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Abstract:

The outbreak of the Greek crisis has revived the literature on the sovereign debt spreads. Recent evidence has shed new lights on the main determinants of interest rates spreads. The sharp increase of government bond yields cannot be entirely attributed to changes in macroeconomic fundamentals. Contagion effects can occur and self-fulfilling speculation may arise. Yet, this literature has been mainly empirical and needs sound theoretical foundations. The aim of this paper is to fill in this gap. We develop a simple model in the spirit of second generation currency crises models developed by (Obstfled, 1996). The model describes a strategic game between governments and financial markets. Eurozone countries face a trade-off as governments may either commit and implement restrictive fiscal policies or default on debt. The cost of the commitment strategy increases when interest rates increase or when the fiscal multipliers are high. This leaves the opportunity for speculators to drive the economy towards a bad equilibrium, forcing the government to renege its commitment. We introduce a source of uncertainty about the cost of default in the model. By this way, we may introduce the possibility that governments do not default although risk premiums on bond yield is high.

JEL Codes: H63, E44, E61.

Key Words: Sovereign default, risk premium, multiple equilibria, asymmetric information.

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Introduction

From 2011, EMU countries have engaged a strategy of frontloaded fiscal consolidation. Despite evidence that such a strategy could be self-defeating (Holland and Portes 2012) and detrimental to growth, fiscal stance has remained restrictive in 2012, in 2013 and it will still be in 2014. This strategy is clearly sub-optimal (see OFCE, IMK, & ECLM, 2012) regarding new evidence on the size of fiscal multipliers. [3] From there, the strategy of frontloaded fiscal consolidation may be justified either by a misperception of the size of fiscal multipliers[4] or by a credibility argument. Some countries (mainly from the South of Europe: Greece, Spain, Italy and Portugal) faced a rapid surge in the risk premium on sovereign bond yields. Whereas spreads did not exceed 60 basis points in the pre-crisis period, they had started to rise moderately with the outbreak of the financial crisis. Then, a break occurred at the autumn 2009 after the newly elected Greek government revised strongly upward the deficit figures for 2009 and after the Gulf emirate of Dubai asked for moratorium on the debt of a public conglomerate. These news triggered a regime-switch as financial markets suddenly realized that a default on public debt might not be excluded Gibson et al. (2012). Therefore, they started to cast some doubt on the credibility of the Greek government to deal with the need to restore public finances sustainability. Contagion rapidly gained other EMU countries which then felt urged to reduce public deficits to show their commitment to fiscal sustainability. Fiscal consolidation was perceived as the only solution to enforce credibility and to step down the risk premium. Conversely, it was thought that delaying the adjustment would have led to an explosion of the spreads and eventually to the split up of the Eurozone.

Credibility has yet not improved despite the measures taken *ex-ante* by governments to improve their fiscal position. Spreads have still increased in 2011 and 2012. Even if it is almost impossible to assess what would have been the development of spreads if countries had considered an alternative fiscal strategy, it remains that consolidation has not been a sufficient condition to ensure credibility. The deterioration of other macroeconomic fundamentals (notably the output gap) may also matter as less future growth resulting from harsh consolidation reduces the ability to repay debt in the future. Credibility and risks of default result from complex interactions where self-fulfilling prophecies or contagion effects also matter. The aim of the paper is then to provide a theoretical analysis of credibility and default in a model with multiple equilibria. Fundamentals matter but, self-fulfilling prophecies may drive up risk premium and force the government to default.

Following the outbreak of the Greek crisis, literature on sovereign debt and risk of default has resurfaced. Empirical analyses have notably developed shedding new lights on the main determinants of interest rates spreads. It has been suggested that a reduction of the fiscal space (through changes in fiscal variables flows or stocks) have a positive impact on government bond yields, [5] imposing by this way discipline on governments (Schucknecht et al. 2009). But other fundamentals may also matter. External imbalances (Alessandrini et al.

[3] Recent literature has indeed showed that fiscal multipliers depend on the macro-financial environment. They are higher when the unemployment rate is high and when a banking crisis occurs. See notably Auerbach and Gorodnichenko (2012); Corsetti et al. (2012); DeLong and Summers (2012); Hall (2009); Karras (2013).

[4] The IMF (see Blanchard & Leigh, 2013) has indeed recognised that fiscal multipliers had been underestimated.

[5] See Haugh et al. (2009) for a synthesis on the impact of increases of debt or deficit on the interest rates.

2012) or business cycles (Grandes 2007) have been found to have an impact of bond yields spreads. However, recent evidence points out that the dynamic of the sovereign spreads during the crisis is hardly explained by fundamentals. Substantial mispricing is often highlighted as notably emphasized by De Grauwe and Ji (2012) though estimated misalignments depend on the choice of the estimated model, the sample... (de Haan et al. 2013). Two alternative interpretations may then be provided (Aizenman et al. 2013). Mispricing is either the result of multiple equilibria where financial markets have jumped from the "good" equilibrium to the "bad" equilibrium, or it may also mirror a future deterioration of fundamentals expected by investors. Yet, members of monetary union are notably more prone to liquidity squeeze as they are indebted in a money over which they do not have complete control (De Grauwe 2012). As financial markets are aware that these countries cannot force the central bank to act as a lender of last resort for public debt, they may be more incline to test the ability of members of monetary union to redeem their debt. These countries face the "original sin" problem emphasized by Eichengreen et al. (2005). Hence, the relation between sovereign debt spreads and fundamentals may be nonlinear reflecting changes in the mood of financial markets and notably the general risk pricing (Bernoth et al. 2006). Arghyrou and Kontonikas (2011) illustrate for example the contagion effect by suggesting that the rise in the spread on Greek bonds has been passed through the sovereign debt spread for most EMU countries during the crisis. Finally Bruneau et al. (2012) suggest that the probability of default is a nonlinear function of fundamentals and driven by self-fulfilling speculation. [6] They highlight the market perception of risks influenced notably by the sovereign CDS market. The perception of risks would then be a key point for explaining sovereign spreads though in practice government defaults are extremely rare events for advanced economies (Buiter and Rahbari 2013).

Yet, this literature has been mainly empirical and needs sound theoretical foundations. The aim of this paper is to fill in this gap. Amid the different features highlighted by the empirical literature, we consider that a model describing the determinants of sovereign debt spread should encompass the following characteristics: the role of macroeconomic fundamentals (not only fiscal position but also business cycle), risk pricing (related to risk aversion of market) and self-fulfilling prophecies. Some tentative have already been proposed. Early literature on sovereign debt default from Calvo (1988), Cole and Kehoe (1996) and Cole and Kehoe (2000) already emphasized that credibility matters and that the choice of default result from a strategic game. Some more recent papers also draw on exchange rate crises literature (Bruneau et al. 2012) or (Arghyrou and Tsoukalas 2011). The second generation currency crises models are notably well suited as they are based on the interactions between expectations of market participants and decisions made by the central bank regarding the peg (Obstfled, 1996) or (Sachs et al. 1996). Central banks face a trade-off between unemployment and devaluation. The incentive to exit the peg increases with the unemployment rate. Speculators are aware of this trade-off and may ask for higher interest rates to offset the risk of devaluation which raises the unemployment rate and triggers the collapse of the exchange rate regime.

In the current context of a monetary union, Eurozone countries face the same kind of trade-off. Governments may either commit and implement restrictive fiscal policies or default on debt. The cost of the commitment strategy increases when interest rates increase or when the fiscal multipliers are high. This leaves the opportunity for speculators to drive the economy

[6] See also De Grauwe and Ji (2013).

towards a bad equilibrium in which the sovereign debt risk premium increases forcing the government to renege its commitment. The analogy with exchange rate crisis models is clearly made by De Grauwe and Ji (2014) who emphasize that speculative attacks occur on the bond market in monetary union while they would occur in the foreign exchange markets in fixed-exchange rate regimes like the EMS. The optimal decision of investors arises endogenously in the model (see Cornand et al. (2014) for another example), contrary to most second-generation models of crises. Compared to Sachs et al. (1996), we also introduce information asymmetries in the model considering that the cost of default is not perfectly known by financial markets or that information on public finances may be asymmetric. In this sense, signaling can have an impact on the equilibrium outcome as in Angeletos et al. (2006). By this way, we may introduce the possibility that governments do not default although risk premium on bond yield is high. [7] Then, the model is able to explain, through numerical simulations, why Greece has de facto defaulted on public debt (fundamentals were deteriorated) and why some other countries have been driven in the zone of increased risk without default (Spain or Italy for example).

The rest of the paper is organized as follows. The model without asymmetric information is described in the first section. The introduction of information asymmetries is made in the second section. Numerical simulations are presented in the third section to see whether the model may fit with data on Greece and Italy. Section four concludes. The portfolio choice of investors is shortly described in appendix.

A model of default on public debt with self-fulfilling prophecies

We develop a simple static macroeconomic model, encompassing the effect of fiscal policy on economic activity and public debt dynamics. The model also takes into account the interest rates (including a risk premium – see Appendix I) effects on debt and economic dynamics. Besides, the policy-maker, here the government, has the choice to default partly on public debt.

Consider an economy in which the output gap og positively depends on the fiscal impulse FI, interest rate and a random demand shock. The impact of fiscal policy depends on the size of the fiscal multiplier identified by the parameter k. The interest rate $r + \phi$ – including the risk premium – has a negative effect on output gap.

(1.1) $$og = k.FI - \alpha(r + \phi) + \mu$$

Debt dynamics depends on past debt including debt burden, cyclical component of taxes and public expenditures $\varphi.og$, the fiscal impulse and the value of repudiated debt $d.b_{-1}$ in case of default:

(1.2) $$b = (1 + r + \phi)b_{-1} - \varphi.og + FI - d.b_{-1}$$

[7] Which is not the case in a one-period model with rational expectations. Once the random shock has occurred, government and speculators may compute the cost of the commitment strategy and the cost of default and choose accordingly the optimal strategy.

We assume $0 \leq d \leq 1$. It is supposed here that the haircut is chosen once the default is announced although history of defaults shows that it may be the result of a long bargaining between creditors and the debtor, which may last several years (Oosterlinck 2013).

Based on the previous portfolio choice model, the risk premium is a positive function of default probability, expected discount and risk aversion:

(1.3) $$\phi = Prob_d . d^e + \tilde{\phi}$$

where $\tilde{\phi} = \frac{1}{\upsilon}\left[\frac{{\gamma_n}^2 \sigma_R^2}{(1-\gamma_n)^2} + (Prob_d)^2 . \sigma_{d^e}^2\right]\omega_1$ increases with risk aversion and uncertainty.

The authorities' objective is to minimize the following quadratic loss function:

(1.4) $$\frac{1}{2}.\left(og^2 + \lambda\left(b-\bar{b}\right)^2 + \delta . d^2\right)$$

According to equation (1.4), the government's objective is to stabilize output and public debt, reflecting notably existing fiscal rules. We may then think of the 60% target for EMU member countries. As inflation is not described in the model, it is not included in the objective function of government. It is otherwise supposed that government dislikes repudiation. Under discretion the government sets FI and d to minimize (1.4) subject to equations (1.1), (1.2) and (1.3) and given the market's expectation of the extent of default on public debt d^e. The solution is given by the following equation:

(1.5) $$og = \frac{-\lambda . z . (1-\varphi . k)}{k} B,\ \left(b-\bar{b}\right) = z . B \text{ and } d = \frac{\lambda . z . b_{-1}}{\delta} B$$

where $z = \frac{\delta . k^2}{\delta . k^2 + \lambda . k^2 . b_{t-1}^2 + \lambda . \delta . (1-\varphi . k)^2}$

and $B = \left(b_{-1} - \bar{b}\right) + \left(r + Prob_d . d^e + \tilde{\phi}\right)\left(b_{-1} + \frac{\alpha}{k}\right) - \frac{\mu}{k}$.

Using (1.1) the fiscal impulse is then $FI = \frac{-\lambda . z . (1-\varphi . k)}{k^2} B + \frac{\alpha\left(r_t + Prob_d . d^e + \tilde{\phi}\right) - \mu}{k}$

The policymaker sets the fiscal impulse to minimize its loss function: a high level of debt, a high debt burden (enlarged by high risk premium) and positive shocks on the output gap imply negative fiscal impulses. Conversely, a negative shock on output gap implies a positive fiscal impulse to stabilize it.

Using (1.5), the loss for government is

(1.6) $$L^d = \frac{1}{2} . \lambda . z . B^2$$

where the subscript d stands for "default" (or discretion regime). Under rational expectations, the following condition $d^e = d$ is verified and from (1.5) we obtain:

(1.7) $$d^e = \frac{\lambda . z . b_{-1}}{\delta - \lambda . z . b_{-1} . Prob_d . \left(b_{-1} + \frac{\alpha}{k}\right)}\left(\left(b_{-1} - \bar{b}\right) + \left(r + \tilde{\phi}\right) . \left(b_{-1} + \frac{\alpha}{k}\right) - \frac{\mu}{k}\right)$$

Now suppose that the policymaker is able to commit to a no default solution. Then, $d = 0$ and we have:

(1.8) $$\left(b - \bar{b}\right) = \tilde{z}.B \text{ and } og = \frac{-\lambda.\tilde{z}.(1-\varphi.k)}{k} B$$

where $\tilde{z} = \frac{k^2}{k^2+\lambda.(1-\varphi.k)^2}$

The corresponding loss, under the "commitment solution" is:

(1.9) $$L^c = \frac{1}{2}.\lambda.\tilde{z}.B$$

The government always have an incentive to default, as $\tilde{z} > z$ since $\lambda.k^2.b_{-1}^2 > 0$. Next, we assume that the government faces an additional fixed private cost $c > 0$ when it defaults on its debt.

Given the expected haircut (d^e) on debt fixed on the financial markets, the government finds it optimal to default if and only if $L^d + c < L^c$. Using (1.6) and (1.9) we get:

(1.10) $$\left(b_{-1} - \bar{b}\right) + \left(r + Prob_d.d^e + \tilde{\phi}\right)\left(b_{-1} + \frac{\alpha}{k}\right) - \frac{\mu}{k} > K$$

where $K = [2c]^{1/2}[\lambda(\tilde{z} - z)]^{-1/2}$.

A default will then occur in equilibrium if:

- inherited debt is sufficiently high relative to the debt target,
- debt burden is sufficiently high, meaning notably that a restrictive monetary policy, by increasing the risk-free interest rate, increases the risk of default,
- a sufficiently high negative shock on output gap occurs, especially when the fiscal multiplier is low,
- risk aversion is sufficiently high, which would be translated in higher risk premium
- expectations of default and are sufficiently high.

Financial markets understand the temptation summarized by equation (1.10) and then rationally determine expectations of default. The left hand-side on the inequality depends on the fundamentals (debt, interest rate and the random shock), on the probability of default, the expected haircut and risk-aversion.

A rational equilibrium with no expectation of default ($d^e = 0$) may be reached if and only if:

(1.11) $$\left(b_{-1} - \bar{b}\right) + \left(r + \tilde{\phi}\right)\left(b_{-1} + \frac{\alpha}{k}\right) - \frac{\mu}{k} \leq K$$

When condition (1.11) is not satisfied, it is never rational not to expect a default. Combining equations (1.7) and (1.10), we may identify a second condition where agents may expect a default. It is given by:

(1.12) $$\left(b_{-1} - \bar{b}\right) + \left(r + \tilde{\phi}\right)\left(b_{-1} + \frac{\alpha}{k}\right) - \frac{\mu}{k} > \frac{K}{1+n.Prob_d.\left(b_{-1}+\frac{\alpha}{k}\right)}$$

where $n = \frac{\lambda.z.b_{-1}}{\delta - \lambda.z.b_{-1}.Prob_d.\left(b_{-1}+\frac{\alpha}{k}\right)}$

Thus $\frac{K}{1+n.Prob_d.\left(b_{-1}+\frac{\alpha}{k}\right)}$ and K defines two thresholds value that may illustrate three cases[8] according to the fundamentals. These three areas are illustrated in figure 1 below.

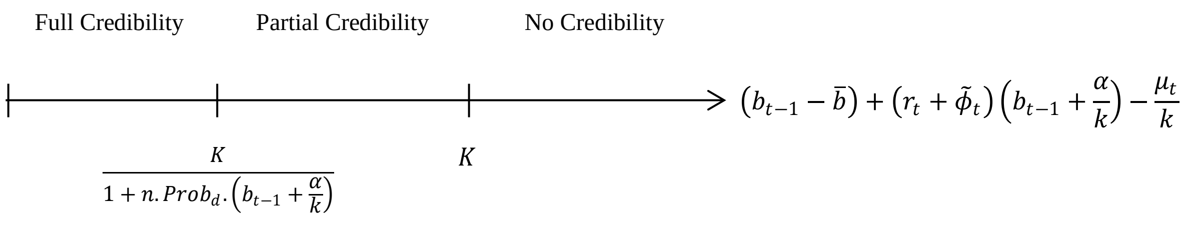


Figure 1. Level of fundamentals and multiple equilibria.

On the left hand side (below $\frac{K}{1+n.Prob_d.\left(b_{-1}+\frac{\alpha}{k}\right)}$), fundamentals (inherited debt, debt burden and shock) are sound. It would not be rational to expect a positive probability to default since no default will occur whatever agents expected. The government enjoys full credibility. This may characterize the situation of Germany. On the right hand side (above K), fundamentals are deteriorated and the government will always default whatever is the expected haircut. The Government enjoys no credibility at all, as it may have been the case for the Greek government in the recent sovereign debt crisis. Once the public information about the true value of the public deficit was revealed, fundamentals have exceeded the threshold and default has become unavoidable.

Finally, for levels of fundamentals in the range $\left[\frac{K}{1+n.Prob_d.\left(b_{-1}+\frac{\alpha}{k}\right)}; K\right]$, there are multiple equilibria. If agents do not expect default ($d^e = 0$), no default occurs (the good equilibria). If financial markets expect a default, then the size of default is given by $d^e = \frac{\lambda.z.b_{-1}}{\delta-\lambda.z.b_{-1}.Prob_d.\left(b_{-1}+\frac{\alpha}{k}\right)}\left(\left(b_{-1} - \bar{b}\right) + \left(r + \tilde{\phi}\right).\left(b_{-1} + \frac{\alpha}{k}\right) - \frac{\mu}{k}\right)$ and the default will be validated by the policymaker (the bad equilibria). Credibility of the government is partial and depends on expectations. Default, in this intermediary area result from self-fulfilling prophecies.

The partition of the state space depends on the value of K, which increases with the cost of default, the impact of interest rate and fiscal impulses on the output gap, the respective weights of public debt and haircut in the loss function of the government. Then for higher cost of default, the area of full credibility increases and the zone where the government has no credibility is moved to the right. Nonetheless, it must also be noted that the area of partial

[8] There is then a clear connection with the cases highlighted by (Sachs et al. 1996).

credibility widens meaning that the risk of multiple equilibria becomes larger. Concerning the value of fiscal multiplier, it can be shown that K decreases with k. Then, in time of crisis, recent evidence showing that fiscal multiplier are higher imply that the government may lose credibility and enter the intermediate area where expectations of defaults can force the government to default on public debt even if the fundamentals have not deteriorated. This model is then insightful to illustrate the role of self-fulfilling prophecies in driving the government into default. It may explain why in the recent crisis, some countries have enjoyed full credibility (fundamentals were sound) or why sovereign risk premium have emerged pushing countries into the partial or no credibility area.

Finally, it must be stressed that once the shock is revealed here, there is no more uncertainty. Financial markets are fully aware of fundamentals and of the trade-off faced by government. If they expect capital losses, $d^e > 0$, the rational solution leads to a default on public debt. Yet, the model may not fully capture the different situations of Eurozone countries during the recent crisis since risk premium have sometimes strongly increased (for Spain or Italy for instance) without triggering a default. To consider these cases, we extend the model in two ways. First, we suppose that the cost of default is unknown and may be assessed differently by the government and financial markets. Second, asymmetric information on the value of debt is introduced.

Sovereign default and government's credibility with information asymmetries

Though, the model sketched above highlights interesting features of sovereign default, it may not capture all the situations observed in reality. There are notably periods of high risk premium which are not followed by default (Cottarelli et al. 2010). Yet, the model does not account for this possibility of false signals or "type I error" where the null hypothesis of "no default" is true but rejected. In the recent sovereign debt crisis, the sovereign debt spreads have suddenly and strongly increased for Spain or Italy but default did not occur so far and spreads have now slowly vanished. Besides, situation where default is not expected – the warning signal has not been sent – but occurs (as for a "type II error"). The model is then extended through the introduction of uncertainty. Then, markets may suddenly shift from equilibrium with full credibility to equilibrium with partial credibility, forcing the government to default when economy is in the partial credibility zone. The introduction of uncertainty may indeed fit better with reality notably because the cost of default is certainly unknown resulting from several complex factors.

First, the access to international capital markets may be restricted for a significant period in case of default. This period of restricted access to financial markets is certainly related to the time needed to resolve public debt default. Pitchford and Wright (2013) reminds that it takes on average 6-8 years to restructure public debt. The government will also suffer from a loss of credibility and may also incur international sanctions (trade restrictions or freezing assets as emphasized by Pitchford & Wright, 2013[9]). Finally, costs of default may become high notably if the default triggers a banking crisis forcing the government to bail-out the banking sector. Those costs may not be proportional to the extent of the default or to any other

[9] It has yet been extremely rare.

macroeconomic variable. Conversely, governments have indeed always the ability to increase taxes or cut public expenditures to honour its debt. But the needed consolidation may not be socially or politically accepted as emphasized by (Buiter and Rahbari 2013) who introduce the maximum feasible primary surplus, reflecting economic, socially and political limits to consolidation measures. There is then a trade-off between economic, social and political costs of consolidation and the cost of default. This trade-off is highly uncertain as neither the social or political cost of consolidation, nor the costs in terms of reputation are observable. We will next consider the case where the cost of default is precisely known by the government only as default is eventually a political choice made by the government. Another source of uncertainty during the crisis was also related to the level of public debt. The Greek crisis has illustrated that public debt could be significantly revised upward. Then, we propose an extension of the model to account for uncertainty on the value of debt. Financial markets may indeed not always have full information when forming their forecasts. Definitive figures for debt at year (t) are only revealed at the beginning of year (t+1) and even if some intermediary information is provided during the year, surprises may occur leading to a new revised assessment on the risk of default.

The case of unknown default

Let's consider a situation where the cost of default is unknown and written in the following way:

$$\hat{c} = c + \Theta$$

where Θ is the random shock on the cost of default. We suppose hereafter that $E(\Theta) = 0$. The cost of default for the government is still c, whereas it is now $\hat{c}$ for the financial markets . Hence, we assume here that the cost of default is fixed for the government and may jump to higher or lower levels for financial markets. Shocks on the assessment of cost of default may well capture some features of the recent financial crisis. Before the outbreak of the Greek crisis, default was not considered as an option and financial markets suddenly realized that it was a possibility and that Eurozone countries were more prone to liquidity crises (De Grauwe and Ji 2012). This new assessment of default also certainly embraced the exit cost of euro area. It may have been infinite before 2010 and has turned to a finite value as some European political leaders openly evoked the possibility for EMU countries to split up.

The resolution of the model is unchanged and default conditions do not change for the government, but do change for financial markets. Now financial markets always expect a default if

(2.1) $$\left(b_{-1} - \bar{b}\right) + \left(r + Prob_d . d^e + \tilde{\phi}\right)\left(b_{-1} + \frac{\alpha}{k}\right) - \frac{\mu}{k} > \widehat{K}$$

where $\widehat{K} = [2\hat{c}]^{1/2}[\lambda(\tilde{z} - z)]^{-1/2}$

Equations (1.11) and (1.12), defining the thresholds are rewritten so that, financial markets grant no credibility to government when fundamentals lie above $\widehat{K}$:

(2.2) $$\left(b_{-1}-\bar{b}\right)+\left(r+\tilde{\phi}\right)\left(b_{-1}+\frac{\alpha}{k}\right)-\frac{\mu}{k}>\hat{K}$$

Credibility is full – expected haircut is null ($d^{e}=0$) – for financial markets when:

(2.3) $$\left(b_{-1}-\bar{b}\right)+\left(r+\tilde{\phi}\right)\left(b_{-1}+\frac{\alpha}{k}\right)-\frac{\mu}{k}<\frac{\hat{K}}{1+n.Prob_{d}.\left(b_{t-1}+\frac{\alpha}{k}\right)}$$

As before, credibility is partial in the intermediary area and risk premium may either be positive or null.

Starting from this, two cases deserve attention according to the difference in the cost of default. Considering a negative shock on the cost of default for financial markets, we would have $c>\hat{c}$. This may result from contagion effect or from a political shock (see Cole and Kehoe 1996). In the case of contagion, the cost of default is reassessed because a default has been observed in a country sharing some common troubles. The shock may result from a change of government if the newly elected government has call for a debt restructuration during the political campaign. A first situation emerges where the government would remain in the full credibility area whereas financial markets expecting a lower cost of default are in the partial credibility area. This situation may arise if (see figure 2):

$$\frac{\hat{K}}{1+n.Prob_{d}.\left(b_{-1}+\frac{\alpha}{k}\right)}\leq\left(b_{-1}-\bar{b}\right)+\left(r+\tilde{\phi}\right)\left(b_{-1}+\frac{\alpha}{k}\right)-\frac{\mu}{k}\leq\frac{K}{1+n.Prob_{d}.\left(b_{-1}+\frac{\alpha}{k}\right)},$$

In such a situation, it is never optimal for the government to default. The loss of the government is indeed lower when not defaulting than when defaulting. Fundamentals are not sufficiently "deteriorated" to force the government to default, but as the expected cost of default is weaker for financial markets, they would wrongly expect a default and send a false signal, increasing the risk premium. Spain and Italy have certainly been in such a situation during the sovereign debt crisis. The outbreak of the Greek crisis has triggered a shock in the expectation of default, a possibility that was overlooked before and became suddenly possible. Financial markets have then underestimated the cost of default relative to the government's assessment of default. This would explain why sovereign spread had risen although the Spanish government did not consider the opportunity to default.

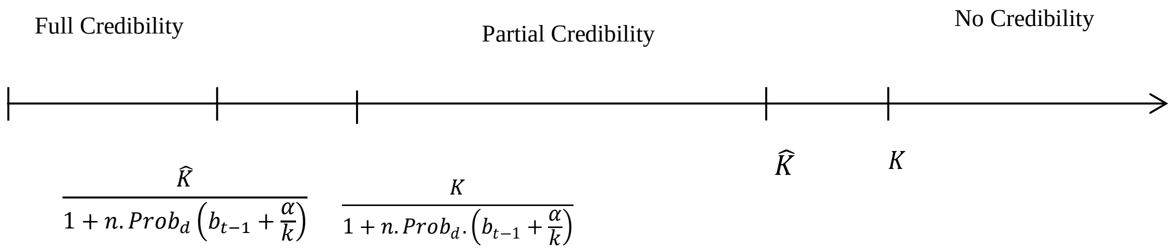


Figure 2. Risk premium and no default.

A second situation arises when:

(2.4) $$\left(b_{-1}-\bar{b}\right)+\left(r+\tilde{\phi}\right)\left(b_{-1}+\frac{\alpha}{k}\right)-\frac{\mu}{k}\geq\hat{K}$$

In that case, the market always expects default, which occurs as soon as:

(2.5) $$\left(b_{-1}-\bar{b}\right)+\left(r+\tilde{\phi}\right)\left(b_{-1}+\frac{\alpha}{k}\right)-\frac{\mu}{k}>\frac{K}{1+n.Prob_{d}.\left(b_{t-1}+\frac{\alpha}{k}\right)}.$$

Here the government is partially credible but forced into default sooner when financial markets consider that the cost of default is weak. The range of multiple equilibria has been consequently reduced, meaning that, for the same value of fundamentals, the possibility that financial markets systematically expect default is increased. Default would occur more frequently or rapidly since financial investors consider that the cost of default is weaker. Let's suppose for example that we start from a situation of sound macroeconomic fundamentals so that the government has no incentive to default and default is not expected by financial markets. Consider then the occurrence of a strong adverse shock deteriorating the economic situation so that the government is now in the partial credibility area. But, with $\hat{c}<c$ the governement has lost its credibility for financial investors and lies is the "no credibility" area. Default becomes the optimal solution since it is expected by financial markets. The default scenario may become certain if at the same time, the adverse shock push fundemandals such that condition (2.4) is met.

Alternatively, we may also consider the polar case where $c<\hat{c}$ so that $K<\hat{K}$. The expected cost of default is higher for financial markets than for financial markets. In such a situation, a default may occur even if it is not expected by financial markets. Here, financial markets have failed to send the warning signal. There is no risk premium but the value of fundamentals for which the government considers that it is preferable to default is so low that it chooses to default. This situation arises for:

(2.6) $$K<\frac{\hat{K}}{1+n.Prob_{d}.\left(b_{-1}+\frac{\alpha}{k}\right)}$$

It would represent a situation where the financial investors attribute a very high value to the public default. It is then not rational to ask for a risk premium. But if the real cost of default is significantly lower, the government may choose to. Actually, history of default shows that financial markets may not expect default even if fundamentals are deteriorated for governments.

The case of asymmetric information on public finances

This situation may be alternatively captured by introducing uncertainty on the value of debt. Instead of considering that the cost of default is estimated differently, we suppose that information on the situation of public finances is asymmetric so that only the government knows perfectly and timely the amount of public debt. Public information on debt is revealed with a lag and may then trigger information surprises if debt is higher (or lower) than expected. The expected value of public debt is such that:

$$\widehat{b_{-1}} = b + \Theta$$

with Θ depending on private information collected by investors on public debt. For the sake of simplicity, we suppose that Θ is a random shock. Equations (1.11) and (1.12), defining the thresholds are rewritten with $\widehat{b_{-1}}$. Starting from this, the default occurs if:

(2.7) $$\left(b_{-1} - \bar{b}\right) + \left(r + \tilde{\phi}\right)\left(b_{-1} + \frac{\alpha}{k}\right) - \frac{\mu}{k} > K$$

whereas:

(2.8) $$\left(\widehat{b_{-1}} - \bar{b}\right) + \left(r + \tilde{\phi}\right)\left(\widehat{b_{-1}} + \frac{\alpha}{k}\right) - \frac{\mu}{k} < K$$

or

(2.9) $$\left(\widehat{b_{-1}} - \bar{b}\right) + \left(r + \tilde{\phi}\right)\left(\widehat{b_{-1}} + \frac{\alpha}{k}\right) - \frac{\mu}{k} < \frac{K}{1 + n.Prob_{d}.\left(b_{-1} + \frac{\alpha}{k}\right)}$$

Here, the default had not been expected, so that risk premium was null. Given the value of public debt, which is known precisely and timely by the government only, it is optimal to default. According to the private information, the market's perception was that government was fully credible or partially credible. Financial markets are then surprised by default. It may yet be acknowledged that this situation may not be usual as the default is often triggered following a period of panic where risk premium reaches a peak. In the Greek crisis, the revelation on the deficits in October 2009 by the newly elected government of Georgios Papandréou has changed financial markets' perception on the ability of the government to meet its commitments so that the situation described above may fit with this reality.

The model can now account for this if the news is interpreted as a signal (random shock) increasing $\widehat{b_{-1}}$. Before the announcement, the assessment made by financial markets on public debt was such that $\widehat{b_{-1}} < b$, with b only known by the government and with condition (2.9) fulfilled and with:

(2.10) $$\left(b_{-1} - \bar{b}\right) + \left(r + \tilde{\phi}\right)\left(b_{-1} + \frac{\alpha}{k}\right) - \frac{\mu}{k} > \frac{K}{1 + n.Prob_{d}.\left(b_{-1} + \frac{\alpha}{k}\right)}$$

The news suddenly changed the market's perception and credibility became partial. It might even be argued that public debt has reached a level such that Greek government had directly jumped to the "no credibility" area. It was clear now that it was insolvent and default was the only option. The sudden increase in the risk premium only reflected this situation.

Sovereign default and governments' credibility in the euro area: some illustrations

Finally, we provide numerical illustrations of the model emphasizing notably the dynamic of credibility, given fundamentals and given a consistent parametrization. To this end, the model is calibrated with Greek and Italian data. The aim is notably to assess whether the model may provide insightful explanations of the recent episode of sovereign crisis in the euro area.

When did Greece lose credibility?

The two situations described by the theoretical model above may be illustrated with numerical simulations. The aim is notably to assess the dynamic of government's credibility according to the level of public debt or the estimated cost of default. The numerical application is first drawn on the Greek data. Values of the parameters are presented in table 1 below.

Table 1. Calibration

λ	1
δ	4
α	0.93
φ	0.43
k	1
$\bar{b}$	1.2
μ	0

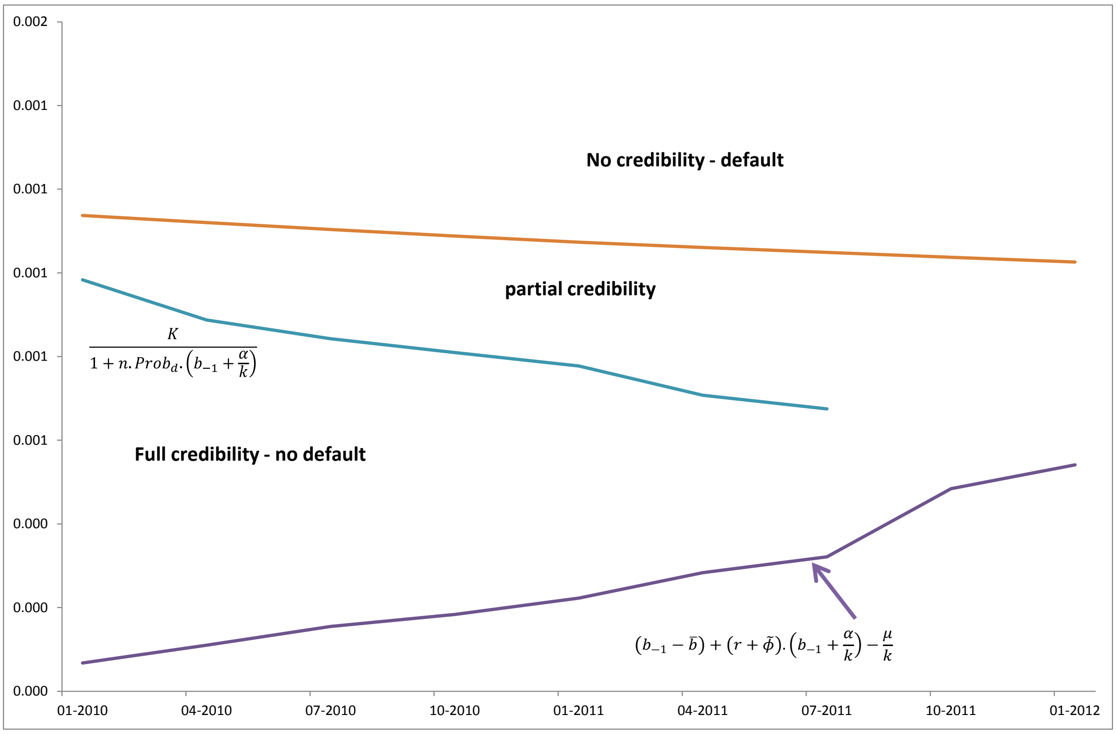
We assume that the government puts the same relative weight on stabilising activity and achieving the debt target ($\lambda = 1$), but that it is reluctant to implement any haircut on the debt ($\delta = 4$). For the sensitivity of the output gap to the real interest rate we retain the value 0.93 estimated by Coenen and Wieland (2005) for the euro area. The sensitivity of taxes to activity comes from European Commission (2005), and the fiscal multiplier is set to 1 following Blanchard and Leigh (2013). The past debt is the Maastricht debt given by OECD economic outlook, and the target debt is the level of debt that Greece engaged to achieve in 2020 when dealing financial help with the IMF and eurozone countries in 2010. In the case of Greece, news about the situation of public finances has led to a sudden increase in public debt after 2010. This increase may have been seen as a signal for financial markets. New information were provided, which modify the perception of the risk of default and of the credibility of the Greek government. OECD database clearly shows that debt was revised upward after 2010. In the Economic Outlook (EO thereafter) released in November 2010 (EO88), public debt for 2010 was estimated to amount to 96 % of GDP. One year after, in the EO90 database, public debt was at 107.4 %. $Prob_d$ stems from (Camba-méndez and Serwa 2014), who estimate the probability of default (PD) and the loss given default (LGD) as perceived by financial markets for several euro area countries. For sake of simplicity, we have supposed that $\mu = 0$ since we don't know the size of shocks that hit the Greek economy during the crisis.

A first way to compute $\tilde{\phi}$ would be $\tilde{\phi} = \phi - Prob_d . d^e$. $Prob_d$ can be computed from CDS with an assumption on d^e. But as CDS may overestimate default probability during the crisis (see Aizenman et al. 2013), we have to make another assumption to compute $\tilde{\phi}$. For sake of simplicity, risk premium is assumed to be half the difference between the Greek 10 year interest rate and the German rate.

Chart 1a and 1b shows the dynamics of fundamentals and, for given thresholds corresponding to conditions (1.11) and (1.12). They indicate whether the government is fully credible, partially credible or has no credibility, for an expected cost of default of 10% of GDP. Yet, fundamentals and thresholds depend on public debt at time (t-1) and it appears that the upward revision of public debt has led Greece to the partial credibility area (chart 1b) whereas it would not have been the case if debt figures had not been revised upward (chart 1a). It may be observed that with this set of parameters, Greece would have entered relatively later in the partial credibility area. Yet, the calibration is based on relative prudent hypotheses as Greece has been hit by negative shocks, which is not taken into account here. The size of fiscal multiplier may also be above unity. We should also stress that with our calibration, we show that Greece would lose all credibility at the end of 2011 (2011-Q3), precisely at the time when the implicit default on debt occurred.

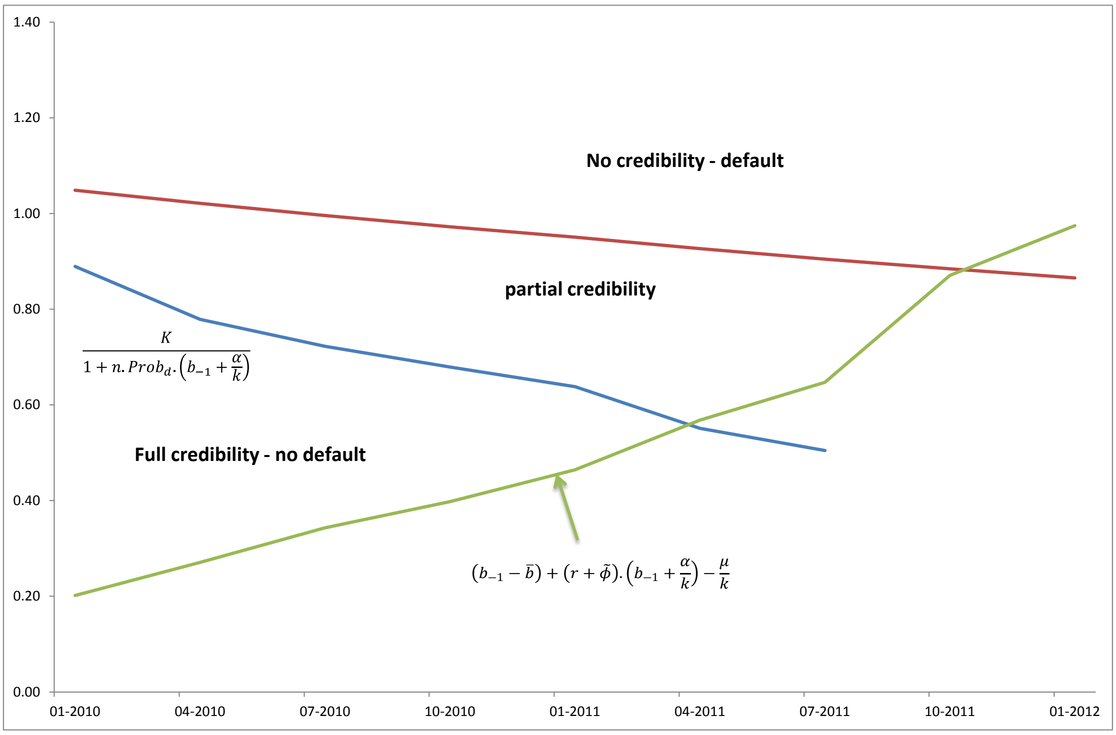
Besides, it may then be shown that the area of partial credibility gets larger with the cost of default. At the same time, fundamentals have to be worse and worse for the government to enter in it (chart 2). Then for a lower expected cost of default, the higher is public debt (or the more deteriorated are fundamentals) the more rapidly do governments fall in partial credibility or no credibility areas. In this sense, uncertainty on the cost of default, and low market expectations on it may have triggered the default. Considering the situation of Greece at the beginning of 2011, the model's simulations highlight that Greece would have been in the partial credibility zone if the cost of default had been below 5% of GDP. It may also be shown that the partial credibility area becomes larger when the probability of default increases (chart 3). These expectations may have pushed the market to speculate against Greek debt via the CDS market. As the partial credibility area is reached more rapidly when $Prob_d$ is higher, financial markets may have expected higher default probability, pushing Greece in the partial area or the default area, which would have triggered the default in 2012.

Chart 1a. Fundamentals (before revision) and credibility in Greece between 2010 and 2012



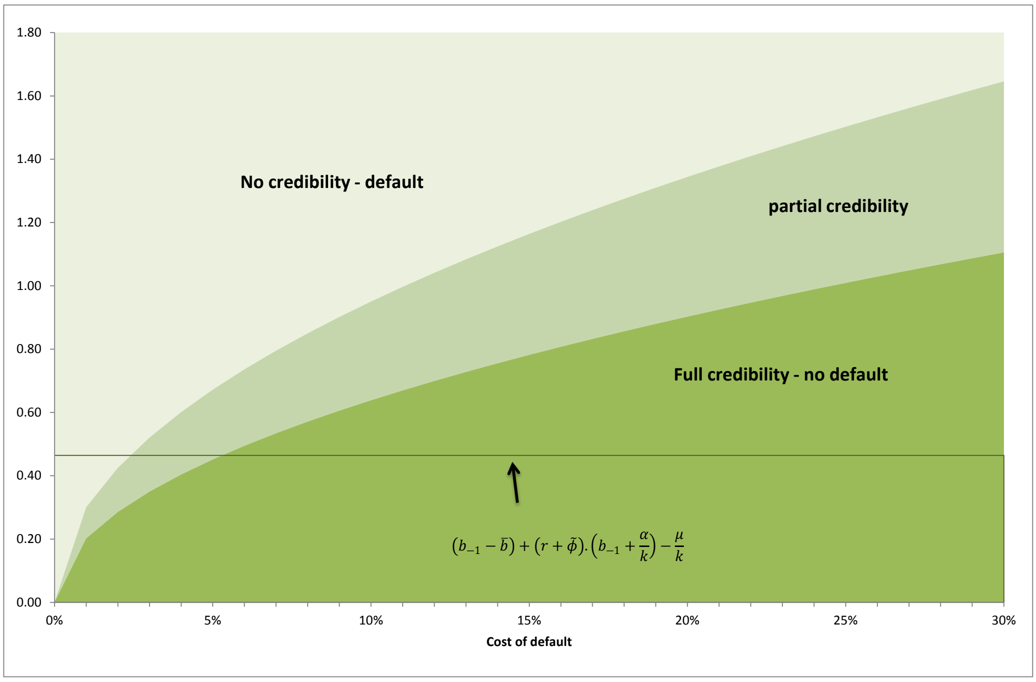
Sources: author's calculations, OECD EO88, Camba-méndez and Serwa (2014).
Note: we assume c=10% of GDP

Chart 1b. Fundamentals (after revision) and credibility in Greece between 2010 and 2012



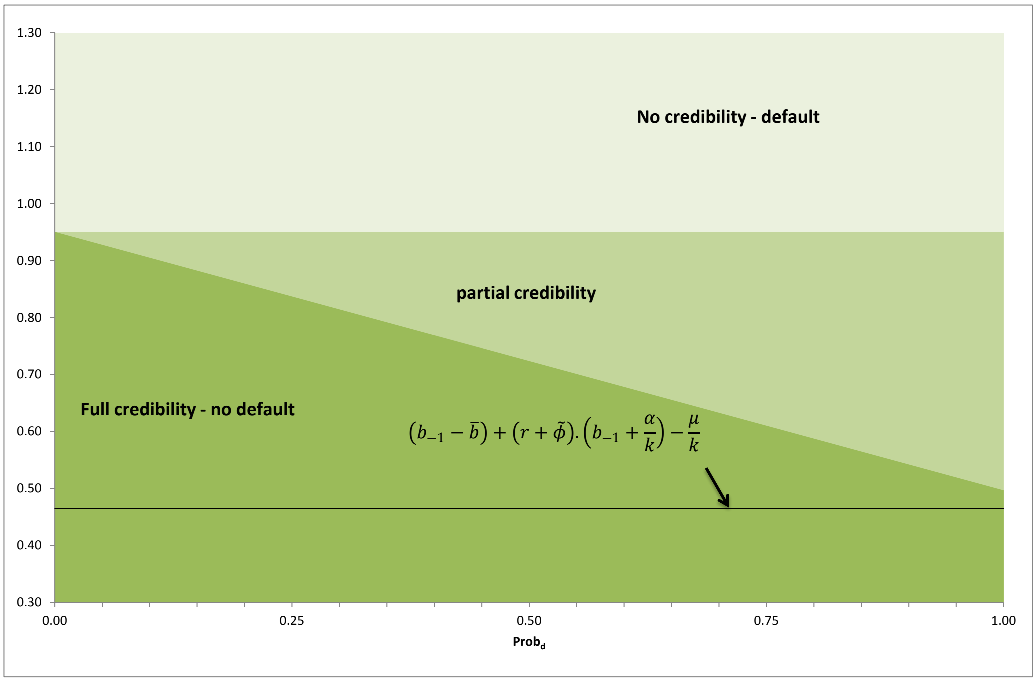
Sources: author's calculations, OECD EO93, Camba-méndez and Serwa (2014).
Note: we assume c=10% of GDP

Chart 2. Credibility areas for Greece and cost of default in 2011Q1



Source: author's calculations, Camba-méndez and Serwa (2014)

Chart 3. Credibility areas for Greece and probability of default in 2011Q1



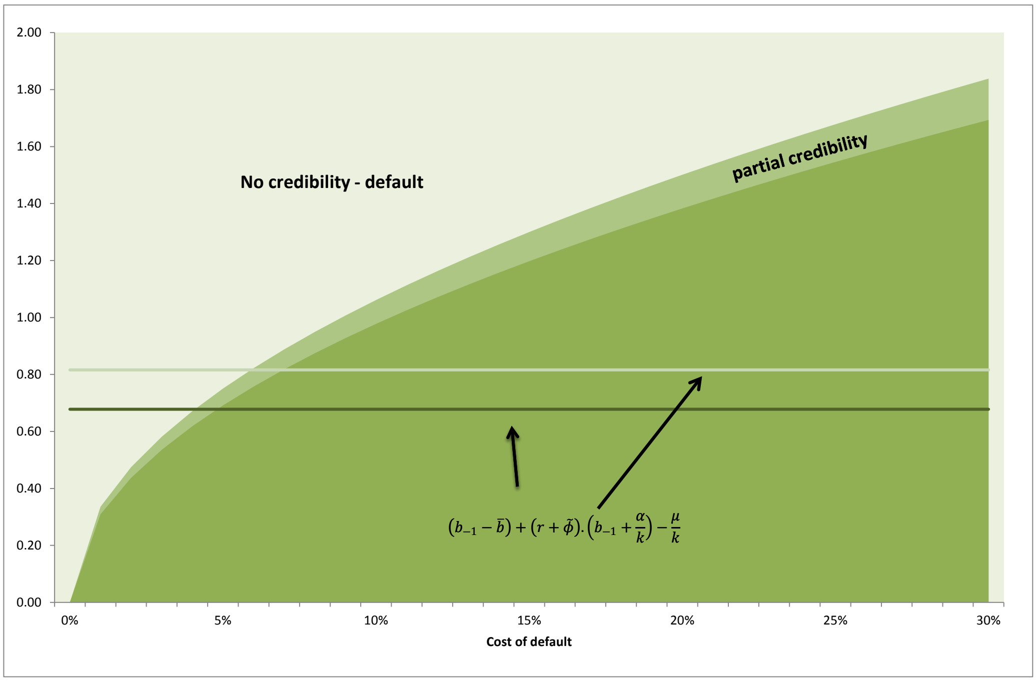
Source: author's calculations, Camba-méndez and Serwa (2014)
Note: we assume c=10% of GDP

The cost of default and the Italian credibility

Finally, the model may also help to understand at which conditions Italy (or Spain, Ireland or Portugal) has become partially credible. As emphasized in the theoretical model developed above, it may be the consequence of a reduction in the estimated cost of default. Then, we determine the area of full partial and no credibility according to the cost of default as in chart 3. To make the reading of the chart easier, we only consider the cost estimated by financial markets. Then, in the partial credibility area, a risk premium may appear but it will not automatically trigger a default if the cost of default for government is higher.

The calibration of the model is the same as in table 1. Fiscal multiplier is supposed to be equal to 1 and the role of macroeconomic shocks is overlooked. The only difference stem from macroeconomic fundamentals and more precisely from the level of public debt. As for Greece, the partial credibility area widens for higher cost of default. But at the same time, fundamentals should be more deteriorated to enter this area. Consider the level of public debt in 2010, Italy would have been in the partial credibility area for a cost of default between 4% and 5% of GDP (chart 4). With higher public debt, as in 2013, partial credibility area would start for a cost of default of 6 %. Here it must be stressed that a small deterioration of fundamentals either through an increase in debt or through a negative shock hitting the economy, would push Italy to the area where it has no credibility for financial markets. A risk premium would then appear. Nonetheless, the default would not automatically occur, as emphasized in the theoretical model with asymmetric information, since the cost of default for the government may be higher. This simple numerical exercise helps to highlight the main feature of the model and shows that for consistent hypotheses regarding the macroeconomic parameter, the model may replicate what has been observed in during the European sovereign debt crisis.

Chart 4. Credibility areas for Italy and cost of default



Source: author's calculations

Conclusion

In this paper, we have developed a simple analytical model to illustrate the interconnections between fundamentals, risk premium on sovereign debt and risk of default. The model is inspired by exchange rate crises models of Obstfled (1996) and Sachs et al. (1996) and adapted to a situation where government decide rationally to default or not taking into account the risk premium, the output gap and the cost of implementing restrictive fiscal policy. The situation of some European countries during the recent sovereign debt crisis is clearly well illustrated by this kind of models suggesting that self-fulfilling prophecies may force the government to default. Here, it is worth reminding that default is always and finally the consequence of a political choice where the government face a trade-off between implementing more austerity to reduce public debt or defaulting. In situations where fundamentals are deteriorated, either because the debt burden has increased, the risk aversion of financial markets have increased or a negative shocks has occurred, the credibility of the government may decrease and become partial. There are then multiple equilibria and the government is forced to default if this is the situation expected by financial markets. This kind of vicious circle is clearly representative of what occurred for European countries in the recent period. Yet, default is costly. The reputation of government may seriously be undermined in case of default. The access to financial markets funding may be restricted for a sustained period. Besides, the threshold over which austerity measures become socially unsustainable is clearly unknown. The ability to raise taxes or to cut spending is part of the trade-off but is uncertain and can only be expected. There is then no reason that expectation of financial investors matches with the supposed cost of default used by the government when the rational choice is made. By the same way, pubic debt is not always perfectly (in real time) known by financial markets. The government has then certainly private information on the real value of debt up to a certain moment when signals are sent to give information on the true value of debt. This situation may then change financial market's perception on credibility. The model is then extended to account for asymmetric information on the cost default or on public debt. Multiple situations may occur, enriching the conclusions of the model of Sachs et al. (1996) as we illustrate cases where financial markets expect default and capital losses whereas no default occurs since the cost of default is high for government and the value of fundamentals is good enough so that the government does not find it optimal to default. In the same way, news on fundamentals – signals – on public debt may suddenly drive fundamentals on area where credibility is partial and where multiple equilibria arise.

Clearly, considering asymmetric information opens new issues regarding the possibility to influence private expectations of the cost of default. There is an incentive to convince financial markets that the cost of default is high. Besides, the role of institutions clearly matters. In the European context, the sovereign spreads were certainly fuelled by the debate on a possible exit of countries from the monetary union. The adoption of the TSCG has contributed to reinforce the viability of EMU. The role of the ECB and the announcement of the OMT have probably played an even greater role to lessen the sovereign spreads. By convincing the financial markets that it would stand ready to intervene to purchase public debt, it has clearly sent a signal that default was less likely and more costly as it would have denied the ability of government to benefit from the support of the central bank. More research is yet needed to formalize more thoroughly this issue.

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Appendix I. A model of portfolio choice with default

We develop a model to describe the main components of the risk premium. This is a simple model of portfolio choices where investors allocate their wealth between a risk-free asset and a risky asset which is a domestic asset issued by the government[10]. It takes into account risk aversion and a probability that the government default on public debt. Such kind of models can be based on term structure models of interest rates to account for long maturity issuances. A general model of term structure is developed following Shiller (1979), and completed with a portfolio choice model.

The expected one-period holding yield H_t^n of a risky asset, a bond, maturing in n periods, given by equation (A.1) is equal to expected price of the bond at time $t+1$, which amounts to P_{t+1}^{n-1} with a probability $1 - Prob_d$ that the government do not default on debt, and amounts to the expected value of the bond if the government defaults at time $t+1$, approximated by $(1-d^e)P_t^n$ plus a coupon payment C at the end of the period, minus the price of the bond at time t, divided by the P_t^n. We assume that d^e is the expected capital loss – or discount – on the bond price and $Prob_d$ is with the probability of default. Both terms are supposed to be exogenous for individual investors. It is clear that the capital loss is not known a priori. In the case of sovereign default, once it is announced by the government, an open discussion is generally opened between the borrower and its creditor to fix the amount of the discounted payment[11]. The exogeneity of the probability of default may be justified either by the atomistic weight of an individual investor or by the fact that this probability simply reflects the historical probability of default, which is very low for advanced economies.

(A.1) $$H_t^n = \frac{C + (1-Prob_d).(E_t P_{t+1}^{n-1}) + Prob_d.(1-d^e)P_t^n - P_t^n}{P_t^n}$$

The yield to maturity R_t^n on an n-period bond is determined by the requirement that the price P_t^n of the bond is the present value of coupons and principal discounted by R_t^n:

(A.2) $$P_t^n = \frac{C}{R_t^n} + \frac{R_t^n - C}{R_t^n.(1+R_t^n)^n}$$

Substituting (A.2) in (A.1), linearizing around $C = R_t^n = R_{t+1}^{n-1} = \bar{R}$ and $Prob_d = \overline{Prob_d}$, simplifying and rearranging gives:

(A.3) $$H_t^n = \frac{R_t^n - \gamma_n.E_t R_{t+1}^n}{1-\gamma_n} - Prob_d.d_{t+1}^e$$

With $\gamma_n = \frac{\gamma\left[\frac{1-\overline{Prob_d}}{\gamma(1-\overline{Prob_d}+\bar{R})} - \gamma^{n-1}\right]}{1-\gamma^n}$ and $\gamma = \frac{1}{1+\bar{R}}$.

$E_t R_{t+1}^n$ is the expected yield to maturity at time $t+1$, with variance σ_R^2. The investor divides his investment fund into fractions ω_1 which is allocated to the purchase of long-term risky bonds and ω_2 allocated to the purchase of risk-free short-term bonds that pay r_t. The expected return of its portfolio has mean:

[10] Such an approach is close to Bernoth et al. (2006).

[11] It may also be noticed that sovereign default are not ruled out by a supranational authority or court having the ability to resolve failing government and even less to proceed to liquidation. Then, the amount of capital losses may always be disputed.

(A.4) $\rho_t = \omega_1(1 + H_t) + \omega_2(1 + r_t)$

And variance:

(A.5) $S_t^2 = \omega_1 \left[\frac{{\gamma_n}^2 \sigma_R^2}{(1-\gamma_n)^2} + (Prob_d)^2 . \sigma_{d^e}^2\right]$

The risk of the portfolio notably depends only positively on the probability of default and on $\sigma_{d^e}^2$, that can be viewed as an indicator of the uncertainty around the amount of capital loss in case of default. For sake of simplicity we assume that R_{t+1}^n and d_{t+1}^e are not correlated but this hypothesis shall be discussed: in case of default, the price of the bond would fall around $(1 - d^e)P_t^n$ and the yield to maturity would go up. In the worst case, (A.5) would then underestimate the variance of the expected return of the portfolio.

Assume the existence of a utility function U that orders the risk-averse investor's preferences according to the values of the couple (ρ_t, S_t^2). The optimal choice for ω is obtained by maximising $U(\rho_t, S_t^2)$ under the constraint $\omega_1 + \omega_2 = 1$. First order conditions give:

(A.6) $\frac{\partial U}{\partial \rho}(1 + r_t) = \frac{\partial U}{\partial \rho}(1 + H_t) + 2\frac{\partial U}{\partial S^2}\omega_1 . \left[\frac{{\gamma_n}^2 \sigma_R^2}{(1-\gamma_n)^2} + (Prob_d)^2 . \sigma_{d^e}^2\right]$

and

(A.7) $\omega_1 + \omega_2 = 1$

The optimal share of long-term risky bonds is then:

(A.8) $\omega_1 = \left[\frac{\upsilon(1-\gamma_n)^2}{{\gamma_n}^2 \sigma_R^2 + (1-\gamma_n)^2 . (Prob_d)^2 . \sigma_{d^e}^2}\right][H_t - r_t]$

with $\upsilon = -\frac{1}{2}\frac{\partial U/\partial \rho}{\partial U/\partial S^2}$

For $\frac{\partial U}{\partial S^2} < 0$, the investor is risk-averse as utility decreases when the risk of the portfolio increases. The share of the risky asset increases with the return of the risky asset and decreases, for risk-averse investor, with the risk of the risky asset, the probability of default and the variance of the capital loss. The yield to maturity on the risky bond is then given by:

(A.9) $R_t = (1 - \gamma_n)[r_t + \phi_t] + \gamma_n . R_{t+1}$

with $\phi_t = Prob_d . d_{t+1}^e + \frac{1}{\upsilon}\left[\frac{{\gamma_n}^2 \sigma_R^2}{(1-\gamma_n)^2} + (Prob_d)^2 . \sigma_{d^e}^2\right]\omega_1$

Let's consider now that, at the market equilibrium for the bond asset, demand by investors is equal to supply of bond. For sake of simplicity, we may suppose the behaviour of a representative investor whose wealth-to-GDP is w. The demand for the government bond – the risky asset here – is simply : $\omega_1 w$. The total debt-to-GDP ratio is debt is given by b, so that the risk premium is finally given by :

(A.10) $\phi_t = Prob_d . d_{t+1}^e + \frac{1}{\upsilon}\left[\frac{{\gamma_n}^2 \sigma_R^2}{(1-\gamma_n)^2} + (Prob_d)^2 . \sigma_{d^e}^2\right]\frac{b}{w}$

Here it only adds the fact that risk premium also depend on the debt-to-GDP ratio. Iterating (A.9) through the future, it gives:

(A.11) $$R_t = \frac{1-\gamma}{1-\gamma_n}\sum_{j=0}^{n-1}\gamma^j E_t\left[r_{t+j} + \phi_{t+j}\right]$$

The long-term risky bond yield to maturity is equal to a weighted sum of expected future short-term interest rates and a risk premium that depends on the degree of risk aversion, the volatility on the risky bond market, a probability that the government will default in the future and the expected capital loss in case of default. If investors are risk neutral, then the premium only depends on the default and the expected discount. If expected premium and expected short term rate are constant, then equation (A.11) simply becomes $R_t = r_t + \phi_t$.