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Wage Regimes, Accumulation and Finance Constraints: Keynesian Unemployment Revisited

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Abstract

This paper presents a sequential model suited to analyze transitions between equilibria. Disequilibrium dynamics are obtained from a standard monopolistic competition model, by introducing a sequential structure and reasonable hypotheses about technology, finance constraints, expectation formation, and the wage setting mechanism. The response to shocks crucially depends on the institutional features of the economy, and on the monetary policy stance. In particular, some degree of wage stickiness proves necessary to avoid explosive paths. This feature of the model makes it a good candidate for the reappraisal of Keynes’ arguments on wages and unemployment.

Keywords: Disequilibrium, Keynesian Economics, Fix Price Models, Time to Build

JEL Codes: C63, D43, E12, E22, E24

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1 Introduction

This paper develops a sequential macroeconomic model that allows to study the characteristics of transitions between equilibria. Most of the times, economic analysis is carried out by comparison of steady states (or of regular growth paths). Transitional (or out-of-steady-state) dynamics are thus neglected because inherently short term phenomena, and because they are pre-determined from the beginning, and as such not particularly informative (think for instance of the saddle path adjustment in standard growth theory). The main point of this paper, instead, is that the transition may be interesting to the economist because, far from being pre-determined, it is shaped by the interaction of agent’s behaviour, institutional factors, and environmental characteristics. Furthermore, during these transitions disequilibria appear in at least some markets, and adjustments may have permanent effects. This approach, while it constitutes no substitute of standard equilibrium analysis, may complement it and give interesting insights on both theoretical and empirical grounds, while delivering policy prescriptions.

The baseline model is a standard monopolistic competition model, similar to those developed by the New-Keynesian literature (e.g. Woodford 2003), in which I introduce four different hypotheses:

- Production takes time, and is often characterized by complementarity rather than substitutability in the factors. The final goods sector uses capital (produced one period earlier) and labour, with a Leontief production function; further assumptions make this representation analytically equivalent to vertically integrated sector, in which labor is inputted first in construction and then in utilization of productive capacity (as in Hicks, 1973).

- Agents have bounded rationality, especially when facing complex environments.

- No variable may move instantaneously. In particular, I reverse the speed of adjustment of prices and quantities in response to disequilibria. As in the early Postkeynesians (Clower, 1965; Leijonhufvud, 1968) or in temporary equilibrium models (Hicks, 1939; Malinvaud, 1977; Benassy, 1982), prices only adjust between periods. In addition, in this model ex-ante disequilibria (i.e. inconsistency of agent’s plans) within the period are eliminated by rationing and stock accumulation rather than by price adjustments.

- Finally, agents are constrained, in their transactions, by financial availability. This sort of cash-in-advance constraint emerges because markets open sequentially.

These hypotheses have all been extensively used in the literature. But, to the best of my knowledge, never jointly. It is easy to see why they are relevant when we are interested in the transition between equilibria: each period begins with state variables determined in the previous one, and with imbalances that constrain agents in their subsequent decisions. Expectations and the structure of productive capacity further link the periods in a sequence. As a consequence, it is impossible to consider each period in isolation, as for example in the temporary equilibrium literature. A shock (no matter of what type) disrupts the coordination between agents and between phases of production that characterizes equilibrium. The “success” of the subsequent transition lies in the ability of the system in recovering coordination; and in this respect, both the role of monetary policy and the institutional wage setting environment prove crucial. In particular, it will turn out that some degree of wage stickiness may be desirable as a means for dampening disequilibrium fluctuations.
This paper mainly relates to three streams of literature. The first is the New Keynesian literature that is progressively becoming the reference model, especially for the analysis of monetary policy (Gali, 2002; Woodford, 2003). I share with these models the reference to a monopolistic competition sticky prices environment, even if I have no reference to forward looking Calvo (1983) pricing; more importantly, my focus is on disequilibrium dynamics, whereas these papers see fluctuations as equilibrium (technology induced) phenomena like in the RBC literature. I share with the temporary equilibrium literature the fix-price hypothesis, and the possibility of rationing; but by releasing the hypothesis of full rationality, in my model I have to deal with the appearance of unsold stocks\(^1\), that enrich the model by linking the periods in a sequence, even if at the price of notable complication. Finally the stream I am most related to is the Neoaustrian approach, developed by Hicks (1970, 1973) and Amendola and Gaffard (1998). Like them I emphasize the temporal articulation of production, and the irreversibilities linked mainly (but not only) to capital accumulation. On the other hand, this paper abandons the Hicksian representation of technology, and substitutes it with a standard CES production function and time to build. In general, I make an attempt to provide microfoundations to some of the Neoaustrian hypotheses, and to relate this type of work to the mainstream literature.

The outline of the paper is as follows: section 2 will lay down the benchmark model; the following section will show that each taken in isolation, complementarity, adaptive behaviour, time to build and a temporary equilibrium structure do not affect the behaviour of the model other than in the short run. Section 4, introduces the hypotheses jointly, together with the intrainperiod sequencing of events. Section 5 describes, by means of simulations, the behaviour of the economy when hit by nominal or real shocks. The analysis focuses in particular on the most appropriate wage regime and monetary policy stance. Finally, section 6 shows how this model may be appropriate to reappraise Keynes’ intuition about wages and unemployment, and concludes.

2 The Benchmark Model

Consider an economy in which each of \(N\) final goods is produced by a monopolist, using capital and labour hired on perfectly competitive markets. Money is the numeraire, and is used for transactions.

Consumers The \(H\) consumers maximize a Dixit-Stiglitz (1977) utility function in leisure and in the basket of goods. As their choice is made within each period, and there is no savings, the time subscript \(t\) can be omitted.

\[
\max_{\ell,y} u_h = \ell_h^{1-a} V_h \quad 0 \leq a \leq 1, \quad h = 1...H
\]

\[
s.t. \quad V_h = \left( \sum_{n=1}^{N} y_{n,h}^{(b-1)/b} \right)^{b/(b-1)} \quad b > 1
\]

\[
\sum_{n=1}^{N} p_n y_{n,h} + w \ell_h = I_h
\]

\(^1\)Bounded rationality is not the only reason why stocks could appear. Another one could be missing information due for example to stochastic rationing (I owe this remark to Gerd Weinrich).
where \( \ell_h \equiv 1 - \ell_h^* \). Assuming that the endowment of labour is 1, total per capita income is equal to \( I_h \equiv (w + d) \), where \( d \) is dividend to be defined below. Maximization yields

\[
y_{n,h} = a \left( \frac{p_n}{P} \right)^{-b} I_h \quad \text{with} \quad P^{1-b} = \sum_n p_n^{1-b} \tag{2a}
\]

\[
\ell_h^* \equiv 1 - \ell_h = 1 - \frac{(1-a)I_h}{w} \tag{2b}
\]

**Production**  
The production function and the associated cost function are CES:

\[
y_n = \left[ (\alpha_l l_y, n)^{\sigma} + \left( \alpha_k k_n \right)^{\sigma} \right]^{1/\sigma} \quad n = 1, ..., N \tag{3a}
\]

\[
c(y_n, w, p_k) = y_n \left[ \left( \frac{w}{\alpha_l} \right)^{\sigma} + \left( \frac{p_k}{\alpha_k} \right)^{\sigma} \right]^{1/\sigma} \quad \sigma \equiv \rho/(\rho - 1) \tag{3b}
\]

where \( p_k \) is the price of capital (that is assumed to completely depreciate at each period). Notice that all monopolists use the same technology and have access to the same capital and labour markets. As a consequence, \( \alpha_l, \alpha_k, w, p_k \) are not indexed by \( n \). Each monopolist maximizes the expected sum of future profits:

\[
\max_p \mathbb{E}_0 \sum_{t=0}^{\infty} \delta^t (p_{n,t} y_{n,t} - c(y_{n,t})) \tag{4}
\]

\[
s.t \quad y_{n,t} = Ha \left( \frac{p_{n,t}}{P_t} \right)^{-b} I_{t,n} \frac{P_t}{P_t} \tag{5}
\]

In this benchmark case everything is contemporaneous\(^2\), so that the solution to this program boils down to a standard markup equation,

\[
p_n = \beta c(y_n), \tag{5}
\]

where \( \beta \equiv b/(b - 1) > 1 \) is the markup.

**Capital Market**  
In the capital market, technology is linear, and perfect competition equates price and marginal cost,

\[
k_n = \gamma l_{k,n} \tag{6}
\]

\[
p_k = \frac{w}{\gamma} \tag{6}
\]

This simple formulation, and the assumption that capital markets are always in equilibrium, de facto make the capital redundant. This is simply a labour economy, not too different from a standard New Keynesian one (e.g. Woodford, 2003)\(^3\). Later on, when adding time to build, we’ll have production as a function of dated labour, similarly to what happens in Wicksell (1898), Hicks (1970, 1973), and Amendola and Gaffard (1998).

\(^2\)In section 4 the recursive structure of the model will allow to focus on two period problems.

\(^3\)The production function can in fact be written as \( y = \left( (\alpha_l l_y)^{\sigma} + (\alpha_k \gamma l_{k,n})^{\sigma} \right)^{1/\sigma} \).
**Equilibrium**  The fact that all firms utilize the same technology, and face the same factor prices, allows to focus on a representative firm omitting the subscript $n$. Eqs. (5), (3b) and (6) give the equilibrium relative price values,

$$\frac{w}{p} = (\beta \left[ \alpha_i^{-\sigma} + (\alpha_k \gamma)^{-\sigma} \right]^{-\sigma})^{-1}$$  \hspace{1cm} (7a)

$$\frac{p_k}{p} = (\gamma \beta A)^{-1}$$  \hspace{1cm} (7b)

Capital disappears from the cost function (3b) that becomes

$$c(y) = Ay y$$

Once we know the equilibrium prices we can determine produced quantity and profits. The latter are defined as

$$\pi = py - c(y) = (p - Aw)y = \frac{\beta - 1}{\beta}py$$  \hspace{1cm} (8)

When the time structure is not relevant, and markets are opened simultaneously, profit is always positive, budget constraints do not bind, and all profits are distributed. As a consequence,

$$r = \pi = py - c(y)$$  \hspace{1cm} (9a)

$$d \equiv \frac{1}{H} \sum_N r_n = N \frac{\beta - 1}{\beta} py$$  \hspace{1cm} (9b)

Aggregating over the $H$ consumers, demand for the $n^{th}$ firm is

$$y^d = Ha \left( \frac{p}{P} \right)^{-b} \left( w + d \right)$$  \hspace{1cm} (10)

After substituting the expressions for $w$ (eq. 7a), and $d$ (eq. 9b), and with some algebra, we obtain the equilibrium values of $y$ and $\pi$

$$y = \frac{aH}{AN (\beta (1 - a) + a)}$$  \hspace{1cm} (11)

$$\pi = \frac{p}{\beta} \frac{\beta - 1}{\beta} aH \frac{aH}{AN (\beta (1 - a) + a)}$$

Aggregate labour supply, substituting the expressions for prices and $y$ becomes

$$L^s = \frac{aH}{\beta (1 - a) + a}$$  \hspace{1cm} (12)

Finally, the demand functions for factors determine the quantity of capital and the division of labour

$$k = \alpha_k^{-\sigma} p_k^{-1} \left( \alpha_k^{-\sigma} p_k^\sigma + \alpha_i^{-\sigma} w^\sigma \right)^{-\frac{1}{\sigma}} y$$  \hspace{1cm} (13a)

$$l_y = \alpha_i^{-\sigma} w^{-1} \left( \alpha_k^{-\sigma} p_k^\sigma + \alpha_i^{-\sigma} w^\sigma \right)^{-\frac{1}{\sigma}} y$$  \hspace{1cm} (13b)
that at equilibrium (after substituting the expressions for \(y, p_k\), and \(w\)) yield

\[
    k = \alpha_k^{-\sigma} 1 - \sigma A^{-\sigma} \frac{aH}{N (\beta(1 - a) + a)}
\]

\[
    l_y = \alpha_l^{-\sigma} A^{-\sigma} \frac{aH}{N (\beta(1 - a) + a)}
\]

(14)

As a coherence check, notice that \(N(l_k + l_y) = L^s\).

The model just outlined is totally standard: relative prices, and equilibrium quantities are determined exclusively by tastes and technology parameters. As we said, in the benchmark case money only determines the absolute price level, and serves as a numeraire (as in the classical Walrasian system). As long as prices are able to adjust in response to shocks, the baseline economy will always and instantaneously return to equilibrium, once perturbed. In the next section we will progressively introduce the hypotheses laid down in the introduction, in order to see how this conclusion changes.

3 Temporary Equilibrium Expectations and Time to Build

The hypotheses will be introduced in the following order: complementarity (\(\rho \to -\infty\) in the production function); temporary equilibrium; adaptive expectations; and finally time to build.

3.1 Complementarity

Suppose capital and labour are complementary. The most extreme form of complementarity is the Leontief production function

\[
    y = \min[\alpha_l l_y, \alpha_k k] = \lim_{\rho \to -\infty} [(\alpha_l l_y)^\rho + (\alpha_k k)^\rho]^{1/\rho}
\]

(15)

This has no effect whatsoever on the model described in section 2, as price flexibility and the contemporaneous opening of all markets allow equilibrium to be always immediately established. Complementarity \textit{per se} does not change the working of the model, as factors can be freely allocated, and prices are flexible. It will play a role only later on, when time to build and forecast errors will imply that some factors are fixed in the short run.

3.2 Temporary Equilibrium Structure

Here we introduce a first sequential element, borrowed from the temporary equilibrium literature (Clower, 1965; Malinvaud, 1977; Benassy, 1982), with an important difference to be discussed below when we introduce expectations. Prices change between periods, and are fixed when markets open and transactions take place. Thus, we have a \textit{fix-price} (à la Hicks, 1956) model, and when plans are not realized equilibrium within the period is attained by rationing.

In the labour market the wage changes in response to disequilibria in the previous period:

\[
    w_t = w_{t-1} \left(1 + \kappa \frac{L_{d,t-1}^d - L_t^s}{L_{d,t-1}^d}\right)
\]

(16)
Wage stickiness may be the consequence of many factors, ranging from union monopoly power to efficiency wages, from market incompleteness, to fairness considerations\textsuperscript{4}. We will take this feature as given, and reflected in equation (16). The parameter $\kappa \geq 0$ summarizes the numerous institutional features that affect the wage setting mechanism; it will turn out to be crucial in the following pages.

To keep the structure of the model as simple as possible, we’ll assume that perfect competition in the capital market continues to ensure the equality of price and marginal cost through changes in $p_{k,t} = w_t/\gamma$; and that monopolists keep setting their prices following the markup rule, $p_t = A\beta w_t$. These admittedly strong assumptions imply that as long as technology does not change, relative prices are always constant and at their equilibrium value. Hence, we are able to focus on absolute rather than relative price distortions\textsuperscript{5}.

Holding relative prices constant, and focusing on representative consumers and firms, we can write the model in “reduced” form, with wage as the only price variable, and in per capita terms\textsuperscript{6}:

\begin{align}
    l_t^d &= A(y_t^d - o_{t-1}) & (17a) \\
    y_t^e &= y_t^d \\
    y_t^d &= \frac{a}{\beta A w_t} \left( w_t + (\beta - 1)A w_{t-1} y_{t-1} + h_{f,t-1}^d + h_{h,t-1}^h \right) & (17c) \\
    l_t^e &= 1 - \frac{1-a}{w_t} \left( w_t + (\beta - 1)A w_{t-1} y_{t-1} + h_{f,t-1}^d + h_{h,t-1}^h \right) & (17d)
\end{align}

With respect to the benchmark, the only novelty is the appearance of stocks in the income of households: If constrained by labour supply, firms cumulate undesired money balances ($h_f^f$), that they will distribute to households along with profit; otherwise unemployment $u$ appears. Likewise in the goods market $o$ and $h_h^h$ are unsold stocks by firms, and undesired money balances by households respectively:

\begin{align}
    h_f^f &= w_t \max(0, l_t^d - l_t^s) \quad u_t = \max(0, l_t^s - l_t^d)/l_t^s \\
    h_h^h &= \beta A w_t \max(0, y_t^d - y_t^e) \quad o_t = \max(0, y_t^e - y_t^d)
\end{align}

When there is no disequilibrium (and hence no stock accumulation), the equilibrium quantities of the benchmark case emerge.

As prices do not change within the period, adjustment happens through rationing; the short side of the market determines the amount exchanged. As a consequence, agents may own undesired stocks of goods, that are carried over from one period to the other. We then

\textsuperscript{4}Here, as in the Calvo pricing scheme commonly used by NK models, the reason why wages/prices do not change is unexplained (it is simply due to ‘institutional’ factors). The important difference with Calvo pricing is that in the formulation I chose price setting is not forward looking.

The literature on causes and consequences of wage stickiness is enormous. For recent surveys of the theoretical and empirical research, the reader is referred to Campbell, and Kamalani (1997), Bewley, (1998), and to Howitt’s, (2002) review of a book by Bewley (1999).

\textsuperscript{5}This is different from the following chapters, where relative prices may change. I plan to relax this feature of the model in future research.

\textsuperscript{6}In fact, all “per firm” variables are brought to “per consumer” terms, by multiplying them by $N/H$. 


have

\begin{align*}
l_t &= \min(l_t^d, l_t^s) \\
y_t &= \min(y_t^*, y_t^d)
\end{align*}

(19)

**Real shocks.** Thanks to the hypotheses on relative prices, real shocks do not affect this economy. Suppose for example that \( \alpha_k \) and/or \( \alpha_l \) increased, and as a consequence \( A \) decreased to \( A' < A \); nothing prevents relative prices changes, so that

\[ p' = \beta A'w < \beta Aw = p \]

(20)

Labour supply and demand would be unchanged, whereas production would increase (see eqs. 11 and 12).

**Nominal shocks.** We now introduce a nominal shock, namely an increase in money supply at time \( t \). Assume that this extra amount is helicopter-dropped to households, in the form of money balances (so that \( h^h_t > h^h_{t-1} = 0 \)). As is clear from equations (17) the increase in monetary income causes an increase of demand for each of the goods, and a corresponding decrease of labour supply. Firms adapt their desired quantities accordingly:

\begin{align*}
\Delta y_t^d &= \frac{a}{\beta Aw_t}h_t^h \\
\Delta l_t^s &= -\frac{1}{w_t}h_t^h \\
\Delta y_t^d &= \frac{a}{\beta Aw_t}h_t^h \\
\Delta l_t^d &= A\Delta y_t^d = \frac{a}{\beta w_t}h_t^h \\
\Delta y_t^s &= \Delta y_t^d = \frac{a}{\beta Aw_t}h_t^h \\
\Delta l_t^d &= A\Delta y_t^s = \frac{a}{\beta w_t}h_t^h
\end{align*}

(21)

If wages and prices were free to adapt, \( w \) and \( p \) would increase to the point at which the extra money balances were absorbed. There would be no real effects.

The unbalance on the labour market causes instead rationing, both in that market and in the goods market:

\begin{align*}
l_t^d - l_t^s &> 0 \\
y_t^d - y_t^s &> 0
\end{align*}

(22)

Only from the following period \( w \) and \( p \) will increase, at a speed given by \( \kappa \) (eq. 16). Manipulation of the system (17) allows, together with the wage setting equation, to reduce the system to

\begin{align*}
l_{t+1}^d &= \frac{a}{\beta}(1 + \frac{w_t}{w_{t+1}})(l_t^d(\beta + 1) - 2l_t^*) \\
l_{t+1}^s &= 1 - (1 - a)(1 + \frac{w_t}{w_{t+1}})(l_t^d(\beta + 1) - 2l_t^*) \\
\frac{w_t}{w_{t+1}} &= \left(1 + \frac{\kappa l_t^d - l_t^s}{l_t^d}ight)^{-1}
\end{align*}

(23)

The system can be further reduced, by substituting the third equation into the first two, and eliminating \( w_{t+1}/w_t \). The steady state is found by imposing, \( l_{t+1}^d = l_t^d, l_{t+1}^s = l_t^s \). This yields
two solutions:

\[ I^s = \frac{a\kappa}{\kappa + 1 - a}, \quad I^d = \frac{a(1 + \kappa)}{\beta(\kappa + 1 - a)} \]

\[ I^s = I^d = \frac{a}{\beta(1 - a) + a} \quad (24) \]

In the appendix I show that the parameter \( \kappa \) can belong to 4 different regions: \((0, \kappa^m_l), (\kappa^m_l, \kappa^m_h), (\kappa^m_h, \kappa^o_h)\). For \( \kappa \) large enough, (between \( \kappa^m_l \) and \( \kappa^o_h \), the second position (that coincides with the benchmark equilibrium) is reached; this can happen monotonically \( (\kappa \in (\kappa^m_l, \kappa^m_h)) \), or with an oscillatory trajectory \( (\kappa \in (\kappa^o_l, \kappa^o_h)) \). For small values of \( \kappa \) \( (\kappa \in (0, \kappa^m_l]) \) the system converges to the “bad” equilibrium, in which changes in money balances and in wages exactly offset each other, so that labour supply and demand are constant at a disequilibrium level. Finally, for values above \( \kappa^o_h \) the variation of the wage is so wide that the system diverges and sooner or later the economy implodes (some variable goes to 0).

To conclude, a temporary equilibrium structure prevents immediate jumps to the new equilibrium position. Nevertheless, if prices change towards the absorption of disequilibrium, and if they are not too sticky, the system eventually converges to the new equilibrium, as is intuitive.

### 3.3 Adaptive vs Rational Expectations

Up to this point expectation formation has been left in the background. In fact this was done for a reason, as the role of agents’ beliefs in the adjustment process described above is marginal. If agents are fully rational, they embed in their decisions both the constraints imposed by the wage setting mechanism, and the rationing following a disequilibrium. As a consequence, the sequence of disequilibria following a shock is purely notional, in the sense that actual demands and supply take into account the constraints, extra money balances are released only gradually, and markets are always in equilibrium; the gradual wage change induces the convergence of actual demand and supply to the equilibrium notional levels. Such a behaviour is analogous, with the due differences, to that of standard temporary equilibrium models (e.g. Benassy, 1982). If agents do not have fully rational expectations, the difference between notional and actual quantities disappears, so that the adjustment process is characterized by disequilibria and accumulation of stocks by rationed agents.

The only significant difference between adaptive and rational behaviour is that as the threshold values derived in the appendix depend on initial conditions, when facing extreme values of \( \kappa \) (below \( \kappa^m_l \) or above \( \kappa^o_h \)) fully rational agents will be able to alter their initial notional demand in order to change the thresholds and set the economy on a sustainable path. But for most values of \( \kappa \) the difference between rational and adaptive behaviour is not really more than a theoretical *curiosum*, as it has not effect on the path followed by the economy.

### 3.4 Time to Build

In the preceding paragraphs we showed that in general, adaptive behaviour and sticky prices only yield temporary disequilibria following a shock. Here we introduce the last hypothesis -

\[ \text{There are not many models in which disequilibria cause stock accumulation. The interesting exception by Bignami et al (2003), allows for mistakes, actual disequilibria and rationing, even if in a different context with respect to this paper.} \]
Time to build - and we will show that as the others, it alone does not affect the behaviour of the economy. To analyze the hypothesis on technology in isolation, we further assume perfect foresight, so that for any variable \( x \), \( E_t \{ x_{t+1} \} = x_{t+1} \). To simplify the exposition, suppose that \( \alpha_l = \alpha \) and \( \alpha_k = (1 - \alpha) \). Capital has to be accumulated one period in advance, so that

\[
y_t = l_{y,t}^\alpha k_{t-1}^{1-\alpha}
\]

Cost minimization yields conditional demands that are 'intertemporal', in the sense that they contain prices and quantities at \( t \) and \( t + 1 \).

\[
k_t = \left( \frac{p_{k,t}}{w_{t+1}} \frac{\alpha}{1 - \alpha} \right)^{1-\alpha} y_{t+1}
\]

\[
l_{y,t+1} = \left( \frac{p_{k,t}}{w_{t+1}} \frac{\alpha}{1 - \alpha} \right)^{1-\alpha} y_{t+1}
\]

The corresponding cost function is

\[
c(y_{t+1}, w_{t+1}, p_{k,t}) = \left( \frac{w_{t+1}}{\alpha} \right)^{\alpha} \left( \frac{p_{k,t}}{1 - \alpha} \right)^{1-\alpha} y_{t+1}
\]

\[
= \left( \frac{w_{t+1}}{\alpha} \right)^{\alpha} \left( \frac{w_t}{\gamma(1 - \alpha)} \right)^{1-\alpha} y_{t+1}
\]

once we consider the equilibrium in the capital market (eqs. 6).

The steady state of this static economy is characterized by \( x_t = x_{t+1} = x \) for each variable. As a consequence, at the equilibrium the economy is like the benchmark (modified to take into account the Cobb-Douglas hypotheses):

\[
c(y, w) = \alpha^{-\alpha} (\gamma(1 - \alpha))^{-(1-\alpha)} w y = A w y
\]

\[
\frac{w}{p} = (\beta \alpha^{-\alpha} (\gamma(1 - \alpha))^{-(1-\alpha)})^{-1} = (\beta A)^{-1}
\]

as in eqs. (7), with \( A = \alpha^{-\alpha} (\gamma(1 - \alpha))^{-(1-\alpha)} \). As a consequence, \( y, \pi \) and \( L^s \) can be calculated as in eqs. (11-12):

\[
y = \frac{aH}{AN (\beta (1-a) + a)}
\]

\[
\pi = \frac{A}{A z} \frac{aH}{\beta (1-a) + a}
\]

\[
L^s = \frac{aH}{\beta (1-a) + a}
\]
Factor demands in equilibrium are given by

\[
k = \left( \frac{1}{\gamma} \frac{\alpha}{1-\alpha} \right)^{-\alpha} \frac{aH}{AN(\beta(1-a)+a)}
\]
\[
= \gamma(1-\alpha) \frac{aH}{N(\beta(1-a)+a)}
\]

\[
l_y = \left( \frac{1}{\gamma} \frac{\alpha}{1-\alpha} \right)^{1-\alpha} \frac{aH}{AN(\beta(1-a)+a)}
\]
\[
= \frac{\alpha aH}{N(\beta(1-a)+a)}
\]

(30)

Obviously, nominal shocks do not affect the equilibrium. Prices, free to adapt, would jump to the new equilibrium level, and quantities would be unaffected. A real shock instead causes a change in quantities. Consider the same shock that will be analyzed in section 5.2 below, namely an increase in \(a\). The new steady state position will be characterized by higher labour supply and production (this can be checked by substituting \(a' > a\) in eqs. 29). Correspondingly, also capital and labour demand will increase (eq. 30).

**Anticipated Shocks**

Suppose the shock happens at \(t+1\). If it is perfectly anticipated, demand for the factors will be

\[
k_t = \left( \frac{w_t}{w_{t+1}} \right)^{-\alpha} \frac{\gamma(1-\alpha)a'H}{N(\beta(1-a')+a')}
\]

\[
l_{y,t+1} = \left( \frac{w_t}{w_{t+1}} \right)^{1-\alpha} \frac{aa'H}{N(\beta(1-a')+a')}
\]

(31)

while given the inherited stock of capital \(l_{y,t}\) is at its steady state value. Total labour demand will be

\[
N(l_{k,t} + l_{y,t}) = N \left( \frac{\alpha aH}{N(\beta(1-a)+a)} + \frac{1}{\gamma} \left( \frac{w_t}{w_{t+1}} \right)^{-\alpha} \frac{\gamma(1-\alpha)a'H}{N(\beta(1-a')+a')} \right)
\]

(32)

\[
= H \left( \frac{\alpha a}{\beta(1-a)+a} + \left( \frac{w_t}{w_{t+1}} \right)^{-\alpha} \frac{(1-\alpha)a'}{\beta(1-a')+a'} \right)
\]

whereas labour supply is

\[
L_t^* = H \left( 1 - \frac{(1-a)(w_t+d_t)}{w_t} \right)
\]

(33)

\[
= \frac{aH}{\beta(1-a)+a}
\]
so that the required wage change will be

\[
\frac{w_t}{w_{t+1}} = \left( \frac{\alpha'}{\beta(1-a') + a} \right)^{\frac{1}{\alpha'}} = \left( \frac{L^s_{t+1}}{L^s_t} \right)^{\frac{1}{\alpha'}} > 1
\]  \tag{34}

where the inequality comes from the fact that \( \alpha' > a \) and \( \alpha < 1 \).

Unanticipated Shocks If the shock is unanticipated, then the existing capital stock constrains firms, that have to change labour demand accordingly:

\[
l_y, t = \left( \frac{y_t}{k^{1-\alpha}} \right)^{\frac{1}{\alpha'}}
\]  \tag{35}

As a consequence, marginal cost and the price are

\[
c_t' = \frac{w_t k^{\frac{1}{\alpha}} - y_t^{\frac{1-\alpha}{\alpha}}}{\alpha}
\]

\[
p = \beta c_t' = \beta \frac{w_t k^{\frac{1}{\alpha}} - y_t^{\frac{1-\alpha}{\alpha}}}{\alpha}
\]  \tag{36}

Notice that as capital is given in the short run, marginal cost is not any more independent of \( y \). Starting from demand (as in equation 10, and assuming that all firms are alike), we have

\[
y_t^d = \frac{Ha'}{Np}(w_t + d_t)
\]

\[
y_t^d = \frac{Ha'}{Np} \left( w_t + w_{t-1} \frac{(\beta - 1) a}{\beta(1-a) + a} \right)
\]

Substituting the price and solving for \( y \) we obtain labour demand in the goods sectors:

\[
y_t^d = \left( \frac{Ha' \beta k^{\frac{1}{\alpha}}}{N} \left( 1 + \frac{w_{t-1}(\beta - 1) a}{w_t(\beta(1-a) + a)} \right) \right)^{\alpha}
\]  \tag{38}

\[
L_{y, t} = \frac{Ha' \alpha}{\beta} \left( 1 + \frac{w_{t-1}(\beta - 1) a}{w_t(\beta(1-a) + a)} \right)
\]

Notice that the stock of capital disappears from labour demand. We know that \( a_{t+1} = a_t = a' \), so that \( w_{t+1} = w_t \) (from eq. 34). Then,

\[
Nk_t = \frac{\gamma(1-\alpha) a'H}{(\beta(1-a') + a')}
\]

\[
L_{k, t} = \frac{(1-\alpha) a'H}{(\beta(1-a') + a')}
\]  \tag{39}

The equality of labour demand and supply gives

\[
L_{k, t} + L_{y, t} = L^s_t = 1 - \frac{(1-a)}{w_t} \left( w_t + \frac{w_{t-1}(\beta - 1) a}{\beta(1-a) + a} \right)
\]

\[
\Rightarrow \frac{w_{t-1}}{w_t} = \frac{a'(\beta(1-a)] + a)}{a(\beta(1-a') + a')} = \frac{\beta a' + aa'(1-\beta)}{a\beta + aa'(1-\beta)} > 1
\]  \tag{40}
This section has progressively complicated the simple model presented in section 2. We showed that neither complementarity nor quantity adjustment within the period, nor time to build really affect the basic message of the model, i.e. that prices change to absorb shocks. We further argued that adaptive or rational expectations make no essential difference. None of the hypotheses we laid down in the introduction, if taken in isolation, has significant effects on the benchmark model. At the very worst, as in the case of sticky prices, adjustment to the new equilibrium that follows a shock takes some time. The next section will put all these hypotheses together, and it will show that in that case these conclusions may drastically change. We’ll be able to show that whether agents commit systematic mistakes or not, or whether capital and labour are complements or substitutes, will crucially affect the path followed by the economy. We’ll also show that in such an economy some surprising results about the desirability of wage flexibility may emerge.

4 The Sequence

The time to build hypothesis can be rephrased in more general terms with reference to the CES:

\[ y_t = \left[ (\alpha_l l_{t,t})^\rho + (\alpha_k k_{t,t})^\rho \right]^{1/\rho} \]

\[ k_t = i_{t-1} \]

where \(i_{t-1}\) is investment in the previous period.

Assuming that markets open sequentially (credit, then labour, and finally goods), is a simple way to introduce a cash in advance constraint: wages have to be paid before goods are sold, and hence in disequilibrium firms may have a positive demand for external financing.\(^8\)

**Expectations**  Expectations may be backward looking, or take into account the steady state properties of the system: at the beginning of period \(t\) entrepreneurs expect demand in the following period, and unemployment in the current period, to be

\[ y_{t+1|t}^* = \phi \left( \lambda y_{t-1|t}^* + (1 - \lambda) y_{t-1}^d \right) + (1 - \phi) \frac{aH}{AN (\beta (1-a) + a)} \]

\[ u_{t|t}^* = \phi \left( \lambda u_{t-1|t-1} + (1 - \lambda) u_{t-1} \right) + (1 - \phi)0 \]

where \(\phi\) is an indicator function taking value 1 if expectations are backward looking, and \(\lambda\) denotes the (lack of) propensity to change expectations based on past experience.

**Desired quantities**  Given wages, and via the markup (eq. 5) relative prices, firms determine desired supply of goods, and desired investment (eq. 13a):

\[ \hat{y}_t^* = a \left( \frac{p_t}{P_{t-1}} \right)^{-\beta} \frac{H(1 - u_{t|t}^*)w_t + \Pi_{t-1} + H_{t-1}}{P_{t-1}} \]

\[ \hat{i}_t = \alpha_k^{-\sigma} (\gamma A)^{1-\sigma} y_{t+1|t}^* \]

\(^8\)Notice that in the section above this sequential opening would have had no effect at all. Cash in advance constraints play a role only in disequilibrium.
with hats denoting desired quantities. Two important things are worth mentioning: the first is that investment \( \hat{i} \) incorporates the “time to build” assumption, in the sense that capital has to be bought at time \( t \) for utilization at time \( t + 1 \). The second feature of eq. 43 is that the monopolist takes into account all expected income, \( i.e. \) wage by employed workers, aggregate profits distributed in the previous period (to all \( H \) households), and also, if any, involuntary money hoardings (\( H^h = \sum_h h^h \)) derived by rationing in the previous period. Notice also that he takes the aggregate price level as given (at the last known level, \( P_{t-1} \)).

Desired production is determined subtracting from desired quantity unsold stocks carried on from last period (\( o_{t-1} \)) and taking into account the constraint of capital availability:

\[
\hat{k}_t = \alpha_k - \sigma \gamma A \quad (44)
\]

\[
\hat{q}_t = (\hat{y}_t - o_{t-1}) \min(1, \frac{\hat{k}_t}{k_t})
\]

In other words, after taking into account stocks, agents check whether the capital inherited from last period is sufficient to carry on desired production; if it is not the case (\( k_t < \hat{k}_t \)), desired production has to be scaled down.

Labour demand is determined by the needs of the two sectors. In the capital sector, perfect competition guarantees that supply matches desired demand, so that as said before, investment is nothing but demand for “dated labour” (eq. 13b)

\[
\hat{l}_{y,t} = A^{1-\sigma} \alpha_l^{-\sigma} \hat{q}_t
\]

\[
\hat{l}_{k,t} = \hat{i}_t / \gamma
\]

Once we know labour demand, we may finally determine the desired wage fund, and the consequent demand for money.

**Cash-in-advance, budget constraints, and the market for credit** In the baseline model, contemporaneous opening of all markets, and the monopolistic competition structure, guaranteed that the firm’s budget constraints was always satisfied. Once we develop the sequential structure of the model, the cash-in-advance constraint may be binding, and it may cause a positive demand for money. As we anticipated above, when the labour market opens firms have to pay wages with all available cash, \( i.e. \) with revenues from previous periods, and with external funds. The reason is that wages have to be paid in advance, and that as we’ll see, outside the steady state internal funds may not be sufficient. Money demand may then be written as the difference between the wage fund and internal resources.

\[
F^d_t = w_t (\hat{L}_{y,t} + \hat{L}_{k,t}) - (R_{t-1} + H^f_{t-1} - \Pi_{t-1})
\]

where \( R \) and \( \Pi \) are aggregate revenues and distributed profits respectively, and \( H^f \) denotes involuntary monetary hoardings by firms (see eq. 51 below). Equation 46 embeds the cash-in-advance constraint: the firm system needs additional funds for whatever of the wage pool it cannot finance out of internal resources.\(^{10}\)

\(^{9}\) Notice that the unemployment rate reduces demand (via household income). This formulation is \( de facto \) identical to the scaling down of demand due to constrained labour supply that I use in the appendix.

\(^{10}\) This model leaves the ambiguity of \( F \) (money or credit?) unresolved. In fact, money supply is simple outside money (not repaid), whereas money demand is more like demand for credit (but for it to be credit, \( \ldots \) continued\( \))
The behavior of monetary authorities not explicitly modeled. We simply assume that the monetary policy authority follows a rule that weights money demand, inflation, and steady state behaviour.

\[ F^a_t = \mu [\xi F^d_t + (1 - \xi) F^s_{t-1}] (1 - \psi \frac{w_t}{w_{t-1}}) + (1 - \mu) F^a_{t-1} \]  

(47)

The parameter \( \mu \) gives the willingness to accommodate demand. \( \xi \) denotes the degree of foresight of the central bank; with \( \xi = 1 \) we have perfect foresight, whereas \( \xi = 0 \) implies that the central bank only observes demand with a lag. Finally, \( \psi \) denotes the weight given to inflation. Broadly speaking, a “Friedman” rule would have \( \mu = 0 \), while a “Taylor” rule would imply positive \( \psi \) and \( \mu \).

**Households’ demand and supply.** On the household side, we have individual demand for each of the goods and supply of labour (see eq. 2):

\[ \hat{y}^d_t = \frac{a}{H} \left( \frac{p_t}{P_t} \right)^{-b} \frac{H w_t + \Pi_{t-1} + H^h_{t-1}}{P_t} \]

\[ \hat{l}^s_t = 1 - \frac{1 - (1 - a) H w_t + \Pi_{t-1} + H_{t-1}}{w_t} \]  

(48)

4.1 Market outcome and rationing

Once the “notional demands” are formulated, the markets open sequentially. The first market to open is the credit market, where we have

\[ F_t = \min(F^d_t, F^s_t) \]  

(49)

If money demand is rationed, the entrepreneurs will not be able to demand all the labour force they planned to hire, so that

\[ L^d_t = \min(F^d_t, F^s_t) + \frac{(R_{t-1} + H^d_{t-1} - \Pi_{t-1})}{w_t} \]  

(50)

where \( L^d_t \equiv L_{y,t} + L_{k,t} \). We assume proportional rationing between the \( y \) and \( k \) markets. If supply is rationed, we assume that the excess money is distributed to households (see below). Notice that firms rationed by credit constraints, by scaling down labour demand to make it consistent with financial availability, take into account monetary policy when forming their demand in the other markets. Thanks to the fact that the financial market opens first, its constraints are embedded in the agent’s plans similarly to the standard temporary equilibrium literature.

we would need to keep track of repayments adding term like \(-(1 + r)F_{t-1}\) in the budget constraint). The reason why the ambiguity is not resolved is that here I am more interested in observing how money demand emerges, and what are the effects of liquidity constraints on investment and income. In future research I plan to develop the financial market. An interesting attempt in this direction in a standard Hicksian setting is contained in Attar and Campioni (2003).
The second market to open is the labour market. Once again, effective employment (and consequently unemployment) is given by the short side rule

\[ L_t = \min(L_t^d, L_t^s) \]

\[ u_t = \frac{\max(0, L_t^s - L_t^d)}{L_t^s} \]

\[ H_t^f = w_t \max(0, L_t^d - L_t^s) \] \hspace{1cm} (51)

If constrained by labour supply, firms cumulate undesired money balances that they will distribute to households together with profits. The total pool of employed workers produces capital and goods:

\[ q_t = \left[ (\alpha_l l_{y,t})^\rho + (\alpha_k \bar{k}_t)^\rho \right]^{1/\rho} \]

\[ i_t = \gamma l_{k,t} \]

\[ y_t^s = q_t + o_{t-1} \]

where \( \bar{k}_t = \min(\bar{k}_t, k_t) \), and \( l_{j,t} = \dot{l}_{j,t} \frac{L_t}{L_t^s} \), with \( j = y, k \), are the constrained quantities.

After equilibrium in the labour market is attained, wages are paid, and households form their demand

\[ y_t^d = a \left( \frac{p_t}{P_t} \right)^{-b} \frac{H(1 - u_t)w_t + \Pi_{t-1} + H_t^h + \max(0, F_t^s - F_t^d)}{P_t} \] \hspace{1cm} (53)

The final market to consider is the goods one, where we have

\[ y_t = \min(y_t^s, y_t^d) \]

\[ R_t = Np_t y_t \]

\[ o_t = \max(0, y_t^s - y_t^d) \]

\[ h_t^h = p_t \max(0, y_t^d - y_t^s) + \frac{\max(0, F_t^s - F_t^d)}{H} \] \hspace{1cm} (54)

Notice that households carry stocks both because they were rationed in the goods market, and because the excess supply of money was distributed to them.

This marks the end of the period. The economy enters the following period with a stock of capital \( K_{t+1} = Ni_t \) that will be available to firms for production. Other state variables are the stocks of unsold goods \( (o) \), undesired money balances \( (H^h) \), and profits distributed to households \( (\Pi_t = R_t + H_t^f - w_t L_t) \).

Stocks on one side enter into the decision process (via expectations); and on the other contribute to shape the system of constraints faced by agents in the following period. This is a crucial difference with respect to the temporary equilibrium literature, in which each
period was *de facto* considered in isolation. Here the stocks carried from one period to the
other shape a sequence\(^{11}\) of periods that it is impossible to analyze separately.

The model outlined above has complex disequilibrium dynamics, that make an analytical
solution impossible to find. The next section will therefore rely on simulations to trace the
path followed by the economy.

5 The Simulations

This section presents some basic results of the sequential model I described. I will analyze
both nominal and real shocks, and describe how different hypotheses (in particular regarding
wage regimes and monetary policy) affect the final outcome of the disequilibrium path fol-
lowed by the economy. In all cases, the economy is initially in steady state, and is perturbed
by a shock that creates a disequilibrium in one or more markets.

The relevant parameters in the model are not many: the degree of substitutability be-
tween inputs \((\rho)\); the responsiveness of wages to disequilibria in the labour market \((\kappa)\); the
parameters governing expectations \((\phi \text{ and } \xi)\); and finally, the monetary policy stance \((\mu \text{ and }
\psi)\).\(^{12}\) A thorough analysis would nevertheless require too many cases to be analyzed, so
that I opted for some time series examples, complemented by a Monte Carlo investigation of
the parameter space. The parameter values are drawn randomly within the relevant range,
and the response of the economy to the shock is analyzed by looking at the final level of
production, and to its variance over the run.

5.1 Nominal Shocks

Suppose that at time \(t = 10\) the stock of money increases by 10%. This extra money adds to
the nominal income of consumers, and will imply an increase in the demand for goods and
leisure. In the benchmark model, the instantaneous price adjustment would immediately
absorb the nominal excess demand, and quantities will be unaffected. Introduce now the
sequential structure described in section 4, but assume the absence of rigidities. The system
quickly goes back to equilibrium (figure 1). Production initially drops, because the increase
in leisure demand implies a shortage in labour supply. Notice that if production could take
place with capital built in the same period, then we would observe an increase of actual
production in the short run. But in this case, the labour shortage cannot be compensated by
the use of more capital (still to be built), even if the technology allows for the substitution.
In the following periods, the increase of wages triggered by the excess demand for labour
helps reabsorb the disequilibrium, and the nominal shock has no long term effects.

Increasing the degree of complementarity \((\rho \rightarrow -\infty)\) does not change the qualitative be-
avour of the system, but makes the initial disequilibrium deeper, and the recovery longer,
as figure 2 shows, by comparing the deviation of supply from its steady state value in the
three cases of linear \((\rho = 1)\), Cobb-Douglas \((\rho = 0)\) and Leontief \((\rho = -\infty)\) technology.
The standard deviation of supply increases with complementarity, whereas the mean de-
creases. This result confirms and extends the findings of section 3.2: even with time to build

\(^{11}\)“Via expectations, and in the attempt to correct the imbalances between demand and supply, a
‘constraints-decisions-constraints’ sequence sets in, that results in an out-of-equilibrium process” (Amendola

\(^{12}\)The other parameters are: \(\alpha_k = \alpha_l = 0.5, \gamma = 16, a = 0.7, b = 2, H = 1000, N = 10, \lambda = 0.5\). All the
simulations where ran in FORTRAN. The code is available upon request.
Figure 1: Nominal Shock. Linear technology \((\rho = 1)\), flexible prices \((\kappa = 0.5)\) accommodating monetary policy \((\mu = 1)\) and perfect foresight \((\phi = 0, \xi = 1)\).

and complementarity, if agents do not form their expectations backward looking, shocks are reabsorbed.

The next step is to introduce the hypothesis of backward looking behaviour. We assume that agents form their expectations based on past demand behaviour \((\phi = 1)\) in eq. 42; and that the monetary authority is only able to observe demand for credit with a lag \((\xi = 0)\) in eq. 47. We are interested the behaviour of the economy under different wage regimes and monetary policy rules. In figure 3 I plotted the ratio of production over the steady state value, under different hypotheses on wage reactivity \((\kappa = 0)\) or \((\kappa = 0.5)\) and monetary policy stance \((\mu = 0)\) or \((\mu = 1)\). These four extreme cases give some interesting insights: first a shock, - even a positive and nominal one - triggers an adjustment process that sets the economy on a permanently lower production level. The reason has to be found in the difficulty to recover coordination, lost after the shock. In particular, the initial labour supply shortage imposes a constraint on both investment and production decisions that, contrary to the previous cases, now cumulates because of the interaction of production and decision lags. The system settles on a new steady state with no involuntary unemployment, but lower labour force participation (figure 4 depicts the case \(\mu = 0\) and \(\kappa = 0.5\)). On the other hand, the excess of monetary means causes a permanent excess demand in the goods market that is
Figure 3: Monetary policy and wage regimes. Nominal shock with perfect complementarity ($\rho = -\infty$) and backward looking behaviour ($\phi = 1$, $\xi = 0$).

Figure 4: Nominal shock. Goods and labour markets in the case of tight monetary policy ($\mu = 0$) and flexible wages ($\kappa = 0.5$).

never reabsorbed, as the driving force in the price movement is disequilibrium in the labour market.

The second conclusion we can derive from the analysis of figure 3 is that wage flexibility may play a positive role in the response to the shock, but that this role depends crucially on the interaction with monetary policy. In fact, an accommodating monetary policy, coupled with wage changes, may set the economy on an explosive path: agents react to wage changes altering their investment and production behaviour, and consequently the demand for external financing. If these fluctuations are excessive, a tight monetary policy may dampen them, whereas by accommodating they may be further enhanced. However, ranking the four cases by mean and standard deviation, we notice that money creation may partially do the job of wage changes, so that if $\kappa = 0$ an accommodating monetary policy ($\mu = 1$) results in a higher mean and lower variance of output. Notice that in the analysis of section 3.2 there was no room for monetary policy. The importance of credit is clearly linked to the sequential nature of the model and to investment.

To summarize, the analysis of the time series gives some important insights. The first is that the sequential structure of the economy does not matter per se, in the sense that well informed agents, not facing credit constraints, are able to reabsorb a monetary shock pretty rapidly. The loss of coordination entailed by shock is only temporary and the economy goes back to the original steady state path.

The second result is that once we introduce intertemporal complementarities in the pro-
duction process and backward looking behaviour, even a nominal shock has permanent effects. Coordination, if the system is sufficiently flexible may be recovered; but, as a consequence of the disequilibrium path taken by the economy, this will generally happen at a level of output different from the original one. The impossibility of instantaneous adjustment to changes in the environment triggers a sequence of constrained decisions, and has permanent effects on the structure of the economy.

Finally, we showed that the interaction of monetary policy and wage regimes plays a crucial role. Decisions by non fully rational agents entail mistakes that may cumulate; so that if on one side some flexibility in wages and/or in credit creation is necessary in order to allow the reabsorption of the shock, on the other side, if excessive, it may actually amplify the effects of bad decisions, and push the economy on unsustainable paths.

The next step is to verify whether the intuition that comes from the analysis of time series is robust to changes in the parameters. I set up a Monte Carlo experiment consisting in the random draw of nine parameters, and in a run of the economy, recording the ratio between final and initial supply, and the standard deviation of supply over the run. The parameters affect the final outcome of each simulation in a strongly nonlinear way. In order to sort out their effect I regressed the two variables over the nine parameters including powers and cross terms to account for nonlinearities. The full results of the regressions are reported in the appendix. Here I’ll focus on the two parameters I am mainly interested in, the wage regime \( \kappa \) and the monetary policy stance \( \mu \). Figure 5 reports the approximating polynomials in \( \mu \) and \( \kappa \) for \( s \) and for its standard error. It shows that increasing wage flexibility has positive effects both in terms of reduced variability (left panel) and final level of production (right panel). But this is true only up to a point. For too high values of \( \kappa \), further increases of the wage reaction coefficient increase volatility, and reduce the final level of output (notice that when analyzing time series, the flexible wage case had \( \kappa = 0.5 \)). This conclusion is even more evident from the left panel of figure 6, where I plotted (the positive values of) the derivative of the \( s \) surface with respect to changes in \( \kappa \). For values higher than \( \kappa \approx 0.6 \) the derivative is almost always negative, meaning that increasing flexibility will lead to a lower final level of output. Notice finally that this is true for almost any level of \( \mu \). The reason for this result is that when adjustment is not instantaneous (both in the agent’s decision processes and in the technological structure), and actions are irreversible, a

\begin{align*}
p &\in [-35, 1], \\
\kappa &\in (0, 1), \\
\mu &\in (0, 1), \\
\lambda &\in (0, 1), \\
\xi &\in (0, 1), \\
\phi &\in \{0, 1\}, \\
\alpha_k &\in [0.1, 0.9] \text{ (with } \alpha_l = 1 - \alpha_k), \\
N &\in [2, 99], \\
\psi &\in (0, 1).
\end{align*}
frictionless system may become extremely unstable, while what are generally seen as ‘market imperfections’ contribute to smooth the effects of actions based on wrong beliefs, and hence help maintaining the system viable. In a world characterized by adaptive behaviours and by a temporally articulated production structure, the conventional wisdom is reversed: some degree of market imperfection may be a factor of stability of the system rather than an obstacle on the way to fully optimal equilibria. I will return on this result when discussing Keynes’ view on unemployment.

For monetary policy the results are less clear-cut. The right panel of figure 6 shows changes in $s$ when $\mu$ changes, again cutting the figure at the zero level. The only clear conclusion is that a more accommodating monetary policy (increasing $\mu$) is more effective for low levels of $\kappa$, confirming what we said about some degree of substitutability between flexibility given by the wage regime and by monetary policy.

To conclude, the systematic investigation over the parameter space confirms the conclusions reached earlier, namely that some degree of wage flexibility is beneficial in recovering equilibrium after a nominal shock hits the economy; and furthermore, that accommodating monetary policy is desirable especially when wages cannot change to reabsorb the shock. The Monte Carlo experiment gives an additional information, i.e. that excessive wage variability may be harmful. We explained this result with the smoothing role that market imperfections play in this model.

5.2 Real Shocks

In this section we consider a change of tastes; specifically, suppose that consumer switch their preferences from leisure to the goods, increasing labour supply accordingly\footnote{We assume that at time 10 the parameter $a$ changes from 0.7 to 0.75.}. In section 3.2, lacking the intertemporal dimension real shocks had no effects (relative prices are free to adjust in this model). Here time to build induces some real effects. Nevertheless, figure 7 shows that when no complementarities exist, and agents are perfectly informed, the shock is almost immediately reabsorbed. As this was the case for nominal shocks as well (see figure 1), a first result is that in a perfectly smooth setting the nature of the shock is irrelevant, and the system recovers coordination.

Comparing figure 8 with figure 3, on the other hand, we notice that the nature of the shock becomes relevant once we introduce irreversibilities in the production process and in
technology. The most immediate consideration is that in the case of a real shock monetary policy is crucial. The equilibrium position following a shift of preferences towards more of the goods implies a higher level of production and hence of installed productive capacity. This is why a tight monetary policy, by constraining investment, is detrimental, and pushes the system on an explosive path. The role of wage flexibility is more controversial; on one hand, it allows a faster recovery. On the other, it is characterized by higher variability, and by a larger initial recession; this is even more clear in figure 9, that shows total supply and unemployment in the two cases of fixed and flexible wages, with accommodating monetary policy.

The next step is to verify the robustness of these results. The strategy for the investigation of the parameter space is the same as in the section on nominal shocks, and the results are reported in figure 10. The left panel shows that the standard error of aggregate supply is increasing in $\kappa$, and decreasing in $\mu$. This means that both more flexible wages and a tighter monetary policy increase output variability. The right panel depicts the ratio between supply at time $t = 200$ and the equilibrium level. The ratio is strongly increasing in $\mu$, meaning that a more accommodating monetary policy helps the system converge to the new equilibrium. The effect of larger $\kappa$ on final supply is less evident.

The Monte Carlo analysis confirms that real shocks are faster reabsorbed when monetary policy is accommodating, and the financial constraint does not play a role. At the same time, more flexible wages do increase output variability, whereas the effects on final output
Figure 9: Comparison of wage regimes. Real shock with accommodating monetary policy $(\mu = 1)$ and backward looking behaviour $(\phi = 1, \xi = 0)$.

Figure 10: Monte Carlo for real shocks. Final/steady state level (right) and standard error (left) of production as a function of the wage regime and monetary policy stance.

are only marginally positive\textsuperscript{15}.

The analysis of nominal and real shocks has highlighted some interesting features of the sequential economy described above. The interplay of reasonable assumptions (on technology, and on price and expectation formation) produces results that seem robust to the parameter specification. In particular, in such a framework both an accommodating monetary policy (as a way of limiting financial constraints), and some degree of wage rigidity (as a way of dampening fluctuations, once wages are not market clearing) appeared as crucial.

The next chapter will show how a sequential model similar in spirit to this one may shed light on an empirical puzzle on technical progress and productivity growth (the so called ‘productivity paradox’). The present chapter will instead be concluded by some notes on Keynes’ discussion of unemployment and wage regimes, and on its relation with out-of-equilibrium modelling.

\textsuperscript{15}This is also determined by the length of the run (200 periods). If we had a shorter period, the effect of $\kappa$ would be more evident, because recovery is faster with more flexible wages.
6 Wage Flexibility and Disequilibrium Dynamics. The General Theory Reconsidered

The scope of this section is to analyze Keynes’ arguments about wages and unemployment, and to show that the sequential model presented in sections 2 and 4 offers a suitable framework for his analysis. In other words, this paper may be seen as a contribution to the literature claiming that Keynes’ theory may not be reduced to a special (fix price and hence short run) case of the Neoclassical model as claimed by the IS-LM approach. The IS-LM is an equilibrium construct, in which markets are complete, and the price vector conveys all the information necessary to fully coordinate agent’s decisions. In this framework, unemployment can only stem from nominal rigidities in the relevant market, namely the one for labour.

Leijonhufvud (1968), is the first author to oppose the standard textbook interpretation of Keynes, and to read the General Theory as an attempt to introduce problems of coordination in the standard Walrasian framework. He argues that Keynes’ main innovation lies in the method, which is the attempt to see unemployment as a disequilibrium phenomenon linked to the adjustment process following an exogenous disturbance. Furthermore, “the Keynesian [system] adjusts primarily by way of real income movements” (Leijonhufvud, 1968, p.51). The main reason for this shift of focus from prices to quantities in the adjustment process lies in the refusal of the costless role of the auctioneer. Price adjustments require information that is costly and lengthy to acquire. This implies that adjustment is not instantaneous, so that the Walrasian hypothesis of trade only happening at equilibrium prices has to be dropped. The result of trading at disequilibrium prices is the appearance of constraints which hamper the realization of agents’ plans\(^{16}\). Hence “Realized transaction quantities enter as arguments of the excess demand function in addition to prices” (Leijonhufvud, 1968, p.56).

Coordination problems arise in particular in the market for savings and investment, where agents with different time horizons interact. “Financial markets are manifestly incapable of providing for the consistency of long-term production and consumption plans” (Leijonhufvud, 1968, p. 276). The reason is that expected future values of the interest rate play a role even more important than its current value, and may induce speculative behaviour. Trade then takes place at a false price, at which \(ex \ ante\) savings and investment are not equated. It is therefore income that has to change in order to restore this equality. The conclusion, according to Leijonhufvud, is obvious: “It was Keynes’ position that it is the failure of the incomplete market mechanism to reconcile the implied values of forward demand and supplies […] that is the source of the trouble. Unemployment of labor and other resources is a derivative phenomenon” (1968, p. 276).

If the ‘source of the trouble’ does not lie in the labour market, whose disequilibrium is only a ‘derivative phenomenon’, the hypothesis of fixed wages that stirred so much controversy acquires a very precise meaning. Keynes writes that “if money-wages were to fall without limit whenever there was a tendency for less than full employment, […] there would be no resting place below full employment until either the rate of interest was incapable of falling further, or wages were zero. In fact, we must have some factor, the value of which in terms of money is, if not fixed, at least sticky, to give us any stability of values in a monetary system” (Keynes, 1936, p. 303). Keynes reverses the common wisdom on wage rigidity, that in his

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\(^{16}\)The realization of the agent’s plans is actually constrained by the trades that they are able to do and by the sequence of sales and purchases. Clower (1965) introduced the distinction between ‘notional’ and ‘effective’ demand, that only coincide when trade happens at equilibrium prices. Out of equilibrium the standard budget constraint of consumers is replaced by a liquidity (later known as cash-in-advance) constraint, and by an income constraint.
framework becomes a necessary institutional feature to avoid the implosion of the system rather than a source of disequilibrium.

Leijonhufvud’s interpretation of the General Theory subtracts it to the fate of a special case of the Walrasian model. Furthermore, it underlines the attempt to bring to the foreground the problems that (the lack of) coordination poses. This intuition, dynamic in nature, is nevertheless constrained by the choice of an equilibrium framework, and has hence to rely on the persistence of ‘wrong’ prices to explain unemployment. And this has contributed weakening the argument; after all, what would prevent, in the medium-to-long run, the creation of a new market able to coordinate investment and consumption decisions? Or the design of some institutional mechanism able to lift the economy from a low activity equilibrium?

The focus of this paper on disequilibrium dynamics allows instead to highlight two features of trading at disequilibrium prices that are mentioned but not developed by Keynes and Leijonhufvud: the appearance of constraints that at each moment in time affect the agent’s plans, and the sequence of suboptimal choices triggered by these constraints. Figures 11 and 12 show how, in my model, a shock may trigger a cumulative deflationary process, as

![Figure 11: Increase in $\Lambda$ with fixed wages ($\kappa = 0$). Perfect complementarity ($\rho = -\infty$), accommodating monetary policy ($\mu = 1$), and backward looking behaviour ($\phi = 1, \xi = 0$).](image)

originally argued by Keynes. The system is now perturbed increasing by 5% the number of workers (an increase of $H$). Unemployment could be reabsorbed by increasing productive capacity and employment to the new equilibrium level, and in fact this is what happens in the standard case. On the other hand, with complementarity in technology, and irreversibilities in the decision and production process, the shock has permanent effects. Figure 11 shows that with fixed wages unemployment is not reabsorbed, even when the financial constraint is not binding ($\mu = 1$). The complementarity between capital and labour in the production process constrains agents’ choices, and production only increases with one period lag. The loss of coordination implies that wage payment is not synchronized with production and goods availability, so that when workers are paid (during construction) they find themselves with undesired money balances; and the following period, when production is available we observe excess supply and unsold stocks, that are put back on the market in the following

---

17 The intuition that the right framework for this discussion is dynamic has kept reappearing. Drèze has recently tackled the issue within a general equilibrium model with rationing, concluding that “it is not obvious at all that price or wage adjustments susceptible of removing inefficiencies caused by price distortions would also operate in the right direction, or with any effectiveness, to circumvent coordination failures...[the heart of the problem is]...the movement from one supply-constrained equilibrium to another as a topic in dynamics, inviting the study of adjustment processes defined over prices (...), quantities, price expectations and plans” (1997, p.1753). Similar points are made by Stiglitz (1999) and Tobin (1993) as well.
period. The system settles down to an underemployment equilibrium, in which production, and consequently the wage bill and demand, are low, and unemployment remains constant.

Figure 12: Increase in $H$ with flexible wages ($\kappa = 0.5$). Perfect complementarity ($\rho = -\infty$), accommodating monetary policy ($\mu = 1$), and backward looking behaviour ($\phi = 1$, $\xi = 0$).

Figure 12 shows furthermore that allowing for wage variability does not help reabsorb unemployment. Rather, as argued by Keynes, it sets the system on a fluctuating, explosive path. The reason is that wage decreases reduce the wage bill, and hence total demand. Production keeps following with a lag, so that at each period more and more stocks cumulate, and the system explodes.

Notice an important point here. In this model relative prices (the real wage) are constant, and at the equilibrium value (given by technology alone, see eq. 7) so that in principle the purchasing power does not change, and the effect described by Keynes should not take place. What in fact determined the implosion of figure 12 is the lack of intertemporal coordination, further aggravated by wage changes. In the following chapters, when we’ll allow for real wages to change, these effects will be even more evident. But it is worth noticing that when the system lacks intertemporal coordination, changes in nominal wages may be harmful even when real wages take their equilibrium value\textsuperscript{18}.

The model presented in this paper proves to be an useful tool for analyzing transitions between equilibria in economies characterized by irreversibilities and complementarities. In many respects it gives insights that are similar to those of standard Hicksian constructs, as the ones used in the following chapters; as such, it constitutes a bridge between that stream of literature and more standard macroeconomic models.

In this paper the model was shown to be useful for reframing the theoretical debate on wage flexibility that followed the publication of the \textit{General Theory}. The focus on the properties of transitions, and on the conditions to be met for guaranteeing their viability, allow to use it to investigate empirical puzzles, as in the next chapter, or more in general to propose a different perspective on economic policy (Amendola and Gaffard, 1998).

Further research along these lines should investigate the consequences of releasing the price setting mechanism, for example allowing variable mark-ups, and consequently changes in real wages. A Further complication of the baseline model, that would most probably have the effect of smoothing fluctuations, is the introduction of stock management by firms, that would work in the direction of at least partially releasing the constraints. It would also be interesting the introduction of heterogeneity (in technology, in firm size, in consumer preferences).

\textsuperscript{18} Amendola, Gaffard and Saraceno (2004), develop the argument on real and nominal wage flexibility in Keynes and in an Hicksian framework.
Appendix

A Convergence to Equilibrium with Fixed Prices

This appendix shows how to derive the threshold values derived in section 3.2. As in the text, let's start with the case of an excess demand of labour and goods, derived for example by nominal transfers. We need to show for what values of $\kappa$ the economy converges, in the sense that an increase of wages succeeds in absorbing the extra money balances. The system 17 is reproduced here, with minor differences.

\[
l_t^s = \frac{a - (1 - a)\beta y_t^d}{a} \Rightarrow y_t^d = \frac{a(1 - l_t^s)}{A\beta (1 - a)}
\]

\[
l_t^{s+1} = 1 - \frac{1 - a}{w_{t+1}} (w_{t+1} + (\beta - 1)Aw_ty_t^d + w_t(l_t^d - l_t^s) + \beta Aw_t(y_t^d - y_t^s))
\]

\[
y_t^d = A^{-1}l_t^d
\]

\[
y_t^s = A^{-1}l_t^s
\]

We can define

\[
x_t = l_t^d - l_t^s = Ay_t^d - \frac{a - (1 - a)\beta y_t^d}{a} = Ay_t^d \left( \frac{a + \beta (1 - a)}{a} \right) - 1 > 0
\]

so that the steady state of the system is defined by $x = 0$. Labour supply may be rewritten as

\[
l_t^{s+1} = 1 - (1 - a) \left( 1 + \frac{w_t}{w_{t+1}} ((\beta - 1)l_t^s + (l_t^d - l_t^s) + \beta w_t(l_t^d - l_t^s)) \right)
\]

\[
= 1 - (1 - a) \left( 1 + \frac{w_t}{w_{t+1}} R_t \right) \text{ with } R_t = l_t^d (1 + \beta) - 2l_t^s
\]

Monotone convergence requires two conditions: The first is that the absolute value of $x$ decreases over time; the second is that its values keeps the same sign along the path. In this case, with the initial $x_t > 0$, we have

a) \[ \frac{x_{t+1}}{x_t} = \frac{\frac{a}{\beta w_{t+1}} (w_{t+1} + (\beta - 1)Aw_ty_t^d + w_t(l_t^d - l_t^s) + \beta Aw_t(y_t^d - y_t^s)) \left( \frac{a + \beta (1 - a)}{a} \right) - 1}{Ay_t^d \left( \frac{a + \beta (1 - a)}{a} \right) - 1} < 1 \]

b) \[ x_{t+1} \geq 0 \]
Conditions $a) + b)$ imply

\[
\frac{a}{a + \beta (1 - a)} \leq \frac{a}{\beta} \left( 1 + \frac{w_t}{w_{t+1}} \left( (l_t^d - l_t^s) + \beta Ay_t^d - Ay_t^s \right) \right) < l_t^d
\]  
(59)

Then, substituting the values for $y$ and rearranging we obtain

\[
\frac{a}{a + \beta (1 - a)} \leq \frac{a}{\beta} \left( 1 + \frac{w_t}{w_{t+1}} R_t \right) < l_t^d
\]

The wage equation (16) gives

\[
\frac{w_t}{w_{t+1}} = \left( 1 + \kappa \frac{l_t^d - l_t^s}{l_t^s} \right)^{-1} = \frac{l_t^s}{l_t^s + \kappa(l_t^d - l_t^s)}
\]

so that

\[
\frac{a}{a(1 - \beta) + \beta} - \frac{a}{\beta} \leq \frac{a}{\beta} \left( \frac{l_t^s R_t}{l_t^s + \kappa(l_t^d - l_t^s)} \right) < l_t^d - \frac{a}{\beta}
\]

which after some algebra, for the two inequalities gives respectively (substituting back $R_t$)

\[
\kappa > \kappa_{lm}^m = \frac{l_t^s}{l_t^d - l_t^s} \left( \frac{a(l_t^d(1 + \beta) - 2l_t^s)}{\beta l_t^d - a} - 1 \right)
\]

Similarly, the condition $b)$ gives the other extreme:

\[
\kappa < \kappa_{lh}^m = \frac{l_t^s}{l_t^d - l_t^s} \left( \frac{a(l_t^d(1 + \beta) - 2l_t^s)}{\beta l_t^d - a} - 1 \right)
\]

To summarize, for values of $\kappa \in (\kappa_{lm}^m, \kappa_{lh}^m)$, the system converges monotonically to the new equilibrium. For values below $\kappa_*$ the gap diverges $(\frac{x_{t+1}}{x_t} \geq 1)$ until it reaches what we called the "bad" equilibrium. For values above $\kappa_{lh}^m$ the variation of wages is such that the system abandons the portion of the plane characterized by excess demand of labour. This does not mean that it will diverge though, as it may well be the case that convergence is oscillatory.

Oscillatory convergence requires the following conditions:

\[
\begin{align*}
& a) : \frac{-x_{t+1}}{x_t} < 1 \quad \text{with } x_t > 0 \\
& b) : \frac{-x_{t+1}}{x_t} < 1 \quad \text{with } x_t < 0
\end{align*}
\]

In words, the absolute value of excess demand/supply must decrease, and the sign must be different at successive $t$'s. Regarding condition $a)$ we have the same starting system that we had for monotone convergence (system 55); but now condition $b)$ of eqs 58 is of course reversed. It is then of course obvious that

\[
\kappa_{l}^o = \kappa_{h}^m = \frac{l_t^s}{l_t^d - l_t^s} \left( \frac{(1 + \beta)l_t^d - 2l_t^s}{\beta l_t^d - a} - 1 \right)
\]

We only need to show that the oscillatory trajectory does not explode, i.e. that $-x_{t+1} < x_t$:

Manipulations similar to those that have been carried out above yield

\[
2 \left( \frac{\alpha}{\alpha + \beta (1 - \alpha)} \right) - \frac{\alpha}{\beta} \left( 1 + \frac{w_t}{w_{t+1}} R_t \right) < l_t^d
\]
that allows to derive $\kappa_h^o$:

$$\kappa < \kappa_h^o = \frac{l_t^s}{l_t^d - l_t^d} \left( \frac{(a + \beta (1 - a)) a}{(2a - l_t^d(a + \beta (1-a))) \beta - (a + \beta (1-a)) a} R_t - 1 \right)$$

The range over which $\kappa$ allows the new equilibrium to be reached is hence $\kappa \in (\kappa_h^{mn}, \kappa_h^o)$.

### B Simulation Results

The following four tables report the complete results of the regressions that are behind figures 5, 6 and 10. To obtain the values in the tables, the following algorithm was followed\(^{19}\). Starting from a set of regressors (the nine variables, plus their squares, with the exception of the indicator $\phi$), new variables were created (powers and cross products) sequentially. Each new variable was regressed on the set of other regressors, and dropped if its $R^2$ was larger than 0.99 (this to avoid multicollinearity). Otherwise, it was added to the set of regressors, and ran on the endogenous variable. If its Student $t$ was larger than 2, it was kept, otherwise dropped.

Notice that this algorithm does not guarantee a “path independent” selection of regressors, because the order in which variables that may be collinear are extracted matters. This is why that set of regressors was further altered by doing joint significance tests, and adding other variables deemed significant; this arbitrary second round allowed to significantly increase the $R^2$ of the regression, or at least to keep it almost unchanged while reducing significantly the number of regressors.

Notice finally that the benchmark value of production has in fact two different meanings when referring to nominal shocks (table 1), and when referring to real ones (table 3). In the latter case, the shock alters the equilibrium values of labour supply and production, so that $y_{ss} \neq y_0$ is the new equilibrium value. In the case of nominal shocks, on the other hand, the shock does not alter the fundamentals of the economy, so that the new equilibrium value of production is the initial one.

\(^{19}\)The algorithm was implemented in Eviews. The help of Giovanni Marini is gratefully acknowledged.
### Table 1: Coefficients and Student t for nominal shocks (fig. 5)

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<th>t</th>
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<th>t</th>
<th>Name</th>
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$R^2$ | 0.846 |
$Adj. \ R^2$ | 0.841 |
$obs$ | 5064 |

### Table 2: Coefficients and Student t for nominal shocks (figures 5 and 6)

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$R^2$ | 0.791 |
$Adj. \ R^2$ | 0.786 |
$obs$ | 5064 |
Dependent Variable: $y_{200}/y_{s.s.}$

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$R^2$ 0.852  
$Adj.R^2$ 0.851  
$obs$ 5603

Table 3: Coefficients and Student $t$ for real shocks (fig. 10)
Dependent Variable: \( s.e.(y) \)

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Table 4: Coefficients and Student \( t \) for real shocks (fig. 10)
References


