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Doubts and Dogmatism in Conflict Behavior

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Doubts and Dogmatism in Conflict Behavior*†

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Abstract. Conflicts are likely less violent if individuals entertain the possibility that the opponent may be right. Why is it so difficult to observe this attitude?

In this paper, we consider a game of conflict where two opponents fight in order to impose their preferred policy. Before entering the conflict, one opponent (the agent) trusts the information received by his principal. The principal wants to affect the agent’s effort, but he also cares that the agent selects the correct policy and that he has the right incentives to acquire information.

We find conditions under which the principal induces hawkish attitudes in the agent. As a result, the agent has no doubts about the optimality of his preferred policy, conflicts are violent and bad decisions are sometimes made. Under some other conditions, the agent adopts dovish attitudes of systematic doubt and conflicts are less violent.

JEL Classification: D74, D64.
Keywords: Social Conflict, Cultural Transmission, Hawkish and Dovish Bias, Dogmatism, Doubts, Ideology, Altruism.

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“I believe that we can avoid violence only in so far as we practice this attitude of reasonableness in dealing with one other in social life. [This attitude] may be characterized by a remark like this: "I think I am right, but I may be wrong and you may be right.” [...] One of the main difficulties is that it always takes two to make a discussion reasonable.”

Karl Popper (1963, p. 357)

1. Introduction

It is hard to disagree with the view that many ideological conflicts are violent precisely because individuals, often against the evidence, negate that they “may be wrong and that [the opponent] may be right.”

The possibility of beliefs manipulation in situations of conflict has received little scrutiny by economists but has been amply documented by the psychological literature. Several studies point out that conflicts are exacerbated when individuals erroneously believe that the opponents’ interests are directly opposed to their own when, in fact, they might be compatible.\(^1\) A closely related bias is the hawkish bias (Kahneman and Renshon, 2009), which makes individuals see threats as more dreadful than reality would suggest. Such faulty perceptions are usually the result of distortions in the way individuals search and process information (Pinkley et al, 1995) as well as of propaganda campaigns by governments or other political groups.\(^2\)

The hawkish bias is not the only type of bias that is observed in situations of conflict. According to the psychological literature, individuals may sometimes underestimate, instead of overestimating, external threats. Such dovish attitudes are often entertained by minority groups while coping with oppression from socially or economically dominant groups.\(^3\) For instance, various studies argue that especially during slavery, African Americans learned that passivity towards the oppressors was a necessary survival strategy. Similarly, governments may resort to appeasement to defuse conflicts. The most well-known historical example is the appeasement strategy pursued by the British government towards Germany before World

\(^1\)Such bias is known in the psychological literature as fixed-pie perception, (Bazerman and Neale, 1983, and Thompson and Hastie, 2000).


\(^3\)See Lewin (1948) and, more recently, Jost and Thompson (2000). Ferenczi (1932) first studied the phenomenon of “identification with the aggressor” when facing an inescapable threat.
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War II— a strategy that was justified by the conviction that German territorial aspirations were partly legitimate as well as by an underestimation of the German threat.⁴

The goal of this paper is to investigate under what conditions the participants in a conflict adopt either dovish or hawkish attitudes. This subject matter should be of interest to economists for at least two reasons. First, individuals’ attitudes in a conflict affect the amount of wasted resources in the fight and, consequently, may have consequences on economic development (Collier et al., 2003). Second, to the extent that such attitudes stem from distortions in the way individuals process information, we expect them to be associated with bad policy decisions, which are obviously detrimental to welfare.

In this paper, we consider a game of conflict where two individuals (or two groups of individuals) fight in order to impose their preferred policy. A key feature of the model is that one participant in the conflict has state-dependent preferences over policy alternatives. In other terms, his preferred policy depends on the realized state of nature. We assume two possible states: in one state, the policies that maximize the utilities of both opponents are different, while in the other state they are identical (i.e., the opponents’ preferences are aligned). Crucially, we assume that the current state is not observable by the two opponents. This implies that the individual with state-dependent preferences cannot be ex-ante certain about the optimality of the policy that he is trying to impose. Ex ante, he has doubts: he entertains the possibility that the policy preferred by the opponent might be the “right” one.

Before entering the conflict the individual with state-dependent preferences naively relies on the information provided by an advisor (the principal) who shares his same preferences over policy alternatives. The principal (e.g., a parent or a political leader) is characterized by an altruism parameter, which measures the extent to which the principal internalizes the effort cost exerted by the agent in the conflict. The principal is assumed to be better (although not necessarily perfectly) informed than his agent about the current state. In particular, the principal receives one of two signals: one signal is perfectly informative about the current state (hence, it leaves no doubt about the policy that maximizes the agent’s utility), while the other is not. After learning the signal, the principal communicates with the agent by sending a public message.

⁴See Rock (2000) who discusses this and other more successful cases of appeasement, such as Great Britain’s resolution of territorial disputes with the US from 1896 to 1903.
Our results show that whether or not the principal is truthful depends on two key parameters. First, manipulation of information does not take place when the prior probability of being in a state where the opponents’ preferences are not aligned is sufficiently low. Since we expect that prior probability to be high in a heterogeneous society, this suggests that truth telling is more likely in homogenous societies. Second, we show that the principal is truthful when nature’s signals are more precise.

Whenever manipulation of information occurs, it takes two forms. Under some parameter values, the principal induces hawkish attitudes in the agent. That is, he removes any doubt that the agent may have had about the possibility that the opponent’s preferred policy is optimal for him. As a result, the hawkish agent strenuously fights. For other parameter values, we show that the principal induces dovish attitudes in his agent. That is, he always tells the agent that he has received the noisy signal. The principal does so even when the evidence that he has received indicates that the policy that the opponent would choose is certainly not optimal for the agent. As a result, the dovish agent exerts little effort because he entertains the possibility that the policy that the opponent is trying to impose may be optimal.

There are three considerations that determine the choice of the message by the principal. First, removing doubts induces the agent to exert higher effort. Not surprisingly, this motivating effect is more valuable to the principal, the lower is his altruism parameter. Second, due to a strategic interdependence in the game of conflict, instilling doubts in the agent decreases the average effort exerted by both opponents and reduces the conflict’s Pareto inefficiency. This effect is valuable because the two opponents cannot credibly commit to low effort levels. Instilling doubts is then a commitment device for the principal. In contrast to the motivating effect, we show that the moderating effect is more valuable to the principal, the higher is his altruism parameter. It is important to stress that the moderating effect does not arise because the principal over-internalizes the effort exerted by the agent. As will be shown, the moderating effect dominates even when the principal internalizes only half of the cost of effort exerted by his agent. Third, the principal also needs to make sure that the message induces the agent to select the correct policy in case of victory—at least in expectations.

In Sections 5 and 6 we modify two assumptions of the basic model. Section 5 relaxes
the assumption that the agent is naive. Instead, we suppose that the agent understands the principal’s incentive to manipulate information. We find that the message strategy when the agent is naive remains an equilibrium strategy when the agent is sophisticated. The sophisticated agent, however, does not update his prior in the regions of parameters in which the principal pools. Section 6 keeps the hypothesis of naiveté, but makes the assumption that the principal’s message is privately communicated. We show that private communication makes conflicts more violent.

In Section 7, we extend the basic model by supposing that the agent can acquire precise information if he conducts autonomous research. With some positive probability autonomous research is successful and the agent perfectly observes the current state of the world. The principal’s message affects the incentives of the agent to learn by himself. In particular, a hawkish message reduces the agent’s incentive to acquire information that may lead to a revision of his beliefs: in other terms, such message makes the agent more dogmatic. Among other findings, we show that the higher is the probability of successful research, the weaker are the principal’s incentives to induce hawkish attitudes. This suggests that societies (or groups) that have access to efficient ways of doing research (such as, well-supplied libraries, media and a good educational system) are less prone to hawkish attitudes.

The remainder of the paper is as follows. In Section 2, we analyze the related literature. Sections 3 presents the basic setup where the agent is naive and communication is public. The results of the basic model are presented in Section 4. Sections 5 and 6 modify the basic setting by assuming, respectively, a sophisticated agent and private communication. In Section 7 we suppose that the agent can obtain precise information on his own. Section 8 concludes. For ease of exposition, all proofs are in the Appendix.

2. Review of the Literature

First, the paper is related to a growing literature that studies the transmission of preferences, beliefs, and social norms (see the survey by Bisin and Verdier, 2010). In Bisin and Verdier (2000, 2001) cultural transmission is the result of interactions inside the family and in the population at large. When parents are able to influence the probability with which children inherit their parents’ preferences, they show that the distribution of cultural traits in the population converges to a heterogenous distribution. More recently, various papers have looked at intergenerational transmission of norms concerning fertility and female labor supply.
decisions (Fernandez and Fogli, 2009), of values favoring trust and cooperation (Tabellini, 2008a,b and Algan and Cahuc, 2010) and of preferences regarding patience and work ethic (Doepke and Zilibotti, 2006).

This paper is also related to recent literature that deals with various examples of distorted collective understanding of reality, such as anti and pro-redistribution ideologies (Bénabou, 2008, Bénabou and Tirole, 2006), over-optimism (and over-pessimism) about the value of existing cultural norms (Dessi, 2008), contagious exuberance in organizations (Bénabou, 2013), and no-trust-no-trade equilibria due to pessimistic beliefs about the trustworthiness of others (Guiso et al., 2008). In Bénabou (2008, 2013), the individuals themselves distort own processing of information. Here instead we consider a model of indoctrination where one opponent in the conflict receives (possibly manipulated) information from his principal. Contrary to Guiso et al. (2008), where parents can perfectly choose the beliefs of their children, indoctrination possibilities are more limited here because the principal can affect the agent’s beliefs only by misreporting the private signal that he has received. In Bénabou (2013) censorship and denial occur because individuals have anticipatory feelings. In our model the principal may decide to misreport the truth for a different set of reasons: to motivate his own agent (a similar motive is also present in Bénabou and Tirole, 2002, 2006) and also, due to strategic interdependence in the game of conflict, to affect the strategy of the opponent. Notice that the latter motive arises in our model also if the principal is perfectly altruistic.

Finally, we briefly review the vast literature on social conflict. Starting from the classic contributions by Grossman (1991) and Skaperdas (1992), the literature has developed theoretical models to study the determinants of social conflict. Recently, Caselli and Coleman (2013) and Esteban and Ray (2008a, 2008b) have focused on the role of ethnic divisions; Besley and Persson (2008a, 2008b) have investigated the economic determinants of social conflict.

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5 As discussed in Bénabou and Tirole (2006), a model of indoctrination is formally identical to a model where individuals with imperfect willpower distort the information they have received to affect their effort decision in the future.

6 See the pioneering paper of Akerlof and Dickens (1982), where beliefs affect agents’ utilities through anticipation of future payoffs. More recently, among others, see Caplin and Leahy (2001).

7 This is different from Carillo and Mariotti (2000) and Bénabou and Tirole (2002, 2006), where a necessary condition to have strategic ignorance or beliefs manipulation is to have disagreement between the multiples selves (that is, time-inconsistent preferences). See also the classic model of strategic information transmission of Crawford and Sobel (1982), where the sender has no incentive to misreport if he has the same utility of the receiver.

8 See the surveys of Blattman and Miguel (2010) and Jackson and Morelli (2011).
conflict, while Weingast (1997) and Bates (2008) have studied the importance of institutional constraints. It should be noticed that in most papers on the subject, the parties in the conflict fight over a given amount of resources (among the exceptions see Esteban and Ray, 2011). In contrast, we consider a conflict over an ideological dimension, which we expect to be more susceptible to beliefs’ manipulation. In Jackson and Morelli (2007), citizens may strategically delegate the leadership of their country to a more hawkish politician in order to extract more transfers from the other country. Baliga and Sjöström (2012) consider a model of conflict where each opponent has private information about his cost of waging war. In their model, an extremist group, who is able to observe the type of one opponent, may engage in various acts (such as, a terroristic attack) so as to affect the fighting strategies of both opponents. Finally, Anderlini et al. (2010) consider a dynastic game of conflict with private communication across generations and show that destructive wars can be sustained by a sequential equilibrium for some system of beliefs. However, their model is very different from ours along various dimensions. For example, in their setting communication is about past history, which has no direct effect on current payoffs, while in our model it concerns the current state of nature, which directly affects players’ payoffs.

3. The Basic Model

Consider a model with three players: A, B and P. Individuals A (he) and B (she) play a game of conflict. The winner of the conflict is able to impose his or her preferred policy to the loser. We let $x$ denote the policy, where $x \in X$. To streamline the analysis, $X$ includes only two alternatives: $X = \{a, b\}$. The model is sufficiently general to admit various interpretations. For example, it could describe a conflict between two political factions in order to decide the type of economic policy (government intervention vs. laissez faire) or the type of constitution (theocracy vs. secular democracy) to adopt in the country.

Individual A is associated to P. The role of P is to provide information to A prior to the beginning of the game of conflict. Individual P is assumed to be (more or less) altruistic towards A. Throughout this paper, we shall refer to P as the “principal” and to A as the “agent”. Depending on the specific application, the principal can be interpreted in different ways. In a model of intergenerational cultural transmission, we can view P as A’s parent. Alternatively, P could represent a political leader who is able to provide information to A through government controlled media. Finally, one could think of P and A as two multiple
selves that exist at different times within the same individual.

The utility of individual $i$, where $i = A, B$, is

$$U^i(c, x, \theta) = -c_i + u_i(x, \theta),$$

(1)

where $c_i$ is the cost of effort exerted in the conflict and $u_i(x, \theta)$ is a term that depends on policy $x$ and on the current state, denoted $\theta \in \Omega$.

There are only two possible states of the world: $\Omega = \{\theta_a, \theta_b\}$. The state is randomly drawn by nature. In state $\theta_b$ the preferences of $A$ and $B$ are aligned: the policy that maximizes the utility of both individuals is $b$. In state $\theta_a$ we assume instead that individuals disagree on the correct policy to implement: $A$’s preferred policy is $a$, while $B$’s preferred policy is $b$. Throughout the paper we will denote $\theta_b$ as the state of alignment and $\theta_a$ as the state of conflict. The assumption that individuals with different views may sometimes agree seems plausible. For example, in particular conditions an individual who usually supports free-market policies may agree with a left-wing individual about the opportunity of government intervention.

The following matrix summarizes the preferred policies by each individual in each state:

<table>
<thead>
<tr>
<th></th>
<th>$A$’s optimal policy</th>
<th>$B$’s optimal policy</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_b$</td>
<td>$b$</td>
<td>$b$</td>
</tr>
<tr>
<td>$\theta_a$</td>
<td>$a$</td>
<td>$b$</td>
</tr>
</tbody>
</table>

For simplicity, the term $u_i(x, \theta)$ is either zero or one: it is equal to one if the appropriate policy for individual $i$ in state $\theta$ is selected, and zero otherwise. More formally,

$$u_A(b, \theta_b) = u_B(b, \theta_b) = u_A(a, \theta_a) = u_B(b, \theta_a) = 1,$$

$$u_A(a, \theta_b) = u_B(a, \theta_a) = u_A(b, \theta_a) = u_B(a, \theta_b) = 0.$$

We assume incomplete information about the current state of the world. Note that individual $B$, unlike $A$, does not need to know the current state in order to decide which policy to adopt in case of victory. In fact, he has no doubt that $b$ is the appropriate policy. On
the contrary, $A$ needs to know the current state of nature in order to determine which is the appropriate policy to adopt.

As mentioned above, $P$ is assumed to be (more or less) altruistic towards $A$. His utility is

$$U^P(c_A, x, \theta) = -\phi c_A + u_A(x, \theta).$$

Let $0 \leq \phi \leq 1$. When $\phi = 1$, the utility of $P$ coincides with the one of $A$. When $\phi < 1$, the principal is not fully altruistic vis-à-vis his agent: $P$ does not fully internalize the cost of effort exerted by $A$. However, it is important to notice that the principal does not disagree with his agent on the right policy to adopt in each state $\theta$.

The prior probability that all players assign to the state of conflict $\theta_a$ is denoted by $\rho$. We will assume that $\rho \in (1/2, 1)$: that is, the two individuals are (ex-ante) more likely to be in a state of conflict than in a state of alignment. To some extent, $\rho$ can be viewed as a measure of ex-ante societal heterogeneity. In fact, we expect that two randomly selected individuals from a heterogenous society are likely to disagree on various issues; consequently, we expect that the prior $\rho$ will be high.

3.1. Timing and Information Structure

There are three periods: $t = 0, 1, 2$. No discounting is assumed. At $t = 0$, information transmission from $P$ to $A$ takes place. At $t = 1$, $A$ and $B$ play a game of conflict. At $t = 2$, the winner decides the policy. We now discuss each stage in detail.

At $t = 0$, $P$ privately observes a signal $s \in \{\alpha, \beta\}$ which is (not necessarily fully) informative about the current state $\theta$. Signal $\alpha$ (resp. signal $\beta$) increases the probability assessment of being in state $\theta_a$ (resp. $\theta_b$). We assume that signal $\alpha$ is perfectly informative and leaves no doubt that the state is $\theta_a$. Signal $\beta$ is noisy and indicates that the state may not be $\theta_a$. In other terms, $\beta$ makes the principal doubt about the optimality of policy $a$.\footnote{The thrust of most of the results would not change with a more symmetric information structure. What is important is that one signal goes against the prior (and fosters doubts about the optimality of policy $a$), while the other signal reinforces the belief that $a$ is optimal. The assumption that $\alpha$ is perfectly informative, however, simplifies the algebra. Without this assumption, the game of conflict would never be symmetric: $B$ would always be the individual with the highest stakes in the conflict.}
The conditional probabilities of receiving signals $\alpha$ and $\beta$ in state $\theta_b$ are

$$Pr(\alpha \mid \theta_b) = 0 \quad \text{and} \quad Pr(\beta \mid \theta_b) = 1.$$  

(3)

That is, signal $\alpha$ is never received if we are in state of alignment $\theta_b$.

In state $\theta_a$, the conditional probabilities are

$$Pr(\alpha \mid \theta_a) = \gamma \quad \text{and} \quad Pr(\beta \mid \theta_a) = 1 - \gamma,$$  

(4)

where $\gamma \in (0, 1)$.

Let $\rho^s_P$ denote $P$’s posterior probability that the state is $\theta_a$ after signal $s$. Principal $P$ updates his prior according to Bayes’ Rule:

$$\rho^\beta_P = \frac{\rho (1 - \gamma)}{1 - \rho \gamma} \leq \rho,$$  

(5)

$$\rho^\alpha_P = 1.$$  

(6)

The parameter $\gamma$ can be viewed as a measure of the precision of nature’s signals. When $\gamma = 0$ the principal’s posterior after $\beta$ coincides with his initial prior $\rho$. As $\gamma$ goes to one signals become more informative.

After receiving a signal from nature, $P$ sends a message $m$, where $m \in \{\alpha, \beta\}$.10 The posterior belief of player $A$ after message $m$ is denoted by $\rho^m_A$.

The principal’s message is assumed to be public. Whether or not assuming public communication is appropriate depends on the specific situation the model addresses. One instance in which our assumption is more fitting is when we interpret the message strategy as inter-generational cultural transmission or political propaganda. Since education and political persuasion are likely to be continual and long-lasting processes, they can be more easily observed. Section 6 considers the case in which the message by $P$ is privately observed by $A$.

An important assumption of the basic model is that $A$ is naive: $A$ believes the signal that

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10 Notice that the principal cannot fabricate new evidence: the message space and the signal space coincide. A similar assumption is also made in Bénabou and Tirole (2006), Bénabou (2008, 2013), and Dessi (2008).
$P$ sends. In other words, $A$ does not realize that the principal may not always tell the truth. 

We also suppose that the naiveté of $A$ is known to $B$ and to $P$. Upon receiving message $m$, $A$’s posterior is equal to (5) when $m = \beta$ and is equal to (6) when $m = \alpha$. Complete naiveté is also allowed, as a special case, in Bénabou and Tirole (2006, p. 710). Similarly, in models of cultural transmission it is assumed that parents can easily manipulate their children. For instance, in Doepke and Zilibotti (2006) and Guiso et al. (2008) parents can directly choose the preferences or the priors of their children. In our model, the ability of the principal to manipulate the agent’s beliefs is weaker than in those papers. Recall in fact that here the principal cannot fabricate new evidence. As a result, the principal cannot perfectly determine the beliefs of the agent. In Section 5 we will analyze the case of a sophisticated agent. 

Naiveté can be partly justified on the basis of various experimental and behavioral evidence suggesting that individuals who rely on the advise of others do not fully take into account the incentives of the information provider. For instance, Malmendier and Shantikumar (2007) find that small investors follow recommendations by analysts literally and do not discount the bias due to analysts’ affiliation. Della Vigna and Kaplan (2007) argue that Fox News viewers underestimate the bias of the media source and therefore are subject to persuasion. Cai and Wang (2006) test in a controlled laboratory experiment the model by Crawford and Sobel (1982) and find that receivers rely more on the senders’ message compared with what the theory predicts.\footnote{Cain et al. (2005) conduct an experiment where individuals must guess the number of coins in a jar by relying on the advice of experts who can inspect the jar. Even when it is common knowledge that experts are paid for how high the subjects’ guess is, they find that individuals do not discount enough to compensate for the experts’ incentive to exaggerate their advice.}

In some contexts naiveté seems a more natural assumption than full sophistication. For instance, we expect individuals to be especially naive when $P$ coincides with a national government or a parent. In countries where education (at school and home) is hierarchical and children are not taught to think independently, individuals may be induced to naively trust the messages sent by their government and parents.\footnote{Using data from the world value survey, Algan and Cahuc (2005) find that when asked what are the values that children should be taught, there is heterogeneity across countries in the way respondents value promotion of child independence.} In Section 5 we will show that the principal prefers to deal with a naive agent than with a sophisticated one. He is therefore likely to choose to interact with a naive agent or, whenever possible, he is likely to teach the
agent to be naive.

3.2. Game of Conflict

At $t = 1$, we posit the following game of conflict. Individuals $A$ and $B$ simultaneously choose effort levels $c_A$ and $c_B$, where $c_A, c_B \geq 0$. The probability of $i$ winning the contest given the effort decisions of the two opponents is

$$p_i(c_i, c_{-i}) = \begin{cases} 
0 & \text{if } c_i < c_{-i}, \\
1 & \text{if } c_i > c_{-i}, \\
\frac{1}{2} & \text{if } c_i = c_{-i}.
\end{cases}$$

In words, the individual that exerts the highest effort wins with probability one. This technology of conflict, which is extremely sensitive to effort differences, turns out to be analytically tractable for our purposes.\(^\text{13}\)

Finally, at $t = 2$, the winner of the conflict is able to pick his or her preferred policy.

4. Results

In each period, players maximize their expected utility given their beliefs at that stage and given the strategies of the other players. For the principal, a strategy specifies a message for every signal $s$. For $i = A, B$, the effort and the decision strategies specify the effort in the game of conflict and the policy decision in case of victory for every message, respectively.

The model is solved by backward induction, starting from the last period.

4.1. Policy Decisions

At $t = 2$, the decision rule of individual $B$ in case she wins the conflict is immediate: $B$ chooses $b$.

Conversely, $A$ picks $a$ only if his posterior probability of being in a state of conflict is greater than $1/2$, which constitutes the threshold of indifference between the two policy

\(^{13}\)In the social conflict literature, this technology of war is considered, for instance, by Jackson and Morelli (2007, ex. 3). This type of contest, known in the literature as all-pay auction, has also been considered by the lobbying and rent-seeking literature: e.g., Ellingsen (1991), Baye et al. (1993), and Che and Gale (1998). For a survey of other technologies of conflict, see Garfinkel and Skaperdas (2007).
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decisions. Let $D_A$ denote the decision made by $A$ in case of victory:

$$D_A = \begin{cases} 
a & \text{if } \rho^m_A > 1/2, \\
b & \text{if } \rho^m_A \leq 1/2. 
\end{cases}$$

(8)

4.2. The Game of Conflict

At the beginning of $t = 1$, both $A$ and $B$ observe the message $m$ sent by $P$. Individual $B$ knows that $A$ is naive. Consequently, she is able to figure out $\rho^m_A$.

We now determine the effort decisions at $t = 1$. The type of conflict described by (7) is equivalent to an all-pay auction. Note in fact that $A$ and $B$ incur an effort cost that is the same whether they win or lose. The gain from winning is given by the possibility of choosing the most-preferred policy. This possibility is more or less valuable to $A$ depending on his beliefs. More specifically, the gain for $A$ is $(2\rho^m_A - 1)$. This value is obtained by subtracting $1 - \rho^m_A$ (the expected payoff in case $B$ wins) from $\rho^m_A$ (the expected payoff in case $a$ is implemented).

It is well known that the game of conflict analyzed here does not have a Nash equilibrium in pure strategies, but does have a unique equilibrium in continuous mixed strategies (see Hillman and Riley, 1988). To find out the equilibrium effort levels, two cases must be considered: $\rho^m_A \leq 1/2$ and $\rho^m_A > 1/2$. First, suppose that parameters are such that $A$ believes that the current state is more likely to be $\theta_b$ than $\theta_a$. When $\rho^m_A \leq 1/2$, it is immediate from (8) that $A$ has no incentive to fight. Then, $c_A = c_B = 0$ and $b$ is chosen. The second possibility is that $\rho^m_A > 1/2$. In this case, from (8) we obtain that $A$ and $B$ want to pursue different policies: a conflict is then inevitable. Let $G_i(.)$ denote the equilibrium cumulative distribution of individual $i$’s effort. The expected payoff to $A$ from exerting effort $c_A$ is

$$EU^A = [1 - G_B(c_A)](1 - \rho^m_A) + G_B(c_A)\rho^m_A - c_A.$$  

(9)

To obtain (9) note that with probability $G_B(c_A)$ individual $A$ wins and implements policy $a$, which gives $A$ an expected payoff equal to $\rho^m_A$. With complementary probability, $B$ wins and implements $b$, which gives $A$ an expected payoff equal to $1 - \rho^m_A$. We can rewrite (9) as

$$EU^A = (1 - \rho^m_A) + G_B(c_A)(2\rho^m_A - 1) - c_A.$$  

(10)
From expression (10) it is immediate to verify that \( A \) never exerts an effort level strictly greater than his value of winning, which is given by \( 2\rho_A^m - 1 \). Further, note that \( A \)'s maximum effort level goes to zero when \( \rho_A^m \) goes to \( 1/2 \). Intuitively, when the two states become equally likely, \( A \) has no incentive to enter into a conflict.

Note that \( B \)'s valuation is 1, which is weakly greater than \( A \)'s valuation. This is because \( B \) has no doubt that \( b \) is the right policy. The expected payoff to \( B \) of choosing \( c_B \) is instead

\[
EU^B = G_A(c_B) - c_B. \tag{11}
\]

The equilibrium of the game of conflict is characterized by the following proposition. The proof, which is contained in the Appendix, follows Hillman and Riley (1988).

**PROPOSITION 1:** Let message \( m \) be given. If \( 0 \leq \rho_A^m \leq 1/2 \), we have \( c_A = c_B = 0 \) and policy \( b \) is selected.

If instead \( 1/2 < \rho_A^m \leq 1 \), in the unique Nash equilibrium, \( B \) randomizes his effort uniformly on \( [0, 2\rho_A^m - 1] \). Player \( A \) exerts zero effort with probability equal to \( 2(1 - \rho_A^m) \).

Conditional upon exerting positive effort, \( A \) also randomizes uniformly on \( [0, 2\rho_A^m - 1] \).

Proposition 1 establishes that in case of conflict the maximum effort level of both individuals is given by \( 2\rho_A^m - 1 \), the valuation of the lower-valuing individual. Moreover, it states that individual \( A \) exerts zero effort with strictly positive probability, which is increasing in his degree of doubt. In contrast, individual \( B \) (the higher-valuing individual) always enters the conflict.

It follows from Proposition 1 that when \( \rho_A^m = 1 \), the conflict is total: both players enter the conflict with probability one and effort is distributed uniformly on the interval \([0, 1]\).

It is important to notice that the principal’s message affects the effort levels by both opponents in the conflict. Appealing to Proposition 1, for every message \( m \) we can compute the expected sum of effort levels of the two opponents as of time 1:

\[
E(c_A + c_B, m) = (2\rho_A^m - 1)\rho_A^m \tag{12}
\]

It is crucial to observe that (12) is increasing in \( \rho_A^m \): \( A \)'s doubts contain the escalation of violence in the conflict.
4.3. Message Strategies

Going backwards, we analyze the information transmission game at $t = 0$. This is immediate to solve given the simple structure with a binary state of the world and binary signals. Intentionally, we kept the setting as tractable as possible. In fact, our interest here is not to contribute to the information transmission literature but to establish conditions of economic nature under which beliefs are manipulated.

Depending on the underlying parameters (namely, $\phi$, $\gamma$ and the initial prior of being in state $\theta_a$) we will show (see Propositions 2 and 3) that the principal uses one of three message strategies. First, there exists a region of parameter values where the principal reports nature’s signal in a *truthful* manner. Second, for other parameters we obtain that $P$ always sends message $\alpha$ regardless of the actual signal received from nature. In this case, we say that $P$ induces a *hawkish attitude* in his agent. Finally, there exists a third region of parameters where $P$ always sends message $\beta$ regardless of the actual signal. In this other case, we say that $P$ induces a *dovish attitude* of systematic doubt.

To solve for the equilibrium strategies, we compute the payoffs to $P$ for every message and for every signal. Given that we consider a model with two signals and two messages, four cases must be considered. We let $V^P(s, m)$ denote the expected utility of the principal after receiving signal $s$ and after sending message $m$.

First, suppose that nature sends signal $\alpha$ and $P$ is truthful. In this case, $A$ and $B$ play a total war and $P$’s expected payoff is

$$V^P(\alpha, \alpha) = -\frac{\phi}{2} + \frac{1}{2}.$$  

(13)

To explain the first term of (13), recall that in a total conflict the expected effort exerted by $A$ is equal to $1/2$ (see Proposition 1) and that $P$ internalizes only a proportion $\phi$ of the agent’s effort. To explain the second term, note that a total conflict is symmetric and both players win with equal probabilities. Since the actual state is $\theta_a$, $P$ earns a payoff equal to one if $A$ wins and zero if $B$ wins.

Second, suppose nature sends signal $\alpha$ but $P$ sends the false message $\beta$. In this case, $A$ is induced to doubt. Two sub-cases must be considered. If the agents’s posterior after the message is sufficiently low (namely, $\rho_A^\beta \leq 1/2$) we know from Proposition 1 that $A$ does not
participate in the conflict, \( b \) is chosen and consequently the principal obtains a payoff equal to

\[
V^P(\alpha, \beta) = 0, \quad \text{if } \rho_A^\beta \leq 1/2
\]  

(14)

If instead \( \rho_A^\beta > 1/2 \) we have that \( A \) enters the conflict with probability \( 2\rho_A^\beta - 1 \leq 1. \)

Conditional on \( A \) exerting positive effort, we know from Proposition 1 that \( A \)'s expected effort is

\[
\frac{2\rho_A^\beta - 1}{2}
\]  

(15)

and that both players have the same probabilities of winning. Therefore, when the current state is \( \theta_a \), making \( A \) doubt gives the principal an expected payoff

\[
V^P(\alpha, \beta) = \left(2\rho_A^\beta - 1\right) \frac{-\phi\left(2\rho_A^\beta - 1\right)}{2} + \frac{\left(2\rho_A^\beta - 1\right)}{2}, \quad \text{if } \rho_A^\beta > 1/2
\]  

(16)

Third, suppose that nature sends signal \( \beta \) and that \( P \) is truthful. Two sub-cases must again be discussed. When \( \rho_A^\beta \leq 1/2 \), there is no conflict, \( b \) is chosen and, consequently, the principal’s expected payoff is

\[
V^P(\beta, \beta) = 1 - \rho_P^\beta, \quad \text{if } \rho_A^\beta \leq 1/2.
\]  

(17)

If instead \( \rho_A^\beta > 1/2 \) a conflict occurs and the principal’s expected payoff is

\[
V^P(\beta, \beta) = \left(2\rho_A^\beta - 1\right) \frac{-\phi\left(2\rho_A^\beta - 1\right)}{2} + \frac{2\rho_A^\beta - 1}{2} + 2 \left(1 - \rho_A^\beta\right) \left(1 - \rho_P^\beta\right), \quad \text{if } \rho_A^\beta > 1/2.
\]  

(18)

To understand why (18) has an additional term compared to (16), notice that \( P \) now expects to obtain a positive payoff when \( A \) does not fight: since the principal has received signal \( \beta \), his probability assessment of being in state \( \theta_b \) is not zero, but \( 1 - \rho_A^\beta \). Since (18) is computed under the assumption that \( P \) is truthful, we have \( \rho_P^\beta = \rho_A^\beta \).

Fourth, and finally, suppose \( P \) receives signal \( \beta \) but sends the false message \( \alpha \). Then, a total conflict arises. When \( A \) wins (an event occurring with probability \( 1/2 \)), \( a \) is chosen and the principal expects a payoff of \( \rho_P^\beta \). With complementary probability \( B \) wins, \( b \) is chosen and the principal expects a payoff of \( 1 - \rho_P^\beta \). Therefore, the expected payoff to the principal
is
\[ V^P(\beta, \alpha) = -\frac{\phi}{2} + \frac{1}{2}. \] (19)

After computing the payoffs (13)-(19), it is a matter of simple algebra to determine the equilibrium message strategies for all \( \phi \).

Before solving the model, we discuss the three considerations that matter in the message decision. First, \( P \) has an incentive to remove \( A \)'s doubts about the possibility that \( B \) may be right in order to increase \( A \)'s effort in the conflict. This *motivating effect* is present in our model because the principal does not fully internalize the cost of effort of \( A \). Second, \( P \) may want to instill doubts in \( A \) to reduce the inefficiency of the game of conflict. To understand this *moderating effect*, recall from Proposition 1 that if \( A \) has more doubts, conflicts are less violent because the equilibrium effort levels of both players decrease: due to strategic complementarities, instilling doubts moderates the escalation of violence. This effect is valuable because effort is wasteful and because the two opponents cannot credibly commit to low effort levels. Instilling doubts is then a *commitment* device for the principal. Using this device, however, is costly. In fact, when the state is \( \theta_a \), instilling doubts induces \( A \) to exit the conflict with positive probability. This implies that policy \( b \), which is suboptimal for \( A \) in state \( \theta_a \), is more often implemented.\(^{14}\) Third, the principal also needs to make sure that his message induces \( A \) to select the right policy in case of victory–at least in expectations.

The trade-off between motivating and moderating effects depends, among other things, on \( \phi \). Consider, for instance, a principal with high \( \phi \). The motivating effect is not very valuable to him because from equations (13) and (19) we know that his expected payoff from a total conflict is close to zero. Therefore, a principal with high \( \phi \) would rather reduce the conflict’s inefficiency than maximize the probability that \( A \) exerts positive effort. The converse holds true for a principal with low \( \phi \): his expected payoff from a total conflict is sufficiently large that he always prefers to maximize the probability that \( A \) enters the conflict, even at the cost of inducing a total conflict. This is why we may observe dovish (resp. hawkish) attitudes when \( \phi \) is high (resp. low). It is important to notice that the moderating effect does not arise because the principal over-internalizes the effort exerted by the agent (throughout \( \phi \) is

\(^{14}\)The two effects can be appreciated by looking at the expected payoffs (16) and (13). Message \( \beta \) when the signal is \( \alpha \) reduces the agent’s expected effort– the first term of (16) is higher than the first term of (13)–but increases the probability that the correct policy \( a \) is not implemented–the second term of (16) is lower than the first term of (13).
assumed to be less than one). As shown below, the moderating effect dominates even when the principal internalizes only half of the effort cost.

It is possible to show that when $\rho$ is sufficiently low, the principal is truthful because sending false messages would induce $A$ to select the wrong policy at $t = 2$.

**Lemma 1:** When $\rho \leq 1/(2 - \gamma)$ the principal is truthful for any $\phi$.

To understand Lemma 1, suppose that $\rho$ is just above $1/2$. Note that after receiving signal $\beta$, the principal’s posterior $\rho_P^\beta$ would fall below $1/2$: the principal would change his view about the optimality of $a$ and start to believe that $b$ is the correct decision. Then, $P$ has no incentive to send message $\alpha$, which would induce $A$ to enter a total conflict with the goal of imposing policy $a$. Similarly, after receiving signal $\alpha$ the principal has no incentive to send the false message $\beta$ since $A$ would incorrectly believe that $b$ is the right decision and would give up the fight and earn a payoff of zero. As will be shown in Propositions 2 and 3 below, when $\phi$ is either zero or one, the condition of Lemma 1 is also a necessary condition for truth-telling.

Next, in Lemma 2 we discuss the strategy of a fully altruistic principal.

**Lemma 2:** Suppose that $\phi = 1$. When $\rho > 1/(2 - \gamma)$ the principal always sends message $\beta$.

To understand Lemma 2, note from (13) and (19) that the expected payoff to the principal of a total war is zero when $\phi = 1$. As a result, the motivating effect is absent: the principal has no interest in pushing the agent to a total fight. On the contrary, the moderating effect is present: inducing dovish attitudes is a way to credibly commit the agent to exert low (but positive) effort. Provided that the prior is sufficiently high, always sending signal $\beta$ allows the principal to de-escalate the conflict without inducing $A$ to support policy $b$.

As we decrease $\phi$, the motivating effect starts to operate. In Proposition 2, we argue that as long as $1/2 \leq \phi \leq 1$ the moderating effect still dominates and dovish attitudes are sometimes observed. Instead, when $\phi < 1/2$ the motivating effect dominates the moderating one and hawkish attitudes are sometimes observed (see Proposition 3).
PROPOSITION 2: (Dovish Bias) Fix any $\gamma \in (0, 1)$ and suppose that $1/2 \leq \phi \leq 1$. For all $\rho \leq \bar{\rho}$, where

$$\bar{\rho} = \frac{1}{2\phi(1-\gamma) + \gamma},$$

information transmission is truthful. When instead $\rho > \bar{\rho}$, the principal $P$ reports $\beta$ regardless of nature’s signal.

PROPOSITION 3: (Hawkish Bias) Fix any $\gamma \in (0, 1)$ and suppose that $0 \leq \phi < 1/2$. For all $\rho \leq \hat{\rho}$, where

$$\hat{\rho} = \frac{1}{2(1-\phi)(1-\gamma) + \gamma},$$

information transmission is truthful. When instead $\rho > \hat{\rho}$, the principal $P$ reports $\alpha$ regardless of nature’s signal.

Figure 1: Beliefs Manipulation in the $(\rho, \phi)$ space with $\gamma = 0.6$

In Figure 1, for a given $\gamma$, we draw the parameter regions in the $(\rho, \phi)$ space where we observe the three types of equilibria of our model: dovish, hawkish and truthful. As stated in Propositions 2 and 3, $P$ sends truthful reports when $\rho$ is sufficiently low. When instead $\rho$ is large, the agent holds either hawkish attitudes (in the lower-right region) or dovish attitudes (in the upper-right region). When $\rho$ is high, truthtelling is less likely because the principal
can affect the effort of the agent (by either motivating or moderating him) without distorting the agent’s decision in case of victory.

Interestingly, notice that when $\phi \approx 1/2$ the conditions for the existence of a truthful equilibrium are more likely to be satisfied. In this range, in fact, the principal’s altruism parameter is not too low (so that inequality (18) $\geq$ (19) is satisfied) but not too high (so that inequality (13) $\geq$ (16) is also satisfied). In words, when $\phi$ is closed to $1/2$, $P$ is sufficiently altruistic to avoid a total conflict when nature’s signal is $\beta$ but not too altruistic to prevent a total conflict when nature’s signal is $\alpha$.

If nature’s signals become more precise (i.e., $\gamma$ increases), it is easy to verify that both cutoffs $\hat{\rho}$ and $\hat{p}$ increase: both the dovish and hawkish regions shrink. More precision reduces the incentives to manipulate beliefs. Graphically, this can be appreciated by comparing Figure 1 (where $\gamma$ is fixed at 0.6) and Figure 2 (where $\gamma$ has been increased to 0.85). When $\gamma$ is high, after a false message the posteriors of the principal and of the agent would likely lie on different sides of $1/2$, the threshold of indifference discussed in Section 4.1. In this case, the principal tells the truth in order to avoid wrong policy decisions. In the limit, when both signals become perfectly informative ($\gamma = 1$), the principal is truthful for all parameter values.

![Figure 2: Beliefs Manipulation in the $(\rho, \phi)$ space with $\gamma = 0.85$](image_url)
4.4. Incidence and Intensity of Conflicts

Using the results of Propositions 1-3, we now investigate how the degree of societal heterogeneity affects the likelihood that a conflict occurs (or incidence of conflict) and the total effort levels exerted in the conflict. In Figure 3 we summarize the implications of each message strategy on conflict behavior. The vertical dashed line is drawn at $\rho = 1/(2 - \gamma)$.

We now state Proposition 4.

PROPOSITION 4: The incidence of conflict is increasing in $\rho$.

The intensity of conflict is weakly increasing in $\rho$ when $\phi < 1/2$ and non-monotone in $\rho$ when $\phi \geq 1/2$.

To understand the first part of the proposition, notice that when $\rho$ is below the vertical dashed line in Figure 3, we know from Lemma 1 that $P$ is truthful. In this case, conflicts occur only when the principal truthfully sends $\alpha$.\(^\text{15}\) When instead $\rho$ is above the dashed line, $\rho^A_\alpha > 1/2$ for all $m$: conflicts always occur, regardless of nature’s signal. Since $\rho$ is likely to be high in heterogeneous societies, this suggests that the probability that a conflict occurs is lower in homogenous societies.

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\(^\text{15}\)When instead the principal truthfully sends message $\beta$, the agent’s posterior falls below 1/2. In this case, player $A$ favors $b$ and no conflict occurs.
The second part of Proposition 4 establishes that when \( \phi \geq 1/2 \) the intensity of conflict may not be monotone in \( \rho \). The latter result occurs because, as described in Proposition 2, in more divided societies individuals may adopt dovish attitudes. This generates a discontinuous drop of the overall effort levels when \( \rho \) is equal to \( \bar{\rho} \).

It is beyond the scope of this paper to test the theoretical predictions of our stylized model. However, it is possible to relate our results to some of the findings obtained by the empirical literature on civil and interstate conflicts. One result that emerges from that literature is that the incidence of conflict is positively correlated with ethnic polarization (Montalvo and Reynal-Querol, 2005). To the extent that ethnic polarization is a good proxy for \( \rho \), this result is coherent with the first part of Proposition 4. Second, various papers have looked at the relation between ethnicity diversity and civil wars’ duration. The relation found in the data is either positive (see Montalvo and Reynal-Querol, 2010) or not monotone. For instance, Collier et al. (2004) show that the duration of a conflict is at its maximum for intermediate values of ethnic fractionalization. While not perfectly, conflict duration is likely to be related to its intensity. Therefore, the latter empirical results are not in contradiction with the second part of Proposition 4.\textsuperscript{16}

5. Naiveté vs Sophistication

In this section, we consider the basic model of Section 3 but remove the assumption that the agent is naive. Instead, the agent is assumed to be sophisticated, i.e. Bayesian. We obtain that a truthtelling equilibrium exists in the same region of parameters described in Figure 1. However, in this region there is another equilibrium in pure communication strategies, where players \( A \) and \( B \) simply ignore the principal’s message: a babbling equilibrium, which is a common feature of all cheap talk games. In this equilibrium, \( A \)'s probability assessment of being in state \( \theta_a \) coincides with his prior.

In the lower-right and upper-right regions described in Figure 1, the babbling equilibrium is the unique equilibrium in pure communication strategies. We summarize these observations in the following proposition.

\textsuperscript{16}Finding an empirical proxy of \( \phi \) is more challenging. One could argue that leaders of full democracies have higher \( \phi \) (on this, see Jackson and Morelli, 2007). In our model, conflict’s intensity is lowest when \( \phi \) is close to one (see Figure 3). Indeed, there is evidence that full democracies fight less (e.g., Maoz and Russett, 1993).
PROPOSITION 5: (Sophisticated Agent, Equilibria in Pure Communication Strategies)

Assume that the agent is sophisticated. A truthtelling equilibrium exists for the same parameters as in the case where the agent is assumed to be naive. In addition, for all parameters a babbling equilibrium exists, where both players A and B ignore the principal’s message. In this equilibrium, A and B play the conflict phase as in Proposition 1, with posterior \( \rho_A^m = \rho \) for all \( m \in \{\alpha, \beta\} \).

Proposition 5 states that the message strategy when the agent is naive, which we described in Propositions 2 and 3, remains an equilibrium strategy when the agent is sophisticated. Moreover, the information transmitted is the same in the two cases, for all parameters. The main difference between the two cases is that the sophisticated agent does not update his prior in the regions of parameters in which the principal pools.\(^{17}\)

Another implication of Proposition 5 is that under full sophistication, the likelihood of conflict is still increasing in \( \rho \), while the intensity of conflict is increasing in the prior for all \( \phi \). In particular, at the thresholds \( \hat{\rho} \) and \( \bar{\rho} \) (as soon as truthful communication is not possible) total effort goes up.

It is possible to compare the expected payoff of the principal under various outcomes. Let \( W^T, W^H, W^D \), and \( W^B \), denote, respectively, the expected payoff of the principal under a truthful strategy (whether the agent is naive or sophisticated), a hawkish strategy (with a naive agent), a dovish strategy (with a naive agent), and a babbling strategy (with a sophisticated agent). By revealed preferences, we know that in the truthful parameter region we have \( W^T \geq \max\{W^D, W^H\} \), in the hawkish parameter region we have \( W^H \geq \max\{W^T, W^D\} \) and in the dovish parameter region we have \( W^D \geq \max\{W^T, W^H\} \). Moreover the inequalities hold strictly for parameters away from the boundaries between regions.

In order to compare \( W^T \) and \( W^B \), it is convenient to introduce the expected value \( U(\rho_P) \) of the principal when his belief is \( \rho_P \) under the assumption that he publicly shares his infor-

\(^{17}\)It is equally easy to see that Proposition 2 and 3 also describe the message of the principal in the following intermediate case between full sophistication and naiveté. Suppose that the principal faces a continuum of agents, where \( \nu \) and \( 1 - \nu \) are the measures of naive and sophisticated agents. At \( t = 1 \) a conflict arises between a randomly selected agent and the opponent. If the type of the agent is observed by the opponent, the message strategy of \( P \) is the one described in Propositions 2-3. Moreover, to the extent that \( \nu \) is strictly positive, we obtain that the expected intensity of a conflict is higher when \( \phi \approx 0 \) than when \( \phi \approx 1 \), exactly as in the basic model.
information with the other two players. We have

\[
U(\rho_P) = \begin{cases} 
\frac{1}{2} (2\rho_P - 1) + 2 (1 - \rho_P)(1 - \rho_P) - \frac{\phi}{2} (2\rho_P - 1)^2 & \text{if } \rho_P \geq 1/2 \\
1 - \rho_P & \text{if } \rho_P < 1/2.
\end{cases}
\]

In the case where \( \rho_P \geq 1/2 \), the first term of the expression is the return obtained when the agent exerts effort, the second one is the return obtained when the agent exerts no effort and the last term is the cost to the principal of the agent’s effort. The function \( U \) is strictly convex in \( \rho_P \) if \( \phi < 1 \) and linear in \( \rho_P \) if \( \phi = 1 \). This implies that the principal always prefers ex ante to release more public information.\(^{18}\) Thus, the following inequality holds: for any parameter, we have \( W^T \geq W^B \). Moreover the inequality is strict as long as \( \phi < 1 \), \( \rho_P \in (0, 1) \) and \( \gamma \in (0, 1) \).

Collecting these observations, we obtain the following result.

**PROPOSITION 6:** (Welfare Comparisons)

1. For all parameters, the principal prefers the truthful outcome to the babbling outcome.

2. In both the hawkish and the dovish regions, the principal prefers the agent to be naive rather than sophisticated.

Point 2 in Proposition 6 provides some justification for our assumption that the agent is naive: the principal prefers to deal with a naive agent than with a sophisticated one. He is therefore likely to choose to interact with a naive agent or he is likely to teach the agent to be naive.

6. Private Communication

In this section, we relax the assumption that communication is public. We assume instead that communication is private, in the sense that the principal’s message \( m \) is observed only by the agent \( A \), but not by \( B \).

We study how private communication affects the three different types of equilibria we previously identified: dovish, hawkish and truthful. We obtain that: (i) there are no dovish

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\(^{18}\)This fact is exploited in particular by Kamenica and Gentzkow (2011).
equilibria, for any value of the parameters; (ii) the parameter region for which hawkish equilibria exist expands. These findings indicate that when communication is private, conflicts are more violent.

To help understand the intuition behind these results, two observations are in order.

First, conditionally on the principal playing either a dovish or a hawkish strategy, the equilibrium conditions of players A and B in the conflict stage are exactly the same as in the public communication case. As a result, the equilibrium effort choices are determined as in Proposition 1. This is not true for a truthful equilibrium. In the private case, in a truthful equilibrium, the agent A receives information from P that B does not possess. As a result, in the conflict stage the information of the two opponents is asymmetric in favor of A in the private communication case, whereas it is symmetric in the public communication case.

Second, for all three types of equilibria, the principal’s incentive to deviate to other messages is not the same in the public and private cases. In the public case, any deviation from either a dovish or hawkish strategy is observed by both players A and B. In the private case, the deviation is observed only by the agent A.

From these two observations, we can easily deduce our first result in Proposition 7, which says that in the private case, there is never a dovish equilibrium for any parameter value. Indeed, suppose a hypothetic dovish equilibrium. Since A is expected to exert low effort, B is also relatively less aggressive in the conflict. Consider a deviation by the principal to a truthful strategy. By the second observation, the effort level of player B is unchanged. For agent A, the information obtained from the truthful principal is valuable and results in a welfare increase. This is achieved because an increase in A’s effort level when receiving message α leads to higher probability of victory by A. Since effort is less costly to the principal than to the agent, this deviation is even more beneficial to the principal than it is to the agent. As a result, it is always profitable for the principal to deviate from a dovish strategy to a truthful one. We conclude that no dovish equilibrium exists when communication is private.

The second observation is also the driving force behind our second result in Proposition 7: the expansion of the hawkish region. The deviation to a truthful strategy is less tempting since the deviation would not be observed by B and, consequently, would not reduce B’s effort.

**PROPOSITION 7:** Suppose that communication is private.
i. (No Dovish Bias) For any value of the parameters $\gamma \in (0, 1)$, $\phi \in [0, 1]$ and $\rho \in (0, 1)$, there is no equilibrium in which the principal always sends message $\beta$.

ii. (Hawkish Bias) Fix any $\gamma \in (0, 1)$ and any $\phi \in [0, 1]$. For all $\rho \geq \rho^H$ where

$$\rho^H = \frac{\phi + 1}{2 - \gamma + \gamma \phi},$$

there is an equilibrium in which the principal always sends message $\beta$. For all $\rho < \rho^H$, there is no such equilibrium.

Before examining the conditions for the existence of a truthful equilibrium, in Proposition 8 we characterize the equilibrium in the game of conflict between players $A$ and $B$, when $P$ plays a truthful strategy and communication is private. Note that this case is not covered by Proposition 1, since now player $A$ has private information. The proof follows directly from Proposition 1 in Siegel (2014) and is therefore omitted.

**PROPOSITION 8:** (Game of Conflict in a Truthful Equilibrium) Fix any $\gamma \in (0, 1)$, any $\phi \in [0, 1]$ and any $\rho \in (0, 1)$. Suppose that $P$ plays a truthful strategy and that he communicates privately with $A$. Then $\rho_A^\alpha = 1$ and $\rho_A^\beta = \frac{\rho(1-\gamma)}{1-\rho\gamma}$.

i. If $\rho_A^\beta \leq 1/2$, in the unique Nash-Bayesian equilibrium player $B$ exerts zero effort with probability $1 - \rho\gamma$. Conditional on exerting positive effort, player $B$ randomizes his effort uniformly on $(0, \rho\gamma]$. Player $A$ exerts no effort upon receiving message $\beta$ and also randomizes his effort uniformly on $(0, \rho\gamma]$ upon receiving message $\alpha$.

ii. If $\rho_A^\beta > 1/2$, in the unique Nash-Bayesian equilibrium player $B$ randomizes his effort on the interval $[0, (2\rho_A^\beta - 1)(1 - \rho\gamma) + \rho\gamma]$ with density equal to $\frac{1}{2\rho_A^\beta - 1}$ on the interval $[0, (2\rho_A^\beta - 1)(1 - \rho\gamma)]$ and with density equal to 1 on the interval $[(2\rho_A^\beta - 1)(1 - \rho\gamma), (2\rho_A^\beta - 1)(1 - \rho\gamma) + \rho\gamma]$. Upon receiving message $\beta$, player $A$ exerts no effort with probability $2 \left(1 - \rho_A^\beta\right)$ and, conditionally on exerting positive effort, randomizes uniformly his effort on the interval $[0, (2\rho_A^\beta - 1)(1 - \rho\gamma)]$. Upon receiving message $\alpha$, player $A$ randomizes uniformly his effort on the interval $[(2\rho_A^\beta - 1)(1 - \rho\gamma), (2\rho_A^\beta - 1)(1 - \rho\gamma) + \rho\gamma]$.
This result enables us to study the set of parameters for which a truthful equilibrium exists. When $P$ receives from nature signal $\alpha$, there is no reason for $P$ to misreport. Such a deviation can only decrease $P$’s expected payoff: it does not affect $B$’s effort, but it misleads $A$ into reducing his effort, which from $P$’s point of view is already too low. Under what condition does $P$ not have an incentive to misreport when receiving signal $\beta$? This deviation cannot possibly be profitable for the principal if $\rho_A^\beta \leq 1/2$. This is because in this case, $P$ believes that the optimal policy is $b$ and therefore does not want $A$ to exert effort, which would be the consequence of misreporting. Therefore, when $\rho_A^\beta \leq 1/2$, there always exists a truthful equilibrium. Suppose now that $\rho_A^\beta > 1/2$. In this case, when truthfully sending message $\beta$, the principal’s expected payoff is

$$
\left( \rho \gamma \left( 1 - \rho_A^{\beta} \right) + \frac{(1 - \rho \gamma)}{2} \right) - \frac{\phi}{2} \left( 2\rho_A^{\beta} - 1 \right) \left( 1 - \rho \gamma \right) \left( 2\rho_A^{\beta} - 1 \right) + 2 \left( 1 - \rho_A^{\beta} \right)^2.
$$

(22)

If the principal deviates to $A$, his expected payoff is

$$
\frac{\rho \gamma}{2} + (1 - \rho \gamma) \rho_A^{\beta} - \phi \left( 2\rho_A^{\beta} - 1 \right) \left( 1 - \rho \gamma \right) + \frac{\rho \gamma}{2}.
$$

(23)

We thus obtain the following result:

**PROPOSITION 9:** *(Truthful Equilibrium)* Suppose that communication is private. Fix any $\gamma \in (0, 1)$ and any $\phi \in [0, 1]$. A truthful equilibrium exists if and only if expression (22) is weakly greater than (23).

The region of parameters where a truthful equilibrium exists is characterized in the Appendix and is shown in Figure 4. For a given $\gamma$, this region consists of all couples $(\rho, \phi)$ to the left of the dashed curve in Figure 4. The continuous curve is given by $\rho^H$, which was defined in Proposition 7. The comparison of Figure 4 with Figure 1 shows that under private communication the hawkish region is strictly larger (one can verify that $\rho^H \leq \hat{\rho}$), truth-telling is more likely when $\phi$ is high, and is less likely when $\phi$ is close to $1/2$. Finally, note that between the hawkish and truthful regions there is an intermediate region where no pure equilibrium exists.\(^{19}\)

\(^{19}\)In this region, one can show that an equilibrium in mixed strategies exists, where the principal reports $\alpha$ when he observes $\alpha$, but randomizes between $\alpha$ and $\beta$ when he observes $\beta$. 
To summarize, we have shown that under private communication the principal cannot commit to a message strategy that instills doubts in $A$ — this explains why private communication makes conflicts more violent. Since public communication makes such commitment possible, the principal would generally benefit from it.

![Figure 4: Private Communication ($\gamma = 0.6$)](image)

7. **Independent Information Acquisition by the Agent**

In Section 3, we have shown that beliefs manipulation distorts effort decisions, but it does not distort policy making. In fact, $A$'s decision at $t = 2$ on the basis of $m$ coincides with the decision that $A$ would make had the true signal been known. This result occurs because the principal does not disagree with the agent on the correct policy to implement in each state. As a result, he does not manipulate information to the point of inducing the wrong policy decision in the final stage.

However, it is reasonable to expect that beliefs manipulation may also lead to inefficient decision-making. A simple extension of the basic setting allows to capture this additional cost. We examine the effect of the option for agent $A$ to independently acquire public information, i.e. information that is also observed by player $B$, after receiving the principal's message. As in Section 3, we assume that $A$ is naive and that the message is public.

We study the following game. The timing of events is drawn in Figure 5. As before, at
At $t = 0$ principal $P$ observes signal $s \in \{\alpha, \beta\}$ and sends a message to $A$. After receiving the message, the agent is now able, if he decides so, to conduct research in order to discover the current state. Research costs $k \geq 0$ and is not manipulable by $A$ himself (or by $P$). If the research cost is incurred by the agent, the principal internalizes a proportion $\phi$ of this cost. With probability $\pi \in [0, 1]$ research is successful and the state becomes common knowledge. With complementary probability $1 - \pi$, research is not successful. We assume that the probability of success is independent from the state $\theta$. As a result, nothing is learned when research is not successful and the information structure remains unchanged.

At $t = 1$ with exogenous probability $(1 - \epsilon) \in [0, 1]$ the conflict phase unfolds exactly as in the basic model: $A$ and $B$ simultaneously choose the effort level and the conflict’s winner is able to select his or her preferred policy. However, with probability $\epsilon$ the two parties are able to avoid the conflict phase. In this case, the decision maker is decided by tossing a fair coin without any of the two players having exerted any effort. A positive $\epsilon$ captures the possibility that the two opponents might be able, possibly through negotiation, to avoid the inefficiency of a conflict.\textsuperscript{20}

Notice that the setting that we have just described is a generalization of the basic model of Section 3. When $\epsilon = 0$ and $k$ is arbitrarily large (or, alternatively, $\pi = 0$) the model studied in this section coincides with the one studied before: as in the basic model, the agent relies exclusively on the information transmitted by the principal and the two players cannot resolve their disagreement without fighting.

\textsuperscript{20}As will be discussed shortly, $\epsilon > 0$ gives $A$ an incentive to acquire costly information.

Figure 5: Timeline with Autonomous Research
The model is solved by backward induction, starting from \( t = 2 \). In the final stage, a player is able to choose his (or her) preferred policy if that player wins the conflict or if the result of the coin toss is favorable to him (or her). In the decision rule (8), the agent’s posterior is computed after observing \( m \) and the outcome of his research effort (if any). At \( t = 1 \), in case the conflict phase starts, the effort strategies of the two opponents are given by Proposition 1.

We now move to \( t = 0 \) and study the agent’s choice of whether or not to acquire information. It is immediate that upon receiving message \( \alpha \), the agent does not acquire any information since he is convinced that the state is \( \theta_a \). Suppose instead that the agent has received message \( \beta \). We compute the expected payoff to the agent if he incurs the research cost. With probability \( 1 - \pi \) the research effort is not successful, and the expected value for the agent is

\[
\frac{\varepsilon}{2} \left( 1 - \rho_A^\beta \right) + \frac{\varepsilon}{2} \max \left\{ \rho_A^\beta, 1 - \rho_A^\beta \right\} + (1 - \varepsilon)(1 - \rho_A^\beta).
\]

(24)

The first term of the expression is the return obtained by the agent when there is no conflict and \( B \) decides, the second one is the return obtained when there is no conflict and \( A \) decides and the last term is the return of entering the conflict stage.

With probability \( \pi \) the research effort is successful, and the expected value for the agent is

\[
\frac{\varepsilon}{2} \left( 1 - \rho_A^\beta \right) + \varepsilon + (1 - \varepsilon)(1 - \rho_A^\beta).
\]

(25)

The difference between (25) and (24) is given by the second term: if the coin toss is favorable to \( A \) and research is successful, the agent is able to choose the right policy and obtain a payoff equal to 1. Using (5) it is immediate to obtain that the agent acquires information after receiving message \( \beta \) if and only if

\[
\frac{\pi \varepsilon}{2} \left( 1 - \max \left\{ \frac{\rho (1 - \gamma)}{1 - \rho_\gamma}, 1 - \frac{\rho (1 - \gamma)}{1 - \rho_\gamma} \right\} \right) \geq \kappa.
\]

(26)

A necessary and sufficient condition on \( \varepsilon, \pi \) and \( \kappa \) for the existence of parameters \( \rho, \gamma \) and
such that it is optimal for the agent to conduct research after message $\beta$ is that

$$ \frac{\pi \varepsilon}{4} \geq \kappa. \quad (27) $$

When this condition holds strictly, there are two thresholds $\rho^+$ and $\rho^-$ such that $\rho^- \leq \frac{1}{2-\gamma} \leq \rho^+$ and inequality (26) holds if and only if $\rho \in [\rho^-, \rho^+]$. This result is intuitive: $A$ does not incur the research cost if he is sufficiently convinced that the state is either $\theta_a$ or $\theta_b$.

After solving (26) we obtain

$$ \rho^+ = \frac{\pi \varepsilon / 2 - \kappa}{\pi \varepsilon / 2 - \gamma \kappa}, \quad \rho^- = \frac{2 \kappa}{\pi \varepsilon - \pi \varepsilon \gamma + 2 \gamma \kappa}. \quad (28) $$

The comparative statics of both thresholds with respect to $\pi$, $k$ and $\varepsilon$ are straightforward. When $\pi$ decreases from 1 to $(4k) / \varepsilon$, the threshold prior belief $\rho^-$ increases, while $\rho^+$ decreases. In other terms, when research is less successful, the agent incurs the research cost only when $\rho$ is close to $\frac{1}{2-\gamma}$ or, equivalently, when $\rho_A^\beta$ is close to 1/2. This is when the agent most values information. Similarly, when research is more costly ($k$ increases) the agent has weaker incentives to conduct research: $\rho^-$ increases and $\rho^+$ decreases. Finally, note that a higher $\varepsilon$ makes information acquisition more valuable. In the limit, when $\varepsilon = 0$, the agent conducts research only when $k = 0$. Because the agent’s expected utility of entering the conflict stage is linear in his own posterior belief (it equals $1 - \rho_A$), the value of public information for the agent is null when $\varepsilon = 0$: as long as this information is shared with $B$, the agent is indifferent between all information structures before starting the conflict phase. The reason for this is that the value of public information is dissipated in effort. The principal, on the other hand, is not indifferent and values information even when $\varepsilon = 0$.

We now study the principal’s message strategy when $k = 0$ and when $k > 0$.

### 7.1. Message Strategy with Costless Research

Suppose $k = 0$. In this case, the agent always conducts research when he receives message $\beta$, but not after message $\alpha$.\footnote{Given that research is costless $A$ is actually indifferent between doing and not doing research when $m = a$. To make $A$ strictly prefer the option of not conducting research, it would be enough to suppose that $k$ is infinitesimal.}
When the agent is allowed to acquire information, $P$ must take into account that his message will affect the agent’s incentives to conduct research. Compared with the basic model, sending message $\alpha$ when nature’s signal is $\beta$ has an additional cost: a hawkish message induces the agent not to acquire information. In other terms, besides making the agent hawkish, message $\alpha$ also makes him more dogmatic: the agent is more likely to disregard evidence that may induce him to revise his beliefs.

The incentives to induce hawkish attitudes are affected by $\pi$ and $\varepsilon$. To understand the role of $\pi$, consider the extreme case $\pi = 1$ and suppose $\varepsilon \in (0,1)$. Does $P$ have an incentive to send message $\alpha$ when nature’s signal is $\beta$? It is immediate that when $\pi = 1$ the answer is negative. To see this, we analyze the consequences for $P$ of sending message $\beta$ and thus inducing $A$ to conduct perfectly revealing research. Notice that if the agent discovers that the state is $\theta_b$, the principal obtains a payoff equal to 1, which is strictly greater than the payoff of sending message $\alpha$. If instead $A$ discovers that the current state is $\theta_a$, $A$ would fight very hard to impose $a$. In the latter case, the principal obtains the same payoff that he would have obtained by sending the false message $\alpha$. Therefore, research provides valuable information to the principal and there is never a hawkish equilibrium when $\pi = 1$. In order to understand how $\varepsilon$ affects the hawkish region, suppose $\pi \in (0,1)$ and consider the extreme case $\varepsilon = 1$. When a conflict never arises, it is also immediate to see that the principal has no incentive to prevent the agent from acquiring information.

Figure 6 illustrates the message strategies in the $(\rho, \phi)$ space for an intermediate value of $\pi$ and $\varepsilon$. There exists a hawkish equilibrium when $\phi$, $\pi$ and $\varepsilon$ are sufficiently low and $\rho$ sufficiently large, but the region of parameter values where the hawkish equilibrium exists has shrunk compared to Figure 1. The higher $\pi$ or $\varepsilon$, the smaller the hawkish region.

It is interesting to note that when research is costless the dovish region is not affected by either $\pi$ or $\varepsilon$. In other terms, the region of parameter values where dovish attitudes occur is identical to the one characterized in Proposition 2. Overall, this suggests that societies that have access to efficient ways of doing research (such as, well-supplied libraries, media and an advanced educational system) and societies where conflicts can be more easily avoided are more prone to either truthtelling or systematic doubts rather than to hawkish attitudes.

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22 To see this, notice that allowing research does not change the incentives to send $\beta$ in state $\theta_a$. In fact, $P$’s expected payoff of sending $\beta$ and inducing $A$ to conduct research is greater than the payoff of telling the truth if and only if (16) is greater than (13), which is the same condition obtained in the basic model.
Proposition 10 describes the equilibrium message strategy when $A$ is able to conduct costless research.

PROPOSITION 10: Suppose $k = 0$. The parameter space $(\rho, \phi)$ is divided in three regions that describe the equilibrium strategy of the principal: truthful, dovish and hawkish.

The boundary between the dovish and the truthful regions does not depend on $\pi$ or $\varepsilon$ and is characterized by Proposition 2.

The incentives to induce hawkish attitudes are decreasing in $\pi$ and $\varepsilon$. They completely vanish as either $\pi$ or $\varepsilon$ gets sufficiently close to 1.

Since inducing hawkish attitudes prevents the agent from conducting potentially successful research, Proposition 10 establishes that in case of victory the agent may make mistakes that could have been avoided if information had been truthfully transmitted.

7.2 Message Strategy when $k > 0$

When $k > 0$, the agent may not conduct research even after message $\beta$. As discussed above, the agent does not find it profitable to acquire information when $\rho \notin [\rho^-, \rho^+]$. It is immediate
to see that for all \( \rho \notin [\rho^-, \rho^+] \) the principal faces the same trade-off as in the basic model: the equilibrium message strategy is therefore described by the cutoffs of Propositions 2 and 3.

Consider now the region of parameters where \( A \) is willing to acquire information after message \( \beta \). When \( k > 0 \) both the hawkish and the dovish strategies become less profitable relative to the basic model, but for different reasons.

To understand why the dovish strategy is less appealing to the principal, first notice that when \( P \) observes \( \alpha \), the dovish message \( \beta \) has no effect on payoffs if conflict is avoided or research is successful. The principal is able to manipulate the agent only when research is unsuccessful and conflict takes place. Positive values of \( \varepsilon \) and \( \pi \) do not affect the incentive to play dovish when \( k = 0 \), but they make the dovish strategy look expensive and ineffective when \( k > 0 \). In contrast to the case of costless research, increasing \( \pi \) and \( \varepsilon \) moves the boundary of the dovish region (the curve \( VU \)) up to the right. Similarly, as \( k \) increases, the incentive of the principal to play the dovish strategy becomes weaker. Thus, a higher \( k \) shifts to the right the boundary between the dovish and the truthful regions. Note however, that increasing \( k \) has another countervailing effect: it shifts \( \rho^+ \) to the left, thus decreasing the parameter region where the agent conducts research. If \( k \) increases enough for the agent to stop conducting research, the dovish strategy may become optimal again for the principal.\(^{23}\)

The hawkish strategy is also less appealing to the principal, because when the principal observes \( \beta \), he wants the agent to conduct research. Since the principal internalizes only a fraction of \( k \), research is more valuable to him than it is to the agent. Sending the hawkish message \( \alpha \) discourages the agent from conducting research, which the principal values, and this makes the hawkish strategy less profitable relative to the basic model. This effect is weaker as the cost of research \( k \) increases, and is stronger as research becomes more effective (higher \( \pi \)) and more useful (higher \( \varepsilon \)).

The equilibrium strategy of the principal is shown in Figure 7 for a given set of parameters. The vertical dashed line in Figure 7 is drawn at \( \rho^+ \). Proposition 11 describes the equilibrium message strategy when research is costly.

**Proposition 11:** Suppose \( k > 0 \). For all \( \rho \notin [\rho^-, \rho^+] \), the agent does not acquire infor-

\(^{23}\)We have the largest dovish regions for extreme values of \( k \): when \( k = 0 \) (see Figure 6) and when \( k \) is so large that research is never conducted (see Figure 1).
Doubts and Dogmatism in Conflict Behavior

Information in equilibrium and the equilibrium message strategy is described by Propositions 2 and 3.

For all $\rho \in [\rho^-, \rho^+]$, the incentives to induce a dovish attitude are decreasing in $\pi$, $k$ and $\varepsilon$.

For all $\rho \in [\rho^-, \rho^+]$, the incentives to induce a hawkish attitude are increasing in $k$ and decreasing in $\pi$ and $\varepsilon$.

![Figure 7: Costly Research ($\pi = 0.4, \gamma = 0.6, \varepsilon = 0.3, k = 0.01$)](image)

8. Conclusions

As argued by Karl Popper (1963), conflicts are less violent when individuals entertain the possibility that the opponent may be right. Why is it so difficult to observe this attitude? To answer this question, we consider two opponents who participate in a game of conflict. One opponent trusts the information received by his principal.

In our model, the principal wants to affect the agent’s effort, but he also cares that the agent selects the correct policy and that he has the right incentives to acquire information.

In the context of our model, information is sometimes manipulated. In some cases, as a result of the principal’s message, the agent never doubts about the possibility of being wrong, although all available information suggests otherwise. The agent is motivated to exert high effort: this leads to excessive violence. Moreover, we show that hawkish attitudes make the
agent more dogmatic: the agent disregards evidence that may induce him to revise his beliefs. In other cases, the agent believes that his opponent may be right even when all the evidence indicates beyond any doubt that the policy preferred by the opponent is suboptimal. In this case, doubts moderate the escalation of violence in the conflict but the agent often loses.

We argue that manipulation of information (in both directions) is more likely to occur in heterogeneous societies and when nature’s signals are less precise. Hawkish attitudes are less likely to be observed when the agent is able to conduct autonomous research and when the principal’s altruism is low. When instead altruism is high, we obtain that the agent is induced by his principal to always doubt.

Moreover, we show that conflicts are more likely in heterogeneous societies. However, the intensity of a conflict is not necessarily at its maximum in very heterogeneous societies.

An interesting extension that we leave to future research would be to consider other forms of naïveté on the part of the agent. For instance, we could suppose that the agent misestimates the precision of the signal received by the principal or is excessively confident about his/her ability to win the conflict. We believe that even in these alternative settings the actions of the principle would be driven by similar considerations. We expect, for instance, that a principal with low altruism would motivate the agent by boosting his overconfidence (a similar motive is described by Charness et al., 2011), while a more altruistic principal would tend to discourage overconfidence in order to de-escalate the conflict.
Appendix

PROOF OF PROPOSITION 1

Let message $m$ be given. Suppose first that $1/2 < \rho^n_A < 1$. We proceed by steps. We first show that the equilibrium expected payoff of $B$ is strictly positive. To see this, notice that $A$ never exerts an effort level higher than his valuation, $2\rho^n_A - 1$, because he would earn a return below $1 - \rho^n_A$. This implies that $B$ can guarantee for himself a strictly positive payoff by exerting an effort level just above $2\rho^n_A - 1$.

We now show that the effort strategies of both players are mixed, with no mass points at a strictly positive effort level. By way of contradiction, suppose that player $j$ has a mass point at a particular effort $c_j > 0$. Then, the payoff of the other player would increase discontinuously at $c_j$. It then follows that there exists a $\varepsilon > 0$ such that the other player exerts effort on the interval $[c_j - \varepsilon, c_j]$ with zero probability. However, if this were the case, $j$ would increase his payoff by bidding $c_j - \varepsilon$ instead of $c_j$.

We now argue that the maximum effort level of the two players is the same. To see this, notice that since the effort strategies are mixed, if one individual has a maximum effort level, the other individual would win with probability one by just exerting that effort level.

Next, we now show that the minimum effort level is zero. By way of contradiction, suppose that an individual has a minimum effort level $c_2 \in (0, 2\rho^n_A - 1)$. Then the other player would not exert effort in the interval $[0, c_2)$ because by doing so he would lose with probability one. But this implies that the first individual would rather exert an effort level lower than $c_2$.

Individual $B$’s expected payoff from exerting effort $c_B$ is

$$EU^B = G_A(c_B) - c_B,$$

while $A$’s expected payoff from exerting effort $c_A$ is

$$EU^A = (1 - \rho^n_A) + G_B(c_A)(2\rho^n_A - 1) - c_A.$$

Noticing that $B$ must be indifferent among all the effort levels in the set and recalling that the equilibrium expected payoff for $B$ is strictly positive, we evaluate $EU^B$ when $c_B = 0$. It follows that $G_A(0) > 0$.

We now show that $B$ cannot put positive mass at zero. If this were the case, there would be a tie with some positive probability. But $B$ would be better off increasing his effort just above zero. This implies that $G_B(0) = 0$ and $A$’s expected payoff is $1 - \rho^n_A$. Then,

$$G_B(c_A) = \frac{c_A}{2\rho^n_A - 1}.$$  \hspace{1cm} (A.3)

When $B$’s effort is $2\rho^n_A - 1$,

$$EU^B = G_A(2\rho^n_A - 1) - (2\rho^n_A - 1),$$

$$EU^A = (1 - \rho^n_A) + G_B(c_A)(2\rho^n_A - 1) - c_A.$$
or
\[ EU^B = 1 - (2\rho_A^m - 1). \quad \text{(A.5)} \]

Then,
\[ G_A(c_B) = 1 - (2\rho_A^m - 1) + c_B. \quad \text{(A.6)} \]

When \( \rho_A^m = 1 \) the equilibrium strategies can be obtained by taking the limit of the equilibrium strategies described above.

Finally, when \( \rho_A^m < 1/2 \) it is immediate that the agent does not enter the conflict: \( c_B = c_A = 0 \). Thus, policy \( b \) is chosen.

This concludes the proof of Proposition 1. \( \square \)

**PROOF OF LEMMA 1**

Two cases must be considered. First, suppose that nature sends signal \( \beta \). Using Bayes’ Rule, we obtain that
\[ \rho^\beta_P = \frac{\rho(1 - \gamma)}{1 - \rho + \rho(1 - \gamma)}. \quad \text{(A.7)} \]

If the condition on \( \rho \) in the statement of Lemma 1 is satisfied, this implies that \( \rho^\beta_P \leq 1/2 \). Suppose that \( P \) is truthful and sends message \( \beta \). Then, by the naivete assumption \( \rho^\beta_A \) is equal to (A.7). Since \( \rho^\beta_A \leq 1/2 \), \( A \) exerts no effort and \( B \) picks policy \( b \). The expected payoff to the principal is then

\[ 1 - \rho^\beta_P \geq \frac{1}{2}. \quad \text{(A.8)} \]

Suppose instead that the principal sends the false message \( \alpha \). In this case, a total conflict arises and, using (19), the principal’s expected payoff would be

\[ -\frac{\phi}{2} + \frac{1}{2}, \quad \text{(A.9)} \]

which is lower than \( 1/2 \). This implies that a deviation from a truthful report is not profitable when the actual signal is \( \beta \).

Second, suppose that \( s = \alpha \). If the principal sends message \( \alpha \) his expected payoff is

\[ -\frac{\phi}{2} + \frac{1}{2}, \quad \text{(A.10)} \]

which is greater than zero, the payoff obtained by sending message \( \beta \) which shifts \( \rho^\beta_A \) below \( 1/2 \) and thus induces \( A \) to exert no effort. This implies that a deviation from a truthful report is also not profitable when the actual signal is \( \alpha \). \( \square \)

**PROOF OF LEMMA 2**
First note, using Bayes’ Rule (5), that when $\rho > \frac{1}{2 - \gamma}$ we have $\rho_i^\beta > 1/2$, for $i = P, A$. To prove Lemma 1, we show that when the signal is $\alpha$ the principal has a strict incentive to send message $\beta$. This can be understood by noticing that being truthful gives $P$ the expected payoff (13), which is equal to zero when $\phi = 1$. Instead, inducing a dovish attitude gives $P$ the expected payoff (16), which is strictly positive when $\phi = 1$. It is thus straightforward to conclude that sending message $\beta$ when $s = \alpha$ is preferable for a fully altruistic principal.

Second, we show that when the signal is $\beta$ the principal has a strict incentive to send message $\beta$. Sending message $\alpha$ gives the payoff (19), which is zero when $\phi = 1$. Sending message $\beta$ gives the payoff (18), which is strictly positive when $\phi = 1$. It is thus straightforward to conclude that sending message $\beta$ is preferable for a fully altruistic principal also when $s = \beta$.  

**PROOF OF PROPOSITION 2**

**Step 1:** When

$$\rho \leq \frac{1}{2 - \gamma},$$  

(A.11)

$P$ is truthful.

See Lemma 1.

**Step 2:** When

$$\frac{1}{2 - \gamma} < \rho \leq \frac{1}{2\phi(1 - \gamma) + \gamma},$$  

(A.12)

$P$ is also truthful.

First, suppose that $s = \beta$ and that the principal is truthful. If the condition in the statement of Step 2 is met, $\rho_A^\beta > 1/2$. Then, a conflict arises. The principal’s expected utility of sending a truthful message is given by (18). Since $\rho_p^\beta = \rho_A^\beta$ when reporting is truthful, we can rewrite (18) as

$$\left(2\rho_A^\beta - 1\right)\frac{1 - \phi}{2} + 2\left(1 - \rho_A^\beta\right)^2.  

(A.13)

To see whether $P$ has an incentive to deviate and send message $m = \alpha$ when the actual signal is $\beta$, we compare (A.13) to (19), the expected utility after the deviation. To show that (19) is lower than (A.13) when the condition in the statement of Step 2 is met, take the derivative of (A.13) with respect to $\rho_A^\beta$:

$$-2\phi(2\rho_A^\beta - 1) + 1 - 4(1 - \rho_A^\beta).  

(A.14)$$

This derivative can be written as

$$(1 - 2\phi)(2\rho_A^\beta - 1) + 2(\rho_A^\beta - 1).  

(A.15)$$
Knowing that $1 \geq \rho_A^\beta > 1/2$ and that $1 \geq \phi \geq 1/2$, one can verify that the derivative is always negative. Since (19) is equal to (A.13) when $\rho_A^\beta = 1$, we have proved that (19) is lower than (A.13). Therefore, $P$ has no incentive to send message $\alpha$ when $s = \beta$.

To conclude the proof of Step 2, we have to show that the principal does not want to deviate even when $s = \alpha$. The principal utility from truthful reporting is (13) while the utility of sending message $\beta$ is (16). One can show that when

$$\rho_A^\beta \leq \frac{1}{2\phi},$$

(A.16)

the principal has no incentive to misreport. In fact, when $\rho_A^\beta = 1/(2\phi)$ and $\rho_A^\beta = 1$ expressions (13) and (16) coincide. Between the two roots, (13) is greater than (16). When $\rho_A^\beta \leq 1/(2\phi)$ we have that (13) is lower than (16): $P$ has no incentive to misreport when $s = \alpha$. Knowing that $\rho_A^\beta$ is given by (5), it is easy to show that $\rho_A^\beta \leq 1/(2\phi)$ if and only if

$$\rho \leq \frac{1}{2\phi(1 - \gamma) + \gamma}.$$  

(A.17)

Step 3: When

$$\rho > \frac{1}{2\phi(1 - \gamma) + \gamma},$$

(A.18)

$P$ sends message $\beta$ regardless of nature’s signals.

Following the algebra of Step 2, we obtain that when the condition in the statement of Step 3 is satisfied, $P$ has an incentive to send message $\beta$ when the actual signal is $\alpha$. When instead $s = \beta$ the report is truthful. It then follows that regardless of $s$, $P$ always sends message $\beta$.

This concludes the proof of Proposition 2. \qed

**PROOF OF PROPOSITION 3**

Step 1: When

$$\rho \leq \frac{1}{2 - \gamma},$$

(A.19)

$P$ is truthful.

See Lemma 1.
Step 2: When

\[
\frac{1}{2 - \gamma} < \rho \leq \frac{1}{2(1 - \phi)(1 - \gamma) + \gamma},
\]

\(P\) is truthful.

First, suppose that \(s = \beta\). Since

\[
\frac{1}{2 - \gamma} < \rho,
\]

we have that \(\rho_A^\beta > 1/2\). Then, a conflict arises. The principal’s expected utility of sending a truthful message is given by (A.13). To see whether \(P\) has an incentive to deviate and send message \(m = \alpha\) when the actual signal is \(\beta\), we compute his utility after this deviation. This is given by (19). In comparing (A.13) to (19), one can show that when \(\phi < 1/2\) it may be the case that (19) is greater than (A.13). However, when

\[
\rho_A^\beta \leq \frac{1}{2(1 - \phi)},
\]

(19) is lower than (A.13). Then, \(P\) has no incentive to send message \(\alpha\) when he receives signal \(\beta\). Knowing that \(\rho_A^\beta\) is given by (5), it is easy to verify that (A.22) is satisfied if and only if

\[
\rho \leq \frac{1}{2(1 - \phi)(1 - \gamma) + \gamma}.
\]

Finally, suppose that the actual signal is \(s = \alpha\). The principal’s utility from truthful reporting is (13), while the utility of sending message \(\beta\) is given by (16). One can show that when \(\phi < 1/2\) the principal has no incentive to misreport.

Step 3: When

\[
\rho > \frac{1}{2(1 - \phi)(1 - \gamma) + \gamma},
\]

\(P\) sends message \(\alpha\) regardless of nature’s signals.

This follows from the algebra in the previous step.

This concludes the proof of Proposition 3. \(\square\)

PROOF OF PROPOSITION 4

Step 1: We show that the incidence of conflict is increasing in \(\rho\).
First, we compute the probability that a conflict occurs:

\[
Pr(\text{conflict}) = \begin{cases} 
\gamma \rho & \text{if } \rho \leq \frac{1}{2-\gamma}, \\
1 & \text{if } \rho > \frac{1}{2-\gamma}.
\end{cases}
\] (A.25)

To understand (A.25), notice that for all \(m\) we have that \(\rho^m_A > 1/2\) when \(\rho > 1/(2-\gamma)\). This implies that regardless of \(P\)'s message strategy, conflicts always occur when \(\rho > 1/(2-\gamma)\). When instead \(\rho \leq 1/(2-\gamma)\), one can verify from Propositions 2 and 3 that \(P\) is truthful. Since \(\rho^b_A \leq 1/2\), a conflict arises only when \(P\) sends message \(\alpha\), an event occurring with probability \(\gamma \rho\).

Note that the probability of observing a conflict is obviously increasing in \(\rho\).

We now move to the proof of the second part of Proposition 4. As a measure of the intensity of conflict, we compute expected total effort by taking expectations over the space of possible signals. Let \(\phi(s)\) denote the probability of observing signal \(s\), which can be derived from (3) and (4). Expected total effort as of time 0 is then given by

\[
E(c_A + c_B) = \phi(\beta)E(c_A + c_B; \beta) + \phi(\alpha)E(c_A + c_B; \alpha).
\] (A.26)

First, knowing the conditional probabilities (3) and (4), we derive the probabilities of the two signals.

\[
\phi(\beta) = 1 - \gamma \rho \quad \text{and} \quad \phi(\alpha) = \gamma \rho.
\] (A.27)

From (A.26), (12), and the results of Proposition 3, we write the expression for \(E(c_A + c_B)\) when \(\phi < 1/2\):

\[
E(c_A + c_B) = \begin{cases} 
\gamma \rho & \text{if } \rho \leq \frac{1}{2-\gamma}, \\
\gamma \rho + (1 - \gamma)(2\rho^b_A - 1)\rho^\beta_A & \text{if } \frac{1}{2-\gamma} < \rho \leq \hat{\rho}, \\
1 & \text{if } \rho > \hat{\rho}.
\end{cases}
\] (A.28)

Using the results of Proposition 2, we write the expression for \(E(c_A + c_B)\) when \(\phi \geq 1/2\):

\[
E(c_A + c_B) = \begin{cases} 
\gamma \rho & \text{if } \rho \leq \frac{1}{2-\gamma}, \\
\gamma \rho + (1 - \gamma)(2\rho^b_A - 1)\rho^\beta_A & \text{if } \frac{1}{2-\gamma} < \rho \leq \bar{\rho}, \\
(2\rho^b_A - 1)\rho^\beta_A & \text{if } \rho > \bar{\rho}.
\end{cases}
\] (A.29)

**Step 2:** We show that \(E(c_A + c_B)\) is weakly increasing in \(\rho\) when \(\phi < 1/2\).

To see this, we first show that

\[
\gamma \rho + (1 - \gamma)(2\rho^b_A - 1)\rho^\beta_A
\] (A.30)

is increasing in \(\rho\). Knowing (5), we find the derivative of (A.30) with respect to \(\rho\):

\[
\gamma + (1 - \gamma)(2\rho^b_A - 1) + \rho\frac{2(1 - \gamma)^2}{(1 - \gamma \rho)^2}
\] (A.31)
which is positive since \((2\rho_A^\beta - 1)\) is positive, \(\rho \in (1/2, 1)\), and \(0 \leq \gamma \leq 1\). Moreover, note that (A.30) is equal to \(\gamma \rho\) when \(\rho = 1/(2 - \gamma)\), and that (A.31) is greater than \(\gamma\), the slope of \(E(c_A + c_B)\) when \(\rho \leq 1/(2 - \gamma)\). Finally, note that (A.30) is lower than one: that is, right after \(\rho = \tilde{\rho}\), total effort jumps.

**Step 3:** We show that \(E(c_A + c_B)\) is not monotone in \(\rho\) when \(\phi > 1/2\).

It is enough to show that right after \(\rho = \bar{\rho}\), total effort drops. This is obvious since

\[
(2\rho_A^\beta - 1)\rho_A^\beta < 1. \tag{A.32}
\]

This concludes the proof of Proposition 4. \(\Box\)

**PROOF OF PROPOSITION 5**

First, for a candidate truthful equilibrium, the equilibrium conditions for all three players are the same regardless of whether the agent is naive or sophisticated. Therefore the parameter regions for which this strategy profile is an equilibrium is the same in the sophisticated agent case, as it is in the naive agent case. Second, when the agent is sophisticated, the information transmission game falls in the category of cheap talk games. It is well known that these games always have a babbling equilibrium, where the receivers (in this case players \(A\) and \(B\)) ignore the sender’s message (in this case the principal) and where the sender does not provide any information. See for example Crawford and Sobel (1982). \(\Box\)

**PROOF OF PROPOSITION 7**

**Proof of (i): Non existence of a dovish equilibrium.**

Suppose, by contradiction, that a dovish equilibrium exists. Then on the equilibrium path, the principal always sends message \(\beta\), regardless of what he observes. In case of conflict, the strategies of players \(A\) and \(B\) are as described in Proposition 1, with the posterior belief \(\rho_A^\beta\). We now distinguish two cases, depending on whether \(\rho_A^\beta \leq 1/2\) or \(\rho_A^\beta > 1/2\).

**Case 1:** Suppose first that \(\rho_A^\beta \leq 1/2\). Then none of the players exerts any effort on the equilibrium path and policy \(b\) is selected. Consider now the information set of the principal where he has observed signal \(\alpha\) from nature. His interim expected payoff on the equilibrium path is 0. If he deviates by sending instead message \(\alpha\), the effort of player \(B\) remains the same. The agent \(A\) will now exert a positive, but almost null, level of effort \(c > 0\), and win the conflict with probability 1. The principal’s interim expected payoff from deviating is now \(1 - \phi c > 0\). This contradicts that the dovish profile is an equilibrium.
Case 2: Suppose now that $\rho_A^\beta > 1/2$. Then on the equilibrium path, player $B$ randomizes his effort uniformly on $[0, 2\rho_A^\beta - 1]$ and player $A$ randomizes on the same interval, but with an atom at 0. Moreover the expected payoff of agent $A$ given his posterior $\rho_A^\beta$ and as a function of his choice of effort $c_A$ is given by

$$EU^A = \left(1 - \rho_A^\beta\right) + G_B(c_A) \left(2\rho_A^\beta - 1\right) - c_A,$$

which is constant for all $c_A \in [0, 2\rho_A^\beta - 1]$. Consider now the information set of the principal where he has observed signal $\alpha$ from nature. His expected payoff if the agent chooses effort $c_A$ is

$$EU^P = G_B(c_A) - \phi c_A,$$

which is strictly increasing in $c_A$. If the principal deviates and sends message $\alpha$, agent $A$’s expected payoff is now

$$EU^A = G_B(c_A) - c_A$$

which is strictly increasing in $c_A$ on $[0, 2\rho_A^\beta - 1]$. In fact, the optimal choice for the agent is now the effort $c_A = 2\rho_A^\beta - 1 + \varepsilon$, which gives the principal a higher expected payoff $EU^P$ than he had on the equilibrium path. The contradicts that the dovish profile is an equilibrium.

We conclude that there is never a dovish equilibrium when communication is private.

**Proof of (ii): Conditions for a hawkish equilibrium.**

In a hawkish equilibrium, on the equilibrium path, the principal sends message $\alpha$, regardless of what he observes. In case of conflict, the strategies of players $A$ and $B$ are as described in Proposition 1, with the posterior belief $\rho_A^\alpha = 1$. Then both $A$ and $B$ randomize their effort level uniformly on $[0, 1]$. Consider now the information set of the principal where he has observed signal $\beta$ from nature. His expected payoff if the agent chooses effort $c_A$ is

$$EU^P = \left(1 - \rho_P^\beta\right) + G_B(c_A) \left(2\rho_P^\beta - 1\right) - \phi c_A = \left(1 - \rho_P^\beta\right) + c_A \left(2\rho_P^\beta - 1\right) - \phi c_A.$$

By deviating to message $\beta$, the principal could induce the agent to exert no effort. Such a deviation is not profitable for the principal if and only if his expected payoff is nondecreasing in effort, which is the case if and only if

$$2\rho_P^\beta - 1 - \phi \geq 0,$$

which is equivalent to $\rho \geq \rho^H$, with

$$\rho^H = \frac{\phi + 1}{2 - \gamma + \gamma \phi}.$$

This concludes the proof of (ii) and the proof of Proposition 6. □
The effort choices in a truthful equilibrium are characterized in Proposition 8. To prove the proposition, it only remains to show that the truth-telling constraints of the principal hold if and only if expression (22) is weakly greater than expression (23). We do this in two steps.

First consider the principal’s information set where he observes the signal $\alpha$ from nature. If he truthfully reports $\alpha$, the agent is indifferent between effort levels in $(c_A^*, c_A + \rho \gamma]$, where

$$c_A^* = \max \left\{ 0, \left(2\rho_A^\beta - 1\right) (1 - \rho \gamma) \right\}$$  \hspace{1cm} (A.37)

Because the principal’s subjective marginal cost of effort is lower than the agent’s cost of effort, the principal’s preference is weakly increasing in these effort levels. Consequently, he cannot possibly gain by deviating to message $\beta$ which would induce the agent to exert less effort.

Second, consider the principal’s information set where he observes the signal $\beta$ from nature. If $\rho_A^\beta \leq 1/2$, truthfully reporting $\beta$ induces the agent not to exert any effort and policy $b$ gets selected, which gives the principal an expected payoff $1 - \rho_A^\beta$, which cannot be improved upon by sending message $\alpha$, which would induce the agent to exert effort that the principal would consider wasteful. Let us then restrict attention to the case where $\rho_A^\beta > 1/2$. If the principal truthfully reports $\beta$, his expected payoff is given by (22). If he deviates, his expected payoff is given by (23). Therefore a truthful equilibrium exists if and only if expression (22) is weakly greater than expression (23). Since this inequality contains the case $\rho_A^\beta \leq 1/2$, it is necessary and sufficient for a truthful equilibrium, for all values of $\rho_A^\beta$.

After some algebra, we obtain the following characterization of the truthful region: a truthful equilibrium exists if and only if $\rho \leq \rho^T (\phi, \gamma)$, where

$$\rho^T (\phi, \gamma) = \begin{cases} \frac{3\gamma - 4 + 4\phi - \gamma\phi}{2(2 - \gamma - 2\phi)} - \sqrt{\left(\frac{3\gamma - 4 + 4\phi - \gamma\phi}{2}\right)^2 - 6(2 - \gamma - 2\phi)(1 - \phi)} & \text{for } \phi \neq 1 - \gamma/2 \\ \frac{3}{4 - \gamma} & \text{for } \phi = 1 - \gamma/2. \end{cases}$$

This concludes the proof of Proposition 9. □

**PROOF OF PROPOSITIONS 10 AND 11**

**Step 1:** No research region: $\rho \notin [\rho^-, \rho^+]$.

When $\rho \notin [\rho^-, \rho^+]$, the agent does not conduct research, following the principal’s message. For this reason, the precise values of parameters $\pi$, $\varepsilon$ and $k$ have no impact on equilibrium (as long as $\rho \notin [\rho^-, \rho^+]$ holds). The equilibria are as in the main model. A higher $k$ increases this region in the sense of inclusion by increasing $\rho^-\gamma$ and decreasing $\rho^c$. Increasing $\pi$ or $\varepsilon$ has the opposite effects.

We now analyze the model when $k \geq 0$ and $\gamma, \pi, \varepsilon \in (0, 1)$ and $\rho \in [\rho^-, \rho^+]$. In particular, we assume that $\frac{\pi}{\varepsilon} \geq k$. For these parameters, the agent conducts research, following message $\beta$. 
Step 2: No counter signalling.

The principal can play four different strategies, depending on whether he sends $\alpha$ or $\beta$ when observing $\alpha$ or $\beta$. Which strategy arises in equilibrium depends on parameters.

However, there are no parameters for which the counter-signalling strategy of sending $\alpha$ when observing $\beta$ (and vice-versa) is a best response for the principal.

To see this, note that the principal’s interim expected payoff of sending $\alpha$ is always the same, regardless of what he observes. On the other hand, his interim expected payoff of sending $\beta$ depends on what he observed: it is strictly higher when he observed $\beta$. In fact, whether or not information acquisition is successful, the principal strictly prefers to send $\beta$ when he actually observes $\beta$ than when he observes $\alpha$. This rules out counter-signalling.

In the continuation, we characterize the parameter regions for which each of the three remaining strategies is optimal. More precisely, as we did in the main model, we characterize the boundary of the dovish-research region, and the boundary of the hawkish-research region. The complement of the union of these two regions is the truthful-research region.

Step 3: The dovish-research boundary.

Fixing $\gamma$, $\pi$ and $\varepsilon$, in $(\rho, \phi)$ space, this boundary separates, on one side, the dovish-research region $D_r$ and on the other hand, the union $T_r \cup H_r$ of the truthful-research and hawkish-research regions. When the principal observes signal $\alpha$, so that his own belief is 1, if he sends message $\alpha$ to the agent, the agent updates his belief to 1. This gives the principal the expected payoff

$$\frac{1}{2} (1 - \phi + \phi \varepsilon).$$

If he sends message $\beta$, the agent conducts and with probability $\pi$, updates his belief to 1 as well. With probability $1 - \pi$, the agent keeps the belief $\rho_A^\beta$.

If $\rho_A^\beta \leq 1/2$, this gives the principal the interim expected payoff

$$\frac{\pi}{2} (1 - \phi + \phi \varepsilon) - k \phi,$$

which is lesser than the expected payoff he can obtain by sending $\alpha$. Because $\rho_A^\beta \leq 1/2$ iff $\rho \leq \frac{1}{2 + \varepsilon}$, we conclude that the set $[\rho^-, \frac{1}{2 + \varepsilon}]$ is included in the truthful region $U^{TH}$.

If $\rho_A^\beta > 1/2$, the principal’s interim expected payoff equals

$$\left(1 - \pi\right)\left(1 - \varepsilon\right) \left(-\frac{\phi}{2} \left(2 \rho_A^\beta - 1\right)^2 + \frac{1}{2} \left(2 \rho_A^\beta - 1\right)\right) + \pi \left(\frac{1}{2} (1 - \phi + \phi \varepsilon)\right) + (1 - \pi) \frac{\varepsilon}{2} - k \phi.$$
Therefore the principal sends message $\beta$ (dovish-research region) if and only if the net gain $\Delta^{D_r}$ of sending $\alpha$ over $\beta$

$$\Delta^{D_r} \equiv (1 - \phi) + \phi \left(2\rho^\beta_A - 1\right)^2 - \left(2\rho^\beta_A - 1\right) + \frac{2k\phi}{(1 - \pi)(1 - \varepsilon)}$$

is nonpositive. The boundary is defined by the equation $\Delta^{D_r} = 0$.

When $k = 0$ (as in Proposition 10), this equation does not depend on $\pi$ or $\varepsilon$ and coincides with the dovish boundary characterized in Proposition 2.

When $k$ increases above 0, because $\frac{\partial \Delta^{D_r}}{\partial k} > 0$, the region $D_r$ decreases in the sense of inclusion. It decreases within the entire research region, but also because the entire research region itself decreases. The overall effect on the dovish region is ambiguous, because as $k$ increases, the shrinking dovish-research region is at the same time replaced by the dovish-no-research region, which is larger.

When $k > 0$, because $\frac{\partial \Delta^{D_r}}{\partial \varepsilon} > 0$ and $\frac{\partial \Delta^{D_r}}{\partial \pi} > 0$, as either $\varepsilon$ or $\pi$ increases, the region $D_r$ decreases in the sense that it looses field to the union $T_r \cup H_r$. It grows in another way, which is that the entire research region itself increases. The overall effect on the dovish region is a decrease in the sense of inclusion. As either $\varepsilon$ or $\pi$ increases, the dovish-research shrinks and at the same time replaces the dovish-no-research region, which is larger.

**Step 4: The hawkish-research boundary.**

Fixing $\gamma$, $\pi$ and $\varepsilon$, in the $(\rho, \phi)$ space, this boundary separates, on one side, the hawkish-research region $H_r$ and on the other hand, the union $T_r \cup D_r$ of the truthful-research and dovish-research regions. When the principal observes signal $\beta$, so that his own belief is $\rho^\beta_P$, if he sends message $\alpha$ to the agent, the agent updates his belief to 1. This gives the principal the expected payoff

$$\frac{1}{2} \left(1 - \phi + \phi\varepsilon\right).$$

If he sends message $\beta$, the agent conducts research and with probability $\pi\rho^\beta_P$, updates his belief to 1 as well, and with probability $\pi \left(1 - \rho^\beta_P\right)$, he updates his belief to 0. With probability $1 - \pi$, the agent keeps the belief $\rho^\beta_A$.

If $\rho^\beta_A \leq 1/2$, this gives the principal the interim expected payoff

$$\pi\rho^\beta_P \left(\frac{1}{2} (1 - \phi + \phi\varepsilon)\right) + \pi \left(1 - \rho^\beta_P\right) + (1 - \pi) \left(1 - \rho^\beta_A\right) - k\phi$$

$$= \pi\rho^\beta_P \left(\frac{1}{2} (1 - \phi + \phi\varepsilon)\right) + \left(1 - \rho^\beta_P\right) - k\phi.$$

Therefore the principal sends message $\alpha$ (dovish-research region) if and only if the net gain $\Delta^{H_r}$ of sending $\beta$ over $\alpha$ is nonpositive. We have

$$\Delta^{H_r} \equiv \left(1 - \rho^\beta_P\right) + \left(\pi\rho^\beta_P - 1\right) \left(\frac{1}{2} (1 - \phi + \phi\varepsilon)\right) - k\phi$$
Because $\rho_A^\beta \leq 1/2$ iff $\rho \leq \frac{1}{2-\gamma}$, we conclude that the set of pairs $(\rho, \phi)$ such that $\rho \in \left[\rho^-, \frac{1}{2-\gamma}\right]$ is included in region $T_r \cup H_r$. From Step 2, we already know that it is included in the region $T_r \cup D_r$, therefore it is included in the region $T_r$.

If $\rho_A^\beta > 1/2$, the principal’s interim expected payoff when sending message $\beta$ equals

$$
(1 - \pi) \left( \left(1 - \varepsilon \right) \left( -\frac{\phi}{2} \left(2\rho_A^\beta - 1\right)^2 + \frac{1}{2} \left(2\rho_A^\beta - 1\right) + 2 \left(1 - \rho_A^\beta\right) \left(1 - \rho_A^\beta\right) \right) + \varepsilon \right)
$$

$$
+ \pi \left( \frac{1}{2} \left(1 - \phi + \phi\varepsilon\right) \rho_A^\beta + 1 - \rho_A^\beta \right) - k\phi.
$$

Therefore the principal sends message $\alpha$ (hawkish-research region) if and only if the net gain $\Delta^{H_r}$ of sending $\beta$ over $\alpha$

$$
\Delta^{H_r} = (1 - \pi) \left( \left(1 - \varepsilon \right) \left( -\frac{\phi}{2} \left(2\rho_A^\beta - 1\right)^2 + \frac{1}{2} \left(2\rho_A^\beta - 1\right) + 2 \left(1 - \rho_A^\beta\right) \left(1 - \rho_A^\beta\right) \right) + \varepsilon \right)
$$

$$
+ \pi \left( \frac{1}{2} \left(1 - \phi + \phi\varepsilon\right) \rho_A^\beta + 1 - \rho_A^\beta \right) - k\phi - \frac{1}{2} \left(1 - \phi + \phi\varepsilon\right).
$$

$$
= \left(1 - \rho_A^\beta\right) \left[ \frac{\pi}{2} \left(1 + \phi \left(1 - \varepsilon\right)\right) + (1 - \pi) \left(1 - \varepsilon\right) \left(1 - 2\rho_A^\beta\right) + 2 (1 - \pi) (1 - \varepsilon) \phi \rho_A^\beta \right] - k\phi.
$$

$$
= \left[ \frac{\pi}{2} \left(1 + \phi \left(1 - \varepsilon\right)\right) + (1 - \pi) \left(1 - \varepsilon\right) \left(1 - 2\rho_A^\beta\right) + 2 (1 - \pi) (1 - \varepsilon) \phi \rho_A^\beta \right] - k\phi.
$$

is nonpositive. The boundary is defined by the equation $\Delta^{H_r} = 0$.

When $k = 0$, we obtain the hawkish boundary’s equation is

$$
\rho_A^\beta = \frac{1}{2 \left(1 - \phi\right)} + \frac{\pi}{2} \left(1 + \phi \left(1 - \varepsilon\right)\right) \frac{1}{2 \left(1 - \phi\right) \left(1 - \pi\right) \left(1 - \varepsilon\right)}
$$

$$
\rho \left(1 - \gamma\right) = \frac{1}{2 \left(1 - \phi\right)} + \frac{\pi}{2} \left(1 + \phi \left(1 - \varepsilon\right)\right) \frac{1}{2 \left(1 - \phi\right) \left(1 - \pi\right) \left(1 - \varepsilon\right)}
$$

which coincides with the hawkish boundary characterized in Proposition 3 in the main model, iff $\pi = 0$. This gives us a strictly increasing function $\rho \left(\phi\right)$.

When $k = 0$, because $\frac{\partial \Delta^{H_r}}{\partial \varepsilon} > 0$ and $\frac{\partial \Delta^{H_r}}{\partial \pi} > 0$, when either $\varepsilon$ or $\pi$ increases, the region $H_r$ decreases in the sense of inclusion. If either $\varepsilon$ or $\pi$ is close enough to 1, the region $H_r$ is empty.

As $k$ increases above zero, holding the other parameters fixed, and with $\varepsilon > 0$ and $\pi > 0$, the region $H_r$ increases in the sense that it gains field against the truthful region, but decreases due to the decrease of the entire research region, because $\rho^+$ decreases as $k$ increases. As $k$ increases, the overall effect on the hawkish
region is an increase in the sense of inclusion, because the growing hawkish-research region is progressively replaced by the hawkish-no-research region, which is larger.

When $k > 0$, the equation $\Delta H_r = 0$ implies that $\frac{\partial \Delta H_r}{\partial \varepsilon} > 0$ and $\frac{\partial \Delta H_r}{\partial \pi} > 0$. Therefore when either $\varepsilon$ or $\pi$ increases, the region $H_r$ decreases in the sense that it loses field against the truthful region, but increases due to the increase of the entire research region, because $p^+$ decreases as $k$ increases. If either $\varepsilon$ or $\pi$ is close enough to 1, the region $H_r$ is empty. As either $\varepsilon$ or $\pi$ increases, the overall effect on the hawkish region is a decrease in the sense of inclusion, because the shrinking hawkish-research region progressively replaces the hawkish-no-research region, which is larger.

This concludes the proof of Propositions 10 and 11. □

Bibliography


