq t\}$, where $\lambda(t)$ is the set of messages that the sender of payoff type t understands. That is, the sender's language and payoff types are perfectly correlated, and a sender of higher payoff type understands all the messages that a sender of lower type understands. For such a language system, we will show that Nash equilibrium outcomes in which the sender uses pure strategies can always be obtained as LBEE outcomes. Notice that Example 5 demonstrates that full language competence of the receiver is not enough to sustain every Nash equilibrium outcome as a LBEE outcome.

4.2 LBEE, Communication Equilibrium and Bayesian Solution

In the former subsection, we have shown that LBEE and Nash equilibrium can lead to different outcomes. We now try to relate LBEE to the concepts of communication equilibrium and Bayesian solution. We first recall that a Nash equilibrium outcome of the cheap talk game is always a communication equilibrium outcome (but the reverse is not true).

An outcome $\mu : T \to \Delta(A)$ is a communication equilibrium outcome (Forges, 1986; Myerson, 1982, 1986) iff it satisfies the following incentive constraints for the sender and receiver:

$$\sum_{a \in A} \mu(a \mid t) u(a; t) \geq \sum_{a \in A} \mu(a \mid t') u(a; t), \quad \text{for every } t, t' \in T; \tag{SIC}$$

$$\sum_{t \in T} \mu(a \mid t) p^0(t) v(a; t) \geq \sum_{t \in T} \mu(a \mid t) p^0(t) v(a'; t), \quad \text{for every } a \in \text{supp}[\mu] \text{ and } a' \in A. \tag{RIC}$$

Note that (RIC) can be rewritten as

$$\sum_{t \in T} \Pr(t \mid a) v(a; t) \geq \sum_{t \in T} \Pr(t \mid a) v(a'; t), \quad \text{for every } a \in \text{supp}[\mu] \text{ and } a' \in A,$$

where the probability Pr is computed according to μ and the prior p^0.

We can now use the previous Example 4 to show that LBEE outcomes cannot always be obtained as communication equilibrium outcomes. Indeed, in the game of this example, there exists a LBEE in which the sender fully reveals his payoff type while the unique communication equilibrium is the babbling one in which $\mu(t) = a_2$ for every t.

The outcome $\mu : T \to \Delta(A)$ is a Bayesian solution (Forges, 1993, 2006) iff it satisfies (RIC). Notice that a fully revealing outcome (which gives the first best to the receiver), i.e., satisfying $\mu(t) \in \arg\max_{y \in \Delta(A)} v(y; t)$, for every $t \in T$, is always a Bayesian solution. The solution of the Bayesian persuasion problem (Kamenica and Gentzkow, 2011) is also a Bayesian solution (it is the Bayesian solution that maximizes the sender's ex-ante expected payoff).

Proposition 2 *For every language system $L = (M, \Lambda_S, \Lambda_R, \pi_S, \pi_R)$, a LBEE outcome of (G, L) is a Bayesian solution.*

Proof. Consider a LBEE (σ_S, σ_R) of (G, L). In what follows, all probabilities without subscript are computed with respect to (σ_S, σ_R) and the prior probability distribution of language and payoff types. If the perceived strategy $\tilde{\sigma}_S^{\lambda_R}$ is used then it is written explicitly as subscript. By definition of a LBEE, we know that the receiver's strategy satisfies

$$\sum_t \Pr_{\tilde{\sigma}_S^{\lambda_R}}(t \mid m) v(a; t) \geq \sum_t \Pr_{\tilde{\sigma}_S^{\lambda_R}}(t \mid m) v(a'; t),$$

for every $m \in \text{supp}[\sigma_S]$, $\lambda_R \in \Lambda_R$, a such that $\sigma_R(a \mid m, \lambda_R) > 0$, and $a' \in A$. These inequalities can be split into two groups depending on whether m is in λ_R or not. Using Lemma 1, if $m \in \lambda_R$ we get

$$\sum_t \Pr(t \mid m) v(a; t) \geq \sum_t \Pr(t \mid m) v(a'; t). \tag{2}$$

If $m \notin \lambda_R$ we get

$$\sum_t \Pr(t \mid m \notin \lambda_R) v(a; t) \geq \sum_t \Pr(t \mid m \notin \lambda_R) v(a'; t). \tag{3}$$

Observe that $\Pr(t \mid m) = \Pr(t \mid m, a, \lambda_R)$ and $\Pr(t \mid m \notin \lambda_R) = \Pr(t \mid m \notin \lambda_R, a, \lambda_R)$. Multiplying (2) by $\Pr(m \mid a, \lambda_R)$ for every $m \in \lambda_R$ and multiplying (3) by $\Pr(m \notin \lambda_R \mid a, \lambda_R)$, and then taking the sum, we get:

$$\sum_t \left(\sum_{m \in \lambda_R} \Pr(m \mid a, \lambda_R) \Pr(t \mid m, a, \lambda_R) + \Pr(m \notin \lambda_R \mid a, \lambda_R) \Pr(t \mid m \notin \lambda_R, a, \lambda_R) \right) \times$$

$$(v(a; t) - v(a'; t)) \geq 0.$$

This inequality can be rewritten as

$$\sum_t \Pr(t \mid a, \lambda_R)(v(a; t) - v(a'; t)) \geq 0.$$

This implies

$$\sum_{\lambda_R} \Pr(\lambda_R \mid a) \sum_t \Pr(t \mid a, \lambda_R)(v(a; t) - v(a'; t)) \geq 0,$$

and thus

$$\sum_t \Pr(t \mid a)(v(a;t) - v(a';t)) \geq 0,$$

which is (RIC). ∎

Proposition 3 *If $|M| \geq |A|$, $\Lambda_S = \{\emptyset\}$ and $\Lambda_R = \{M\}$ (it is commonly known that the sender is incompetent and the receiver is fully competent), then a Bayesian solution is a LBEE outcome.*

Proof. Let μ be a Bayesian solution. Since $|M| \geq |A|$ we can associate to each action $a \in A$ a different message $m_a \in M$. Consider the sender's strategy σ_S such $\sigma_S(m_a \mid t) = \mu(a \mid t)$ for every a and t, and an obedient receiver's strategy satisfying $\sigma_R(m_a) = a$ for every a. By construction, since (RIC) is satisfied, σ_R is a best response to σ_S for the receiver. In addition, since $\lambda_S = \emptyset$, the strategy of the receiver perceived by the sender is given by

$$\tilde{\sigma}_R^{\lambda_S}(a \mid m) = \sum_{m' \in M} \Pr_{\sigma_S}(m')\sigma_R(a \mid m') = \sum_{a \in A} \Pr_{\sigma_S}(m_a)\sigma_R(a \mid m_a) = \sum_{a \in A} \Pr_\mu(a),$$

which is independent of m. Hence, the sender's perceived expected payoff is independent of the message he sends, and he thus has no incentive to deviate from σ_S. Therefore, μ is a LBEE outcome. ∎

From the last two propositions we get:

Corollary 1 *The union of all LBEE outcomes of (G, L) for all possible language systems L coincides with the set of Bayesian solutions of G.*

Hence, for every language system L, the set of LBEE outcomes of (G, L) is non-empty and always included in the set of Bayesian solutions.

5 Applications

In this section we apply the LBEE concept to pure persuasion problems and to the uniform-quadratic model of Crawford and Sobel (1982). We assume that there are n ordered payoff types $T = \{t_1, \ldots, t_n\}$, where $0 = t_1 < \cdots < t_n = 1$, and make the following assumptions about the language system. First, the set of messages available is $M = T$. Second, it is commonly known that the receiver is fully competent (i.e., $\Lambda_R = \{M\}$). Third, for each payoff type t, there is only one possible language type for the sender (the sender's language type is perfectly correlated to his payoff type) and it is given by

$$\lambda(t) = \{s \in T : s \leq t\}.$$

Hence, the sender's language competences are ordered according to his payoff type, $\lambda(t_1) \subseteq \lambda(t_2) \subseteq \cdots \subseteq \lambda(t_n)$, with the interpretation that a sender of a given payoff type understands all the messages that senders of lower payoff types understand. As an example, consider that the sender's payoff type represents his true technical ability, such as being able to use more or less complex softwares or cameras. To communicate about his ability to a receiver, the sender can use a technical jargon which he understands better when of greater true ability.

We start by establishing that, with the language system considered, the set of Nash equilibrium outcomes of the cheap talk game in which the sender plays in pure strategies is always included in the set of LBEE outcomes. The following two subsections will show how partial language competence of the sender can lead to equilibrium outcomes which are impossible to implement with fully competent players.

Proposition 4 *Assume that the receiver is fully competent, $M = T \subseteq [0, 1]$ and $\lambda(t) = \{s \in T : s \leq t\}$, where $\lambda(t)$ is the set of messages that the sender of payoff type t understands. Then, every outcome of a Nash equilibrium in which the sender plays in pure strategies is a LBEE outcome.*

Proof. Consider a Nash equilibrium (σ_S, σ_R) in which every payoff type t sends a message $m(t) \in M$ with probability one, i.e., $\sigma_S(m(t) \mid t) = 1$ for every t. We construct a LBEE (σ^*_S, σ^*_R) in which every sender type t sends a message $m^*(t) \in \lambda(t)$ that he understands with probability one, i.e., $\sigma^*_S(m^*(t) \mid t) = 1$ for every t, and which implements the same outcome as (σ_S, σ_R). For every $t \in T$, let $m^*(t) = \min\{s \in T : m(s) = m(t)\}$ be the smallest payoff type who sends the same message as type t according to the strategy σ_S. Let $\sigma^*_R(m^*(t)) = \sigma_R(m(t))$. By construction, σ^*_R is a best response to σ^*_S because σ_R is a best response to σ_S and σ^*_S induces the same beliefs for the (fully competent) receiver as σ_S:

$$\Pr_{\sigma_S}(s \mid m(t)) = \Pr_{\sigma^*_S}(s \mid m^*(t)), \text{ for every } s, t \in T.$$

Also, for every $t \in T$ we have $m^*(t) \in \lambda(t)$ so along the equilibrium path of (σ^*_S, σ^*_R) the perceived expected utility of the sender is the same as the expected utility of the sender in the Nash equilibrium (σ_S, σ_R). A sender of type t with partial language competence perceives correctly the strategy of the receiver that follows any message that he understands. Hence, a sender of type t with partial language competence does not deviate from $m^*(t)$ to another message $m' \in \lambda(t)$. If he deviates to a message that he does not understand, $m' \notin \lambda(t)$, then his perceived expected utility is a convex combination of expected utilities he would get by deviating from the Nash equilibrium (σ_S, σ_R), and is therefore not profitable. ■

Remark 1 It is clear from the proof of this proposition that the result can be extended to more general language systems and to mixed strategies for the sender as long as a strategy σ_S^* with the following properties could be constructed: (i) it induces the same beliefs as σ_S for the receiver, and (ii) every sender type t only sends messages that he understands to the receiver.[3]

In the two following subsections, we assume that the receiver estimates the expected value of the payoff type of the sender given the priors, the sender's strategy, and the message. This can be represented by the set of actions $A = [0, 1]$, and the receiver's utility function $v(a; t) = -(a-t)^2$, so that his optimal action corresponds to the expected value of the state given his belief. Hence, the receiver's strategy is always a pure strategy, that we denote by $\sigma_R : M \to A$. The two applications differ with respect to the sender's utility. In addition, we focus on equilibria in which each sender type t sends the messages that he does not understand with the same (possibly null) probability: $\sigma_S(m \mid t) = \sigma_S(m' \mid t)$ for every $t \in T$ and m, $m' \notin \lambda(t)$. Notice that the equilibria constructed in Proposition 4 satisfy this property.

5.1 Pure Persuasion

In the pure persuasion case, the sender's utility is independent of the state and can be written as $u(a; t) = u(a)$.[4] We further assume that $u(a)$ is non-decreasing, meaning that the sender always wants the receiver's action to be as high as possible. Hence, every Nash equilibrium outcome of the cheap talk game is the babbling outcome.

The optimal action of the receiver is uniquely defined along the equilibrium path as the expected value of the type given the message and the sender's strategy:

$$\sigma_R(m) = \sum_{t \in T} \Pr_{\sigma_S}(t \mid m)\, t, \text{for every } m \in \text{supp}[\sigma_S], \tag{4}$$

where $\Pr_{\sigma_S}(t \mid m) = \frac{p^0(t)\sigma_S(m|t)}{\sum_s p^0(s)\sigma_S(m|s)}$. We also fix $\sigma_R(m) = 0$ for $m \notin \text{supp}[\sigma_S]$ and $\tilde{\sigma}_R^{\lambda(t)}(m) = 0$ for $m \notin \lambda(t)$ if $\Pr(m' \notin \lambda(t)) = 0$; this is w.l.o.g. since action 0 is the most severe punishment for every sender type t.

No Separation at the Top. We first mention that there is no LBEE in which the two highest types t_{n-1} and t_n get different payoffs (and hence send distinct messages when $u(a)$ is

[3]For instance, these properties do not hold in Example 5 in which we cannot implement the Nash equilibrium outcome as a LBEE outcome.

[4]This setting is sometimes called "cheap talk with transparent motives" (Lipnowski and Ravid, 2018)

strictly increasing in a). By assumption about the sender's language competences, the sender of type t_n understands every message in T and the sender of type t_{n-1} understands every message in $T \backslash \{t_n\}$. Since there is a unique message that the sender of type t_{n-1} does not understand, he perceives correctly the receiver's reaction to this message despite it being out of his language type. In the end, it is as if types t_{n-1} and t_n had the same language competence. It follows that, if these two sender's types get distinct equilibrium payoffs, one of them has an interest in mimicking the other one. In particular, this implies that there does not exist a LBEE where the sender fully reveals his payoff type when $u(t_{n-1}) < u(t_n)$. In the next paragraphs, we however demonstrate that some separation is always possible in equilibrium.

One-Threshold LBEE. The following proposition establishes that, for every threshold type level $t^* \in T$, there exists a LBEE in which (i) each sender type t below t^* over-reports his type by sending every message $m > t$ with the same probability; (ii) each sender type t above t^* truthfully reports that his type is above t^* by reporting $m = t^*$. In such an equilibrium, low types use all messages they do no understand while high types use the highest message they commonly understand. The intuition of this equilibrium is simple: when sender types below the threshold uniformly lie by over-reporting their types, then the receiver's expectation about the state (and therefore his action) is increasing with the message he gets, except when he gets the message t^* corresponding to the threshold, for which his expectation is the highest (see Equation (10) in the proof below). Hence, sender types above the threshold report the threshold because they commonly understand this message and it induces the highest equilibrium action of the receiver. On the other hand, types below the threshold understand that reporting the truth or under-reporting induces a lower action than over-reporting. However, since they do not understand the meaning of the over-reporting messages, they are unable to identify the best message (the message which consists in reporting the threshold t^*) and it is a best response for them to over-report uniformly. Notice that it is not crucial that types below the threshold precisely use a uniform over-reporting strategy. The uniform strategy is simple and natural, and it guarantees that the ordering of the receiver's strategy (10) is satisfied whatever the prior distribution of types.

Proposition 5 *For every threshold $t^* \in T$, the following strategy for the sender constitutes a LBEE:*

$$\sigma_S(m \mid t_k) = \begin{cases} \frac{1}{n-k} & \text{if } m > t_k \\ 0 & \text{if } m \leq t_k, \end{cases} \quad \text{for every } t_k < t^*, \tag{5}$$

$$\sigma_S(m \mid t_k) = \begin{cases} 1 & \text{if } m = t^* \\ 0 & \text{if } m \neq t^*, \end{cases} \quad \text{for every } t_k \geq t^*. \tag{6}$$

Proof. If $t^* = t_1$ we have a trivial pooling equilibrium, so let $k^* \in \{2, \ldots, n\}$ be such that $t_{k^*} = t^*$. From the sender strategy σ_S and the priors, we can compute the total probabilities of the different messages sent by the sender, and then get the conditional probabilities (beliefs of the receiver) for every message sent with positive probability. Simplifying, we get the following optimal actions for the receiver as a function of the message m he receives:

- For $t_1 < m = t_{k'} < t_{k^*}$,

$$\sigma_R(m) = \frac{\frac{1}{n-1}p^0(t_1)t_1 + \frac{1}{n-2}p^0(t_2)t_2 + \cdots + \frac{1}{n-(k'-1)}p^0(t_{k'-1})t_{k'-1}}{\frac{1}{n-1}p^0(t_1) + \frac{1}{n-2}p^0(t_2) + \cdots + \frac{1}{n-(k'-1)}p^0(t_{k'-1})}. \tag{7}$$

- For $m > t_{k^*}$,

$$\sigma_R(m) = \frac{\frac{1}{n-1}p^0(t_1)t_1 + \frac{1}{n-2}p^0(t_2)t_2 + \cdots + \frac{1}{n-(k^*-1)}p^0(t_{k^*-1})t_{k^*-1}}{\frac{1}{n-1}p^0(t_1) + \frac{1}{n-2}p^0(t_2) + \cdots + \frac{1}{n-(k^*-1)}p^0(t_{k^*-1})}. \tag{8}$$

- For $m = t_{k^*}$,

$$\sigma_R(m) = \frac{\frac{1}{n-1}p^0(t_1)t_1 + \cdots + \frac{1}{n-(k^*-1)}p^0(t_{k^*-1})t_{k^*-1} + p^0(t_{k^*})t_{k^*} + p^0(t_{k^*+1})t_{k^*+1} + \cdots + p^0(t_n)t_n}{\frac{1}{n-1}p^0(t_1) + \frac{1}{n-2}p^0(t_2) + \cdots + \frac{1}{n-(k^*-1)}p^0(t_{k^*-1}) + p^0(t_{k^*}) + \cdots + p^0(t_n)}. \tag{9}$$

From Equation (7), we get $0 = \sigma_R(t_2) = t_1 < \sigma_R(t_3) < \cdots < \sigma_R(t_{k^*-1})$. Indeed, for $t_{k'+1} < t_{k^*}$, $\sigma_R(t_{k'+1})$ puts less weight than $\sigma_R(t_{k'})$ on each $t = t_1, \ldots, t_{k'-1}$, but puts some weight on $t_{k'}$. Similarly, $\sigma_R(t_{k^*+1}) > \sigma_R(t_{k^*-1})$. From Equation (8), we get $\sigma_R(t_{k^*+1}) = \sigma_R(t_{k^*+2}) = \cdots = \sigma_R(t_n)$. Finally, comparing Equations (8) and (9), we get $\sigma_R(t_{k^*}) > \sigma_R(t_{k^*+1})$. Hence we have:

$$0 = \sigma_R(t_2) < \sigma_R(t_3) < \cdots < \sigma_R(t_{k^*-1}) < \sigma_R(t_{k^*+1}) = \sigma_R(t_{k^*+2}) = \cdots = \sigma_R(t_n) < \sigma_R(t_{k^*}) \tag{10}$$

From this ordering we can observe that the sender has no profitable deviation. Indeed, for every type $t_k < t^*$, the equilibrium distribution of actions he perceives by sending a message he does not understand is a distribution over $\{\sigma_R(m) : m > t_k\}$, which is always higher than $\sigma_R(t_{k+1})$; if he deviates to a message that he understands he gets at most $u(\sigma_R(t_k)) \leq u(\sigma_R(t_{k+1}))$, so he does not deviate. Any type $t_k \geq t^*$ gets a payoff equal to $u(\sigma_R(t_{k^*}))$, which is the maximum payoff the sender can get given the strategy of the receiver. ∎

If we consider standard cheap talk (i.e., cheap talk bewteen fully competent players) in this pure persuasion game, there is no informative equilibrium whenever $u(a)$ is strictly increasing. A particular case of the one-threshold LBEE we just described is one in which the threshold is $t^* = t_1$, which is equivalent to pooling. For all other possible thresholds, some information is transmitted. We can note that $\sigma_R(t^*)$ is increasing in t^*, so when t^* increases sender types $t \geq t^*$ are better off.

Status Quo vs Alternative. Consider now a particular case in which the sender cares about whether the action taken by the receiver is above a threshold or not: $u(a) = 0$ if $a < \bar{a}$ and $u(a) = 1$ if $a \geq \bar{a}$, where $\bar{a} \in (0, 1)$. An interpretation is that the receiver has two actions and chooses the *alternative* instead of the *status quo* if his estimate of the expected value of the state is higher than $\bar{a}$. The sender always wants the alternative to be chosen. We assume that $E(t) < \bar{a}$, implying that the alternative is never chosen in the absence of information transmission.

As demonstrated above, there exists a t^*-threshold equilibrium for every $t^* \in T$. We can therefore characterize the best threshold for the sender, that is, the threshold that maximizes the probability that the alternative is chosen. This optimal threshold is the smallest type in the set $\{\tilde{t} \in T : E(t \mid m = \tilde{t}) \geq \bar{a}\}$ if this set is non-empty (otherwise, the alternative is never chosen whatever the threshold).

Notice that in the limit case in which the set of payoff types is $T = [0, 1]$ we have

$$E(t \mid m = t^*) = E(t \mid t \geq t^*),$$

and therefore t^* is the unique solution of $E(t \mid t \geq t^*) = \bar{a}$. For example, if payoff types are uniformly distributed, then $t^* = 2\bar{a} - 1$. It is interesting to observe that this LBEE coincides with the ex-ante optimal Bayesian solution of the sender. Indeed, in this example, the optimal Bayesian solution for the sender is given by a threshold information structure in which the receiver learns that $t \geq t^\#$ and is indifferent between the status quo and the alternative whenever $t \geq t^\#$. Hence, $t^\#$ solves $E(t \mid t \geq t^\#) = \bar{a}$, so $t^\# = t^*$.

Equilibrium with multiple thresholds. The previous proposition shows that there always exist equilibria with a single threshold. Equilibria with multiple thresholds could exist, but the existence of such equilibria depend on the specific parameters of the game. As an illustration, assume that $u(a) = a$. Let $T = \{t_1, t_2, t_3, t_4\}$, and assume that $p^0(t_1) = p^0(t_2) = p^0(t_3) = 1/3$ and $p^0(t_4) = 0$ (the example is robust to any small perturbation of the prior probability distribution). Consider the following 2-threshold strategy for the sender, where the thresholds are $t^* = t_2$ and $t^{**} = t_3$:

$$\sigma_S(m \mid t_1) = \begin{cases} \frac{1}{3} & \text{if } m > t_1 \\ 0 & \text{if } m = t_1, \end{cases}$$

$\sigma_S(t_2 \mid t_2) = 1$, $\sigma_S(t_3 \mid t_3) = 1$, and $\sigma_S(t_3 \mid t_4) = 1$. Using Bayes rule, the following strategy is a best response for the receiver: $\sigma_R(t_1) = \sigma_R(t_4) = 0$, $\sigma_R(t_2) = \frac{3}{4}t_2$, and $\sigma_R(t_3) = \frac{3}{4}t_3$. Clearly, the sender types t_1, t_3 and t_4 have no incentive to deviate. Consider the sender type t_2. By sending message $m = t_2$, his perceived (and correct) expected utility is $\sigma_R(t_2) = \frac{3}{4}t_2$. If he deviates to a message he does not understand ($m = t_3$ or $m = t_4$), then his perceived expected utility is

$$\frac{\Pr(m = t_3)}{\Pr(m = t_3) + \Pr(m = t_4)}\sigma_R(t_3) + \frac{\Pr(m = t_4)}{\Pr(m = t_3) + \Pr(m = t_4)}\sigma_R(t_4) = \frac{3}{5}t_3.$$

Hence, the 2-threshold strategy above constitues a LBEE whenever $\frac{3}{4}t_2 \geq \frac{3}{5}t_3$, i.e., $5t_2 \geq 4t_3$.

5.2 Biased Sender

In this subsection we consider the uniform-quadratic cheap talk model of Crawford and Sobel (1982). As in the previous section, the set of actions is $A = [0, 1]$ and the fully competent receiver estimates the expected value of the payoff type of the sender. Contrary to the pure persuasion case, the sender's preference over the estimates of the receiver depends on his payoff type and is given by

$$u(a; t) = -(a - (t + b))^2,$$

where $b > 0$ is a bias parameter. For tractability, we assume that the set of payoff types is $T = [0, 1]$, with a uniform prior probability distribution. This assumption, common in the literature, allows us to compare our findings with existing results.

Partitional LBEE. As shown in Crawford and Sobel (1982), this cheap talk game has informative Nash equilibrium outcomes if and only if $b < 1/4$. Such equilibria are described by a partition of T into K intervals, $[t^*_{k-1}, t^*_k]$, $k = 1, \ldots, K$, such that the sender reveals to the receiver to which of these intervals his payoff type belongs. There is a unique K-partitional equilibrium outcome for every $K \geq 2$ satisfying $b \leq \frac{1}{2K(K-1)}$. In addition, every K-partitional equilibrium satisfies $t^*_K \leq 1 - 4b$, that is, there exists a pooling interval of size at least $4b$ at the top of the type space. From an ex-ante point of view, the best equilibrium is the same for the sender and the receiver and is given by the K^*-partitional equilibrium where K^* is the highest integer satisfying $b \leq \frac{1}{2K^*(K^*-1)}$.

From Proposition 4, these partitional Nash equilibrium outcomes are also LBEE outcomes for the language system assumed in this section. A LBEE equilibrium implementing a partitional Nash equilibrium outcome is constructed as follows: for every $k = 1, \ldots, K$, each type $t \in [t^*_{k-1}, t^*_k]$ sends message t^*_{k-1}, and the receiver responds optimally (i.e., chooses action $\frac{t^*_{k-1}+t^*_k}{2}$

after the message t^*_{k-1}), and plays action 0 off the equilibrium path. Consider a sender type $t \in [t^*_{k-1}, t^*_k]$ for some $k = 1, \ldots K$. In a Nash equilibrium, this type prefers to reveal that his type belongs to the interval $[t^*_{k-1}, t^*_k]$ than to any other interval. Hence, a sender with partial language competence does not under-report because he perceives the strategy of the receiver correctly by under-reporting, and he does not over-report because his perceived expected utility by over-reporting is a convex combinaison of the utilities he would get by over-reporting in a Nash equilibrium.

One-Threshold LBEE. The informative threshold LBEE identified in Proposition 5 are never LBEE in the current framework for $b \leq 1/4$ because type $t = 0$ strictly prefers action 0 (what he induces by sending the message $m = 0$) to a distribution of actions whose expectation is $1/2$ (what he perceives to induce by sending a message $m > 0$). Also observe that there is no fully revealing LBEE for $b > 0$: a high enough type $t < 1$ would deviate and over-report because by over-reporting he would perceive to induce a distribution actions slightly above t, and hence closer to his ideal action $t + b$.

The next proposition shows that the following strategy for the sender always constitutes a LBEE for some $t^* \in (0, 1)$. If the sender's type t is strictly below t^*, then he fully reveals the state by sending the highest message he understands, i.e., by sending message $m = t$. Otherwise, if the sender's type is above t^*, then he sends message $m = t^*$.

Proposition 6 *The following pure strategy for the sender*

$$\sigma_S(t) = \begin{cases} t & \text{if } t < t^* \\ t^* & \text{if } t \geq t^*, \end{cases} \quad \text{for every } t_k < t^*, \tag{11}$$

constitutes a LBEE if and only if $b \leq \frac{1}{4}$ *and* $t^* = 1 - 4b$.

Proof. Consider a strategy for the sender as stated in the proposition. The best response of the receiver is:

$$\sigma_R(m) = \begin{cases} m & \text{if } m < t^* \\ E(t \mid t > t^*) = \frac{1+t^*}{2} & \text{if } m \geq t^*. \end{cases}$$

To constitute a LBEE, the sender type t^* must be indifferent between sending message $m = t^*$ and sending a message $m = t^* - \varepsilon$ when $\varepsilon \to 0$, i.e.,

$$-(t^* - (t^* + b))^2 = -(\frac{1 + t^*}{2} - (t^* + b))^2 \iff b^2 = (\frac{1 - t^*}{2} - b)^2$$

$$\iff b = \frac{1 - t^*}{2} - b \iff t^* = 1 - 4b.$$

A sender of type $t > t^*$ prefers higher actions than the sender of type t^*, so the former prefers to pool on $m = t^*$ rather than send a message $m < t^*$. A sender of type $t < t^*$ does not deviate iff

$$-(t-(t+b))^2 \geq -\frac{1}{1-t}\left(\int_t^{t^*}(m-t-b)^2 dm + (1-t^*)(\frac{1+t^*}{2}-t-b)^2\right),$$

where the LHS is the utility of type t if he sends message $m = t$ and the RHS is the perceived expected utility of type t if he over-reports, i.e., if he sends a message he does not understand. A sufficient condition for this inequality to be satisfied is

$$b^2 \leq \frac{1}{1-t}\left((1-t^*)(\frac{1+t^*}{2}-t-b)^2\right).$$

Replacing t^* by $1-4b$ and simplifying we get

$$4(1-t)^2 - 25b(1-t) + 36b^2 \geq 0.$$

The LHS is equal to zero at $t = 1-4b$ and is decreasing in t for $t \leq 1-\frac{25}{8}b$. Since $t < t^* = 1-4b < 1-\frac{25}{8}b$, we conclude that the LHS is positive for every $t < t^*$. We conclude that the sender has no profitable deviation and therefore the strategy profile described above for $t^* = 1-4b$ and $b \leq 1/4$ is a LBEE. ■

In the LBEE of the previous proposition, the strategy of the sender becomes more informative when the conflict of interest b decreases, and converges to full revelation when b tends to 0. The threshold $t^* = 1-4b$ is higher than the highest possible threshold t_K^* of a Nash equilibrium. Hence, this LBEE is more informative and therefore Pareto superior (at the ex-ante stage) to *all* Nash equilibria of the standard model of Crawford and Sobel (1982). For small biases, this LBEE also ex-ante Pareto dominates all (mediated) communication equilibria. This assertion can be verified easily by using the upper bounds of the ex-ante expected communication equilibrium payoffs in Goltsman, Hörner, Pavlov, and Squintani (2009, Lemma 2) and checking that, when b is small enough, the ex-ante expected payoffs at the LBEE constructed above are strictly higher than these upper bounds.

6 Conclusion

In this paper, we propose a novel way to model the fact that communicating agents may not understand all messages available to them. In contrast to existing works (Blume and Board, 2013 and Giovannoni and Xiong, 2019), we assume that agents can practically send and receive all messages, relying on the observation that words can sometimes be used and distinguished without being perfectly understood. Precisely, when an agent uses a message that is out of his language type, we assume that he has an imperfect understanding of the consequences of his message. When an agent receives a message that is out of his language type, we assume that he has an imperfect understanding of the meaning of this message.

To study the effect of partial language competence on communication, we develop an equilibrium concept, Language-Based Expected Equilibrium, based on the Analogy-Based Expected Equilibrium initially proposed in Jehiel (2005). This concept falls into the category of works which consider that the bounds of rationality are not on agents' ability to optimize their strategy given beliefs, but rather on agents' ability to form correct beliefs. In particular, this concept assumes that bounded rational players do not perceive the strategy of other players as finely as rational players do. In our communication context, a receiver does not fully understand how the sender's strategy maps each payoff type into messages, and a sender does not fully understand how the receiver's strategy maps each message received into actions. Instead, agents bundle sets of messages into analogy classes, and expect the same average mapping for every message within each such class. In our work, agents bundle all messages they do not understand, which makes a natural link between agents' analogy classes and their language competence.

Mainly, we show that LBEE is always a Bayesian Solution, providing a tight bound on the outcomes that partial language competence permit to achieve. We also apply the LBEE concept to two well-known communication settings, namely pure persuasion and the uniform quadratic model of Crawford and Sobel (1982). In both settings, partial language competence of the sender improves communication compared to the case of cheap talk between fully competent players. The qualitative properties of the equilibria in the two settings are however quite different: in the pure persuasion case, low types lie about their types by systematically over-reporting (because they have nothing to lose by doing so), and high types pool by truthfully reporting that they are higher than some threshold. This threshold could be any payoff type on which players coordinate on, and can be interpreted as a norm used by high type senders to distinguish themselves from lower types. In the (uniform-quadratic) model of Crawford and Sobel (1982), low types truthfully and perfectly reveal their types (because over-reporting is too risky), and high types pool. In the latter case, the threshold level above which high types pool is not arbitrary and decreases with the conflict of interest between the sender and the receiver.

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