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On the Political Economy of Adverse Selection

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Abstract

We consider a standard insurance economy where consumers are supposed to vote over menus of insurance contracts: A menu of contracts is majority stable if there does not exist another menu which is supported by an appropriate majority of consumers. We compute the smallest level of super majority for which there always exists a stable menu of contracts, and such that all stable menus of contracts are Pareto optimal. Lower super majority voting rules may ensure existence of stable menus if individual states and/or types of consumers are aggregated, but then stable menus of contracts need not be Pareto optimal: hence a trade-off between Pareto optimality and conservativeness of the voting rule is exhibited.

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1 Introduction

We consider a standard insurance economy à la Rothschild & Stiglitz (1976) with S individual states and I types of consumers, where types are characterized by their probability distributions over the individual states. The type of a consumer is private information. Since Rothschild & Stiglitz it is known that there need not exist competitive equilibria where (i) consumers choose between a menu of insurance contracts in order to maximize utility and (ii) firms produce insurance contracts in order to maximize profit. Moreover if equilibria exist then they need not be Pareto optimal relative to the available information (constrained Pareto optimal for short). The source of the market failure is discussed in Rothschild & Stiglitz and Prescott & Townsend (1984).

In order to get existence and optimality of equilibrium allocations different modifications of the market structure have been considered by Bisin & Gottardi (2000) and Rustichini & Siconolfi (2003). In both latter papers consumers declare their type (they may lie) and trade state-contingent goods *at type-dependent prices*. In Rustichini & Siconolfi there are no firms and a notion of weak equilibrium is introduced for which existence is proven; but weak equilibria need not be constrained Pareto optimal.

In Bisin & Gottardi the problem is modelled as a consumption externality that comes through the admissible consumption set: the set of feasible net-trades for consumers is constrained by incentive compatibility conditions and therefore by the net-trades of all types. The externalities are internalized through an expansion of the commodity space in the spirit of Arrow-Lindahl : on top of state contingent commodities, agents trade 'coasian' property rights on each other's consumption. Bisin & Gottardi show that equilibria exist and are all constrained optimal. Assuming a continuum of consumers in each type, the Arrow-Lindahl approach to the adverse selection problem is immune to the classical critic about the validity of assuming price-taking behavior on the markets for rights on external consumption. But a large number of such markets have to be created (actually $SI(I-1)$); moreover, since the set of admissible net-trades for a consumer depends on the net-trades of all types, information about the trades of all types needs to be publicly available.

The present note investigates how this minimal, yet important, complexity of the market mechanism needed to decentralize constrained Pareto-optimal allocations (in the presence of adverse selection) translates when the alternative route of a voting mechanism is followed. Here consumers are supposed to vote over menus of insurance contracts: A menu of contracts is stable if there does not exist another menu which is supported by an appropriate majority of the consumers. In general super majority voting rules are needed to ensure existence of stable menus of contracts and stable menus of contracts need not be Pareto optimal. However we find that when the rate of super majority is high enough (i.e., when I contracts with S states are being offered, a rate larger than $1 - 1/I$), then

a stable menu exists and all stable menus are Pareto optimal.

Lower super majority voting rules may ensure existence of stable menus if individual states and/or types of consumers are aggregated (i.e., when there are $K(\leq I)$ menus of contracts insuring $T(\leq S)$ groups of individual states, the rate decreases to $1-1/KT$), but at the expense of Pareto optimality. Hence the tradeoff between Pareto optimality and trading possibilities exhibited in Bisin & Gottardi and Rustichini & Siconolfi is reflected here in a tradeoff between Pareto optimality and the conservativeness (level) of the needed super majority voting rule: the 'price' to pay for the first welfare theorem is either the construction of $SI(I-1)$ new markets for trading external consumptions or the establishment of a conservative rate of super majority. The higher the number of missing markets, the higher the needed rate of super majority, a finding that was already made in Tvede & Crès (2005) in the case of incomplete markets.

The paper is organized as follows: In Section 2 the model is outlined; in Section 3 two slightly different notions of equilibria are introduced; finally Section 4 discusses the optimality properties of equilibria.

2 Setup

Consider a standard insurance economy: There is a finite set of individual states $\mathcal{S} = \{1,\ldots,S\}$; a consumer in state s has the endowment ω^s. There is a finite number of types of consumers $\mathcal{I} = \{1,\ldots,I\}$ and a continuum of each type. The fraction of consumers of type i is e_i, where $e_i > 0$ and $\sum_i e_i = 1$. The type of a consumer is private information.

Consumers have the same state utility function $u : \mathbb{R}_+ \to \mathbb{R}$, so each type is characterized by a probability distribution $\pi_i = (\pi_i^1,\ldots,\pi_i^S)$ over the set of individual states, where $\pi_i^s > 0$ and $\sum_s \pi_i^s = 1$. The fraction of consumers of type i who are in state s is assumed to be π_i^s. The consumers are supposed to maximize their expected utility, so the utility of a consumer of type i choosing the insurance contract $x = (x^1,\ldots,x^S)$ (where the dividend in state s is non-negative: $x^s \geq 0$) is $\sum_s \pi_i^s u(x^s)$. Dividends of contracts x^s are supposed to belong to a compact set $C \subset \mathbb{R}_+$, and the full insurance contract $x_f = \sum_s \omega^s \sum_i e_i \pi_i^s$ belongs to C.

Both states and types of consumers may be aggregated in order to reduce the complexity of menus of contracts. Let $\mathcal{P}_\mathcal{S} = \{\mathcal{S}_1,\ldots,\mathcal{S}_T\}$ be a partition of $\mathcal{S}$, and suppose that there is a menu x of K insurance contracts: $x = (x_1,\ldots,x_K)$, where $x_k = (x_k^1,\ldots,x_k^T)$ and $x_k^t \in C$ for all k and t. Contracts are exclusive in the sense that consumers may hold one and only one contract. Furthermore suppose that there is a partition of $\mathcal{I}$: $\mathcal{P}_\mathcal{I} = \{\mathcal{I}_1,\ldots,\mathcal{I}_K\}$ such that contracts are *incentive compatible* in the sense that con-

sumers in $\mathcal{I}_k$ weakly prefer contract k, so for all $i \in \mathcal{I}_k$ and k'

$$U_i(x_k) \;=\; \sum_t \sum_{s \in \mathcal{S}_t} \pi_i^s u(x_k^t) \;\geq\; \sum_t \sum_{s \in \mathcal{S}_t} \pi_i^s u(x_{k'}^t) \;=\; U_i(x_{k'})$$

and that the contracts are *feasible*, so

$$\sum_k \sum_t x_k^t \sum_{i \in \mathcal{I}_k} e_i \sum_{s \in \mathcal{S}_t} \pi_i^s \;\leq\; \sum_s \omega^s \sum_i e_i \pi_i^s.$$

For a partition of states and a partition of types $\mathcal{P} = (\mathcal{P}_\mathcal{S}, \mathcal{P}_\mathcal{I})$ and two menus of contracts x and x', let $\mathcal{I}(x, x', \mathcal{P}) \subset \mathcal{I}$ be the set of types which prefer x to x', so

$$\mathcal{I}(x, x', \mathcal{P}) \;=\; \cup_k \{i \in \mathcal{I}_k | U_i(x_k) > U_i(x'_k)\}\;;$$

and let $\rho(x, x', \mathcal{P}) \in [0, 1]$ denote the fraction of consumers who prefer x to x', so

$$\rho(x, x', \mathcal{P}) \;=\; \sum_{i \in \mathcal{I}(x, x', \mathcal{P})} e_i\;.$$

3 Equilibrium

We first consider an equilibrium notion where the relative support for different menus of contracts matter, so if x and x' are compared and the support for x' is sufficiently large compared with the support for x, then x' defeats x, even if the support for x' is small compared with the total population.

Definition 1 *A menu of contracts x is a* **relative δ-majority stable equilibrium** *for $\mathcal{P}$ if:*

- *x is incentive compatible and feasible, and;*
- *for all incentive compatible and feasible menus of contracts x'*

$$\rho(x', x, \mathcal{P}) \;\leq\; \delta \rho(x, x', \mathcal{P}).$$

Theorem 1 *Suppose that*

$$\delta \;>\; \min\left\{KT - 1, \max_i \frac{1}{e_i} - 1\right\}.$$

Then there exists a relative δ-majority stable equilibrium for $\mathcal{P}$.

Proof: Contracts are in C^{KT}, but clearly if a menu of incentive compatible contracts satisfies the feasibility contraint with ">", then there exists another menu of incentive compatible contracts such that all types of consumers are better off. Therefore the relevant set of contracts $F \subset C^{KT}$ is

$$F \;=\; \{x \in C^{KT} | \textstyle\sum_k \sum_t x_k^t \sum_{i \in \mathcal{I}_k} e_i \sum_{s \in \mathcal{S}_t} \pi_i^s \;=\; \sum_s \omega^s \sum_i e_i \pi_i^s\}.$$

The dimension of F is equal to or less than $KT - 1$.

Artificial agents are introduced: incentive compatibility agents (ic-agents for short). Let x_f be the menu of identical full insurance contracts, so $x_{fk}^s = \sum_i e_i \sum_{s'} \pi_i^{s'} \omega^{s'}$ for all k and s. The preference correspondence of the identical ic-agents $P_{ic} : F \to F$ is defined as follows:

$$P_{ic}(x) \;=\; \begin{cases} \emptyset & \text{for } x \text{ incentive compatible} \\ \text{int } B(x_f, \|x - x_f\|) \cap F & \text{otherwise,} \end{cases}$$

where int $B(x_f, \|x - x_f\|)$ is the open ball with center x_f and radius $\|x - x_f\|$. Since the set of incentive compatible contracts is closed, the graph of P_{ic} is open. Then, according to Theorem 3 in Greenberg (1979), there exists a δ-relative equilibrium for the extended economy as soon as $\delta > KT - 1$ (see Greenberg (1979) for a definition of a δ-relative equilibrium).

Let e_{ic} be the 'number' of ic-agents. Consider the extended economy with consumers and artificial agents, so the total 'number' of agents in the extended economy is $\sum_i e_i + e_{ic} = 1 + e_{ic}$. Take $e_{ic} > KT - 1$, then the δ-relative equilibrium for the extended economy is incentive compatible, so it is a relative δ-majority stable equilibrium for $\mathcal{P}$ for the insurance economy.

Clearly, if some consumer prefers x' to x, then all consumers of the same type prefer x' to x. Therefore if x is a relative δ-majority stable equilibrium for $\delta > KT - 1$, then x is also a relative δ-majority stable equilibrium for $\delta > \max_i 1/e_i - 1$.

Q.E.D

Next, we consider an equilibrium notion where the absolute support for different contracts matters, so if x and x' are compared and the support for x' is sufficiently large compared with the total population, then x' defeats x.

Definition 2 *A menu of contracts x is a* **ρ-majority stable equilibrium** *for $\mathcal{P}$ if:*

- *x is incentive compatible and feasible, and;*
- *for all incentive compatible and feasible menus of contracts x'*

$$\rho(x', x, \mathcal{P}) \;\leq\; \rho.$$

Corollary 1 *Suppose that*

$$\rho \ > \ \min\left\{1 - \frac{1}{KT}, \max_i 1 - e_i\right\}.$$

Then there exists a ρ-majority stable equilibrium for $\mathcal{P}$.

4 Optimality of equilibria

Clearly the fact that types of consumers is private information should be taken into account in the notion of Pareto optimality.

Definition 3 *A menu of incentive compatible and feasible contracts x is* **constrained Pareto optimal** *if there is no other menu of incentive compatible and feasible contracts x' where $x' = (x'_1, \ldots, x'_I)$ and $x'_i = (x'^1_i, \ldots, x'^S_i)$, such that no type of consumers is worse off and at least one type of consumers is better off, so $U_i(x'_i) \geq U_i(x_i)$ for all i with "$>$" for at least one i.*

If neither individual states nor types of consumers are aggregated, then equilibrium menus of contracts are constrained Pareto optimal. Intuitively: if x is not constrained Pareto optimal, then there exists a menu of contracts x' such that all types are better off with x' than with x, and agents would unanismously support the change from x to x'.

Corollary 2 *If neither states nor types of consumers are aggregated, so $T = S$ and $K = I$, then all equilibria are constrained Pareto optimal.*

Reciprocally, if x is constrained Pareto optimal, then for all other menus of incentive compatible and feasible contracts x' at least one type is better off with x than with x'; therefore x is ρ-majority stable as soon as $\rho \ > \ \max_i 1 - e_i$.

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