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ABSTRACT

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JEL Classification: C73, D23, L24, O31 and O34
Keywords: contracting, knowledge trading, patents and war of attrition

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Emeric Henry and Carlos J. Ponce*

September, 2009

Abstract

We study the problem of an inventor who brings to the market an innovation that can be legally copied. Imitators may 'enter' the market by copying the innovation at a cost or by buying from the inventor the knowledge necessary to reproduce and use the invention. The possibility of contracting affects the need for patent protection. Our results reveal that: (i) Imitators wait to enter the market and the inventor becomes a temporary monopolist; (ii) The inventor offers contracts which allow resale of the knowledge acquired by the imitators; (iii) As the pool of potential imitators grows large, the inventor may become a permanent monopolist.

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1. Introduction

The economics of innovation revolve around the design and analysis of incentive schemes for inventors under the threat of imitation. The main premise is that, when knowledge is cheap to imitate, innovative rewards are vulnerable to ex-post expropriation by imitators. Imitators immediately copy the innovation, dissipating the rents of inventors and thus discouraging costly research. This reasoning justifies the need for patents and other legal means of protecting the inventors' profits.

Controversies, however, about the strength of intellectual property rights, the set of technologies that should be protected by intellectual property law and even the need for such a system have been recurrent over time. The essence of these debates is expressed in the following question raised by Gallini and Scotchmer (2001):

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Are there *natural market forces* that protect inventors so that formal protections or other incentives are *not* necessary?[1]

This paper answers this question by providing theoretical foundations for a natural, intuitive and market-based mechanism which yields substantive rents for inventors in the absence of patent protection. Specifically, we reveal that the possibility of trading the knowledge which is necessary to develop and use the innovation fundamentally alters the 'conventional wisdom' on the need for patent protection.

This natural market force is based on the dynamic trading of knowledge. As a preview of our main results, we note that, in equilibrium, potential imitators will 'imitate' the innovation by buying knowledge rather than by spending duplicative resources. Furthermore, the inventor offers to sell knowledge through contracts which allow subsequent reselling by the buyers. Thus, the first buyer will compete with the inventor to sell his acquired knowledge to the remaining imitators. Therefore, initially, imitators have an incentive to delay their entry with the hope that some of their rivals will trade with the inventor before them, anticipating as a consequence a future reduction in the price of knowledge. Temporarily the inventor becomes a monopolist and might receive a reward arbitrarily close to monopoly profits, even for relatively small imitation costs.

Thus this paper suggests that the lack of appropriability typically associated with inventions holds only under important restrictions about trading opportunities between inventors and imitators. Whereas the previous literature has focused on the profits after imitation (i.e., ex-post profits) as the main determinant of the inventors' payoff, we underline that the payoff received by inventors is determined by: (i) The difference between the ex-post profits and the imitation cost in an economy with a small number of potential imitators; and: (ii) By the limiting ratio between the ex-post profits and the imitation cost in an economy with a large number of potential imitators.

Discussion of Main Results

We examine the following environment. Potential imitators may legally copy an innovation at a certain imitation cost.[2] Copying is profitable: Even if all potential imitators copy, the equilibrium market profits which they collect are sufficient to cover the imitation cost. Within the limits of this problem, our point of departure is to consider the dynamic trade of knowledge between the inventor and potential imitators. Knowledge may be traded through contracts in two distinct situations. First, the inventor, under the threat of profitable imitation, may sell to the imitators the knowledge sufficient for them to reproduce and use the

[1] (pp. 52; italics added)

[2] The cost of reverse engineering the product which can be very small.

invention. Second, those imitators who initially bought knowledge from the inventor might also subsequently resell it to the remaining imitators.

Let us emphasize that the knowledge contracts we refer to are different from traditional licensing contracts that are familiar to us in our world with patents.[3] Knowledge contracts are an engagement to transfer the knowledge necessary to develop and use the innovation in exchange for a fee. A buyer of such a contract thus saves the cost of reverse engineering the product. Licensing contracts, on the contrary, rely on patents: They grant the buyer the right to use the patented innovation. Note nevertheless that licensing contracts are often accompanied by know-how transfer clauses where the seller takes for instance the engagement of training the buyer's employees.[4] Furthermore, there is evidence, reviewed in section 6, that pure know-how contracts unrelated to patents, are growing in importance.

The elements previously discussed are initially captured in a model with one inventor and two potential imitators. Its extensive form is as follows. Time is divided into an infinite sequence of periods. The innovation is introduced into the market at period zero. In each period, imitators can 'enter' the market (start using the innovation) either by imitating at a commonly known imitation cost or by buying knowledge (through a knowledge contract) from the inventor. If, at any period, only one of the potential imitators has entered the market, he might then compete in subsequent periods with the inventor to sell knowledge to the remaining imitator.[5]

As a benchmark result, we establish that, when knowledge cannot be traded, both imitators enter the market immediately, at period zero, by copying the innovation. By contrast, our main result reveals that, when knowledge can be traded, potential imitators wait before entering the market. Thus, *the inventor becomes a temporary monopolist even in the absence of patent protection*.

The intuition for this result lies in the dynamics of the equilibrium price of knowledge. The inventor initially offers knowledge for sale, through two transferable contracts, at a posted price equal to the imitation cost. The contracts are called transferable because the buyer may then resell his acquired knowledge to the remaining imitator without any restriction. So, once an imitator enters the market by buying knowledge, through what we call the first contract, he subsequently competes with the inventor on the 'knowledge market'.[6]

[3]Note that the lack of patent protection is *not* an obstacle to sell knowledge in our environment. It has been argued, in a different framework, that when buyers cannot assess the value of an idea they may be reluctant to sign contracts (See Anton and Yao (1994)). In our environment, however, the innovation is already on the market, success is publicly observable and these concerns can be safely ignored.

[4]Arora et al (2002) state that 'over two thirds of a sample of UK firms reported that most or all of their licensing agreements had know-how provisions'

[5]With simultaneous entry, the game ends.

[6]For the sake of simplicity we call the knowledge market the set of all potential or actual trades or

In the unique (symmetric) Markov Perfect Equilibrium (MPE), after the first knowledge contract is sold, the sellers immediately sell knowledge, through the second contract, to the remaining imitator at a price equal to its marginal cost: zero.[7] These dynamics in the price of knowledge ensure an equilibrium payoff for the second entrant strictly higher than the equilibrium payoff for the first entrant. Thus, each potential imitator waits a random amount of time before entering the market with the hope that his rival will enter before him and thus trigger competition in the knowledge market sufficient to decrease its price to zero.

Two features of this market mechanism are remarkable. First, even though the inventor could offer contracts forbidding resale, she strictly prefers not to do so. If the inventor offered such non-transferable knowledge contracts, potential imitators would realize that the price of knowledge would not fall in the future and hence they would immediately enter to avoid sacrificing current market profits. The inventor would collect twice the contract fee but her market profits would immediately be reduced to triopoly rents. We show that the expected length of monopoly time when transferable contracts are offered is sufficiently long to dominate lost contract revenues. Thus, *the transferability clause acts as a commitment tool that provides credibility to the future price reductions of knowledge.*

The second remarkable feature is revealed by a simple formula that we obtain for the equilibrium payoff of the inventor. This expression uncovers that the inventor's payoff is strictly increasing in the level of the imitation cost. Moreover, the expected length of monopoly time may be considerable, even when the imitation cost is small. For instance, suppose that the imitation cost is small and close to, but smaller than, the present value of triopoly profits. Then if knowledge trades were not feasible the inventor would receive the present value of triopoly profits, a potentially very small amount. However, when knowledge is traded, the expected length of monopoly time goes to infinity and *the inventor receives a payoff arbitrarily close to the present value of monopoly profits.*

We also examine two further issues. First, we extend our model to an arbitrary number of potential imitators. We show that, under some conditions, as the number of potential imitators increases, the equilibrium reward of the inventor becomes arbitrarily close to monopoly profits. Last, we contrast the predictions of the MPE with those of Subgame Perfect Equilibrium (SPE). We show that a condition on market profits is necessary and sufficient for the MPE to be the unique SPE. If this condition is not satisfied, multiple equilibria exist. In some, entry might take place immediately and an appropriation failure might reappear. However, it is not a consequence of the lack of intellectual property rights per se but rather a problem caused by a slow diffusion of knowledge.

transactions in which knowledge is involved.

[7] The cost of transferring knowledge is normalized to zero.

Although the prevalence of patents in most industrial sectors in developed countries makes it difficult to find evidence of the use of this market mechanism in practice, anecdotal evidence from countries in which patent protection is weak or non-existent fits well with our results. We describe (see Section 6) how a mechanism surprisingly similar to ours can explain the observed behavior in the market for generic drugs in India. We also provide more systematic suggestive evidence. Further empirical work on this topic is undoubtedly needed and our work can provide a suitable theoretical framework to conduct it.

Organization of this Paper

In Section 2 we describe the model. In section 3, we present our main findings. We show how our mechanism leads to a natural protection for inventors and discuss the importance of transferable knowledge contracts. In Section 4 we extend our model to an arbitrary number of potential imitators. Section 5 discusses uniqueness and contrasts the predictions of MPE with those of SPE. Section 6 presents suggestive evidence and section 7 discusses the related literature. Finally, section 8 concludes. All proofs are presented in the Appendix.

2. The Model

An inventor ('she'), denoted by s, has developed an innovation (process or product) that is *not* legally protected against imitation. Two imitators, indexed by $g \in \{j, l\}$, may 'adopt' the innovation by either: (i) Using a costly imitation technology (henceforth, imitating); or by: (ii) Buying knowledge from the inventor.

Time is broken into a countable infinite sequence of periods of length $\Delta > 0$. Each period is indexed by t ($t = 0, 1...$). The innovation is introduced into the product market at period zero. At that period, the imitators might already be producing in the market. Their profits if they do not use the innovation are normalized to zero.[8]

The following terminology will be used throughout the paper. When an imitator adopts the innovation at period t, we say that he *enters* the market regardless of his mode of entry. Also we describe him as *active* in the market from that period on. The inventor and each active imitator obtain the same equilibrium profit flow independently of how the imitators entered. The profit flow received by each individual firm when $n-1$ other firms are also active is denoted by π_n.[9]

All parties are risk neutral and maximize the sum of their expected discounted payoffs (profits plus potential contract payments). Agents discount the future exponentially with a

[8]This is without loss of generality. The model can encompass either a drastic or non-drastic innovation.

[9]To make our arguments most general profits are specified in reduced form.

per-period discount factor equal to $\delta \equiv e^{-r\Delta}$, where $r > 0$ is the discount rate.[10] So, the profits received by each individual firm during a period in which $n-1$ other firms are also active is $\int_0^\Delta \pi_n e^{-rt} dt = (1-\delta)\,\Pi_n$; where $\Pi_n \equiv r^{-1}\pi_n$ is the present value of market profits per firm when n firms are active. We assume that profits satisfy the following standard condition.

Assumption 0: $\Pi_1 > 2\Pi_2 > 3\Pi_3$

An imitator by spending, at any period t, an amount of resources $\kappa > 0$ obtains instantaneously (at the same period) a perfect version of the innovation. We view κ as a one-time sunk cost that must be incurred to reverse engineer the fine details of the innovation. An alternative to imitation is to enter the market by buying knowledge through contracting. The inventor, being the creator of the innovation, possesses the required (indivisible) knowledge to transfer the innovation. If an imitator buys this piece of knowledge at t, he will be able to instantaneously obtain a perfect version of the innovation at zero added cost.[11]

Contracting takes place as follows. At any t, before entry, a fix-fee contract between the inventor (seller) and imitator g (buyer) is a pair $(p_{sg}^t, \theta_{sg}^t) \in [0,\infty] \times \{0,1\}$.[12] $p_{sg}^t \geq 0$ is the price at which the inventor offers a contract of type θ_{sg}^t to imitator g at t. Two types of contracts can be offered: Non-transferable, $\theta_{sg} = 0$, and transferable contracts $\theta_{sg} = 1$. A transferable contract allows an imitator to resell the knowledge acquired from the inventor to the other imitator in subsequent periods. The following convention is adopted: Offering no contract to imitator g, at period t, is equivalent to offering a contract at $p_{sg}^t = \infty$. Last, we also assume that if an imitator enters by imitating, he will also become a competitor of the inventor in the knowledge market.[13] Specifically, at each t, in which only one imitator, say j, is active in the market and entered either by imitation or by buying a transferable contract, he and the innovator offer a contract at a price p_{jl}^t and p_{sl}^t to imitator l respectively.

Potential knowledge exchanges occur within the framework of the following extensive form game. Consider any period t in which no imitator has entered yet. Then:

(i) The inventor announces, on a take-it-or-leave-it basis, a pair of contracts, $\left\{p_{sg}^t, \theta_{sg}^t\right\}$ for $g \in \{j, l\}$. Then:

(ii) The imitators simultaneously decide whether to enter the market -either by imitating, i_g, or by buying knowledge through contracting, c_g- or not to enter, w_g.

[10]Note that when $\Delta \to 0$, our model converges to a continuous time model with impatient decision makers.

[11]Transferring knowledge has no real cost.

[12]A previous version of the paper showed that the main results remain the same if we allow for two part tariffs and linear demand but at the expense of simplicity.

[13]This assumption considerably simplifies many of our proofs and the number of continuation games that need to be considered.

The game continues in this manner as long as no imitator enters the market. If, at period t, both imitators enter, the game formally ends and all players collect triopoly profits from that period on. But if only one of them enters, say j, from the beginning of period $t+1$ until entry of the second imitator the game continues as follows:

(i) The sellers simultaneously announce prices for knowledge: $p_{jl}^{t+\tau}$ and $p_{sl}^{t+\tau}$ respectively, for all $\tau = 1, 2...$ If imitator j bought a non-transferable contract at t, the convention is that $p_{jl}^{t+\tau} = \infty$ for all $\tau = 1, 2...$Then:

(ii) Imitator l decides whether to enter the market -either by imitating, i_l, or by buying knowledge from one of the sellers, c_{jl} or c_{sl}- or not to enter, w_l.

All parties observe the history of the game up to the beginning of period t and the buyer(s) observe the contract(s) offered by the seller(s) at the beginning of period t.

A description of pure strategies follows.[14] Let H^t be the set of all histories up to, but not including period t. A history $h^t \in H^t$ includes all the past offers by the seller(s) and all the decisions taken by the buyer(s). Let $A_s(H^t)$ be the set of contracts for the inventor at t. A *contracting strategy* of s is a sequence $\sigma = \{\sigma^t\}_{t=0}^{\infty}$ of functions, each of which assigns to each history a (possible pair of) contracts. Thus $\sigma^t : H^t \to A_s(H^t)$.[15]

For simplicity, we only define the strategy of, say, imitator j. Let $\mathcal{H}^t$ be the set of all histories, at any period t, after s has offered a pair of contracts. Let $\mathcal{H}_j^t$ be the set of all histories, at any period t, after the sellers (s and l) announced knowledge prices. An *imitation strategy* of j is a sequence $\varphi_j = \left\{e_j^t, d_j^t, p_{jl}^t\right\}_{t=0}^{\infty}$ of three functions. Specifically $e_j^t : \mathcal{H}^t \to \{i_j, c_j, w_j\}$ is an *entry* function which assigns to each history an entry decision before entry of any imitator. Similarly $d_j^t : \mathcal{H}_j^t \to \{i_j, c_{js}, c_{jl}, w_j\}$ is a *decision* function that allocates to each history an entry decision after imitator l has entered. Last $p_{jl}^t : H^t \to [0, \infty]$ is a *pricing* function which maps each history to a price offer for imitator l.

We use Subgame-Perfect equilibria (SPE) and Markov Perfect Equilibria (MPE) as solution concepts. A SPE is a $\{\sigma, \varphi_j, \varphi_l\}$ that forms a Nash equilibrium after every possible history. In MPE strategies are functions only of payoff-relevant histories which are determined in our model by the number of buyers and sellers in the market for knowledge contract. So we cluster all possible histories, according to the market structure, into three disjoint and exhaustive subsets: (i) The subset of all histories in which the inventor is the unique seller and the imitators are the buyers. Such a subgame is called the *monopoly* game; (ii) The subset of all those histories in which the inventor and one of the imitators are the sellers. Such a subgame is called the *competitive* game; and last: (iii) The remaining subset of histories in which the inventor is the unique seller and there is a unique buyer.[16] Such a subgame is

[14] The extension to behavioral strategies is direct

[15] To ease notation, we do not index strategies, as we should, by Δ.

[16] This corresponds to the case in which only one imitator previously entered by buying knowledge through

called the *bilateral monopoly* game. Then a MPE is a SPE in which the contracting strategy of the inventor is only a function of the market structure and the entry-decision functions of the imitators depend only on the prices and type of contracts being offered.

3. Main Results: Appropriation without Patents

We introduce our main results in this section. Proposition 1 formalizes the lack of appropriability and fast imitation in the absence of knowledge trades, features which are typically associated to inventions. This result should be contrasted with our main finding, in Proposition 2: When knowledge trades are feasible, imitators strategically delay their entry to the market and so the inventor becomes a temporary monopolist. Finally, in Proposition 3, we show that the strategic delay in imitation is generated by the use of transferable contracts which the inventor strictly prefers to the alternative of non-transferable contracting.

3.A. Innovative Rents without Knowledge Trading

Here, we analyze the dynamics of entry when contracting is *not* feasible. Thus, if potential imitators choose to enter they must do so by imitating. We focus on pure strategies SPE. We find that both imitators enter the market, without delay, at period zero.

Although we consider an economy without patent rights, the imitation cost works as an entry barrier determining a natural measure of protection for the inventor. A value of κ such that $\kappa > \Pi_2$ is sufficient to protect the inventor from imitation: If $\kappa > \Pi_2$ no imitator copies and the inventor receives monopoly profits Π_1. Thus, to make our problem interesting, Assumption 1 is imposed throughout Sections 3 and 5.[17]

Assumption 1: $0 < \kappa < \Pi_3$

Assumption 1 ensures that copying is profitable for both imitators. Proposition 1 is our benchmark result.

Proposition 1 *Suppose knowledge cannot be traded. Then: (i) There is a unique SPE in which both imitators imitate at period $t = 0$; and: (ii) The equilibrium payoffs for the inventor and the imitators are Π_3 and $\Pi_3 - \kappa$ respectively.*

Proposition 1 shows that both imitators enter the market immediately. Indeed there is no benefit from delaying entry since the entry cost will remain fixed at the level of the imitation cost. Moreover, by delaying entry imitators sacrifice profits during the periods in

a non-transferable contract.

[17] In section 4 we extend Assumption 1 to the case of a large number of imitators.

which they do not use the innovation. Hence, if entry occurs it will take place at period zero. Assumption 1 ensures that entry does occurs as it is profitable for both imitators.

Proposition 1 summarizes the 'conventional wisdom' justifying the need for patent protection. In the absence of such protection, imitators enter immediately and compete away the rents of the inventor. Foreseeing the risk that their reward might be insufficient to cover their research costs, inventors might thus shy away from initially investing in research.

3.B. Appropriation with knowledge trading

We show that the results of Proposition 1 justifying patent protection are fundamentally altered when trades in knowledge are feasible. We start by focusing our attention on transferable contracts and next we demonstrate that, indeed, the inventor strictly prefers to sell knowledge through transferable rather than non-transferable contracts.

Knowledge trading

We focus on MPE. As we restrict our attention to the case of transferable contracts, after entering, an imitator competes with the inventor to sell knowledge to the remaining imitator. Thus, we only need to examine the competitive and monopoly subgames.[18] First, we focus on the competitive game. Suppose that imitator j has entered at period t. The sellers (j and s) may, in subsequent periods, offer contracts to the buyer (l). The competitive game has a unique MPE that we call the no-delay contracting equilibrium. Lemma 1 presents the equilibrium outcome of the no-delay contracting equilibrium.

Lemma 1 *In the unique MPE of the competitive subgame (that we call the no-delay contracting equilibrium), knowledge is sold to imitator l immediately at period $t + 1$ at a zero price.*

In this equilibrium knowledge is sold immediately to the remaining imitator at a price equal to its marginal cost. The sellers offer knowledge at a zero price at each period if entry has not occurred yet. The buyer then has no incentive to delay his purchase since knowledge is offered at its minimal price. For future reference, we call the contract through which knowledge is offered after the first entry the *second* contract. We note that this equilibrium is not necessarily the unique SPE of this continuation game. Section 6 examines the potential multiplicity of SPE and provides a condition under which this equilibrium is indeed the unique SPE of the competitive subgame.

[18] If both imitators enter simultaneously the entry game ends and players collect triopoly profits at every period.

We now determine the expected payoffs of the imitators. Imitator l, the follower imitator, enters at $t+1$ by buying knowledge at a zero price. His expected equilibrium payoff in period t is

$$\delta\Pi_3 \tag{1}$$

The expected payoff of imitator j depends on his mode of entry. If he enters the market by imitating, his expected payoff in period t is:

$$(1-\delta)\Pi_2 + \delta\Pi_3 - \kappa$$

as: (i) He obtains a flow of duopoly profits in period t; and: (ii) Since the no-delay equilibrium is played, imitator l immediately enters at period $t+1$ and thus imitator j's profits decrease to triopoly profits π_3 thereon. If he instead enters the market by buying knowledge, his expected payoff in period t is:

$$(1-\delta)\Pi_2 + \delta\Pi_3 - p_{sj} \tag{2}$$

The only distinction between the payoff of imitator j if he enters by imitating rather than by contracting resides in the entry cost. In particular, in neither case, does he expect to obtain future profits from selling knowledge. Hence, to determine the mode of entry of the imitators we need to examine the prices at which the inventor offers to sell knowledge. We thus turn our attention to the monopoly game. We prove that the inventor offers to sell knowledge to both imitators at prices less than or equal to the imitation cost. So, imitators always enter the market by buying knowledge from the inventor rather than by imitating.

Lemma 2 *In the monopoly game, the inventor offers knowledge to both imitators at prices* $p_{sg} \leq \kappa$ *for* $g \in \{j, l\}$.

The intuition is the following. The inventor can always do better by offering to sell knowledge to an extra imitator at a price equal to κ at every period than by offering to sell knowledge to only one of them. In this manner, she does not change the entry costs and thus the entry decision of the imitators (the previously excluded imitator could always enter by imitating and paying κ). However, she collects revenues from the execution of this knowledge contract when the imitator enters the market.[19]

We still need to establish the optimal price for knowledge. First, however, we summarize the payoffs of the imitators. If the leader imitator (the one who enters first and denoted by superscript 1), enters at t, then, according to equations (1), (2) and Lemma 2, the payoffs for him and the follower imitator, in period t, are

[19] The same idea applies to show that it is preferable to offer two contracts rather than no contract at all.

$$V_g^1 = (1-\delta)\Pi_2 + \delta\Pi_3 - p_{sg};\ V_g^2 = \delta\Pi_3 \tag{3}$$

for $g \in \{j,l\}$. If both imitators enter simultaneously, payoffs in period t are

$$V_g^b = \Pi_3 - p_{sg} \tag{4}$$

Lemma 2 and some properties of these payoffs lead to our main results. Lemma 2 allows us to view the imitators as choosing a time period at which to buy knowledge. So our reduced form game closely resembles a 'traditional' timing game. The main difference is, however, that the inventor, by choosing the price of knowledge affects the equilibrium structure of the imitators' payoffs and hence the nature of the timing game.

The Main Result

Now we characterize the equilibrium price of knowledge and the distribution of entry times of the imitators. Our analysis is formalized by examining the equilibrium of a sequence of games of period length Δ that shrinks to zero.[20] Proposition 2 presents one of the main results of this paper.

Proposition 2 *As Δ shrinks to zero, there exists a unique symmetric MPE in which: (i) The inventor sets a price for knowledge $p_{sg} = \kappa$ for $g \in \{j,l\}$; (ii) The equilibrium distribution of entry times of each imitator converges to an exponential distribution with hazard rate equal to $\lambda = r(\Pi_3 - \kappa)/\kappa$; and (iii) The inventor's equilibrium expected payoff is $V_s = \mu(\kappa)\Pi_1 + (1-\mu(\kappa))(\Pi_3 + \kappa)$; where $\mu(\kappa) := r/(r+2\lambda) \in (0,1)$.*

Result (ii) shows that potential imitators wait a random length of time before entering the market. The intuition is as follows. According to result (i), the price of the knowledge sold through the first contract equals the imitation cost, κ. After the first sale, however, the equilibrium price of knowledge drops to zero due to competition in the knowledge market. Thus, as Δ shrinks to zero, the payoff of the follower imitator becomes strictly greater than the payoff of the leader. As a result, both players have an incentive to delay their entry. Delay is, however, costly as both imitators sacrifice current market profits. Thus, in equilibrium, potential imitators choose their entry times randomly (i.e., play a behavioral strategy) and

[20] There are however two degenerate, asymmetric pure strategy equilibrium in which imitator l (j) never enters before j (l) and where imitator j (l) enters at date zero. Nonetheless, as an extensive previous literature that has been concerned with similar issues, (see, for instance, Bolton and Farrel (1990) and some of the references cited there), we believe that asymmetric pure strategy equilibria are both implausible and unsuitable to examine imitation in a *decentralized* market environment.

result (ii) indicates that the limiting distribution is exponential with hazard rate equal to λ.[21]

The hazard rate, λ, must be interpreted as the instantaneous entry rate of each imitator at any point in time. In a behavioral equilibrium, the benefit and cost, for each imitator, of waiting an infinitesimal amount of time to enter must be the same. The cost of waiting corresponds to the lost payoff during that infinitesimal amount of time: $r\left(\Pi_3 - \kappa\right)$. The benefit of waiting is the avoided entry cost were his rival to enter first: $\lambda\kappa$. So, we obtain the expression reported in result (ii): $\lambda = r\left(\Pi_3 - \kappa\right)/\kappa$.

It naturally follows from result (ii) that the inventor will hold a monopoly in the market for a random time period until the time of the first entry. The expected length of this monopoly time equals $(2\lambda)^{-1}$. Indeed, since imitators use behavioral strategies, the time of the first sale, $t_1 := \min\{t_i, t_j\}$, is a random variable. Moreover, t_1 is exponentially distributed with a hazard rate equal to 2λ and thus with expectation equal to $(2\lambda)^{-1}$.[22]

Although our game resembles a classical timing game, the speed of entry is actually endogenously determined by the pricing decision of the inventor. Result (i) sheds light on this aspect. It describes the optimal pricing decision of the inventor, in other words the choice of p_{sg} that maximizes her expected payoff given the equilibrium strategies of the imitators.[23] If she sets a unique price p_s, the equilibrium hazard rate is $\lambda(p_s) = r\left(\Pi_3 - p_s\right)/p_s$ and the expected payoff for the inventor can be written as

$$V_s\left(p_s\right) = \mu\left(p_s\right)\Pi_1 + \left(1 - \mu\left(p_s\right)\right)\left(\Pi_3 + p_s\right)$$

In this synthetic form, the above expression shows that the price of the knowledge (sold through the first contract) p_s has several effects. First, a higher price of knowledge raises the revenues that the inventor collects when she sells it to the first buyer. Second, a higher price for knowledge increases $\mu\left(p_s\right)$, the discounted expected fraction of monopoly time. Indeed, as p_s increases, the benefit of waiting to enter increases and the imitators delay their entry even more. There is nevertheless a third countervailing effect: As $\mu\left(p_s\right)$ increases and the imitators delay their entry times, the revenues from selling knowledge and the triopoly profits are obtained later, potentially decreasing the present value of the expected payoff. But because monopoly profits are larger than triopoly profits plus the largest revenue that

[21]This is a typical result in war of attrition games. War of Attrition games were first introduced by Maynard Smith (1974). See, for example, Fudenberg and Tirole (1991) for a formal definition of a war of attrition game and Hendricks et. al. (1988) for a full characterization.

[22]The minimum of two independent exponentially distributed random variables, t_i and t_j, is also exponentially distributed with parameter $(\lambda_i + \lambda_j)$. So its expectation is $(\lambda_i + \lambda_j)^{-1}$.

[23]Note that she optimally sets the same price for both contracts and thus induces the two imitators to play the same strategy.

the inventor may receive from selling knowledge, this third effect is always dominated by the second one. Thus, the optimal choice for the inventor is to set the maximum price that imitators will accept: The imitation cost.

The essential message of Proposition 2 is that, when knowledge can be traded, potential imitators wait to enter the market and the innovator collects monopoly profits for a random time period.

On The Optimality of Transferable Contracts

In Proposition 2 only transferable contracts were considered. This is restrictive since the inventor might prefer to sell knowledge through non-transferable contracts that prevent competition in the knowledge market. However, we show that the inventor always strictly prefers to trade knowledge through transferable rather than non-transferable contracts. We follow two steps. First, we obtain the inventor's payoff when knowledge is sold through non-transferable contracts. Second, we prove that this equilibrium reward is strictly smaller than the equilibrium reward reported in result (iii) of Proposition 2.

We focus on MPE. In the case of non-transferrable contracts, competition in the market for knowledge occurs only when an imitator enters by copying. The unique MPE of that competitive game is still the no-delay contracting equilibrium of Lemma 1. Next we examine the bilateral monopoly game that follows any history in which one of the imitators, say j, has bought, at period t, knowledge through a non-transferable contract. This continuation game exhibits a unique SPE.

Lemma 3 *In the unique SPE of the bilateral monopoly game that starts at period $t+1$, the inventor offers knowledge at a price $p_{sl}^{\tau} = \kappa$ for all periods $\tau \geq t+1$ and imitator l buys it immediately at period $t+1$.*

The inventor would like to promise the buyer to lower the price in the future to delay his entry into the market. This promise is, however, not credible as once that period comes, she has an incentive to keep the price high, and it is optimal for the buyer to accept such a high offer rather than to incur the imitation cost. Using the results of Proposition 2 and Lemma 3, we obtain our second main finding.

Proposition 3 *In the unique MPE when non-transferable contracts are used: (i) Both imitators enter at period $t = 0$ by buying knowledge at a price equal to κ; and: (ii) The inventor's equilibrium payoff $V_s^{\mathbf{n}} = \Pi_3 + 2\kappa$ is strictly smaller than her equilibrium payoff V_s when transferable contracts are used.*

As in the case without contracting (Proposition 1), the imitators perceive that their entry cost will remain fixed through time and thus they decide to enter at period zero. However, in the present case, the entry cost remains constant over time due to the non-transferability clauses contained in the contracts. When using non-transferable contracts, the inventor cannot commit to lower the price of knowledge in the future and so, from the point of view of the imitators, she replicates the same 'pricing' environment as if knowledge could not be traded. The inventor, however, obtains higher rents: She appropriates, in the form of contracting revenues, what before were lost imitation costs.

Result (ii) shows that the inventor always prefers to use transferable rather than non-transferable contracts. The intuition is as follows. Non-transferable contracts yield higher revenues (two contracts, instead of one, are sold at a price of κ) and, moreover, these revenues are received earlier (at period zero). However, the rents of the inventor are immediately reduced to triopoly profits. Transferable contracts, on the other hand, allow the inventor to commit to a lower future price of knowledge by introducing competition in the knowledge market. As a result, potential imitators delay their entry. Hence, Proposition 3 stresses that the extra profits due to the strategic delay in entry are larger than the lost contracting revenues.

It is important to understand this trade-off more formally. According to result (iii) of Proposition 2, the inventor's equilibrium reward is $V_s = \mu\Pi_1 + (1-\mu)(\Pi_3 + \kappa)$; for $\mu = r/(r+2\lambda)$. Her equilibrium reward when non-transferable contracts are used is $V_s^{\mathbf{n}} = \Pi_3 + 2\kappa$. Not forbidding resale is profitable if and only if $\mu(\Pi_1 - \Pi_3) > (1+\mu)\kappa$. That is, the extra expected discounted amount of money collected during her monopoly time, $\mu(\Pi_1 - \Pi_3)$, must be large enough to compensate her for the sum of: (i) The lost contracting revenues due to the fact that the price of knowledge is driven to zero due to competition: κ; and: (ii) The lost contracting revenues due to the imitators' delay: $\mu\kappa$.

But μ is determined by the equilibrium incentives of the imitators. In particular, the payoff corresponding to entering at any time in the support of their randomization must be equal to their expected payoffs if they follow their behavioral strategies. So: $\Pi_3 - \kappa = (1-\mu)\left(\Pi_3 - \frac{\kappa}{2}\right)$, where the left hand side is the payoff from buying knowledge at time zero and the right hand side is the expected payoff from playing their corresponding behavioral strategies. This equality implies that $(1+\mu)\kappa = 2\mu\Pi_3$, which in turns determine that transferability is optimal if and only if $\Pi_1 > 3\Pi_3$, which is satisfied by Assumption 0.[24]

Some empirical evidence suggests the importance of imposing less restrictive clauses when patent protection is weak. Anand and Khanna (2000) report the percentage of non-

[24] An alternative to non-transferrable contracts would be for the inventor to commit to a decreasing price schedule. The inventor would do weakly better. However, such commitments are not easy to put in place.

exclusive licenses signed in their sample of contracts.[25] For chemicals (mostly drugs in the sample), the percentage of non-exclusive licenses is 12.36%, for computers 28.48% and for electronics 30.35%. This evidence can be confronted to the data collected in the Carnegie Mellon Survey, reported by Cohen, Nelson and Walsh (2000), that asked managers about the effective mechanisms to appropriate returns from innovative activities. For drugs, 50% of managers reported that patents were effective, for computers 41% and for electronics 21%.[26] So the sectors least likely to use patents are also those in which non-exclusive licenses are most prevalent. Admittedly, exclusive licenses are not strictly equivalent to our non-transferrable knowledge contracts but they are similar in the sense that they slow the speed at which prices decrease in the future. This evidence therefore suggests that, in the spirit of our mechanism, contracts that impose less restrictive terms and don't prevent competition are beneficial for the inventor when patent protection is weak.

Sources of rents and Comparative Statics

Having established the optimality of transferable contracts, it follows, from Propositions 1 and 2, that the extra reward received by the inventor is

$$\begin{array}{rcccc} V_s - \Pi_3 & = & \mu(\kappa)\,(\Pi_1 - \Pi_3) & + & (1-\mu(\kappa))\,\kappa \\ & & \text{[Rewards from Delay]} & + & \text{[Revenues from Knowledge Sale]} \end{array} \tag{5}$$

The sources of rents come from both monopoly profits accumulated before entry of the first imitator and from contracting revenues at the entry date. We are of course mostly interested in the first source of rents.

We examine some comparative statics on these results. When κ increases, the expected duration of monopoly time and the rents of the inventor increase. As $\kappa \to \Pi_3$, the cost of waiting goes to zero and, in the limit, entry never happens: $\mu(\kappa) \to 1$. So, due to the possibility of trading knowledge, the inventor becomes a permanent monopolist but precisely because a trade occurs with an increasingly small probability on the equilibrium path. We note that this can be true for a very small imitation cost. If the imitation cost and triopoly profits are close and negligible, the innovator would obtain very small profits in the absence of knowledge trading but collects profits close to monopoly rents if contracting is possible.

Corollary 1 *In the unique limiting symmetric MPE: (i) The expected duration of monopoly time and the inventor's expected equilibrium payoff are strictly increasing in the imitation*

[25]See Table III (i) in their paper.
[26]See Table I in their paper.

cost, κ; and: (ii) The inventor's expected equilibrium payoff converges monotonically to the present value of monopoly profits, Π_1, as κ converges to Π_3.

We finish this discussion with an illustrative example. Consider a product that generates monopoly profits of $\Pi_1 = \$1M$ and triopoly profits of $\Pi_3 = \$0.1M$. We vary κ between 0 and Π_3. We present the results in the following table. In the first column, we report the expected duration of monopoly time. In the second, we report the expected discounted profits of the inventor derived from Result (iii) in Proposition 2. In the last three columns, we decompose the percentage contributions of the different revenue streams: (i) Percentage coming from monopoly profits before entry ($\mu(\kappa)\Pi_1$); (ii) Percentage coming from triopoly profits after entry ($(1-\mu(\kappa))\Pi_3$); and: (iii) Percentage obtained from the sale of knowledge ($(1-\mu(\kappa))\kappa$).

The results indicate that if the cost of reverse engineering the process is \$10000, the inventor expects to retain monopoly profits for half a year and overall to obtain profits of \$160000 (compared to \$100000 without trading of knowledge). If the cost of reverse engineering is \$30000 (resp \$50000), entry would be prevented on average for more than 2 (resp. 5 years) and the innovator would obtain profits of \$360000 (resp \$430000), more than three (resp. four) times what she would obtain in the absence of contracting. We see that even for relatively small imitation costs, the rents of the innovator can be quite substantial.

κ ($\$M$)	Dur. Mon. Time (years)	Discounted Profits of innovator ($\$M$)	% Before Entry	% After Entry	% Contracting Revenues
0.01	0.56	0.16	34	60	6
0.02	1.25	0.22	51	41	8
0.03	2.14	0.36	62	29	9
0.05	5	0.43	77	15	8
0.07	12	0.62	88	7	5
0.09	45	0.85	96	2	2
0.095	95	0.92	98	1	1
0.099	495	0.98	100	0	0

4. Large Number of Imitators

After having examined the essential economics of strategic delay in imitation and the value of transferable contracts, we extend the model of Section 3 to any number of potential imitators, $\mathbf{N} \geq 2$. To simplify the exposition, we directly focus on the continuous time version of our model and without loss of generality we assume that the inventor offers transferable contracts.[27]

For any number of active imitators $n \leq \mathbf{N}$, we posit that, in the product market, there is a unique level of equilibrium profit per firm denoted (in present value terms) by Π_{n+1} for $n = 0, 1...$We assume that $\{\Pi_{n+1}\}_{n=0}^{\infty}$ is a strictly decreasing sequence that converges to zero. Example 1 illustrates this idea.

Example 1. Consider a 'Cournot' market with $n+1$ active rivals, marginal cost equal to zero and (inverse) linear demand equal to $p = 1 - X$ if $X \leq 1$ and 0 otherwise. In this case there is a unique symmetric Nash equilibrium in which profits are $\Pi_{n+1} = (n+2)^{-2}$.

Assumption 1, imposed in the previous sections, ensured the profitability of imitation for the case of two potential imitators, $\mathbf{N} = 2$. However, for a number of potential imitators $\mathbf{N}$ sufficiently large, individual market profits $\Pi_{\mathbf{N}}$ will be smaller than any fixed imitation cost κ. To retain the spirit of Assumption 1, and the idea that imitation is ex-ante profitable for *all* potential imitators, we depart from the assumption of a fixed cost and we assume that the imitation cost, $\kappa_{\mathbf{N}}$, is also a decreasing function of the number of *potential* imitators.[28] Thus, in a similar vein to Assumption 1, we impose:

Assumption 2: $\forall \mathbf{N} \geq 2\colon\ (\Pi_{\mathbf{N}+1} - \kappa_{\mathbf{N}}) > 0$.

Under Assumption 2, Proposition 1 generalizes for any number of potential imitators $\mathbf{N}$: The unique SPE is such that (i) All potential imitators imitate immediately at $t = 0$; and: (ii) As $\mathbf{N} \to \infty$, the inventor's equilibrium payoff converges to zero. Thus, the inventor would not invest in research. This appears to be the perfect justification of the need for patent protection.

We discover that, as in the previous sections, the possibility of contracting affects this reasoning as shown in the following generalization of Proposition 2:

Proposition 4 *Under Assumption 2, there is a MPE such that: (i) The optimal price of knowledge is $p_{\mathbf{N}} = \kappa_{\mathbf{N}}$ for a given $\mathbf{N}$; (ii) The distribution of entry times of each imitator is exponential with hazard rate equal to $\lambda_{\mathbf{N}} = r\left(\Pi_{\mathbf{N}+1} - \kappa_{\mathbf{N}}\right) / \left(\mathbf{N}-1\right)\kappa_{\mathbf{N}}$; and (iii) The inventor's equilibrium payoff is $V_s = \mu_{\mathbf{N}}\Pi_1 + (1-\mu_{\mathbf{N}})\left[\kappa_{\mathbf{N}} + \Pi_{\mathbf{N}+1}\right]$; where $\mu_{\mathbf{N}} \equiv r/\left[r + \mathbf{N}\lambda_{\mathbf{N}}\right]$.*

[27] It is without a loss of generality that we analyze directly the continuous time version of our game since, as shown in the proof of Proposition 2, it corresponds to the limit of the discrete version when $\Delta \to 0$

[28] In a previous version we showed that when $\Pi_{\mathbf{N}+1} < \kappa_{\mathbf{N}}$ there is a SPE in which no imitator ever enters because after entry, competition on the knowledge market leads to full diffusion, and the profits $\Pi_{\mathbf{N}+1}$ obtained by the initial imitator are not sufficient to cover the imitation cost $\kappa_{\mathbf{N}}$.

Proposition 4 shows that, when contracting is possible, the inventor retains a positive equilibrium payoff since imitators strategically wait before entering the market. We now examine the limit results when the inventor faces a very large pool of potential imitators.

Note that, unavoidably, when $\mathbf{N}$ becomes large, both profits in the product market and the optimal price of knowledge become negligible. Hence, this suggests that when the number of potential imitators increases the inventor's rents dissipate. However this reasoning is correct only if the *length* of the delay in imitation remains small. In other words, the traditional held belief that innovative rents evaporate is true if and only if the discounted monopoly time, $\mu_{\mathbf{N}}$, goes to zero, when the pool of potential imitators grows large.

We show in the following proposition, that what matters is not the limit value of profits $\lim_{\mathbf{N}\to\infty}[\Pi_{\mathbf{N}+1}]$ but the speed of convergence or the ratio between these profits and the imitation cost at the limit. We impose an extra assumption that states that $\{\Pi_{n+1}\}_{n=0}^{\infty}$ and $\{\kappa_{\mathbf{N}}\}_{\mathbf{N}=2}^{\infty}$ are of the same order of magnitude at the limit.

ASSUMPTION 3: $\lim_{\mathbf{N}\to\infty}[\Pi_{\mathbf{N}+1}/\kappa_{\mathbf{N}}] = b$ for b *finite* and *weakly larger* than 1.

Proposition 5 reveals that the key determinant of the innovator's rents is the limit ratio b. Furthermore, it shows that in the case in which $b = 1$, the inventor's payoff converges to monopoly profits.

Proposition 5 *Let Assumptions 2 and 3 hold. As* $\mathbf{N} \to \infty$

$$V_{\mathbf{N}} \to \frac{r\Pi_1}{r + b - 1}$$

and if $b = 1$*, then* $V_{\mathbf{N}} \to \Pi_1$*.*

In our view, Proposition 5 fundamentally challenges the traditional belief that innovation cannot occur in a world with a large number of imitators. To shed light on the mechanism behind the result notice that the limiting discounted fraction of monopoly time is

$$\lim_{\mathbf{N}\to\infty} \mu_{\mathbf{N}} \equiv \lim_{\mathbf{N}\to\infty} \frac{r}{[r + \mathbf{N}\lambda_{\mathbf{N}}]} = \frac{r}{r + b - 1}$$

Thus, at the limit, the length of monopoly time depends on the expected date of the first sale (i.e, first entry) and hence on, what we loosely call, the limiting 'aggregate' hazard rate: $\lim_{\mathbf{N}\to\infty} \mathbf{N}\lambda_{\mathbf{N}}$. Proposition 5 shows then that as $\mathbf{N} \to \infty$, $\mathbf{N}\lambda_{\mathbf{N}} \to b - 1$. The intuition of this result is as follows.

First, as a direct consequence of Assumption 3, as $\mathbf{N} \to \infty$, the ratio $r\left(\Pi_{\mathbf{N}+1} - \kappa_{\mathbf{N}}\right)/\kappa_{\mathbf{N}}$ converges to $b - 1$. This, however, is *not* sufficient for $\mathbf{N}\lambda_{\mathbf{N}}$ to converge to a *finite* number.

However, as shown in Proposition 4, $\lambda_{\mathbf{N}}$ is divided by $\mathbf{N}-1$ and so overall, the whole expression, $\mathbf{N}\lambda_{\mathbf{N}}$, also converges to $b-1$. The fact that $\lambda_{\mathbf{N}}$ is divided by $\mathbf{N}-1$ clarifies the role that the free-riding effect between potential imitators has on entry times. The probability that each potential imitator has of obtaining a knowledge contract for free rises with the size of the pool of potential rivals, $\mathbf{N}-1$, as each potential imitator will receive the required knowledge for free if at least one of the other $\mathbf{N}-1$ potential imitators signs a contract with the inventor first. So, the incentives of each potential imitator to be the first in buying knowledge are dramatically diminished as $\mathbf{N}$ grows large. Even at the limit, non-negligible externalities among potential imitators are present in the equilibrium and the inventor exploits them to earn substantial rewards.

5. Subgame Perfect Equilibria: Further Results

In this section we answer the following question: Is the no-delay contracting equilibrium the unique SPE of the competitive game? A negative answer to this question reveals that the 'speed' at which the second contract determines the inventor's reward.

Uniqueness

To simplify, we use continuous time and, without loss of generality, we assume that knowledge is sold through transferable contracts. Competition to sell knowledge to the follower imitator (i.e., the second contract) starts instantaneously at time t, with the entry of the first imitator, say j. Let $t_2 \geq t$ be the time at which the second contract is sold and thus $d_2 \equiv (t_2 - t) \geq 0$ is the delay in trading the second contract. The no-delay contracting equilibrium, which is the unique MPE corresponds to $d_2 = 0$. We will see in this section that some SPE might actually be characterized by a strictly positive delay in trading the second contract. Note that, $\overline{d}_2$, characterized by $\Pi_3 - \kappa = \Pi_3 e^{-r\overline{d}_2}$, is the maximum length of time that imitator l is willing to wait to buy knowledge through the second contract at a zero price rather than imitating immediately at t. Proposition 6 describes the set of pure strategy SPE (outcomes) of the competitive game.

Proposition 6 *Consider the competitive game. Then: (i) If $2\Pi_3 > \Pi_2$ the no-delay contracting equilibrium is the unique SPE; and (ii) If $2\Pi_3 \leq \Pi_2$, for each $d_2 \in \left[0, \overline{d}_2\right]$ there exists a SPE in which imitator l enters the market at $t_2 = t + d_2$ by buying knowledge at a zero price.*

Result (i) shows that, for some environments, the unique SPE is the no-delay contracting equilibrium. For linear demand and Cournot competition, this condition is satisfied. The intuition is as follows. When $2\Pi_3 > \Pi_2$, the market is such that for any candidate equilibrium with delay in the knowledge market, a profitable deviation exists for one of the sellers. Indeed, suppose knowledge is believed to be sold at a price of zero with a delay $d > 0$. Selling knowledge to imitator l at the highest acceptable price, $\zeta\Pi_3$, allows the deviator to collect extra profits of $2\zeta\Pi_3$ rather than $\zeta\Pi_2$ in the market (where $\zeta = [1 - e^{-rt}]$).[29]

Result (ii) stresses that, if this condition is not satisfied, the competitive game might admit a multiplicity of SPE. In all of these SPE the price of knowledge sold through the second contract is zero. This is because the seller with the lowest price will serve the entire market and so each seller has an incentive to undercut his rival. However, all of these SPE are characterized by different delays to trade knowledge. An outcome in which the knowledge is sold through the second contract with a delay $0 \leq d_2 \leq \overline{d}_2$ is a SPE for two reasons. First, imitator l, given that $d_2 \leq \overline{d}_2$, (weakly) prefers to wait d_2 to buy knowledge through the second contract at a zero price rather than imitating immediately. Second, the sellers, when $2\Pi_3 \leq \Pi_2$, (weakly) prefer collecting duopoly profits rather than deviating and receiving the contracting revenues.

Delay in contracting and Appropriation

The existence of delay to trade knowledge through the second contract brings one conceptual novelty: The imitator who enters first (the leader) collects duopoly profits in the market up to the time of the second entry. It may become worthwhile, if the delay is sufficiently long, for imitators to become leaders rather than followers. To simplify the presentation of our next result, we define a critical time delay to trade knowledge through the second contract as $\widetilde{d}_2 \equiv r^{-1} \left[\ln\left(\Pi_2 / \left(\Pi_2 - \kappa\right)\right)\right]$ and note that $\widetilde{d}_2 \in \left(0, \overline{d}_2\right)$. Also for any $d_2 \in \left[0, \overline{d}_2\right]$, let $\alpha_2 \equiv (1 - e^{-rd_2})$. We obtain the following results

Proposition 7 *Suppose that* $2\Pi_3 \leq \Pi_2$. *Then: (i) If* $0 \leq d_2 < \widetilde{d}_2$, *the inventor offers to sell knowledge to both imitators at a price* $p_s = \kappa$; *the distribution of entry times for each imitator is exponential with hazard rate* $\lambda = r\left[(\Pi_3 - \kappa) + \alpha_2\left(\Pi_2 - \Pi_3\right)\right] / \left[\kappa - \alpha_2\Pi_2\right]$; *and the inventor's equilibrium payoff is* $V_s = \mu(\kappa)\Pi_1 + (1 - \mu(\kappa))\left[(\Pi_3 - \kappa) + \alpha_2\left(\Pi_2 - \Pi_3\right)\right]$, *where* $\mu(\kappa)$ *is defined as in Proposition 2; (ii) If* $\widetilde{d}_2 \leq d_2 < \overline{d}_2$, *as* $\Delta \to 0$, *the unique SPE outcome is that entry happens for sure at* $t = 0$; *the inventor offers to sell knowledge to both imitators at a price* $p_s = \kappa$; *and her equilibrium payoff is* $V_s^d = \phi V_s^{\mathbf{n}} + (1 - \phi)V_0$, *where* $\phi(\kappa, d_2)$ *is the probability of simultaneous entry at* $t = 0$, $V_s^{\mathbf{n}} \equiv 2\kappa + \Pi_3$; *and* $V_0 \equiv \left[\kappa + \alpha_2\Pi_2 + (1 - \alpha_2)\Pi_3\right]$.

[29] Note that this informal argument is valid if $\zeta\Pi_3 < \kappa$ but the result holds generally as shown in the proof of Proposition 6.

Proposition 7 proves that, depending on the length of the delay to trade knowledge through the second contract, imitators will follow very different strategies. Result (i) shows that when the second contract is traded relatively 'quickly', the dynamics of entry is still governed by a waiting process between the imitators. This result may be considered as a generalization of Proposition 2. The main difference is that the inventor now collects duopoly profits after the first imitator enters the market. However, we cannot conclude that the inventor strictly prefers this case to the case where $d_2 = 0$. Indeed, as d_2 increases, the equilibrium hazard rate of each imitator also increases and thus the inventor accumulates monopoly profits for a shorter time period.[30]

Result (ii) reveals a new aspect of the appropriability problem. It states that when the delay to exchange the second contract is 'too long' the dynamics of entry is completely reversed: Entry of at least one imitator takes place for sure at time $t = 0$. The game becomes a preemption game: Both imitators want to be leaders but they do not enter at date zero with probability one because the payoff from simultaneous entry is strictly smaller than the payoff from being the follower imitator.

Result (ii) also specifies the equilibrium payoff of the inventor in this second case. It can be shown that her equilibrium payoff is strictly smaller than the equilibrium payoff that she obtains when $d_2 = 0 : V_s > V_s^d : \forall d_2 \in \left[\widetilde{d}_2, \overline{d}_2\right]$.[31] Proposition 7 can be summarized as follows. When potential imitators correctly believe that knowledge will be diffused slowly in the knowledge market, entry will be inevitably fast. Thus, if an appropriation failure is likely to exist, it is not caused by a lack of intellectual property rights per se but rather by a different kind of problem coming from a slow diffusion of knowledge.

6. Some Evidence

In this section we provide suggestive evidence which reveals both the importance of knowledge trading and the potential relevance of the appropriation mechanism highlighted in this paper. In our theoretical exercise we underlined the existence of a natural and intuitive market force which generates innovative rents in the absence of patent protection. Finding empirical evidence of such forces is not an immediate exercise since patents are so prevalent

[30]In which SPE is the inventor's reward the highest? Several offsetting forces makes it difficult to answer this question. The higher d_2, the longer the period for which she retains duopoly profits after the first entry. Also, the higher d_2, the higher λ and so duopoly profits and contracting revenues are received earlier. However, the first entry occurs, on average, earlier and the inventor obtains monopoly rents for a shorter period.

[31]Result (ii) indicates that the reward of the inventor is a convex combination of $V_s^{\mathbf{n}}$, the same reward as that of Proposition 3, and V_0, the reward that she obtains when only one of the imitators enters at $t = 0$ and the other follows after a delay of at least $\widetilde{d}_2$. From Proposition 3 we know that: $V_s > V_s^{\mathbf{n}}$. Besides, direct calculations show that $V_s > V_0$ for all values of $d_2 \in \left[\widetilde{d}_2, \overline{d}_2\right]$. So $V_s > V_s^d : \forall d_2 \in \left[\widetilde{d}_2, \overline{d}_2\right]$.

in many industrial sectors of developed countries. We nevertheless identify two classes of situations where our mechanism is most likely to play a key role. First, our mechanism might play an important role in countries where intellectual property protection is weak or non-existent. Second, for those countries in which legal protection is stronger, our mechanism may rationalize the use of secrecy which does not shield inventors from reverse engineering by imitators.[32]

We start by briefly discussing the evidence which suggests the existence of an active knowledge market. There appears to exist a robust market for technological know-how that does *not* involve patents (see Contractor (1985), Rostoker (1983) and Bessen, (2005)). According to the European Commission, pure know-how licensing agreements, that involve secret information, are playing an increasingly key role in the transfer of technology (Harris, (1997)). Moreover, as Gallini (2002) has also pointed out, components of technical know-how absent from patent applications are often transferred through licensing contracts in high technology industries, such as software and biotechnology (See, also Arora, 2002).

Although these types of contracts appear to be relevant in practice, detailed data is much scarcer than the data on licensing agreements, for which more extensive information on contract terms and timing is available. Thus we initially resort to indirect evidence which suggest that our mechanism could be important in practice. Two very influential surveys (Yale Survey (1983) and Carnegie Mellon Survey (1994)) have asked managers to rank the most effective means of protecting their innovations. The Yale Survey, conducted in 1983, reports that for both product and process innovations, secrecy was consistently ranked as one of the worst methods to protect an innovation. The Carnegie Mellon Survey, conducted ten years later, reports, on the contrary, that secrecy was consistently ranked first. As Cohen et.al. (2000) point out, there is no apparent explanation for the 'growth in the importance of secrecy as an appropriability mechanism'. This fact is particularly surprising, since the period between 1983 and 1994 was one in which patent protection tended to strengthen.

We believe that the mechanism highlighted in this paper offers a potential explanation for this puzzle. Indeed, a firm choosing secrecy is not protected against reverse engineering by rivals.[33] However, as emphasized in this paper, in the presence of active markets for knowledge, imitators might delay entry. Thus, more active knowledge markets might increase the relative returns that firms can expect from choosing secrecy. Did the markets for knowledge indeed increase in importance during that period?

[32] An inventor can choose trade secret as an alternative to patenting. She is thus protected against illegal copying of the invention (such as corrupting an employee) but not against reverse engineering. However, unlike in the case of patenting, she does not need to reveal the details of the invention.

[33] In the case of secrecy, reverse engineering is allowed although obviously paying the competitor's employees for the secret is illegal.

As pointed out earlier, there is no systematic data on contracts to transfer knowledge or know-how in the absence of intellectual property rights. There is however more systematic information about licensing markets. We believe that the trading activity in the licensing market (agreements to use patent rights) and in what we call the knowledge market (agreements to exchange knowledge services, designs, codes, etc.) should be positively correlated. An indication that our belief is well founded can be found in Arora et. al. (2002) who states that licensing contracts, based on patents, often also include transfer of know how, such as sending teams to explain the technology to the buyer. Licensing activity did indeed intensify in the period 1983 to 1994. Arora et.al. (2002) using data compiled by the Securities Data Company report that the total number of disclosed licensing deals during the period 1985 to 1989 was 1130 while for the years 1993 and 1994, 2009 and 2426 deals were signed respectively.[34] If the increase in licensing activity is a good indicator of an increase in trading of knowledge, our model provides a convincing theoretical foundation for the apparent puzzle of the rise in popularity of secrecy.

To find more specific anecdotal evidence of the importance of our mechanism, it is natural to search for case studies in countries in which patent protection is weak or even non-existent. Until the recent TRIPS agreements, India did not grant product patents but process patents. This had important practical consequences, for instance, for the pharmaceutical sector: It meant that Indian generic producers could reverse engineer drugs from western big pharma companies and sell them in India as long as they used a slightly different process of production. Lanjow (1998) notes that the average delay between the date of world introduction by the inventor and the date of introduction in India is between 3 to 5 years.[35] Although the author mentions that this unusually long delay might be due to regulatory deferrals, some circumstantial evidence suggests that a mechanism such as the one described in this paper could also be at play.

Consider the story of the compound oseltamivir (marketed by Roche as the famous Tamiflu). It was approved in the US in 1999. In October 2005, the Indian generic producer Cipla announced a plan to begin manufacture of generic oseltamivir without a license from Roche.[36] In December 2005, presumably as a response, Roche granted a sub license to another Indian generic manufacturer, Hetero Drugs, for the production of osteltamivir. A similar pattern is observed for the production of HIV antiretroviral drugs in South Africa. In February 2001, Cipla announced plans to sell aids antiretroviral drugs to sub Saharan Africa and in September 2001 GSK, the patent owner, granted rights free of charge to Aspen, a

[34]The Securities Data Company contains data on licensing deals and joint ventures

[35]This data is obtained from a sample of 'blockbuster drugs' marketed in 1993.

[36]Even though this occurred after the TRIPS agreement was signed, the production of oseltamivir by Cipla was allowed by an Indian court that judged that the patent was not infringed.

local generic producer. In both cases, Hetero Drugs and Aspen obtained the knowledge at a smaller cost than Cipla and one reason for the delayed entry could be that, for some period of time, both generic producers preferred to wait, hoping that their rival would move first.[37]

These examples and the data on delay presented in Lanjow (1998) correspond to well-known successful drugs. However more examples of less popular drugs, or other inventions, may also exist. The difficulty is that these cases are not well documented. In general, we believe that our theoretical work opens the way for further empirical analysis and could provide a framework to conduct it.

Finally, note that our mechanism highlights an important and subtle connection between secrecy and lead time, two appropriation mechanisms that might not actually be independent as the formulation of the Yale and Carnegie Mellon surveys suggests.[38] When inventors do not patent their technologies and imitators strategically delay their entry, an endogenous lead time emerges as the equilibrium outcome. So our findings also shed light on a potential interesting relationship between secrecy and lead time.

7. Related Literature

Our results show that there exists previously ignored sources of rents for inventors even in the absence of patent protection. Our paper is thus connected with a literature that argues in favor of weakening intellectual property rights (Boldrin and Levine (2002, 2005, 2008a) and Maurer and Scotchmer (2002)). Our intended goal is to depart in a minimal way from the 'conventional' model that justifies patent protection, by only introducing the possibility of trading knowledge.

In Boldrin and Levine (2002, 2005, 2008a), an inventor creates an initial prototype which can be used for both producing additional copies of the innovation and also for consumption. The inventor competes directly with the buyers of her copies. She can therefore sell the initial prototype at a price that reflects the future utility and revenue stream that it will generate.[39] We highlight, in particular, an important difference between our and their approach. We clearly separate two markets: The product market, from which the rents of the inventor come from, and the market for knowledge. In our model, the inventor does not compete with her own consumers but rather with potential imitators who might 'steal' the inventor's

[37] We point out that one feature of the model is not reflected in this example but does not matter for the results: The first imitator, Cipla, entered in both cases by imitating and not by contracting with the innovator. Note however that our model would yield exactly the same results if the innovator was assumed not to offer a contract before the entry of at least one imitator; the cost of entry would still fall from κ (cost of imitating) to zero after the first entry due to competition on the market for knowledge.

[38] Both Arundel (2001) and Cohen et al (2000) found that in the majority of manufacturing industries innovators considered secrecy and lead time as the crucial appropriation mechanisms.

[39] This price is positive and potentially substantial due to the initial scarcity of the innovated commodity.

customers. So, the inventor uses the dynamics of the trading of knowledge to delay the entry of potential imitators and remain a monopolist for a longer time period.

Maurer and Scotchmer (2002) also indirectly argue for a change in the patent system by analyzing the effects of introducing independent invention defence. The authors show, in an static model, that the inventor can use licensing to deter duplication if the imitation cost is sufficiently high. We focus, instead, on the dynamics of knowledge trading and show that relatively 'small' imitation costs can lead to a permanent monopoly position for the inventor.[40] Moreover, our results stress the importance of using transferable contracts as a tool that provides credibility to future price reductions.

The mechanism we study leads to delay in imitation. Other explanations for the existence of such delays have also been proposed.[41] Benoit (1985) shows that a unique imitator might want to delay entry on the market if the profitability of her innovation is uncertain and gradually revealed over time. In Choi (1998) endogenous delay also occurs as a consequence of the strategic interaction between imitators in a context of patent infringement suits brought by the patent holder.[42] Note also that in static frameworks, some papers have shown that licensing can serve as a barrier to entry (Gallini (1984) and Rockett (1990)). In these papers, licensing is used to deter the development of a superior technology and to crowd out the market to prevent entry of superior rivals.

The market for knowledge is also the focus of Muto (1986). This paper shares with ours the focus on resale of information. However there are several major differences. First, in Muto (1986), an important restriction is imposed: If no sale occurs in a period, the game ends. This removes the possibility of waiting to enter. Second, imitation is not a possibility in that paper. As a consequence, naturally, it is optimal to restrict resale, both in terms of profits of the monopolist and in terms of diffusion rates.

Selling ideas in the absence of patent protection is a subtle issue as Anton and Yao (1994, 2002) have underlined. If buyers cannot assess the value of an idea they may be reluctant to pay the sellers, but when the idea is revealed buyers might potentially steal it. Anton and Yao (1994, 2002) propose subtle solutions to that problem. In our environment, the innovation is already commercialized in the market and its success is publicly observable.

[40] In contrast, the condition in Maurer and Scotchmer (2002) states that the imitation cost needs to be more than half the initial invention cost.

[41] Scherer and Ross (1980) suggest that technological constraints generate 'natural lags' in imitation. This explanation does not depend on the strategic responses of imitators.

[42] Bernheim (1984) also examines the dynamics of entry deterrence. The dynamics is however very different than in our model. In particular, Bernheim (1984) assumes that entrants are ordered in an exogenously given sequence. Arora and Fosfuri (2003) highlights that it may be optimal for a firm to license out its technology to a rival. This paper shares some similarities with what we have called the competitive game: The trade-off considered in that paper is comparable to the one that guarantees uniqueness of the no-delay contracting equilibrium in the competitive game.

So, we believe that asymmetries of information are minimal and the concerns studied by Anton and Yao (1994, 2002) can be left aside.

Last, we highlight a technical point. Our game is shown to be a timing game and in certain circumstances corresponds to a war of attrition (Maynard Smith (1974), Fudenberg and Tirole (1991), Hendricks et. al. (1988)). However, in our context, the speed of entry is not exogenously given by the parameters of the game but is endogenously controlled by the pricing decisions of the inventor. We thus solve simultaneously for the entry rate in a war of attrition and the pricing decision of an inventor facing multiple imitators.

8. Concluding Remarks

The main goal of this paper was to study natural market forces that protect rents of innovators faced with easy imitation. We show that the introduction of a market for knowledge fundamentally affects the traditional view on the need for patent protection. Even in the absence of such protection, imitators strategically delay their entry and the inventor accumulates monopoly profits for a random time period. In essence, we have shown that potential knowledge trades between inventors and imitators serve as good substitutes for patents in terms of guaranteeing rents for the inventor.

To examine the appropriability problem in the presence of knowledge trading and revisit the conventional wisdom on intellectual property rights, we built the simplest possible model. We made abstraction of certain issues which could be the object of future research. We mention two of those. First, we assumed that the imitation cost is commonly known. It would be interesting to presume that imitators have private information about their imitation costs and to examine how this affects the inventor's equilibrium payoff. Second, our model does not consider the possibility of sequential invention. Contracting in that case might provide not only knowledge to reproduce the current innovation but also to discover future improvements.

We believe, however, that the theory constructed in this paper opens a central question for further research: Without intellectual property rights, do inventors appropriate an equilibrium reward equal to the social value of their contribution (see Makowski and Ostroy (1995))? Shapiro (2007) has argued that under the current patent system inventors receive private rewards that exceed their social contributions. Could the dynamic pricing of knowledge appropriately tailor these rents to their social contributions?

Appendix

Proof of Proposition 1. A pure strategy for imitator g prescribes, at each t, whether to imitate i_g or to wait w_g. Consider the decision problem following a history in which only one imitator, say l, has imitated at period $t-1$. Then: j's unique best response is to imitate at t. By imitating at any $t_j \geq t$ he obtains a payoff in period t of $V_j(t_j) = \delta^{(t_j-t)}(\Pi_3 - \kappa)$ and $t = \arg\max_{t_j \geq t} V_j(t_j)$. We now turn our attention to those histories starting at t and in which no imitator has imitated yet. Given the symmetry of the game, we study j's best response to the following two strategies of l. First, suppose that l's strategy is i_l at t. Then: j's unique best response is i_j at t. This follows because if he waits until $t_j \geq t$ he receives a payoff in period t equal to $V_j(t_j) = \delta^{(t_j-t)}(\Pi_3 - \kappa)$ and $t = \arg\max_{t_j \geq t} V_j(t_j)$. Second, suppose that l's strategy dictates to wait until $t_l > t$. Then: j's unique best response is also i_j at t. To see this, recall that we showed that if j imitates at $t_j < t_l$, l's best response is to imitate at $t_j + 1$. Thus, if j chooses i_j at t_j such that: $t \leq t_j < t_l$ he obtains a payoff in period t of $V_j(t) = \delta^{(t_j-t)}[(1-\delta)\Pi_2 + \delta\Pi_3 - \kappa]$. But if he chooses i_j at $t_j \geq t_l$ he receives a payoff in period t of $V_j(t) = \delta^{(t_j-t)}[\Pi_3 - \kappa]$. Comparing these payoffs, it follows that j's unique best response is to imitate at t.

So, this analysis reveals that there is a unique SPE in which the imitators choose to imitate immediately at all periods in which imitation has not occurred yet. Thus, the unique equilibrium outcome is both imitators choosing to imitate at period zero and the equilibrium payoff for the inventor and the imitators are Π_3 and $\Pi_3 - \kappa$ respectively. $\square$

Proof of Lemma 1. A (Markovian) strategy for the sellers (j and s) specifies a price at which they offer knowledge through a contract (hereafter, a contract) if l has not entered yet. Imitator l's decision rule dictates whether to enter or not and how to enter as a function of the prices. We follow a sequence of steps. In step 1, we show that the unique pair of stationary prices that can be part of an equilibrium is $p_{sl} = p_{jl} = 0$ as claimed in Lemma 1. In step 2, we show that the buyer has also a unique best response and thus that the no-delay contracting equilibrium is the unique MPE.

Step 1. First: l will enter for sure. This follows since he can imitate immediately at $t+1$ and obtain a payoff in period t of $\delta(\Pi_3 - \kappa) > 0$. Second: Entry will occur by contracting. Suppose it were not the case. Then one of the sellers (j or s) might deviate, at the time of entry, and offer a contract at a price equal to κ that would be accepted by l. This deviation is payoff profitable since: (i) It increases the contracting revenues of the seller from zero to a strictly positive amount; and: (ii) It does not affect the present value of the market profits of the deviant seller as entry would have taken place at that period anyway. Thus we can rule out as equilibrium candidates any pair of stationary prices above κ. Furthermore, any pair of stationary prices at which one or both sellers get positive contracting revenues is not immune to a profitable deviation. At least one of them might decrease his/her price and increase his/her contracting revenues without affecting the present value of his/her market profits; and: (iii) Any pair of stationary prices at which one seller sets a zero price and the other a positive price is not resistant to a profitable deviation by the lowest price seller. Indeed, he or she can find a higher price to increase his or her contracting revenue without affecting his or her market profits. Thus the unique pair of stationary prices that can be part of an equilibrium is $p_{sl} = p_{jl} = 0$ (Bertrand outcome).

STEP 2. We show that the equilibrium strategy of the buyer is unique. Say that l observes a pair of prices $\{p_{sl}, p_{jl}\}$ different from $p_{sl} = p_{jl} = 0$. Let $m \equiv j, s$ denote the seller who offers the minimal price and $p_m \equiv \min_{m \in \{j,s\}} p_{ml}$. His unique best response is: contract c_{lm} if $p_m \leq \min[\kappa, (1-\delta)\Pi_3]$; imitate i_l if $\kappa \leq \min[p_m, (1-\delta)\Pi_3]$; and wait w_l if $\min[p_m, \kappa] > (1-\delta)\Pi_3$. These strategies follow naturally from the fact that by waiting one period, the imitator can get the contract at a zero price next period but abandons triopoly profits during the current period. Hence, the buyer's decision rule and the price offers at any period $p_{sl} = p_{jl} = 0$ constitute the unique MPE. $\square$

Proof of Lemma 2. A (Markovian) contracting strategy specifies a time-independent pair of prices $p_{sg} \in [0, \infty]$ for $g \in \{j, l\}$ if entry has not happened yet. An optimal contracting strategy must be inclusive: Two knowledge contracts must be offered at prices $p_{sg} \leq \kappa$ for $g \in \{j, l\}$. We prove this lemma by showing that for any (Markovian) contracting strategy which excludes an imitator there exists an inclusive contracting strategy that performs (weakly) better for the inventor.

Consider, first, a strategy which excludes both imitators: Prices are $p_{sg} = \infty$ for $g \in \{j, l\}$. Then, the inventor prefers the contracting strategy in which two contracts are offered at $p_{sg} = \kappa$ for $g \in \{j, l\}$. To see this, observe that:

1. For the imitators both strategies are payoff equivalent since: (i) Their mode of entry, contracting or imitating, does not affect the profits that they collect in the market; (ii) Under both strategies, their entry cost is equal to κ if they enter first. (They can enter by imitating in the first case); and: (iii) The price of knowledge for the second contract will be equal to zero since competition is assured after the first entry.

2. The inventor, however, prefers the second (inclusive) contracting strategy since: (i) The entry times of the imitators have not been affected and so the present value of her market profits are the same under both contracting strategies; and: (ii) When entry happens, it takes places through contracting and the inventor receives strictly positive contracting revenues.

It follows that the same type of argument applies to show that there exists an inclusive strategy that the inventor prefers to a contracting strategy which excludes only one of the imitators. $\square$

Proof of Proposition 2. We follow a number of steps. Step 1 shows that when Δ (length of the period) is small enough there exists a unique symmetric MPE in which the imitators use behavioral strategies. Step 2 obtains the limiting equilibrium distribution as

$\Delta \to 0$. Step 3 determines the inventor's expected payoff for a pair of prices p_{sj} and p_{sl}. Last, step 4 shows that the optimal prices are $p_{sj} = p_{sl} = \kappa$.

STEP 1. If $\delta \geq \delta_u \equiv (\Pi_2 - \kappa)/\Pi_2$ and if $\min_{g\in\{j,l\}} \{p_{sg}\} \in \mathbb{I}(\delta) = (p_\delta, \kappa]$, for $p_\delta \equiv (1-\delta)\Pi_2$ there exists a unique symmetric MPE in which the imitators use behavioral strategies.

If $\min_{g\in\{j,l\}} \{p_{sg}\} > p_\delta$ then for $g \in \{j, l\}$: (i) $V_g^2 - V_g^1 > 0$; (ii) $V_g^2 - V_g^b > 0$ and; (iii) $\delta^t V_g^1$ is strictly decreasing in t.[43] Thus our game closely resembles a war of attrition. Note that $\delta \geq \delta_u$ guarantees that $p_\delta \leq \kappa$ and thus $\mathbb{I}(\delta) \neq \emptyset$.

Notation: $p \equiv (p_{sj}, p_{sl}) \in \mathbb{I}(\delta) \times \mathbb{I}(\delta)$ denotes a given price list offered by s. Let $\psi_g : \mathbb{I}(\delta) \times \mathbb{I}(\delta) \to [0, 1]$ be the probability function used by imitator g at each period t when deciding whether to buy knowledge (a contract from now on) or not, conditional on the game having reached that period.

Definitions: Let $Q_j(\psi_l) \equiv \psi_l V_j^b + (1-\psi_l)V_j^1$ be the value for j of buying a contract in the current period and let $W_j(\psi_l) \equiv \psi_l V_j^2 + (1-\psi_l)\delta Q_j(\psi_l)$ be the value for j of buying a contract in the next period. Thus, the net value of waiting a period is: $\mathcal{W}_j(\psi_l) \equiv W_j(\psi_l) - Q_j(\psi_l)$.

A necessary condition for an equilibrium in behavioral strategies to exist is:

$$\mathcal{W}_j(\psi_l) = \psi_l(V_j^2 - V_j^b) + (1 - \psi_l)(\delta Q_j(\psi_l) - V_j^1) = 0$$

Existence: $\mathcal{W}_j(\psi_l) = 0$ has at least one solution $\psi_l^* \in (0, 1)$. First, note that: $\mathcal{W}_j(0) = \delta Q_j(0) - V_j^1 = -V_j^1(1 - \delta) < 0$. Second, observe that: $\mathcal{W}_j(1) > 0$ as $\mathcal{W}_j(1) = (V_j^2 - V_j^b) = p_{sj} - \Pi_3(1 - \delta) > 0 \iff p_{sj} > (1 - \delta)\Pi_3$ and by assumption $p_{sj} > p_\delta > (1 - \delta)\Pi_3$. So, since $\mathcal{W}_j(0) < 0$, $\mathcal{W}_j(1) > 0$ and $\mathcal{W}_j$ is continuous, it follows that $\mathcal{W}_j(\psi_l) = 0$ has at least one solution. By symmetry $\mathcal{W}_l(\psi_j) = 0$ also has at least a solution. This condition is also sufficient for the existence of a perfect equilibrium: Because of the stationarity of the payoffs all continuation games that start after a history in which no imitator has entered yet are isomorphic.

Uniqueness: It is sufficient to show that $\mathcal{W}_j(\psi_l)$ is strictly increasing on $[0, 1]$. As $\mathcal{W}_j$ and Q_j are C^1 functions

$$\mathcal{W}_j^l \equiv d\mathcal{W}_j/d\psi_l = (V_j^2 - V_j^b) - (\delta Q_j - V_j^1) + (1 - \psi_l)\delta(dQ_j/d\psi_l)$$

Using the definition of Q_j, it follows that: $\mathcal{W}_j^l = (V_j^2 - V_j^b) + V_j^1(1-\delta) + \delta(V_j^1 - V_j^b)(2\psi_l - 1)$.

So a sufficient condition for $\mathcal{W}_j$ to be strictly increasing on $[0, 1]$ is: $\beta(1 - \delta)(1 - 2\delta) + \delta p_{sj} > 0$; where this last inequality is derived from our formulas for V_j^1, V_j^2, V_j^b in the main text and where $\beta \equiv \Pi_2 - \Pi_3$. Then our maintained assumption in step 1 $\inf p_{sj} = p_\delta \equiv (1-\delta)$

[43] We remind the reader that V_g^1 is the payoff of the leader, V_g^2 of the follower and V_g^b of simultaneous entry

can be used to show that the previous inequality is satisfied for all $\delta \geq \delta_u$. Thus, $\mathcal{W}_j(\psi_l)$ is strictly increasing on $[0,1]$. By symmetry, the same arguments apply to j.

STEP 2. As $\Delta \to 0$, the limiting distribution of entry times of each imitator is exponential with hazard rate $\lambda_j = r(\Pi_3 - p_{sl})/p_{sl}$ and $\lambda_l = r(\Pi_3 - p_{sj})/p_{sj}$.[44]

For simplicity, we do not index our variables by Δ. We prove the result for l. Note that $\mathcal{W}_j(\psi_l) = 0$ can be rearranged as

$$\mathcal{W}_j(\psi_l) := a_j\psi_l^2 + b_j\psi_l + c_j = 0$$

for $a_j \equiv \delta(V_j^1 - V_j^b) = \beta(1-\delta) > 0$; $b_j \equiv (V_j^2 - V_j^b) + \delta(V_j^b - V_j^1) + (1-\delta)V_j^1 > 0$ and $c_j \equiv -(1-\delta)V_j^1 < 0$. The unique candidate solution is

$$\psi_l^* = \frac{-b_j + \sqrt{b_j^2 - 4a_jc_j}}{2a_j} = \frac{(z_j - b_j)}{2a_j} = \frac{(z_j - b_j)}{2a_j}\frac{(z_j + b_j)}{(z_j + b_j)}$$

At t, conditional on no imitator having entered before, we examine $\lim_{\Delta\downarrow 0}[\psi_l^*/\Delta]$. Note that $\psi_l^* = (\gamma_1/\gamma_2)$ where $\gamma_1 \equiv -2c_j$ and $\gamma_2 \equiv z_j + b_j$. As $\Delta \to 0$, $a_j \to 0$, $b_j \to p_{sj}$ and $c_j \to 0$. Thus $\gamma_1 \to 0$ and $\gamma_2 \to 2p_{sj}$ determining that $\psi_l^* \to 0$. Thus, by l'Hôpital's rule: $\lim_{\Delta\downarrow 0}[\psi_l^*/\Delta] = \lim_{\Delta\downarrow 0}\left[\frac{\gamma_1'\gamma_2 - \gamma_1\gamma_2'}{\gamma_2^2}\right]$ at $\Delta = 0$. Because at $\Delta = 0$: $\gamma_1' = 2r(\Pi_3 - p_{sj})$ and $\gamma_2' = -2r(\beta + p_{sj})$

$$\lambda_l := \lim_{\Delta\downarrow 0}\frac{\psi_l^*}{\Delta} = \frac{\gamma_1'}{\gamma_2} = \frac{r(\Pi_3 - p_{sj})}{p_{sj}}$$

A distribution has a constant hazard rate iff it is an exponential distribution. Thus, the cumulative distribution function (cdf) of entry times for l is $G_l(t) = 1 - e^{-t\lambda_l}$ for $t \in [0,\infty)$.

STEP 3. Recall that $p \equiv (p_{sj}, p_{sl})$. Then, the inventor's expected payoff is

$$V_s(p) = \frac{r\Pi_1}{(r + \lambda_j + \lambda_l)} + \frac{(\lambda_j + \lambda_l)\Pi_3}{(r + \lambda_j + \lambda_l)} + \frac{\lambda_j p_{sj} + \lambda_l p_{sl}}{(r + \lambda_j + \lambda_l)}$$

Her payoff depends on the time of the first sale, $t_1 \equiv \min\{t_j, t_l\}$, a random variable that takes values in $[0,\infty)$. Since t_j and t_l are independent rv with hazard rates λ_j and λ_l, t_1 has a hazard rate equal to $\lambda_j + \lambda_l$. As the second imitator enters almost instantaneously at time t_1, s obtains: (i) A flow of π_1 up to time t_1; (ii) A flow of π_3 from time t_1 on; and (iii) At time t_1, she receives either p_{sj} or p_{sl} depending on the identity of the first imitator. So, for a given t_1: $V_s(p;t_1) = \Pi_1[1 - e^{-rt_1}] + e^{-rt_1}\Pi_3 + e^{-rt_1}(\frac{\lambda_j}{\lambda_j+\lambda_l}p_{sj} + \frac{\lambda_l}{\lambda_j+\lambda_l}p_{sl})$.

Since t_j and t_l are exponentially distributed, t_1 is also exponentially distributed with

[44] Note that as $\delta \to 1$ $(\Delta \to 0)$ the conditions on δ imposed in Step 1 guaranteeing existence and uniqueness are automatically satisfied.

parameter $\lambda_j + \lambda_l$.[45] Thus, it follows that the inventor's expected payoff is the expression reported above.

STEP 4. The payoff maximizing knowledge prices are $p^*_{sj} = p^*_{sl} = \kappa$.

The inventor must choose a pair $\{p_{sj}, p_{sl}\}$ to maximize $V_s(p)$. The derivative of $V_s(p)$ with respect to p_{sj} is:

$$\frac{\partial V_s}{\partial p_{sj}} \equiv V^j_s = \frac{\lambda_j}{D} - \frac{1}{D^2}\frac{\partial \lambda_l}{\partial p_{sj}}\left[\pi_1 - \pi_3 - p_{sl}r - p_{sl}\lambda_j + \lambda_j p_{sj}\right]$$

where $D \equiv (r + \lambda_j + \lambda_l)$. Using, from step 2, the result for λ_j, we obtain: $V^j_s = \frac{\lambda_j}{D} - \frac{1}{D^2}\frac{\partial \lambda_l}{\partial p_{sj}}\left[\pi_1 - 2\pi_3 + \lambda_j p_{sj}\right]$. Since $\partial\lambda_l/\partial p_{sj} < 0$ and $\pi_1 > 2\pi_3$, it follows that V^j_s is strictly increasing in p_{sj} for all $p_{sl} > 0$. By symmetry, V^l_s is strictly increasing in p_{sl} for all $p_{sj} > 0$. So, $p^*_{sj} = p^*_{sl} = \kappa$.

To conclude: Result(i) is a direct consequence of step 4; Result (ii) is step 2 for the optimal contract prices $p^*_{sj} = p^*_{sl} = \kappa$; and finally Result (iii) is step 3 for $p^*_{sj} = p^*_{sl} = \kappa$. $\square$

Proof of Lemma 3. The strategy for the inventor dictates the price of knowledge at the beginning of each period $\tau \geq t+1$ for every feasible history (i.e., any history for which l has not entered yet), $p^\tau_{sl} : H^\tau \to [0, \infty]$. The strategy of l dictates, at each period $\tau \geq t+1$, whether to enter or not and how to enter, $d^\tau_l : H^\tau \times [0, \infty] \to \{i_l, c_l, w_l\}$. Any SPE in pure strategies must satisfy the following two properties:

P1 (Imitation never occurs). Imitator l enters the market by buying knowledge.

Suppose not and that he were to imitate at period $\tau \geq t+1$. Then the inventor would be strictly better off by selling a contract at any price that imitator l would accept. This implies that the inventor will never offer a contract at a price strictly higher than κ.

P2 (No Delay). Imitator l buys a contract immediately when it is offered if its price is equal or smaller than κ.

By P1 l never imitates in equilibrium. When a contract is offered at period τ at a price smaller or equal than κ he can either accept it or reject it and wait to accept a future offer. Rejecting a current offer is a best response only if l expects to obtain a higher payoff by accepting a future offer. However this is clearly impossible: At any time period at which the contract is sold, its price must be equal to κ. Otherwise s could increase its price and be strictly better off.

So, we have formally shown that the following pair of strategies constitute the unique SPE of the bilateral monopoly continuation game: $p^\tau_{sl} = \kappa$ for all $\tau \geq t+1$ at which l has not entered yet; and $d^\tau_l = c_l$ iff $p^\tau_{sl} \leq \kappa$ for all $\tau \geq t+1$ at which l has not entered yet; and last: $d^\tau_l = i_l$ if $p^\tau_{sl} > \kappa$ for all $\tau \geq t+1$ at which l has not entered yet. $\square$

[45] See, for instance, Proposition C.1, page 302, Marshall and Olkin (2007).

Proof of Proposition 3. A pure strategy for the inventor prescribes to offer, at the beginning of each period for which no imitator has entered yet, a pair of knowledge prices. (Only non-transferable contracts are offered). For all $t = 0, 1, ..$: $\sigma^t : H^t \rightarrow [0, \infty] \times [0, \infty]$. For the imitators, the strategy is a sequence of functions $\left\{e_g^t\right\}_{t=0}^{\infty}$ for $g \in \{j, l\}$ such that $e_g^t : H^t \times [0, \infty] \times [0, \infty] \rightarrow \{i_g, c_g, w_g\}$.

STEP 1. Any SPE must satisfy the following three properties:

P1 (Imitation never occurs). The imitators enter the market by buying knowledge.

The argument is the same as that of Lemma 2 but with an additional subtlety. No imitator has an incentive to deviate in this case, although when he deviates and enters by imitating, competition in the knowledge market follows. The crucial point to notice is that the no-delay contracting equilibrium of Lemma 1 characterizes the competitive game that ensues and the imitator who deviates makes zero profit in the knowledge market. Hence, for any offer such that the fee is less or equal than κ, imitators must enter by buying knowledge.

P2 (Simultaneous Entry). The imitators enter the market at the same time period.

Suppose it were not the case. Then one of them, say j, would enter at period $\tau \geq t$ and l at period $\widehat{\tau} > \tau$. By Lemma 3, in equilibrium, $\widehat{\tau} = \tau + 1$ and l's equilibrium payoff, in period τ units, would be $\delta\left(\Pi_3 - \kappa\right)$. However, by deviating and buying knowledge at period τ, l's worst payoff would be $\left(\Pi_3 - \kappa\right)$ which is strictly higher than $\delta\left(\Pi_3 - \kappa\right)$.

P3 (No Delay). Whenever the innovator offers contracts at prices smaller or equal than κ, the offers will be simultaneously accepted by the imitators.

By P1 and P2, in equilibrium, imitation never takes place and entry occurs simultaneously. Rejecting any current offer for a price equal or smaller than κ cannot be part of an equilibrium. The reason is that by rejecting current offers, the imitators postpone their entry and sacrifice current profits. But then the best offer they can expect from the inventor in the next period is a contract for a price of κ. Otherwise, the inventor could increase her price and be strictly better off.

P1, P2 and P3 together imply that, at any time period and for any history at which entry has not happened yet, the imitators will simultaneously and immediately accept to buy a contract at a price equal or smaller than κ and they will imitate iff the posted price is higher than κ. The unique best response of the inventor is to offer, at any time period and for any history at which a contract has not been bought yet, a pair of contracts at prices equal to κ. So, in the unique MPE, both imitators enter at period zero and the inventor's equilibrium payoff is $\Pi_3 + 2\kappa$.

STEP 2. In the case of non-transferable contracts the inventor's payoff is $V_s^{\mathbf{n}} = \Pi_3 + 2\kappa$. In the case of transferable contracts her payoff is $V_s = \frac{\pi_1}{r+2\lambda} + \frac{2\lambda}{r+2\lambda}\left(\Pi_3 + \kappa\right)$. Let $V \equiv V_s - V_s^{\mathbf{n}}$. Then $V > 0$ iff: $\Pi_1 - \Pi_3 > 2\kappa(r + \lambda)$. Using the expression for λ from Proposition 2 we find: $V > 0$ iff $\Pi_1 - 3\Pi_3 > 0$. This last inequality is satisfied by Assumption 0. □

Proof of Proposition 4. We prove this proposition in a number of steps. After entry of any number of imitators, we concentrate on the MPE such that all active players sell knowledge at a zero price to every inactive imitator. In such an equilibrium, after $n \in \mathbf{N}$ imitators enter the market, $\mathbf{N} - n$ contracts are sold instantaneously at a zero price to the remaining $\mathbf{N} - n$ imitators. Here, we concentrate on the monopoly subgame.

Step 1. The inventor offers to sell $\mathbf{N}$ contracts at a price $p_{s\mathbf{N}} \leq \kappa_{\mathbf{N}}$

This step is a generalization of the proof of Lemma 2. It follows the same argument: No imitator enters by imitating as the inventor would weakly increase his payoff by offering a contract to that imitator at a price equal to the imitation cost $\kappa_{\mathbf{N}}$. In other words, an optimal contracting strategy must be inclusive: The inventor must offer to sell knowledge to the $\mathbf{N}$ potential imitators. The implication of this step is that the leader imitator receives an instantaneous payoff in time t equal to $V_1 = \Pi_{\mathbf{N}+1} - p_{s\mathbf{N}}$.

Step 2. We focus on a symmetric Markov Perfect Equilibrium. Using Step 1we denote by $G_{\mathbf{N}}(t, p_{s\mathbf{N}}) : [0, \infty) \times [0, \kappa] \to [0, 1]$ the distribution function of entry times for the imitators. We assume momentarily that $G_{\mathbf{N}}(t, p_{s\mathbf{N}})$ has a density denoted by $g_{\mathbf{N}}(t, p_{s\mathbf{N}})$. Under this assumption, ties in the buying times can be safely ignored. Since we focus on a behavioral equilibrium, if the first sale of knowledge has not happened up to time t, the buyers must be indifferent between: (i) Buying knowledge at t; and (ii) Waiting dt extra units of time to buy knowledge at price $p_{s\mathbf{N}}$. This indifference condition requires that the opportunity cost of waiting dt extra units of time (MC) be exactly equal to the expected marginal benefit of waiting dt extra units of time (EMB).

The MC is the flow of profits that an imitator obtains if he is the leader at time t. Using Step 1 we have that $MC = rV_1 dt$. The MB is the increase in the payoff that an imitator receives if he is one of the followers rather than the leader. That is $MB = (V_2 - V_1)$. But an imitator receives $(V_2 - V_1)$ iff at least one of the other $\mathbf{N} - 1$ imitators enters first. Since: (i) We focus on a symmetric equilibrium and: (ii) Randomizations by the imitators are independent; it follows that: $EMB = (\mathbf{N} - 1) \lambda_{\mathbf{N}} (t, p_{s\mathbf{N}}) (V_2 - V_1)$ where $\lambda_{\mathbf{N}} (t, p_{s\mathbf{N}})$, the hazard rate, is defined as: $\lim_{\Delta \downarrow 0} \Pr(t < t_g \leq t + \Delta | t_g \geq t)/\Delta = g(t, p_{s\mathbf{N}}) / (1 - G(t, p_{s\mathbf{N}}))$.

In equilibrium: $EMB = (\mathbf{N} - 1) \lambda_{\mathbf{N}} (t, p_{s\mathbf{N}}) (V_2 - V_1) = rV_1 dt = MC$, implying that the distribution of equilibrium entry times is characterized by a constant hazard rate given by

$$\lambda_{\mathbf{N}} (p_{s\mathbf{N}}) \equiv \lambda_{\mathbf{N}} = \frac{rV_1}{(\mathbf{N} - 1)(V_2 - V_1)} = \frac{r(\Pi_{\mathbf{N}+1} - p_{s\mathbf{N}})}{(\mathbf{N} - 1) p_{s\mathbf{N}}}$$

And the cdf for the imitators is $G(p_{s\mathbf{N}}) = 1 - e^{-\lambda_{\mathbf{N}}(p_{s\mathbf{N}})t} : t \in [0, \infty)$ (A proof similar to that of Proposition 2, though longer, is available upon request)

STEP 3. We establish that $p_{s\mathbf{N}}^{*}=\kappa_{\mathbf{N}}$.

Let the time of the first entry be at $t_1 \in [0,\infty)$. Since the follower imitators enter the market with a zero delay, the inventor obtains: (i) A flow of π_1 up to time t_1; (ii) At time t_1, she receives $p_{s\mathbf{N}}$; and (iii) A flow of $\pi_{\mathbf{N}+1}$ from time t_1 on. Hence, her payoff is $V_{s\mathbf{N}}(p_{s\mathbf{N}};t_1) = \Pi_1\left[1-e^{-rt_1}\right]+e^{-rt_1}\left[p_{s\mathbf{N}}+\Pi_{\mathbf{N}+1}\right]$. Since the time of the first entry $t_1 := \min\{t_i,t_j,...t_{\mathbf{N}}\}$ has an exponential distribution with parameter equal to $\mathbf{N}\lambda_{\mathbf{N}}$, it follows that her expected payoff is $V_{s\mathbf{N}}(p_{s\mathbf{N}}) = \mu(\lambda_{\mathbf{N}})\Pi_1+(1-\mu(\lambda_{\mathbf{N}}))\left[p_{s\mathbf{N}}+\Pi_{\mathbf{N}+1}\right]$; where $\mu(\lambda_{\mathbf{N}}) := r/\left[r+\mathbf{N}\lambda_{\mathbf{N}}\right]$. Then: $\partial V_{s\mathbf{N}}(p_{s\mathbf{N}})/\partial p_{s\mathbf{N}} = (\partial\lambda_{\mathbf{N}}/\partial p_{s\mathbf{N}})(r\mathbf{N}/D^2)(p_{s\mathbf{N}}+\Pi_{\mathbf{N}+1}-\Pi_1)+(\mathbf{N}\lambda_{\mathbf{N}}/D)$; where $D\equiv(r+\mathbf{N}\lambda_{\mathbf{N}})$. Since by Assumption 2: $\forall\mathbf{N}\geq 2:(\Pi_{\mathbf{N}+1}-\kappa_{\mathbf{N}})>0$ and $p_{s\mathbf{N}}\leq\kappa_{\mathbf{N}}$: $\sup_{p_{s\mathbf{N}}}(p_{s\mathbf{N}}+\Pi_{\mathbf{N}+1}-\Pi_1)<2\Pi_{\mathbf{N}+1}-\Pi_1<0, \forall\mathbf{N}\geq 2$ by Assumption 0. Last $\partial\lambda_{\mathbf{N}}/\partial p_{s\mathbf{N}}<0$ ensues that $\partial V_{s\mathbf{N}}(p_{s\mathbf{N}})/\partial p_{s\mathbf{N}}>0$ for all $p_{s\mathbf{N}}\in(0,\kappa_{\mathbf{N}}]$ and so $p_{s\mathbf{N}}^{*}=\kappa_{\mathbf{N}}$ as stated.

STEP 4. It follows directly that

$$\lambda_{\mathbf{N}} = \frac{rV_1}{(\mathbf{N}-1)\left(V_2-V_1\right)} = \frac{r\left(\Pi_{\mathbf{N}+1}-\kappa_{\mathbf{N}}\right)}{(\mathbf{N}-1)\,\kappa_{\mathbf{N}}}$$

and that: $V_{\mathbf{N}} = \mu_{\mathbf{N}}\Pi_1+(1-\mu_{\mathbf{N}})\left[\kappa_{\mathbf{N}}+\Pi_{\mathbf{N}+1}\right]$ for $\mu_{\mathbf{N}}\equiv r/\left[r+\mathbf{N}\lambda_{\mathbf{N}}\right]$. □

Proof of Proposition 5. We know, from Proposition 4, that in equilibrium: $V_{\mathbf{N}} = \mu_{\mathbf{N}}\Pi_1+(1-\mu_{\mathbf{N}})(\kappa_{\mathbf{N}}+\Pi_{\mathbf{N}+1})$. So: (i) $\lim_{\mathbf{N}\to\infty}(\kappa_{\mathbf{N}}+\Pi_{\mathbf{N}+1})=0$ by Assumption 2; and (ii) $\lim_{\mathbf{N}\to\infty}\mu_{\mathbf{N}}=\frac{r}{r+b-1}$; since:

$$\lim_{\mathbf{N}\to\infty}\mathbf{N}\lambda_{\mathbf{N}} = \lim_{\mathbf{N}\to\infty}\frac{\mathbf{N}}{[\mathbf{N}-1]}\lim_{\mathbf{N}\to\infty}\frac{r\left[\Pi_{\mathbf{N}+1}-\kappa_{\mathbf{N}}\right]}{\kappa_{\mathbf{N}}} = b-1>0$$

because: $\lim_{\mathbf{N}\to\infty}\frac{\mathbf{N}}{[\mathbf{N}-1]}=1$ and: $\lim_{\mathbf{N}\to\infty}\frac{r[\Pi_{\mathbf{N}+1}-\kappa_{\mathbf{N}}]}{\kappa_{\mathbf{N}}}=b-1$ by Assumption 3. Then

$$\lim_{\mathbf{N}\to\infty}\mu_{\mathbf{N}} = \frac{r}{[r+\mathbf{N}\lambda_{\mathbf{N}}]} = \frac{r}{r+b-1}$$

Thus

$$\lim_{\mathbf{N}\to\infty}V_{\mathbf{N}} = \frac{r}{r+b-1}\Pi_1$$

and hence when $b=1$, it follows that $\lim_{\mathbf{N}\to\infty}V_{\mathbf{N}}=\Pi_1$. □

Proof of Proposition 6. We follow a sequence of steps.

STEP 1. After one of the imitators, say j, enters at time t, the other must also enter after a finite length of time by buying knowledge (a contract) at a zero price.

Imitator l must necessarily enter after some finite length of time because by imitating instantaneously at t he guarantees himself a payoff in time t units equal to $V_l=\Pi_3-\kappa>0$. Imitation, however, cannot occur in a SPE. To see this, suppose that imitation would take place in a SPE at time $t+\tau$ for a finite $\tau \geq 0$. Then one of the sellers (j or s) would deviate and offer a contract at a price equal to κ that would be accepted by l. This deviation would be payoff profitable since: (i) It would increases the contracting revenues of the seller from zero to a strictly positive amount; and: (ii) It would not affect the present value of the market profits of the deviant seller because entry were going to happen at time $t+\tau$ anyway. Last, observe that in any SPE, knowledge is sold to the second buyer (i.e., the second contract) at a zero price and both sellers must offer it at a zero price. To see this, suppose that the second contract were sold at time $t+\tau$ at a strictly positive price. Then one or maybe both sellers would make strictly positive expected profits. But then at least one of them might decrease the price and increase his contracting revenues without affecting the present value of his or her market profits. So when a contract is sold it must be sold at a zero price.

STEP 2. For any $d_2 \in \left[0, \overline{d}_2\right]$ there exists at least a SPE in pure strategies whose outcome is for l to enter the market at time $t_2 = t + d_2$ by buying a contract at a zero price iff $2\Pi_3 \leq \Pi_2$.

This step will prove both Results (i) and (ii). We show that the following strategies support the SPE outcomes described above. However, note that these strategies are *not* unique: They were chosen because of their simplicity. What remains unique is the equilibrium outcome delivered by these strategies.

The sellers (j and s) use the following symmetric pricing strategy: $p_l = \infty, \forall \tau < t_2$ and $p_l = 0, \forall \tau \geq t_2$. We now define the buyer's strategy. Let $m = j, s$ be the seller who offers the minimal price and $p_m \equiv \min_{m \in \{j,s\}} p_m$.

$$d_l = \begin{cases} c_{lr} & \text{if } p_m < \Pi_3 \left[1 - e^{-r(t_2 - t)}\right], \forall \tau < t_2 \\ w_l & \text{if } p_m \geq \Pi_3 \left[1 - e^{-r(t_2 - t)}\right], \forall \tau < t_2 \\ c_{lr} & \text{if } p_m \leq \min\left[\kappa, (1-\delta)\Pi_3\right], \forall \tau \geq t_2 \\ i_l & \text{if } \kappa \leq \min\left[p_m, (1-\delta)\Pi_3\right], \forall \tau \geq t_2 \\ w_l & \text{if } \min\left[p_m, \kappa\right] > (1-\delta)\Pi_3, \forall \tau \geq t_2 \end{cases}$$

We show that for any $d_2 \in \left[0, \overline{d}_2\right]$ these strategies form a SPE and give rise to the equilibrium outcomes described in result (ii). First: The decision function of the buyer and the pricing function of the sellers for $\tau \geq t_2$ are exactly the same as those of Lemma 1 and hence they constitute mutual best responses. We now show that these strategies are also best responses for $\tau < t_2$. By definition of $\overline{d}_2$, l does not have any incentive to deviate and imitate at any $\tau < t_2$. Suppose that one of the sellers deviates and proposes a price p_* at time $\tau < t_2$. If l accepts the offer, he obtains a payoff at time τ equal to $\Pi_3 - p_*$. By rejecting it, l expects a payoff of $e^{-r(t_2-\tau)}\Pi_3$. He therefore accepts the offer iff $p_* \leq \Pi_3 \left[1 - e^{-r(t_2-\tau)}\right]$ as prescribed by the above strategy.

Consider the pricing strategy of the sellers. Let $t_2 = t + d_2$. For $d_2 = 0$ we have already constructed in Lemma 1 the unique MPE. Let d_2 be such that $0 < d_2 \leq \overline{d}_2$. Then for a seller, say j, the best deviation at time $\tau < t_2$ is to offer a contract at a price equal to υ where $\upsilon = \Pi_3 \left[1 - e^{-r(t_2-\tau)}\right]$. The seller will not find such a deviation profitable if $\Pi_3 + \upsilon \leq \Pi_2 \left[1 - e^{-r(t_2-\tau)}\right] + e^{-r(t_2\tau)}\Pi_3$. That is if $2\Pi_3 \leq \Pi_2$. We have therefore shown the second part of the proposition.

To show uniqueness, suppose that $2\Pi_3 > \Pi_2$ and that the second contract is sold at time $t_2 > t$. According to Step 1, the equilibrium price must be zero. Besides we know that for any $\tau \in [t, t_2)$ imitator l will accept to pay at most υ for the second contract. But because $2\Pi_3 > \Pi_2$ offering a contract at a price υ is a profitable deviation for the sellers (see discussion in previous paragraph). Because our argument is valid for any value of $t_2 > t$ it follows that the unique SPE outcome, when $2\Pi_3 > \Pi_2$, corresponds to that of the no-delay equilibrium. □

Proof of Proposition 7. Time is continuous: $t \in [0, \infty)$. We denote by $G(t, p_s) : [0, \infty) \to [0, 1]$ the distribution function of entry times for the imitators. We assume momentarily that $G(t, p_s)$ has a density denoted by $g(t, p_s)$. Under these assumptions ties in the buying times can be safely ignored. To simplify, we also suppose that the inventor does not price discriminate between the imitators. The proof follows a number of steps.

STEP 1. If $d_2 < \widetilde{d}_2 = r^{-1} \ln(\Pi_2/(\Pi_2 - \kappa))$, the distribution of entry times of each imitator is exponential with hazard rate equal to $\lambda(p_s) = r\left[(\Pi_3 - p_s) + \alpha_2 (\Pi_2 - \Pi_3)\right] / \left[p_s - \alpha_2\Pi_2\right]$.

Notice that now the payoff to the leader imitator is $V_1 = \alpha_2\Pi_2 + (1 - \alpha_2)\Pi_3 - p_s$ and to the follower is $V_2 = (1 - \alpha_2)\Pi_3$; where $\alpha_2 \equiv (1 - e^{-rd_2})$. Hence $V_2 > V_1$ iff $\alpha_2\Pi_2 < p_s$. (Note that $\alpha_2\Pi_2 < \kappa$ when $d_2 < \widetilde{d}_2$). We will show in Step 2 that the optimal price chosen by the inventor is κ and thus the condition above is satisfied. For the moment, we assume that $V_2 > V_1$ when $d_2 < \widetilde{d}_2$.

As we focus on a behavioral equilibrium, at time t, if entry has not happened yet, imitators must be indifferent between: (i) Buying knowledge at t; and (ii) Waiting dt extra units of time before buying it. This condition requires that the opportunity cost of waiting dt extra units of time be exactly equal to the expected marginal benefit of waiting dt extra units of time. The opportunity cost is $MC = rV_1dt$. The marginal benefit is $MB(dt) = V_2 - V_1$. However, the marginal benefit is only received if the rival imitator enters first: an event that is determined by the hazard rate $\lambda(t, p_s) \equiv (g(t, p_s)/1 - G(t, p_s))$. So, in a SPE it must be that $\lambda(t, p_s)(V_2 - V_1) = rV_1dt$. Thus for any $d_2 < \widetilde{d}_2$ equilibrium entry times are characterized by a constant hazard rate (and thus by an exponential distribution) given by $\lambda(p_s) = r\left[(\Pi_3 - p_s) + \alpha_2 (\Pi_2 - \Pi_3)\right] / \left[p_s - \alpha_2\Pi_2\right]$.

STEP 2. The optimal knowledge price is $p_s^* = \kappa$.

Like in the proof of Proposition 2, assume that the time of the first sale occurred at

$t_1 \in [0, \infty)$. Since now the follower imitator buys knowledge (i.e., the second contract) with a delay equal to d_2 (i.e., enters at time t_2), the innovator obtains: (i) A flow of π_1 up to time t_1; (ii) At time t_1, she receives p_s; (iii) A flow of π_2 from time t_1 up to t_2; and last: (iv) A flow of π_3 from t_2 on. Hence, her payoff is $V_s(t, p_s, d_2) = \Pi_1 \left[1 - e^{-rt_1}\right] + e^{-rt_1}\left[p_s + \alpha_2\Pi_2 + (1-\alpha_2)\,\Pi_3\right]$. Because the time of the first sale $t_1 := \min\{t_l, t_j\}$ has an exponential distribution with parameter 2λ it follows that her expected payoff is $V_s(p_s, d_2) = \mu(\lambda(p_s))\Pi_1 + (1 - \mu(\lambda(p_s)))\left[p_s + \alpha_2\Pi_2 + (1-\alpha_2)\,\Pi_3\right]$; where $\mu(\lambda(p_s)) := r/\left(r + 2\lambda(p_s)\right)$. To conclude this step it suffices to show that the optimal price chosen by the inventor is equal to κ. To accomplish this, observe that the inventor chooses p_s to $\max V_s(p, d_\tau)$ subject to the hazard rate found in Step 1. Thus: $\partial V_s(p, d_2)/\partial p_s = -\left(2r/D^2\right)\left(\partial\lambda(p_s)/\partial p_s\right)\left(\Pi_1 - \mathcal{M}\right) + \left(2\lambda(p_s)/D\right)$; where $D \equiv r + 2\lambda(p_s)$ and $\mathcal{M} \equiv \left[p_s + \Pi_2 - \alpha_2\left(\Pi_2 - \Pi_3\right)\right]$. Since: (i) $\partial\lambda(p_s)/\partial p_s < 0$; (ii) $\arg\min_{p_s \in (0,\kappa]}(\Pi_1 - \mathcal{M}) = \kappa$; and (iii) The least upper bound of κ is Π_3, it is evident that $\inf_\kappa\left(\Pi_1 - \mathcal{M}\right) = \Pi_1 - \Pi_3 + \Pi_2 + \alpha_2\left(\Pi_2 - \Pi_3\right) > 0$. So, because $\partial V_s(p, d_2)/\partial p_s > 0 \forall p_s > 0$, s chooses $p_s^* = \kappa$ and her equilibrium payoff is obtained by replacing p_s^* into $V_s(p_s; d_2)$. Steps 1 and 2 prove result (i) of Proposition 7.

To prove result (ii) note that for preemption games the limiting distribution of discrete-time games (as Δ goes to zero) may not usually be expressed as an equilibrium in continuous time strategies of the kind used in Step 1. Because of this limitation, we use our discrete time model and then we compute the limiting distribution of the equilibrium outcomes (See, for example, Fudenberg and Tirole (1991)).

Step 3. We obtain for our discrete time model the unique symmetric SPE in behavior strategies.

The equilibrium condition is $\mathcal{W}(\psi) = \psi\left[V_2 - V_b\right] + (1-\psi)\left[\delta Q(\psi) - V_1\right] = 0$, for $Q(\psi) = \psi V_b + (1-\psi)V_1$. This quadratic equation has two roots: $\psi = \left(-b \pm \sqrt{b^2 - 4ac}\right)/2a$; where $a := \delta\left(V_1 - V_b\right) > 0$; $b := \left(V_2 - \delta V_1\right) + (1-\delta))\left(V_1 - V_b\right)$; and $c := -(1-\delta)V_1 < 0$. Since δ depends on Δ, it follows that as $\Delta \to 0$, $\psi \to \left(V_1 - V_2\right) \pm \left(V_2 - V_1\right)/\left[2\left(V_1 - V_b\right)\right]$. We consider the solution corresponding to the positive root $\psi = \left(V_1 - V_2\right)/\left(V_1 - V_b\right)$. By definition of V_1, V_2 and given that $V_b = \Pi_3 - p_s$ we have that $\psi = \left(\alpha_2\Pi_2 - p_s\right)/\alpha_2\left(\Pi_2 - \Pi_3\right)$. Note that $\psi \leq 1$ iff $V_2 > V_b$. Since we consider $\widetilde{d}_2 \leq d_2 \leq \overline{d}_2$, it follows that $V_2 > V_b$ iff p_s satisfies $\kappa\left(\Pi_3/\Pi_2\right) < p_s \leq \kappa$. Thus, at each period, each imitator plans to enter the market by buying knowledge with probability ψ conditional on not having entered before.

Step 4. Assuming that $V_2 > V_b$, we establish the limiting distribution of entry times as $\Delta \to 0$ and show that $p_s^* = \kappa$.

Fix any real time $t > 0$ and observe that the probability that no imitator will have bought a contract by time t is approximately equal to $(1-\psi)^{n(t,\Delta)}$ where $n(t, \Delta) \equiv (2t)/\Delta$ is the number of decision nodes between time 0 and time t when the real time length of a period is

Δ. As Δ goes to zero, $n(t,\Delta)$ increases without bound and hence this probability converges to zero. The conclusion is that at least an imitator will buy a contract and enter the market for sure at time $t=0$. Next, we obtain the probability of simultaneous entry at $t=0$. For that, consider that we must compute the probability of simultaneous entry conditional on the information that the event 'no imitator enters the market at $t=0$' has zero probability. Because both imitators entering simultaneously has probability ψ^2, we can conclude that the probability of simultaneous entry at $t=0$ is $\phi(p_s,d_2)=\psi/(2-\psi)$ and therefore the inventor's expected payoff is

$$V_s(p,d_2)=\phi(p_s,d_2)\left[2p_s+\Pi_3\right]+\left[1-\phi(p_s,d_2)\right]\left[p_s+\alpha_2\Pi_2+(1-\alpha_2)\Pi_3\right]$$

Observe that $\frac{\partial V_s(p,d_2)}{\partial p_s}=(\partial\phi/\partial p_s)\omega(p_s,d_2)+(1+\phi)$, where $\omega(p_s,d_2):=[p_s-\alpha_2(\Pi_2-\Pi_3)]$. We can then show that: $\partial\phi/\partial p_s=2(2-\psi)^{-2}\left[\partial\psi(p_s,d_2)/\partial p_s\right]$; where $\partial\psi(p_s,d_2)/\partial p_s=-1/\alpha_2(\Pi_2-\Pi_3)$ and $(1+\phi)=2/(2-\psi)$. Straightforward mathematical manipulations show that: $\partial V_s(p_s,d_2)/\partial p_s>0$ iff $-\omega(p_s,d_2)/\alpha_2(\Pi_2-\Pi_3)+(2-\psi)>0$. And after replacing ψ and $\omega(p_s,d_2)$ by their values, we finally obtain that: $\partial V_s(p_s,d_2)/\partial p_s>0$ iff $2\alpha_2(\Pi_2-\Pi_3)-\alpha_2\Pi_3>0$. Since by Assumption 0: $2\Pi_2>3\Pi_3:\partial V_s(p_s,d_2)/\partial p_s>0$ for all $\kappa(\Pi_3/\Pi_2)<p_s\leq\kappa$ and the optimal price is $p_s^*=\kappa$ validating our assumption that $V_2>V_b$. The equilibrium payoff for the inventor is directly computed by replacing p_s^* in $V_s(p_s,d_2)$. Thus Steps 3 and 4 complete the proof of Proposition 7. $\square$

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