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# Employment protection and capital-labor ratios\*

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## Abstract

Employment protection (EPL) has a well known negative impact on labor flows as well as an ambiguous but often negative effect on employment. In contrast, its impact on capital accumulation and capital-labor ratio is less well understood. The available empirical evidence suggests a non-monotonic relation between capital-labor ratios and EPL: positive at very low levels of EPL, and then negative.

We explore the theoretical effects of EPL on physical capital in a model of a firm facing labor frictions. Under standard assumptions, theory always implies a monotonic negative link between capital-labor ratios and EPL. For a positive link to arise, a very specific pattern of complementarity between capital and workers protected by EPL (senior workers, as opposed to unprotected new entrants, or junior workers) has to be assumed. Further, no standard production technology is able to reproduce the inverted U-shape pattern of the data.

Instead, endogenous specific skills investment leads to an inverted U-shape pattern: EPL protects and therefore induces investments in specific skills. We develop such a model and calibrate the returns to seniority by using estimates from the empirical literature. Under complementarity between capital and specific human capital, physical capital and senior workers having accumulated specific human capital are *de facto* complement production factors and EPL may increase capital demand at the firm level. The paper concludes that labor market institutions may sometimes favor physical and human capital investments in second-best environments.

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# 1 Introduction

An expanding literature investigates the role of employment regulations on various outcomes such as investment in physical capital, human capital, productivity, innovation and growth. Labor studies have shown that a particular component of these regulations, employment protection, has sizeable effects on unemployment rates, turnover, job flows and unemployment duration. Employment protection legislation (hereafter, EPL) has a well known negative impact on labor flows and an ambiguous but often negative effect on employment.<sup>1</sup>

The effect on investment and the capital-labor ratio is less well understood and, in the empirical literature, the effect of EPL is ambiguous. In articles based on countries characterized by low employment protection stringency, the effect of EPL on capital-labor ratios is found to be positive. For example, Autor et al. (2007) use the adoption of wrongful-discharge protections by U.S. state courts from the late 70s to the late 90s to evaluate the link between dismissal costs and other economic variables. With firm-level data, they find a positive effect of employment protection on the capital-labor ratio. More recently, Cingano et alii (2014) use a labor reform in Italy - known for being a country with a relatively low degree of employment protection in small firms. The authors exploit an interesting quasi-experiment and a regression-discontinuity design, the reform being applied differently below and above the 15-employees threshold. They show that the capital-labor ratio may increase after an increase in firing costs, with however a decline in total factor productivity.<sup>2</sup> On the other hand, papers focusing on countries with strong employment protection document a negative relationship. Cingano et alii (2010) find a negative effect on capital per worker in the case of European firms.<sup>3</sup> Calgagnini et alii (2009) also document a negative effect of employment protection on investment in a sample of firms from ten European countries. Hence, the effect of EPL may therefore be non-linear, and in particular, positive for investment at low values of EPL, and negative for investment at higher values, which would reconcile the various empirical findings of these papers. This paper precisely analyses the effect of employment protection on capital accumulation and capital-labor ratios and attempts to reconcile the various findings in the literature.

Our paper is guided by the intuition that, at low levels, EPL may induce more capital investment and not less capital investment, for theoretical reasons that need

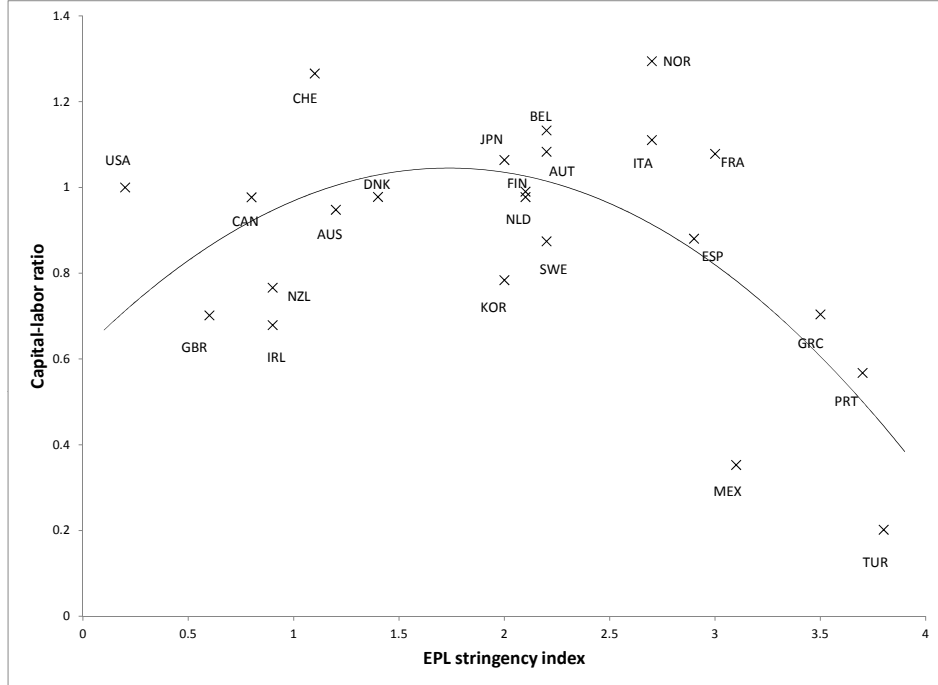
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<sup>1</sup>Influential papers have investigated the role of employment regulations on other dimensions such as productivity and growth: Hopenhayn and Rogerson (1993) and Bertola (1994) argue that productivity is lower because of a misallocation of employment to technologies, favoring less productive structures, leading to reduced incentives for capital accumulation. Bassanini et al. (2009) empirically document the link between employment protection and productivity growth and find that EPL reduces productivity growth in industries where EPL is more likely to be binding. There is also an emerging literature on the pattern of trade specialization: Saint-Paul (1997, 2002) shows that countries with a rigid labor market will tend to produce relatively secure goods, at a late stage of their product life cycle and therefore innovate less, rather immitate. See also the more recent paper by Cuñat and Melitz (2010). There is a very active and broader literature extending models of trade to imperfect labor markets e.g. Costinot (2009).

<sup>2</sup>As the authors argue, their empirical analysis supports several findings of our paper, discussed below.

<sup>3</sup>Their methodology follows Rajan and Zingales (1998): it compares the impact on the demand for capital in sectors requiring large job reallocation with sectors where job reallocation is low.

Figure 1: Capital-labor ratio and employment protection stringency in the OECD



Notes: data on employment protection is from OECD (2004) and data on capital-labor ratios is from Caselli (2005). On the graph, the capital-labor ratio is expressed as a ratio relative to the US ratio.

to be developed. Inspection of data shows, without a claim on causality, Figure 1 at least suggests that an inverted U-shape pattern tends to emerge from the data: the x-axis is the standard OECD stringency index and the vertical axis is the capital-labor ratios. At low EPL level, the correlation is positive, and it becomes negative when the index becomes larger than 1.75. The regression analysis in Table 1 confirms this, but also the fragility of the correlation. The inclusion of a dummy variable for Anglo-Saxon countries leads to a negative and significant coefficient on EPL, while the inclusion of a dummy for “high EPL” countries produces a positive and non-significant correlation. Overall, the correlation coefficient is negative (-0.34) and in the regression, the linear effect is negative but not significant. The correlation coefficient is equal to 0.40 when Greece, Mexico, Portugal and Turkey are removed from the sample and takes the value -0.67 when Anglo-Saxon countries are not considered. To sum up, at low EPL level, the correlation is positive, and it becomes negative when the index becomes larger.

We will indeed show the following results:

i) In the simplest setup with only physical capital and labor, there is a negative link between capital-labor ratios and EPL for standard production functions. When capital and labor are complements, the demand for capital per unit of labor has to decrease if EPL negatively affects labor productivity.

Table 1: Regressions of capital-labor ratio on EPL stringency

Specifications:	(I)	(II)	(III)	(IV)
EPL index	-0.088 (0.053)	-0.253*** (0.069)	0.079 (0.048)	0.490** (0.184)
Squared EPL index				-0.141*** (0.044)
Anglo-Saxon dummy		-0.499*** (0.158)		
High EPL dummy			-0.663*** (0.127)	

Notes: data on employment protection is from OECD (2004) and data on capital-labor ratios is from Caselli (2005). The Anglo-Saxon dummy is equal to one for Australia, Canada, Ireland, New Zealand, the UK and the USA. The high-EPL dummy is equal to one for Greece, Mexico, Portugal and Turkey. \*\* significant at 5% \*\*\* significant at 1%

ii) A positive link may arise due to a specific pattern of complementarity between capital and workers protected by EPL (hereafter we call such workers senior workers, as opposed to unprotected new entrants, or junior workers). The intuition is the following: EPL increases the share of senior workers in employment. When senior workers are complement with capital, this raises the demand for capital. Hence, higher EPL leads to higher investment in physical capital. However, the positive link remains monotonic over the whole range of EPL values: no standard production technology is able to reproduce the inverted U-shape pattern.<sup>4</sup>

iii) When workers invest in specific skills, the impact of EPL is however non-monotonic and leads to an inverted U-shape pattern. The reason is that EPL protects and therefore induces investments in specific skills. Assuming complementarity between capital and specific human capital is natural and therefore, the complementarity between capital and senior workers discussed in ii) above becomes an endogenous outcome of the model: EPL raises the demand for capital through raising the share of senior workers. Further, the size of this effect on investment in physical capital varies with the intensity of employment protection: it is higher at low levels of EPL, and lower at higher levels of EPL. Indeed, given decreasing returns to scale in human capital, at higher levels of EPL, a marginal increase in EPL has little effect of investments and therefore on demand for capital. This generates the hump-shape curve.

iv) For low values of EPL, our model implies both a positive effect of EPL on capital-labor ratios and a negative productivity effect due to greater misallocation of workers, consistent with Cingano et alii (2014).

We base our analysis on a model of a large firm with physical capital, facing labor-market matching frictions and endogenous job destruction. Labor frictions have indeed been shown to be key in understanding the effect of employment protection on labor market flows and the demand for factors (Mortensen and Pissarides, 1999). Starting from their setup, we can explore the effect of EPL on the demand for capital. This requires to extend the benchmark model of EPL to endogenous capital accumulation.

<sup>4</sup>It is always possible to find a more complex production function leading to an inverted U-shape pattern but this would arguably be an artificial way of reproducing the empirical results.

It also requires to introduce these ingredients in the so-called large firm matching model developed initially in Pissarides (1990). The large firm matching model requires the derivation of a set of wage determination rules that are more complex than the conventional Nash-bargaining solution. Indeed, the large firm, when it bargains over wages with its different workers, can exploit the possibility of complex strategic interaction à la Stole and Zwiebel (1996a, 1996b). In their setup, in a frictionless labor market, decreasing returns to scale lead the bargaining firm to raise employment above the competitive level, in order to reduce the marginal product of labor and progressively reduce wages - driven down to the reservation wage at the optimal employment level of the firm. Here, with bargaining over wage and search frictions, the same issue arises because the presence of physical capital in production imposes decreasing returns to scale in labor. Decreasing returns to scale in labor require the firm to take into account that, in over-hiring, it can reduce the marginal product of workers and therefore the bargained wage. This leads to higher profits than if the firm simply ignored these interactions. These effects were analyzed in the context of a matching model in Smith (1999), Cahuc and Wasmer (2001), Cahuc, Marque and Wasmer (2008) and Bagger et alii (2011). Hereafter they are referred to as *intrafirm bargaining*.<sup>5</sup>

Hence, in order to answer the question of the effect of employment protection on capital accumulation, we proceed as follows:

1. We generalize the large firm bargaining model to endogeneous job destruction à la Mortensen and Pissarides (1999), implying in particular to move from a countable number of categories of workers to a continuum of substitutable workers with different productivities.
2. We generalize the intrafirm bargaining model to the existence of a dual employment structure with firing taxes affecting only senior workers and not junior workers.
3. We generalize the large firm matching model to endogenous specific human capital investments and study the fixed skill models as a limiting case.

A literature suggests a positive relation between employment protection and investment in specific human capital. Arulampalam et alii (2004) and Brunello (2004) illustrate with data from the European Commission Household Panel a positive link between EPL and training. Though the cost of training is typically paid by the firm and it is not clear that all components of training reflect specific human capital, this evidence is supportive of the mechanism we propose in this paper. Our paper is also related to the work by Belot et alii (2007). These authors document a hump-shaped relation between employment protection and growth. They then present a three-period model, where the hump is the result of two opposite forces. On the one hand, employ-

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<sup>5</sup>Elsby and Michaels (2013) study the business-cycle and cross-sectional properties of a large-firm model with endogenous job destruction. In their model, job destruction appears because of idiosyncratic shocks to firm-level productivity. In our case, it is the productivity of each individual employee that is hit stochastically. Our model is more tractable because we do not need to consider the presence of an inaction region along the labor demand schedule. The job creation and destruction relations in Mortensen and Pissarides (1994) even correspond to a specific case of the relations presented in Section 3 (when the production function is linear). However, a drawback of our model is that it does not generate zero-employment growth for a subset of firms, as Elsby and Michaels (2013) do.

ment protection increases the cost of job separation. On the other hand, it encourages workers to invest in specific human capital. See also Saint-Paul (2002) who analyzes the political economy consequences of this mechanism. His model generates multiple equilibria, because of the mutual feedback between employee rents and employment protection.

The paper is organized as follows. In Section 2, we present the labor demand block of the model; in Section 3, we derive labor demand, capital demand, the bargaining conditions over wages and the optimal investment in human capital. Section 4 presents simulation results for a large class of production functions. Section 5 concludes that labor market institutions may sometimes favor physical and human capital investments in second-best environments.

## 2 Labor inputs and labor demand

### 2.1 Setup

Time is continuous and discounted at a rate  $r$ . We study the steady state of an economy populated by a representative firm and a unit mass of workers. The firm produces output with labor and capital in quantities  $N$  and  $K$  respectively. The output can either be re-invested, consumed or used to cover other expenses such as vacancy costs and layoff costs.

There are two sources of labor heterogeneity within the firm. First, at the extensive margin, there are two seniority levels within the firm: workers are hired as *junior* and eventually become *senior*. Each junior-senior status implies a specific set of rules and assumptions regarding labor regulation, wage negotiation and productivity, which are fully described below. Second, at the intensive margin and within the senior status, workers have different levels of efficiency.

When hired, a worker starts as a junior and is therefore endowed with one efficiency unit as a normalization. Junior workers subsequently become senior at an exogenous rate  $\lambda$ . When hit by this shock, the amount of efficiency units each worker is endowed with changes too: it is equal to  $h_i z$ , where  $h_i$  is the level of human capital of a senior worker, discussed later in the paper, while  $z$  is a stochastic component specific to that worker. The value  $z$  is drawn for each newly senior worker, independently, from a cumulated distribution  $G$  (and density  $g = G'$ ) defined over the  $[0, 1]$  interval. As time goes, senior workers subsequently face changes in the  $z$  component. Changes occur at a rate  $\lambda$ . The new  $z$  is drawn from the same cumulated distribution  $G$ .<sup>6</sup>

A key assumption is that human capital  $h$  is determined before the productivity  $z$  is known, as the outcome of an ex-ante investment. This will notably imply that, in equilibrium, all senior workers will have a common level of human capital  $h$ .

The firm may endogenously choose to destroy jobs upon the revelation of productivity following an idiosyncratic shock. In particular this occurs when the stochastic

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<sup>6</sup>It is beyond the scope of this paper to study alternative stochasticity assumptions as for instance den Haan et al. (2000), Walsh (2005), or Krause and Lubik (2007). These alternative assumptions are however very useful in DSGE contexts and, for a study of the shorter-run relations between EPL and K/N ratios. Short-run and business cycle implications of the effect of EPL, not studied here, are important topics and should receive some more attention.



component of worker's efficiency drops below a threshold  $R(h) \in (0, 1)$ , the value of which is determined below. Note that  $R$  depends on  $h$ , the level of human capital of senior workers, and this will be taken into account in skill investment decisions. Another important remark is that the firm, having perfect access to capital market, may keep workers with a negative marginal net revenue, since the shock on  $z$  is temporary.<sup>7</sup>

Finally, when fired, a worker joins the pool of unemployed. Leaving the firm implies that, upon re-entry, senior workers will start as junior workers with the entry productivity of junior workers, namely 1: the level of specific human capital  $h_i$  cannot be transferred to another job, by definition.<sup>8</sup>

## 2.2 Labor market frictions and steady-state employment

The labor market is characterized by search and matching frictions. This implies the existence of frictional unemployment. In particular, the firm posts vacancies at a flow cost  $c$  in order to hire workers. The labor force is normalized to 1 and we denote by  $V$  the mass of posted vacancies. Vacancies are filled at a rate  $q(\theta)$  that depends negatively on the labor market tightness  $\theta \equiv \frac{V}{1-N}$ , i.e. the vacancy-unemployment ratio. This rate is derived from a matching function  $m(1-N, V)$  with constant returns to scale, increasing in both its arguments, concave and satisfying the property  $m(1-N, 0) = m(0, V) = 0$ , implying that  $q(\theta) = \frac{m(1-N, V)}{V} = m(\theta^{-1}, 1)$ . Similarly, the rate at which unemployed workers find a job is equal to  $\theta q(\theta)$ .

Denote by  $N = N_J + N_S$  where  $N_J$  and  $N_S \equiv \int_R^1 n_S(z) dz$  the mass of junior and senior workers respectively, and  $n_S(z)$  is the mass of senior workers employed by the firm, who are endowed with  $z$  efficiency units. Their laws of motion are described by the following equations:

$$\dot{N}_J = Vq(\theta) - \lambda N_J \quad (1)$$

and

$$\dot{N}_S = \lambda(1 - G(R))N_J - \lambda G(R)N_S. \quad (2)$$

In the steady state, flows into aggregate employment equal flows out of employment. This leads to the following steady-state level of total employment, itself a sum of senior and junior employment, as:

$$N = N_S + N_J = \frac{\theta q(\theta)}{\theta q(\theta) + \lambda G(R)}, \quad (3)$$

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<sup>7</sup>Note also that the degree of persistence varies with our parameter  $\lambda$  (productivity shock frequency). As a result, high frequency leads to firms keeping their workers even if they have temporarily low productivity and possibly negative cash flows. This productivity parameter and EPL introduced later have similar consequences on labour hoarding. This similarity fully applies in a world of perfect capital markets. See den Haan et alii (2003) for imperfect capital markets. Studying the channels through which credit market imperfections amplifies the effect of EPL is beyond the scope of this paper but they are rich.

<sup>8</sup>Note also, following a discussion with a referee, that our setup does not necessarily imply that senior workers are more heterogenous than junior workers. Our view is that junior workers are sort of temporary workers, that is, ex ante identical ; they may be different but the firm only has the information after the  $\lambda$  shock (acquisition of seniority) is realized ; at this stage it decides to truncate the distribution and only keep good matches. Hence, the pool of junior workers is not necessarily homogeneous. It is just that firms do not know yet and make the optimal vacancy decisions in waiting for the correct signal of their productivity has appeared.

while the unemployment rate is the complement to 1:

$$u = \frac{\lambda G(R)}{\theta q(\theta) + \lambda G(R)}. \quad (4)$$

Given that inflows and outflows into an interval of productivity  $z$  and  $z + dz$  are independent of  $z$ , this implies that the mass  $n_S(z)$  of senior workers at productivity  $z$  is simply equal to:

$$n_S(z) = N_S \frac{g(z)}{1 - G(R)}.$$

The last important stock variable is the mass of senior workers within the firm, and expressed in efficiency units. It is the sum of its individual components. We denote it by  $Z$ , and

$$Z(\mathbf{h}) \equiv \int_R^1 z n_S(z) \left( \int_{\mathcal{C}_{i(z)}} h_i di \right) dz = \int_R^1 z n_S(z) dz \int_{\mathcal{C}_i} h_i di = h \int_R^1 z n_S(z) dz = Z(h),$$

where  $\mathbf{h}$  is the vector of human capital of its workers,  $i(z)$  is an individual in a mass of workers of productivity  $z$ ;  $\mathcal{C}_{i(z)}$  is the subset of these individuals;  $\mathcal{C}_i$  is the subset of all individuals regardless of their productivity  $z$ ; and  $h$  is the total human capital in the firm, equal to  $\int_R^1 \int_{\mathcal{C}_{i(z)}} h_i di dz = \int_{\mathcal{C}_i} h_i di$ . The second equality where  $\int_{\mathcal{C}_i} h_i di$  is taken away from the integral over  $z$  arises from the fact that the distribution of skills  $h_i$  is independent of the level of  $z$  due to the assumption that human capital choices are done ex ante. Further,  $Z(h)$  can eventually be written as a function of the scalar  $h$  since each component of  $\mathbf{h}$ ,  $h_i$ , turns out ex post to be independent of the productivity component of identical marginal workers. In what follows, we write for simplification  $Z(h)$  except when it is necessary to distinguish between individual and aggregate components of productivity  $h_i z$ . Note that our specification excludes heterogeneity in the ability to acquire skills, to preserve the symmetry of the choice of human capital.

Physical capital accumulation follows

$$\dot{K} = I - \delta K, \quad (5)$$

which describes the dynamics of the capital stock. We consider here a simplified capital accumulation and depreciation process and do not consider additional cost of installing physical. See however Yashiv (2014) and the subsequent references for the effect of these additions to the matching model.  $I$  is investment in capital,  $\delta$  is the capital depreciation rate.

Finally, production  $Y$  follows a constant-returns-to-scale production function that is strictly concave in each argument, here the three stock variables:

$$Y = F(N_J, Z(h), K)$$

### 2.3 Bellman equations of workers and firms

Workers invest in *specific* skills at the time of entry. They do so at cost  $C(h_i)$  on the spot, with  $C'(h_i) > 0$  and  $C''(h_i) \geq 0$ . The productivity of the investment in human capital is however differed, and adds up to productivity only when workers

become senior. The  $h$  component is thus endogenous. The special case with fixed human capital is obtained when  $h$  cannot be chosen by workers, e.g. when  $C(h_i)$  becomes vertical around the equilibrium value.

The present discounted value of being unemployed is defined as

$$rU = b + \theta q(\theta) \text{Max}_{h_i} [W_J - C(h_i) - U], \quad (6)$$

where  $b$  is the flow value of being unemployed ;  $W_J$  is the present discounted value of being employed as a junior worker and  $U$  the value of unemployment. Junior workers earn the amount  $W_J$  until they become senior or leave the firm. It takes the following value:

$$rW_J = w_J + \lambda \int_0^1 [\max \{W_S(z, h), U\} - W_J] dG(z) \quad (7)$$

and the present discounted value of being employed as a senior worker with  $z$  efficiency units and  $h$  units of human capital is:

$$rW_S(z, h) = w_S(z, h) + \lambda \int_0^1 [\max \{W_S(z', h), U\} - W_S(z, h)] dG(z'). \quad (8)$$

We assume that when a senior worker is laid off, the firm pays a firing tax  $T$  that is not redistributed to the worker but lost to the match - e.g. an administrative cost as in Mortensen and Pissarides (1999). The total value of the firm is therefore

$$\bar{\Pi} = \max_{V, I} \frac{1}{1 + rdt} \left\{ \left( F(N_J, Z(h), K) - w_J N_J - \int_R^1 w_s(z, h) n_S(z) dz - cV - I - \lambda(N_S + N_J)G(R)T \right) dt + \bar{\Pi}' \right\}, \quad (9)$$

subject to equations (1), (2) and (5). In equation (9),  $dt$  is an arbitrarily small interval of time,

## 2.4 Wages

We restrict the analysis to the solutions in a stationary state in which the mass of senior workers in efficiency units  $Z$  is constant.<sup>9</sup>

Denote by  $\Pi_J = \frac{\partial \bar{\Pi}}{\partial J}$  the marginal value of a junior worker. Entry wages are negotiated *à la* Nash between the worker and the firm. We denote by  $\beta \in (0, 1)$  the bargaining power of workers. Hence, the wage of a junior worker is determined as follows:

$$w_J = \arg \max \Pi_J^{1-\beta} [W_J - U]^\beta, \quad (10)$$

Denote by  $\Pi_z = \frac{\partial \bar{\Pi}}{\partial Z} \frac{\partial Z}{\partial n_s(z)}$  the marginal value of a senior worker endowed with  $z$  efficiency units. The firm applies the following rule at  $z = R$ :

$$\Pi_R + T = 0. \quad (11)$$

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<sup>9</sup>This is not due to the law of large numbers. This is simply a stationary assumption whereby aggregate  $Z(h)$  at the firm level does not change in time. We only focus here on steady-states of this representative firm. One can however not rule out the possibility of having oscillations of  $Z$  for sub-representative firms that would be leaving the aggregate  $Z$  constant. This would require a different model as regards to the aggregation of these sub-firms.

Because the firm pays the firing tax only when a senior worker is dismissed, the firm's threat point is different when bargaining wages with senior workers. This implies the following rule for the wage of senior workers, instead of the solution described by equation (10):

$$w_S(z, h) = \arg \max [\Pi_z - (-T)]^{1-\beta} [W_S(z, h) - U]^\beta. \quad (12)$$

It can be seen that  $T$  positively affects the firm surplus in this case, as  $T$  reduces the outside option of the firm.

### 3 First order solutions and equilibrium

Given that investment in human capital can only be made upon entry, the marginal cost of human capital investment is not considered in Nash bargaining, neither in labor demand equations : it is sunk. We also assume the firm takes  $h$  as pre-determined. Workers have decided about the investment before the firm takes its decisions regarding factor demand.

#### 3.1 First-order conditions and factors demand

We now introduce compact notations for first order derivatives in what follows:  $F_J = \partial F(N_J, Z(h), K)/\partial N_J$ ;  $F_Z = \partial F(N_J, Z(h), K)/\partial Z(h)$ ;  $F_K = \partial F(N_J, Z(h), K)/\partial K$ . We drop the arguments of these derivatives unless they are different from  $(N_J, Z(h), K)$ .

In the Appendix, we show that the first-order conditions of the factor demand problem in (9) together with the envelope theorem imply the following equilibrium conditions:

$$(r + \lambda) \frac{c}{q(\theta)} = F_J - w_J - \frac{\partial w_J}{\partial N_J} N_J - \int_R^1 \frac{\partial w_S(z)}{\partial N_J} n_S(z) dz - \lambda(1 - F(R))T + \lambda \int_R^1 \Pi_z dG(z), \quad (13)$$

and

$$r + \delta = F_K - \frac{\partial w_J}{\partial K} N_J - \int_R^1 \frac{\partial w_S(z, h)}{\partial K} n_S(z) dz. \quad (14)$$

Equation (13) describes the incentives for the firm to open up new vacancies. The left-hand side represents the marginal cost of filling a junior vacancy, while the right-hand side is the marginal revenue the marginal junior worker brings to the firm—equal to  $(r + \lambda)\Pi_J$ , that is, its marginal product (first term) net of the wage (second term) and the effect of this hiring on the wage of the other workers (third and fourth terms). The last two terms are respectively the expected value of the firing tax (the tax itself multiplied by its probability of layoff) and the continuation value of profits. Equation (14) characterizes the firm's capital investment decision. The firm purchases capital such that the opportunity cost of capital (the left-hand side) equalizes the marginal revenue (the right-hand side). The latter is composed by the marginal product of capital and the effect of the capital stock on wages.

To solve for this system, we need to know the value of  $-\lambda(1 - F(R))T + \lambda \int_R^1 \Pi_z dG(z) = -\lambda T + \lambda \int_R^1 [\Pi_z + T] dG(z)$ . In particular, we need to compute  $\Pi_z + T$ . It is given by:

$$(r+\lambda)(\Pi_z+T) = F_Z zh - \frac{\partial w_J}{\partial Z} zh N_J - \int_R^1 \frac{\partial w_S(z')}{\partial Z} zh n_S(z') dz' - w_s(z) + rT + \lambda \int_R^1 (\Pi_{z'} + T) dG(z') \quad (15)$$

Equation (15) can be understood in a similar way as Equation (13).

### 3.2 Strategic bargaining

We have previously illustrated that the employment and capital stocks affect wages. Both workers and firms take this into account when negotiating wages. In particular, firms use hiring decisions as a way to strategically affect the marginal product of labor and in turn, on equilibrium wages. This is the logic of intrafirm bargaining in Stole and Zwiebel (1996a and b), extended to search models in Smith (1999), Cahuc and Wasmer (2001), Cahuc, Marque and Wasmer (2008) and Bagger and alii (2011). In the Appendix, we show that the solutions for wages resulting from these strategic interactions follow a simple rule.

The wage is a weighted average of the reservation value of the worker  $rU$  and of the marginal product of each type of labor, augmented (for senior workers) or diminished (for junior workers) by the value of layoff costs:

$$w_J = (1 - \beta)rU - \beta\lambda T + \beta\Omega_J F_J \quad (16)$$

in the case of the junior wage and

$$w_S(h, z) = (1 - \beta)rU + \beta rT + \beta\Omega_S F_Z zh \quad (17)$$

for senior, where

$$\Omega_J = \frac{\int_0^1 \frac{1}{\beta} x^{\frac{1-\beta}{\beta}} F_J(N_J x, Zx, K) dx}{F_J} \quad (18)$$

and

$$\Omega_S = \frac{\int_0^1 \frac{1}{\beta} x^{\frac{1-\beta}{\beta}} F_Z(N_J x, Zx, K) dx}{F_Z} \quad (19)$$

are *over-employment factors* resulting from strategic interactions. They are derived and discussed in Cahuc et alii (2008, Section 2): when  $\Omega_J$  and  $\Omega_S$  are equal to one, the solution is the one described in Mortensen and Pissarides (1999) when the production technology is such that returns to labor are constant. When they differ from one, e.g. with decreasing returns to scale in labor, we are away from Mortensen and Pissarides' solution: they describe a situation of "over-employment" if they take a value larger than 1 and "underemployment" if their value is below 1. Finally, firing taxes have the standard effect on wages; junior wages decrease with  $T$  and senior wages increase with  $T$ .

### 3.3 Job creation and destruction

By replacing the solution for wages in (13) and (15), we obtain the following job creation rule:

$$\frac{c}{q(\theta)} = (1 - \beta) \left[ \frac{\Omega_J F_J - \Omega_S F_Z R(h)h}{r + \lambda} - T \right]. \quad (20)$$

In the Appendix, we also show that job destruction rule (11) can be written as follows:

$$0 = \Omega_S F_Z h \left( R(h) + \frac{\lambda}{r + \lambda} \int_R^1 (z - R(h)) dG(z) \right) - rU + rT, \quad (21)$$

with  $rU$  defined in (6).

Again, these conditions differ from Mortensen and Pissarides' through the presence of the over-employment factors and the fact that marginal products of labor are not necessarily constant.

### 3.4 Capital demand

Using the wage equations (16) and (17), we can rewrite the capital demand (14) as

$$(1 - \beta)\Omega_K F_K = r + \delta, \quad (22)$$

where

$$\Omega_K = \frac{\int_0^1 \frac{1}{\beta} x^{\frac{1-\beta}{\beta}-1} F_K(N_J x, Zx, K) dx}{F_K} \quad (23)$$

is an over-investment factor that takes the condition away from the neo-classical investment model when its value differs from one, and is identical to the expression in Cahuc et alii (2008, Section 4).

### 3.5 Human capital investment

#### 3.5.1 The case of endogenous human capital

So far we have treated  $h$  as a parameter. However, it is chosen optimally by junior workers prior to knowing their future idiosyncratic component  $z$ . The first-order condition for human-capital investment of the program in (6) reads as

$$(r + \lambda)C'(h_i) = \frac{\partial w_J(h)}{\partial h_i} + \frac{\lambda}{r + \lambda G(R)} \int_R^1 \frac{\partial w_S(h_i, z)}{\partial h_i} dG(z). \quad (24)$$

It is straightforward to see from the equation above that, for a given marginal effect of  $h_i$  on wages and a convex function  $C$ , lower  $R$  implies higher investment  $h_i$ . The reason is because when  $R$  is low, workers anticipate longer tenure on the job, which increases the marginal return on human capital.<sup>10</sup>

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<sup>10</sup>Here we assumed that investment in human capital is paid and decided by workers, in line with Laing et al. (1995) or Acemoglu (1997) in the case of general human capital investment. As regards to specific human capital investments, they are often provided by employers, as argued by Lynch and Black (1998). Wasmer (2006) shows that for specific skill investments, it does generally not matter whether employers or employees pay for the investment, as the outcome of the investment is to create a rent and a surplus to the match employer-employee. Hence the logic and the determinants of the investments are the same for employers and employees. Things would be much more asymmetrical for general skills, since there, indeed, workers benefitiate from them but not firms. We thank a referee for making this point.

We show in the Appendix that condition (24), which describes the incentives of supplying human capital, can be rewritten as<sup>11</sup>

$$C'(h) = \frac{\lambda[1 - G(R)]}{r + \lambda} \frac{\beta\Omega_S}{r + \lambda G(R)} F_Z \int_R^1 z \frac{dG(z)}{1 - G(R)} \quad (25)$$

once wages are replaced by their equilibrium values.

The condition is interpreted as follows. The left-hand side of the equation is the marginal cost of investing in human capital, while the right hand side is the marginal return. Because investment is made upon entry and the increase in wages only takes place once a worker becomes senior, the return has to be discounted by a factor  $\frac{\lambda[1-G(R)]}{r+\lambda}$ , which reflects the average time a junior worker has to wait to become senior. The term  $F_Z \int_R^1 z \frac{dG(z)}{1-G(R)}$  is the expected marginal product that is brought by each unit of human capital a given worker invests in. From this expected marginal product, the worker will receive a share  $\beta\Omega_S$  in wages, which takes into account the bargaining power of the worker and the fact the worker will appropriate part of the decrease in wages of other workers (if he invests in this marginal unit of human capital). Finally, by dividing by the term  $r + \lambda G(R)$ , one obtains the present discounted value of the marginal increase in senior wages.

### 3.5.2 The case of fixed human capital investment

The model can be summarized with the following equations with  $h = \bar{h}$ , respectively a job creation condition, a job destruction condition, an optimal investment equation and the value of outside options of employed workers:

$$\begin{aligned} \frac{c}{q(\theta)} &= (1 - \beta) \left[ \frac{\Omega_J F_J - \Omega_S F_Z R \bar{h}}{r + \lambda} - T \right] \\ \Omega_S F_Z h \left( R(h) + \frac{\lambda}{r + \lambda} \int_R^1 (z - R) dG(z) \right) &= rU - rT \\ (1 - \beta) \Omega_K F_K &= r + \delta \\ rU &= b + \theta q(\theta) \text{Max}_{h_i} [W_J - U - C(\bar{h})]. \end{aligned}$$

## 4 Numerical examples

We resort to numerical examples to illustrate the various effects of employment protection, reflected by the tax on lay-offs  $T$  in our model. Our strategy is as follows: we first find a set of parameters that approximate an economy with labor-market characteristics similar to the United States, with endogenous human capital. In this economy, taxes on lay-offs are absent. We call this the benchmark economy. We then ask how macroeconomic aggregates in this economy are affected by the introduction of a firing tax in the different cases studied in the theoretical part. We also study the case of fixed human capital in fixing the value of human capital to the endogenous value in the benchmark economy.

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<sup>11</sup>Notice that the derivative  $\partial w_S(h_i, z)/\partial h_i$  in equation (24) is calculated with respect to the stock of human capital  $h_i$  of an individual worker, not with respect to the total stock of human capital  $h$ .

## 4.1 Calibration

Our benchmark economy with no firing tax resembles the economy described in Pissarides (2009). The reason is because the theoretical model in Pissarides (2009) is a particular case of the theoretical model we describe in Section 2. It corresponds to the case where the supply of human capital is inelastic, the distribution  $G$  is degenerate in zero, and the production function is linear in each of its arguments, which impedes the firm from over-hiring. Hence, we borrow a great deal from Pissarides (2009) and share many of his parameter values.

We assume that a unit interval of time corresponds to a month. We set the discount rate at a 4% annual rate. The matching function is assumed to be Cobb-Douglas  $m(1 - N, V) = m_0(1 - N)^\eta V^{1-\eta}$ , with unemployment elasticity  $\eta = 0.5$ . We also follow common practice by setting  $\beta = \eta$ . This would internalize the search externalities in the standard one-worker-per-firm model.<sup>12</sup> We follow Pissarides by targeting a labor-market tightness of 0.72, which is consistent with data from the JOLTS and the Help-Wanted Index for the period 1960-2006. This implies a scale parameter of the matching function  $m_0 = 0.7$ .

The production function of the benchmark economy is Cobb-Douglas  $F(N_J, Z(h^*), K) = A(N_J + Z(h^*))^\alpha K^{1-\alpha}$ . We fix the labor share  $\alpha$  to a standard value of two thirds and the total factor productivity  $A = 0.54$  produces a job finding rate equal to 59.4%, which is consistent with Shimer's (2007) monthly transitions data. In the sensitivity analysis of Section 4.4, we consider other specifications for the production function that help understand the economic forces at work.

We fix the parameter  $\lambda$  by targeting a monthly job separation rate of 3.6%, as reported in Shimer (2007). For this exercise, we assume a uniform distribution for  $G$  as in Mortensen and Pissarides (1999). This produces a value for  $\lambda = 0.05$ , implying that junior workers become senior after 20 months on average. The job finding and separation rates in the benchmark economy imply an unemployment rate of 5.7%.

We rely on micro estimations of the returns to seniority to calibrate the equilibrium stock of human capital  $h^*$ . The structure of the cost function of investing in human capital influences this equilibrium value. A flexible parametrization of the cost function is

$$C(h) = \sigma_1 h^{\sigma_2}, \quad (26)$$

$\sigma_1$  being a scale parameter and  $\sigma_2 \geq 1$  influences the elasticity of the supply function of human capital.

Several papers have estimated returns to seniority, including Abraham and Farber (1987), Altonji and Shakotko (1987), Topel (1991) and Altonji and Williams (2005). For our calibration, we consider the recent estimates by Bunschinsky et al. (2010). Their paper reports annual returns to seniority of about 6 percent. Let's denote by  $\Gamma$  the ratio of average senior wages to junior. Since a junior worker becomes senior at a rate  $\lambda$  and our calibration considers a unit interval of time to be a month, the value for  $h^*$  has to be such that  $\Gamma = 1.06^{1/12\lambda}$ .

There are many combinations of the parameters  $\sigma_1$  and  $\sigma_2$  that lead a 6% annual return. Our strategy is to fix  $\sigma_2$  to a specific value and then obtain the value of  $\sigma_1$

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<sup>12</sup>Notice however that the Hosios-Pissarides efficiency rule only applies to the case without intrafirm bargaining. When the firm chooses to over-employ, the rule has to be modified in order to account for this additional externality. See Smith (1999).



Table 2: Parameters: summary of the benchmark calibration

Notation	Value	Parameter	Target/Source
$r$	0.0033	Discount rate	4% annual rate
$b$	0.71	Flow value of unemployment	Hall and Milgrom (2008)
$c$	0.356	Vacancy cost	Hall and Milgrom (2008)
$m_0$	0.7	Scale parameter (matching)	Vacancy-unemployment ratio
$\eta$	0.5	Elasticity (matching)	Pissarides (2009)
$\beta$	0.5	Bargaining power	$\beta = \eta$
$\lambda$	0.05	Productivity shock frequency	Job separation rate
$A$	0.5374	Total factor productivity	Job finding rate
$\sigma_1$	0.55	Human capital cost shift parameter	6% annual return on seniority
$\sigma_2$	7.5	Human capital supply elasticity	Fixed
$\alpha$	2/3	Labor share	Labor share
$\delta$	0.0087	Capital depreciation rate	10% annual rate

that produces a 6% annual rate of return to seniority. We choose  $\sigma_2 = 7.5$ , giving a supply elasticity of human capital equal to 0.15. The resulting value for  $\sigma_1$  is 0.55. To compare our results to an economy without human capital, we will also consider an economy where the cost function is vertical and the equilibrium value of human capital is the same as in the benchmark economy: in this case,  $h^*$  does not vary with  $T$ .

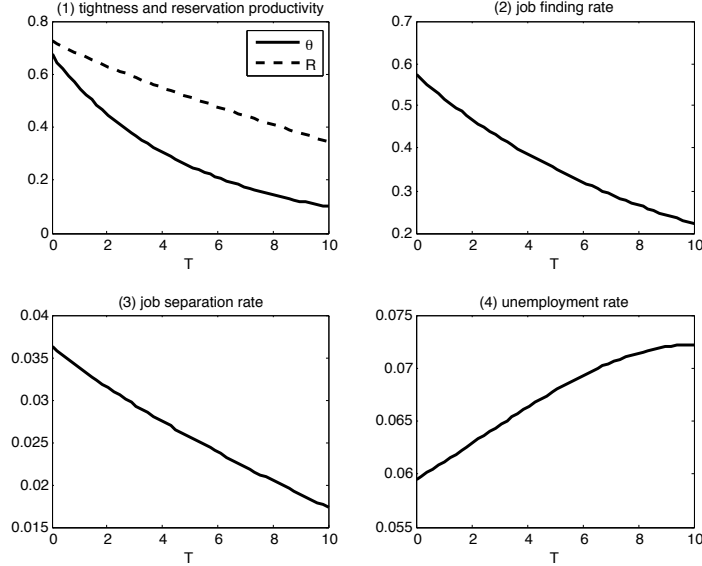
The parameters  $b = 0.71$  and  $c = 0.356$ , which respectively correspond to the flow value of being unemployed and the flow cost of keeping a vacancy posted, are also taken from Pissarides (2009). Those values are consistent with two facts from Hall and Milgrom (2008). First, the flow value of non-work produces a realistic gap between the flow utility of employment and unemployment. Second, the value of  $c$  generates recruiting costs equal to 14 percent of quarterly pay per hire, which is in line with evidence reported in Silva and Toledo (2009).

The capital depreciation rate  $\delta$  is assumed to be 10% annual, consistent with evidence in Gomme and Rupert (2007). They report depreciation rates for different sorts of capital, and ten percent corresponds to the annual rate averaged across all market types of capital in their paper.

## 4.2 Benchmark simulations with endogenous human capital $h = h^*$

The comparative statics exercise in this subsection is close to Ljungqvist's (2002). We compute steady-state equilibria for different values of the firing tax  $T$ : all parameters in the model are as in the benchmark economy but the tax on layoffs, and we consider varying values of  $T$ , that range from 0 to 10. The latter value corresponds to a tax approximately equal to one year of an average worker's salary. Figure 2 confirms earlier findings in the literature on the impact of employment protection in the labor market. Given that these results are well-understood, we will only briefly describe them. First, it is easy to understand that the introduction of a firing tax

Figure 2: Endogenous human capital: unemployment and transition rates as a function of the firing tax  $T$

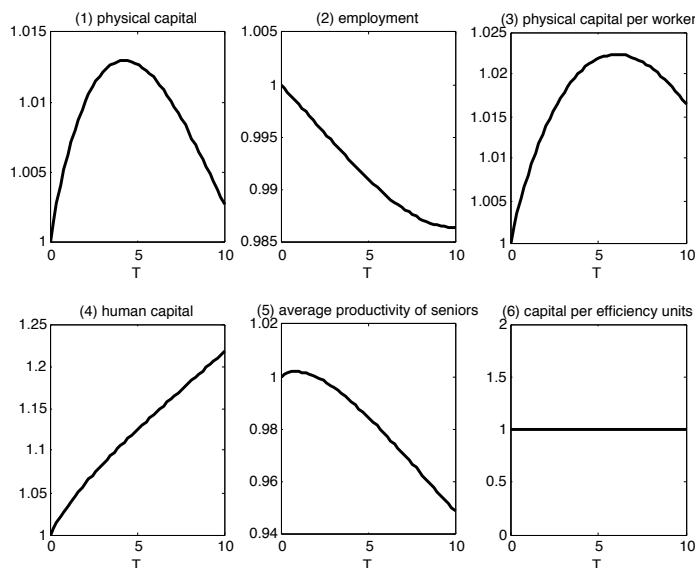


lowers labor turnover.<sup>13</sup> Second, and given this, the effect on unemployment is ambiguous. Unemployment increases if incidence (job flows into unemployment) falls by proportionally less than duration increases. This is discussed in Ljungqvist (2002): on the one hand, workers are locked into their jobs, reducing job separations; on the other hand, the firing tax reduces the expected gain of posting a vacancy, requiring in equilibrium a lower vacancy to unemployment ratio so that the vacancy cost equals the expected gain. For the simulation we propose, it turns out that the firing tax has a positive impact on the rate of unemployment. For a tax as large as a year of an average worker's wage, the associated unemployment rate is about 7.2% (as opposed to 6% in the benchmark economy). We illustrate in Section 4.4 that the unemployment rate may decrease under another parametrization.

Figure 3 illustrates the hump-shaped effect of employment protection. It displays the steady-state values of capital, employment, capital-labor ratio, human capital, average productivity of senior workers and capital per effective labor for each one of the cases respectively. To understand the effect on capital accumulation, remember that capital is a complement of effective workers for the production function of the

<sup>13</sup>As shown on the bottom left panel of Figure 1, a higher firing tax is associated with a lower job separation rate. Because the firm is reluctant to pay the tax, it is willing to keep some low-productivity workers, who would otherwise have been dismissed absent labor regulation. This implies a lower threshold  $R$  (see the first panel) as condition (11) suggests that the marginal value of the least productive worker becomes more negative at larger  $T$ . As a result, the job separation rate is also lower. In turn, employment protection generates lower labor-market tightness and job finding rate (see the first and second panel respectively): because the firm anticipates to pay the firing tax at some moment once a worker is hired, incentives to open up new vacancies are reduced *ex ante*. This negatively affects the probability to find a job for an unemployed worker, which is confirmed by the job creation condition (20): for given values of  $R$  and  $K$ ,  $\theta$  is lower at higher  $T$ .

Figure 3: Endogenous human capital: the impact of a firing tax ( $T$ ) in the model with endogenous specific human capital

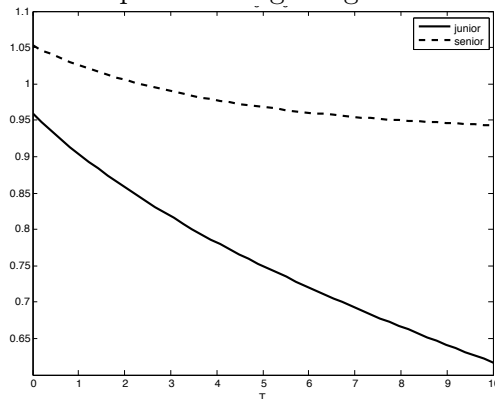


Note: physical capital, employment, the ratio of physical capital to labor, and human capital are all normalized to one for a  $T$  value of zero.

benchmark economy. When the stock of effective workers increases, the stock of physical capital has to increase too and it declines when a decrease in the stock of effective workers occurs. A firing tax thus has two opposite consequences on the demand for capital: the first one, as in Mortensen and Pissarides (1994), is to reduce the reservation threshold of idiosyncratic productivity  $R$  and therefore its average level, and finally the demand for physical capital ; the second one is to reduce labor turnover and thus raise the incentives to invest in human capital, at a constant level of the physical capital. This can be seen from the job creation and destruction conditions (20), (21) and the human capital supply function (25). The increasing part of panels (1) and (3) in Figure 3 is thus due to the fact that the second effect dominates the first one, while, along the decreasing part of the curves, it is the first effect that dominates.

Finally, Figure 4 illustrates the impact on average wages. Wages of senior workers are decreasing in  $T$ . There are two reasons for this. First, the firm is less willing to dismiss low-productivity workers when the firing tax is large. Second, the value of being unemployed decreases with  $T$  too (because it becomes hard to find a job and newly hired workers are willing to pay a large cost  $C(h)$  to obtain human capital). This latter mechanism also explains why the wage of junior workers is decreasing in  $T$  too. The wage of junior workers is more sensitive to  $T$  because, unlike senior workers, junior workers do not benefit from an increase in the threat point as  $T$  increases. Moreover, they do not have human capital.

Figure 4: Endogenous human capital: average wages as a function of the firing tax  $T$



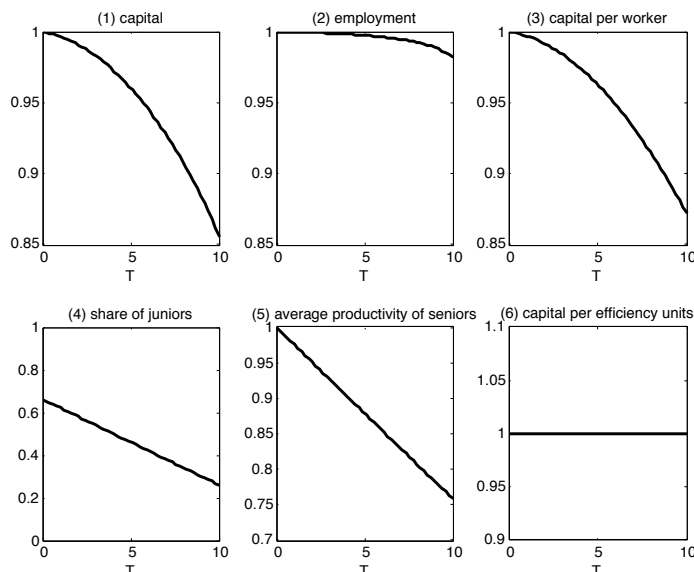
### 4.3 Accumulation of physical capital with fixed human capital

Endogenous human capital generates a positive feedback effect on physical capital investment, due to the ability of workers to respond to incentives in changing their investment in human capital. When human capital changes, so does the demand for capital. What happens if human capital is fixed, at the value  $h^*$  obtain from the benchmark model and  $T = 0$ ?

In this economy, the cost function of investing in human capital is fixed and the equilibrium value for  $h^*$  is fixed too, both at the same level as in the benchmark economy. We denote by  $\bar{h}$  and  $C(\bar{h})$  these fixed quantities. Figure 5 displays the results. Beyond labor variables such as total employment, labor productivity and the share of young workers, we report: i) the effect on the aggregate stock of capital (first panel), ii) the capital-labor ratio (third panel) and iii) the ratio of capital to the whole stock of efficiency units in the firm (sixth panel). The Figure shows that the first two elements fall when the size of the firing tax increases, while the third one is not affected by employment protection.

The intuition for these results is the following. First, we already know from Figure 2 that the employment level drops when a firing tax is introduced. Hence, the second panel in Figure 5 shows the information similar to the fourth panel in Figure 2. Going back to the capital demand equation (22), and with the particular structure imposed by our calibration strategy on the production function, the number of arguments that appear in the marginal product of capital in that equation reduces to two: the total stock of efficiency units of labor ( $N_J + Z(\bar{h})$ ) and capital  $K$ . Moreover, the over-hiring factor  $\Omega_K$  becomes a constant equal to  $(1 - \beta + \beta\alpha)^{-1}$ . Hence, it suffices to follow the total stock of efficiency units of labor: capital adjusts such that its marginal income (the left-hand side of (22)) equals its opportunity cost (the right-hand side). Precisely, in our simulations, a firing tax decreases the total stock of efficiency units for three reasons. First, employment falls, as in the second panel and as already discussed in Section 4.2. Second, the drop in the reservation productivity  $R$  makes senior workers less productive on average. This last effect has been largely studied in the literature, as in Hopenhayn and Rogerson (1993), Mortensen and Pissarides (1999), Veracierto (2001) and Lagos (2006). Third, the composition of employment is affected: as the reservation productivity  $R$  decreases, the share of junior workers in

Figure 5: Fixed human capital: capital and capital-to-labor ratio as a function of the firing tax  $T$



Note: capital, employment, capital-to-labor ratio and the ratio of capital to efficiency units are all normalized to one for a  $T$  value of zero.

employment decreases (see fourth panel).<sup>14</sup>

Since the total stock of efficiency units in the economy decreases, the aggregate stock of capital has to decrease too in order to keep the marginal income of capital equal to its opportunity cost. This is a direct consequence of the constant-returns-to-scale nature of the production function, which implies that labor efficiency units and capital are complements in the production function. The first panel accordingly shows a drop in the aggregate stock of capital following an increase in the firing tax. As an illustration, our simulation suggests that the introduction of a tax equal to a year of an average worker's wage in the benchmark economy implies a fall by 15% in the capital stock. This fall completely reflects the drop in the stock of labor efficiency units.

The fall in the capital-labor ratio comes from the absence of an effect of  $T$  on the ratio of capital to the whole stock of efficiency units. Indeed, the production function is homogeneous of degree one, and its derivative (the marginal product) is homogeneous of degree zero. This implies that the ratio of capital to the stock of labor efficiency units  $\bar{k} = K/(N_J + Z(\bar{h}))$  is constant and only determined by the opportunity cost of capital. Thus,  $\bar{k}$  has to be independent of employment protection for the particular production function assumed in the calibration exercise, as observed on the last panel of Figure 5. The other effect of  $T$  is to decrease the efficiency of workers. Combined, the overall effect is a drop of capital per unit of labor. The observations on the first and last panels of the Figure consequently help the analysis of the third panel, which

<sup>14</sup>This comes from the standard Mortensen-Pissarides (1994) assumption of a higher productivity for junior workers than senior workers: this composition effect negatively impacts the total stock of efficiency units.

displays the impact on the capital-labor ratio. Since  $K$  decreases and  $\tilde{k}$  remains constant, the ratio of  $K$  to  $N$  has to decrease. As an illustration, the Figure shows that the introduction of a tax equal to a year of an average worker's wage generates a decrease by 13% in the ratio. This decrease has to occur to compensate the decrease in the average productivity of workers of a similar size.

## 4.4 Sensitivity analysis on human capital elasticity and production functions

### 4.4.1 Other elasticities for the supply of human capital

In the Appendix, we show that the hump-shape relation between the firing tax and the capital-labor ratio disappears when the elasticity of the supply of human capital is either too large or too small. Section 4.3 already illustrated this for a fixed stock of human capital (inelastic supply). When the elasticity is very large, the human capital stock actually decreases with  $T$ , while the productivity threshold  $R$  is barely affected. Intuitively, when  $T$  increases, the productivity of the least productive job has to decrease. This may occur either through the idiosyncratic component  $z$  or the stock of human capital. When the supply of human capital is very elastic, it is through the latter component that productivity decreases.

### 4.4.2 Alternative production technologies, fixed $h = \bar{h}$

In this subsection, we discuss the robustness of the results displayed on Figure 5 and in particular the absence of hump-shaped pattern. We consider four alternative parametrizations, for which Figure 6 shows the comparative statics: it reports the effect of a firing tax on employment, capital, the capital-labor ratio and capital per efficiency units, the latter being defined as  $\tilde{k} \equiv \frac{K}{J+Z(\bar{h})}$ .<sup>15</sup>

In the first parametrization, which we label “*low matching efficiency*”, we illustrate that the impact of the firing tax on employment can be positive. As emphasized by Ljungqvist (2002), in matching models with highly frictional labor markets, lay-off costs tend to increase employment by reducing labor reallocation. This parametrization accordingly considers an economy with the same production function as in the benchmark economy, but it enhances the degree of search frictions in the labor market. This is done by setting the scale parameter in the matching function equal to half its value in the benchmark economy. Figure 6 confirms this idea: the dashed line shows that employment is increasing in  $T$ . The effects on capital are qualitatively similar as in Figure 5, for the same reasons given in Section 4.3.

The second alternative parametrization, labelled “*Cobb-Douglas  $N_J - Z - K$* ”, considers another Cobb-Douglas production function of the form  $F(N_J, Z(\bar{h}), K) = AN_J^\alpha(Z(\bar{h}))^\gamma K^{1-\alpha-\gamma}$ . It shows that, though the effect of the firing tax on capital and capital per worker is negative, it may be positive for the stock of capital per efficiency units.

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<sup>15</sup>In all the alternative parametrizations, we choose the values of the TFP  $A$  and the arrival rate  $\lambda$  by targeting a job finding rate of 59.4% and a job separation rate of 3.6%. The parameter  $\bar{h}$  is simply fixed to 1.

Third, we consider a situation where the impact on both the capital stock and the capital-labor ratio is positively affected by an increase in the firing tax. We label this parametrization as “*J and Z-K additively separable*”. We consider a production function of the form  $F(N_J, Z(\bar{h}), K) = A [N_J + \gamma (Z(\bar{h}))^\alpha K^{1-\alpha}]$ , which implies that capital and efficiency units provided by senior workers are complements in production as in the benchmark specification, but junior workers do not affect the marginal product of capital anymore. Hence, because an increase in the firing tax generates an increase in the share of senior workers in employment, the firm is given incentives to invest in capital.

Finally, we present the case where separations are *exogenous*. In this situation, the production function only depends on two factors, i.e. labor and capital. As a consequence, the capital-labor ratio is constant and independent of  $T$ . The reason for this is the same reason why the stock of capital per efficiency units is independent of  $T$  in Figure 5: the marginal product of capital is homogeneous of degree zero. Moreover, the aggregate level of employment decreases as in Pissarides (2000) and the capital stock is negatively affected because of its complementarity with labor in the production function. In the Appendix B, we formally show these comparative statics. We also study the case with decreasing returns to scale, where the capital-labor ratio is increasing in  $T$ .

## 5 Conclusions

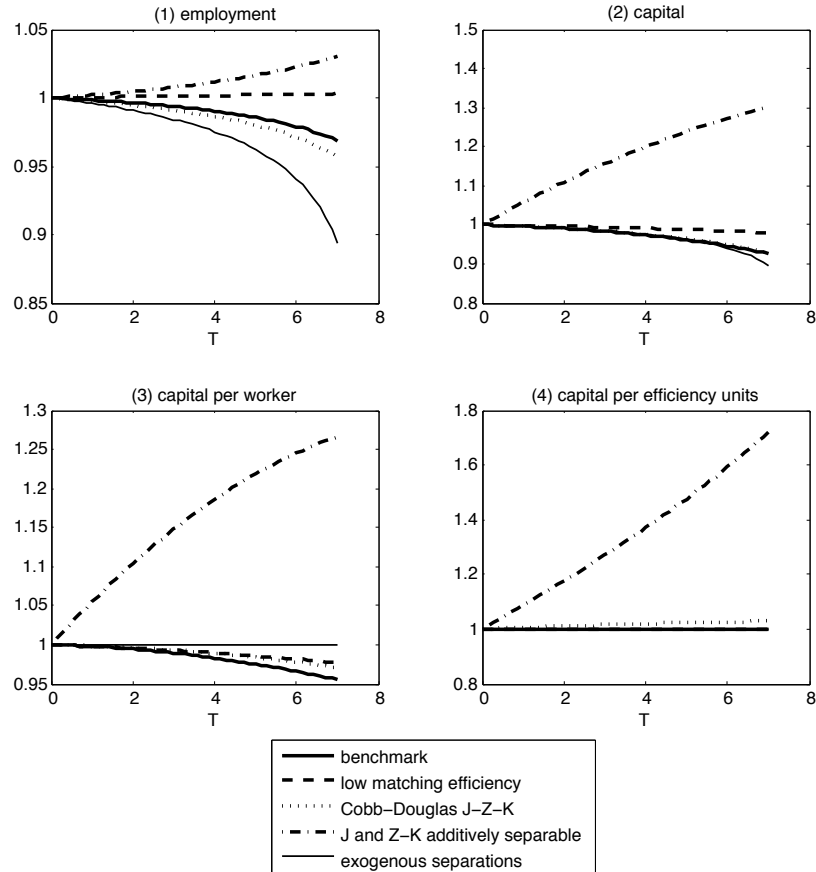
This paper has attempted to clarify the role of employment protection, on economic outcomes such as capital and capital-labor ratio. We have shown that the main effect of employment protection on investment and the capital-labor ratio is a quite robust negative one: because EPL reduces future profits, firms underinvest in physical capital. This is a variant of the hold-up effect. We view this set of results as short-run ones.

However, by protecting skills, employment protection can also increase the investment in human capital and therefore, under the realistic assumption of complementarity between physical and human capital, may also increase the demand for physical capital. Hence, the effect of employment protection can be mitigated and even reversed.

This leads us to two conclusions : first, that specific human capital investments (Becker 1964, Lazear 2009) are a relevant ingredients in many studies of labor market regulations ; second, that in such contexts, labor institutions have ambiguous effects: they are sometimes negative due to restriction they impose to the choice sets of agents ; but they can sometimes be positive when specific investments, that need to be protected, are present. This is a second best result arising from contractibility issues over investment in physical and human capital (hold-up). It however suggests along the line of the new public economics literature that future research should more systematically investigate the positive role of labor market institutions in the presence of specific investments. One should in particular enrich empirical specifications in order to account for such impacts on long-run efficiency of labor as well as short-run efficiency costs.

Finally, although our analysis was applied in a world of perfect capital markets, it should be stressed that the impact of credit market imperfections on the effect of EPL, yet beyond the scope of this paper, is extremely rich and interesting. Recent

Figure 6: Fixed human capital: capital and capital-to-labor ratio as a function of the firing tax  $T$ , alternative parametrizations



Note: capital, employment, capital-to-labor ratio and the ratio of capital to efficiency units are all normalized to one for a  $T$  value of zero.

works such as Lalé's (2013) and subsequent references with heterogenous wealth and imperfect borrowing would be a good starting point for a more general analysis.

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## A Appendix

### A.1 Equilibrium

#### A.1.1 First-order conditions

Differentiating the right hand side of (9) with respect to  $V$  and setting it equal to zero gives

$$-c + \Pi_J q(\theta) = 0, \quad (27)$$

while the condition for  $I$  is

$$\Pi_K = 1, \quad (28)$$

with  $\Pi_K$  being the marginal value of capital.

The marginal return on a junior employee and capital are:

$$(r + \lambda)\Pi_J = F_J - w_J - \frac{\partial w_J}{\partial N_J} N_J - \int_R \frac{\partial w_S(z, h)}{\partial N_J} n_S(z) dz - \lambda T + \lambda \int_R [\Pi_{z'} + T] dG(z') \quad (29)$$

and

$$(r + \delta)\Pi_K = F_K - \frac{\partial w_J}{\partial K} N_J - \int_R \frac{\partial w_S(z, h)}{\partial K} n_S(z) dz. \quad (30)$$

Conditions (13) and (14) are obtained by combining (27) with (29) and (28) with (30) respectively.

#### A.1.2 Wages

The Nash bargaining rule (10), combined with (7) and (29), give the following formulation for wages of junior workers:

$$w_J = \beta \left[ F_J - \frac{\partial w_J}{\partial N_J} N_J - \int_R \frac{\partial w_S(z, h)}{\partial N_J} n_S(z) dz - \lambda T + \lambda \int_R [\Pi_z + T] dG(z) \right] \\ - (1 - \beta) \left[ \lambda \int_R [W_S(z, h) - U] dG(z) - rU \right]$$

The same Nash bargaining rule allows to establish that

$$(1 - \beta)\lambda \int_R [W_S(z, h) - U] dG(z) = \beta\lambda \int_R [\Pi_z + T] dG(z). \quad (31)$$

Hence,

$$w_J = \beta \left[ F_J - \frac{\partial w_J}{\partial N_J} N_J - \int_R^1 \frac{\partial w_S(z, h)}{\partial N_J} n_S(z) dz - \lambda T \right] + (1 - \beta) r U. \quad (32)$$

Similarly, the rule (12) together with (8) and (15), imply that

$$w_S(z, h) = \beta \left[ F_Z z h - \frac{\partial w_J}{\partial n_S(z)} N_J - \int_R^1 \frac{\partial w_S(z, h)}{\partial n_S(z)} n_S(z) dz + r T \right] + (1 - \beta) r U. \quad (33)$$

Given the results by Cahuc et al. (2008), we conjecture that the solution to the system of differential equations described by (32) and (33) is (16) and (17). To verify our conjecture in the case of the wage of junior workers, we derive (16) and (17) with respect to  $N_J$ . We obtain:

$$\frac{\partial w_J}{\partial N_J} = \int_0^1 x^{\frac{1}{\beta}} \frac{\partial^2 F(N_J x, Zx, K)}{\partial(N_J x) \partial(N_J x)} dx \quad (34)$$

and

$$\frac{\partial w_S(z, h)}{\partial N_J} = \int_0^1 x^{\frac{1}{\beta}} \frac{\partial^2 F(N_J x, Zx, K)}{\partial(Zx) \partial(N_J x)} z h dx. \quad (35)$$

Comparing (16) and (32), our conjecture is correct if

$$\beta \Omega_J F_J = \beta \left( F_J - \frac{\partial w_J}{\partial N_J} N_J - \int_R^1 \frac{\partial w_S(z, h)}{\partial N_J} n_S(z) dz \right) \quad (36)$$

Replacing the derivatives of (16) and (17) with respect to  $N_J$  in the equation above indicates that the conjecture can be verified if

$$\beta \Omega_J F_J = \beta \left[ F_J - \int_0^1 x^{\frac{1}{\beta}} \left( \frac{\partial^2 F(N_J x, Zx, K)}{\partial(N_J x) \partial(N_J x)} N_J + \frac{\partial^2 F(N_J x, Zx, K)}{\partial(Zx) \partial(N_J x)} Z h \right) dx \right] \quad (37)$$

Finally, by integrating by parts the right hand side of the equation above, one can verify that the conjecture is correct.

The conjecture can be verified in a similar way in the case of wages of senior workers.

Moreover, notice that (16) and (17) can be rewritten as

$$w_J = \beta \Omega_J F_J + (1 - \beta) b + \beta \theta c - \beta \lambda T - (1 - \beta) \theta q(\theta) C(h) \quad (38)$$

and

$$w_S(z, h) = \beta \Omega_S F_Z z h + (1 - \beta) b + \beta \theta c + \beta r T - (1 - \beta) \theta q(\theta) C(h), \quad (39)$$

in general equilibrium, by use of equation (6) together with (10), (12) and (27).

### A.1.3 Job destruction

The job destruction relation is obtained as follows. First, replace wages and their derivatives in (29) to get

$$(r + \lambda) \Pi_J = (1 - \beta) [\Omega_J F_J - r U - \lambda T] + \lambda \int_R^1 (\Pi_z + T) dG(z). \quad (40)$$

Similarly, from (15), we have

$$(r + \lambda)(\Pi_z + T) = (1 - \beta)[\Omega_S F_Z z h - rU + rT] + \lambda \int_R^1 (\Pi_z + T) dG(z). \quad (41)$$

It follows that

$$(r + \lambda)(\Pi_z - \Pi_R) = (1 - \beta)\Omega_S F_Z(z - R)h. \quad (42)$$

Given that  $\Pi_R + T = 0$ ,

$$\Pi_z = \frac{1 - \beta}{r + \lambda} \Omega_S F_Z(z - R)h - T. \quad (43)$$

Evaluating (41) for  $z = R$  and combining it with (43) yields the job destruction condition:

$$0 = (1 - \beta)[\Omega_S F_Z R(h)h - rU + rT] + \lambda \int_R^1 \frac{1 - \beta}{r + \lambda} \Omega_S F_Z(z - R(h))h dG(z). \quad (44)$$

Finally, replacing  $rU$ , in the equation above, by

$$rU = b + \theta \frac{\beta}{1 - \beta} c \quad (45)$$

gives (21).

#### A.1.4 Job creation

Combining (29) with (15), together with the solution for wages, we have

$$(r + \lambda)(\Pi_J - \Pi_z) = (1 - \beta)(\Omega_J F_J - \Omega_S F_Z z h) + \beta(r + \lambda)T. \quad (46)$$

If we notice that  $\Pi_R + T = 0$  and given the result in (27), we can obtain equation (20) by evaluating this equation at  $z = R$ .

#### A.1.5 Capital

The capital demand (22) is obtained from the marginal value of capital (30). Notice first that the derivatives of wages with respect to capital write as

$$\frac{\partial w_J}{\partial K} = \int_0^1 x^{\frac{1-\beta}{\beta}} \frac{\partial^2 F(N_J x, Zx, K)}{\partial(N_J x) \partial K} dx \quad (47)$$

and

$$\frac{\partial w_S(z, h)}{\partial K} = \int_0^1 x^{\frac{1-\beta}{\beta}} \frac{\partial^2 F(N_J x, Zx, K)}{\partial(Zx) \partial K} z h dx. \quad (48)$$

Replacing them into (30) implies

$$r + \delta = F_K - \int_0^1 x^{\frac{1-\beta}{\beta}} \frac{\partial^2 F(N_J x, Zx, K)}{\partial(N_J x) \partial K} N_J dx - \int_0^1 x^{\frac{1-\beta}{\beta}} \frac{\partial^2 F(N_J x, Zx, K)}{\partial(Zx) \partial K} Z dx, \quad (49)$$

which is equivalent to

$$r + \delta = F_K - \int_0^1 x^{\frac{1-\beta}{\beta}} \left[ \frac{\partial^2 F(N_J x, Zx, K)}{\partial(N_J x) \partial K} N_J + \frac{\partial^2 F(N_J x, Zx, K)}{\partial(Zx) \partial K} Z \right] dx. \quad (50)$$

By integrating by parts the integral in the equation above, one can get equation (22).

### A.1.6 Human capital investment

The worker's maximization program is given by

$$\max_{h_i} W_J(h_i) - C(h_i),$$

with the participation constraint

$$W_J(h_i) - U \geq C(h_i).$$

This leads to the first-order condition:

$$(r + \lambda)C'(h_i) = \frac{\partial w_J}{\partial h_i} + \lambda \int_{R(h)}^1 \frac{\partial W_S(h_i, z)}{\partial h_i} dG(z) - \lambda \frac{\partial R(h)}{\partial h_i} [W_S(h_i, R) - U] g(R).$$

Given that the worker's surplus is zero for  $z = R$ , we have that  $[W_S(h, R) - U] = 0$ . This condition simplifies as

$$(r + \lambda)C'(h_i) = \frac{\partial w_J}{\partial h_i} + \lambda \int_{R(h)}^1 \frac{\partial W_S(h, z)}{\partial h_i} dG(z).$$

Given the formulation in (8), one can calculate

$$(r + \lambda) \frac{\partial W_S(h_i, z)}{\partial h_i} = \frac{\partial w_S(h_i, z)}{\partial h_i} + \lambda \int_{R(h)}^1 \frac{\partial W_S(h_i, z)}{\partial h} dG(z)$$

Similarly,

$$(r + \lambda G(R)) \int_R^1 \frac{\partial W_S(h_i, z)}{\partial h_i} dG(z) = \int_R^1 \frac{\partial w_S(h_I, z)}{\partial h_i} dG(z).$$

Given the wage equation

$$w_S(z, h_i) = (1 - \beta)rU + \beta rT + \beta \Omega_S F_Z z h_i, \quad (51)$$

where

$$\Omega_S = \frac{\int_0^1 \frac{1}{\beta} x^{\frac{1-\beta}{\beta}} F_Z(N_J x, Zx, K) dx}{F_Z}, \quad (52)$$

we can calculate the derivative of the wage of a senior worker with respect to  $h_i$ :

$$\frac{\partial w_S(h_i, z)}{\partial h_i} = \beta \Omega_S F_{ZZ} z + \int_0^1 x^{\frac{1}{\beta}} \frac{\partial^2 F(N_J x, Zx, K)}{\partial(Zx) \partial(Zx)} dx n_S(z) z^2 h_i di \quad (53)$$

Notice that the derivative in the above equation is made with respect to  $h_i$  and not with respect to the total level of human capital  $h$ .

By letting  $di \rightarrow 0$  in (53) and replacing it in the first order condition for human capital, we obtain the condition (25).

## B The model with exogenous separations and fixed $\bar{h} = 1$

In this section we analyze a slightly different version of the model, with exogenous job separations. We show that an increase in the firing tax on the capital-labor ratio is positive or null depending on the form one assumes for the production function.

## B.1 Equilibrium

Instead of assuming that the amount of efficiency units endowed by workers evolves stochastically according to a Poisson process, we consider that all workers own the same unit amount. This implies that the firm does not choose to destroy jobs endogenously as in the main text and labor heterogeneity does not appear anymore. Now separations occur exogenously at a rate  $s$ . We denote here by  $F_N = \partial F(N, K)/\partial N$  and as before we drop the argument unless they are different from  $(N, K)$ :

In this setting the relations describing the equilibrium of the economy are the following:

$$N = \frac{\theta q(\theta)}{\theta q(\theta) + s}, \quad (54)$$

$$\frac{c}{q(\theta)} = \frac{(1 - \beta)(\Omega_N F_N - b - sT) - \beta\theta c}{r + s} \quad (55)$$

and

$$(1 - \beta)\Omega_K F_K = r + \delta, \quad (56)$$

where

$$\Omega_N = \frac{\int_0^1 \frac{1}{\beta} x^{\frac{1-\beta}{\beta}} F_N(Nx, K) dx}{F_N} \quad (57)$$

and

$$\Omega_K = \frac{\int_0^1 \frac{1}{\beta} x^{\frac{1-\beta}{\beta}-1} F_K(Nx, K) dx}{F_K}. \quad (58)$$

Equations (54) and (56) are the counterparts of (3) and (22) when separations are exogenous and (55) is the equivalent of the free-entry condition from Pissarides (1985).

Notice that we consider that workers earn the outside wage as in Pissarides (2000).

## B.2 The impact of a firing tax on the capital-labor ratio

With exogenous separations, the effect of a firing tax on the capital-labor ratio is analogous to the effect of a labor tax in a frictionless framework. Two effects appear. First, firms substitute away from labor because the relative cost of labor increases. This can be shown by manipulating equations (55) and (56), which leads to

$$\frac{(1 - \beta)\Omega_K F_K}{\Omega_N F_N} = \frac{r + \delta}{(r + s)\frac{c}{q(\theta)} + \beta\theta c + (1 - \beta)(b + sT)}. \quad (59)$$

The left hand side of the equation is analogous to the standard marginal rate of transformation that appears in relations describing the equilibrium of a Walrasian economy, while the right hand side is analogous to the relative cost of labor. We see from this equation that an increase in  $T$  leads to an increase in the capital-labor ratio, for a given labor-market tightness  $\theta$ .

Of course, the labor-market tightness also reacts to a change in  $T$ . This observation leads us to a second effect: an increase in  $T$  implies a decrease in  $\theta$ . This effect goes in the opposite direction as it negatively affects the capital-labor ratio.

We now illustrate those two effects through two specific examples: i) the case where the production function displays constant returns to scale and ii) a Cobb-Douglas case with decreasing returns.



Let us first consider the case with constant returns to scale. Under this assumption, it is easy to see from equation (56) that the capital-labor ratio is not affected by an increase in the firing tax  $T$ . Moreover, given this result, the labor-market tightness has to decrease (see equation (55)) as well as employment (see equation (54)). Because the capital-labor ratio is not affected and  $N$  decreases, the aggregate stock of capital has to decrease too.

In the second example, we assume the production function takes the following form:

$$F(N, K) = N^\alpha K^\nu, \quad (60)$$

with  $\alpha > 0$ ,  $\nu > 0$  and  $\alpha + \nu < 1$ .

With this production function, equations (55) and (56) can be rewritten as

$$\frac{c}{q(\theta)} = \frac{(1 - \beta) \left( \frac{\alpha k^\nu N(\theta)^{\alpha+\nu-1}}{1-\beta+\beta\alpha} - b - sT \right) - \beta\theta c}{r + s} \quad (61)$$

and

$$\frac{1 - \beta}{1 - \beta + \beta\alpha} \nu N(\theta)^{\alpha+\nu-1} k^{\nu-1} = r + \delta \quad (62)$$

respectively, where  $k \equiv \frac{K}{N}$  and  $N(\theta)$  is given by equation (54), an increasing relation of  $\theta$ .

Equation (61) describes an increasing relation between the capital-labor ratio  $k$  and the labor-market tightness  $\theta$ , while equation (62) is decreasing in the space  $(k, \theta)$ . Hence, an increase in  $T$  leads to a decrease in  $\theta$  and an increase in  $k$  with this production function.

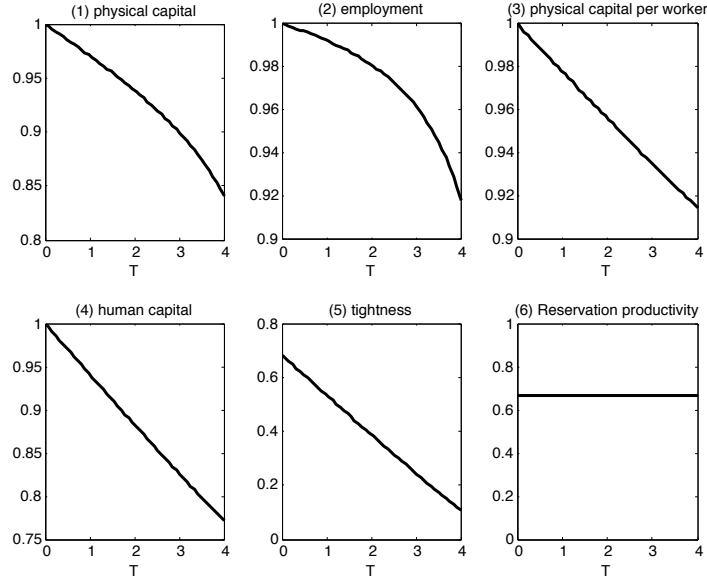
## C Sensitivity : numerical exercises for other elasticities for the human capital supply function

In this Appendix, we illustrate how the comparative statics in Section 4.2 may change when one considers alternative values for the human capital supply function. Specifically, we are interested in understanding under which circumstances, the U-shaped pattern may not appear anymore.

### C.1 The effect of a firing tax when the supply of human capital is imperfectly elastic (high or low elasticities)

The hump-shaped pattern of capital does not appear in the two more extreme situations where the supply elasticity is either large or low, as in Figures 7 and 8 respectively. In Figure 7, which shows the case of an infinitely elastic supply of human capital (i.e. the cost function is linear), the reservation productivity is independent of  $T$  and the stock of human capital is a decreasing function of it. In the Appendix C.2, we formally show that this result always hold when the supply of human capital is infinitely elastic. The intuition is the following. When  $T$  increases, the productivity of the least productive job has to decrease. In this context, productivity has to be understood as a combination of the idiosyncratic component  $z$  and the stock of human

Figure 7: Endogenous human capital: the impact of a firing tax ( $T$ ) with high elasticity of human capital supply



Note: physical capital, employment, the ratio of physical capital to labor, and human capital are all normalized to one for a  $T$  value of zero.

capital. It turns out that when the supply of human capital is very elastic, it is through a change in the latter component that productivity decreases.

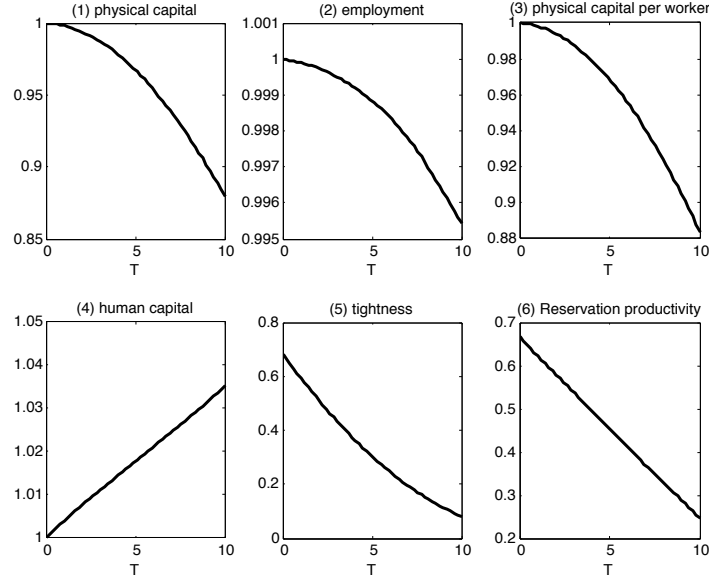
Moreover, Figure 7 shows that employment, capital and the capital-labor ratio decrease with  $T$ . The first outcome is due to the fact that only the job finding rate is affected by  $T$  in this case, the job separation rate being kept constant since  $R$  is not affected. Second, because employment decreases and workers are less productive, capital has to decrease as well: it is a complementary factor. Third, the fall in the capital-labor ratio is due to the fall in productivity as in Section 4.2. The value for  $\sigma_1$  we consider in this graph is 3.09 (it is the value that yields 6% annual rate of return to seniority).

In Figure 8 we consider a low supply elasticity. We fix  $\sigma_2$  at 40. This produces an elasticity of 2.56%. The resulting calibrated value for  $\sigma_1$  is close to zero. Figure 8 and Figure 7 display similar comparative statics with the exception that employment increases in this specific example. In our simulations, for all of the cases where no hump-shaped pattern appears, we found that the stock of capital and the capital-labor ratio are always negatively affected by an increase in  $T$ . This is a natural result as we come back to a situation that resembles the one in Section 4.2 in those cases.

## C.2 The effect of a firing tax when the supply of human capital is perfectly elastic

Here we formally show that, when the production function is the one we consider in the benchmark economy and the cost of investing in human capital is a linear function of  $h$ , a firing tax is always associated with less human capital, less physical capital, less employment, lower capital-labor ratio, lower tightness and leave the reservation

Figure 8: Endogenous human capital: the impact of a firing tax ( $T$ ) with low elasticity of supply of human capital



Note: physical capital, employment, the ratio of physical capital to labor, and human capital are all normalized to one for a  $T$  value of zero.

productivity  $R$  unaffected.

Define

$$\hat{k} \equiv \frac{K}{N_J + Zh} = \frac{K}{N} \frac{1}{\xi_j + (1 - \xi_j)\bar{z}_R h},$$

the stock of capital per effective worker, where  $\xi_j$  is the share of junior workers in employment and  $\bar{z}_R$  is the average value of  $z$  among senior workers, increasing functions of  $R$ .

From (22), one can obtain  $\hat{k}$  immediately. It is independent of  $T$ . This is because  $F$  is homogeneous of degree one in two arguments, physical capital  $K$  and effective workers  $(N_J + Zh)$ .

Given a value for  $\hat{k}$ , (25) allows to get  $R$ . Hence,  $R$  also is independent of  $T$ . The independence of  $R$  implies that  $\xi_j$  and  $\bar{z}_R$  do not depend on  $T$  either.

The rest of the analysis resembles Mortensen and Pissarides (1999), but instead of considering the space  $(\theta, R)$  for the job creation and destruction relations, one considers the space  $(\theta, h)$ . The job creation relation is decreasing in this space and the job destruction is increasing. A firing tax unambiguously decreases  $h$  and  $\theta$  in the same manner as it decreases  $R$  and  $\theta$  in Mortensen and Pissarides (1999).

As a consequence, the effect on employment is negative because  $\theta$  decreases and  $R$  is not affected. Given  $\hat{h}$  is independent of  $T$  and both  $N$  and  $h$  decrease for an increase in  $T$ ,  $K$  has to decrease as well.

From the definition of  $\hat{h}$  above, the capital-labor ratio has to decrease because  $h$  decreases.

## D Online Appendix (not for publication): Determination of equation 15

Deriving the profit function in equation (9) with respect to  $n_S(z)$  we have:

$$\frac{\partial \bar{\Pi}}{\partial Z} \frac{\partial Z}{\partial n_s(z)} = \frac{1}{1+rdt} \left\{ \left( F_Z \frac{\partial Z}{\partial n_s(z)} - \frac{\partial w_J}{\partial Z} \frac{\partial Z}{\partial n_s(z)} N_J - \int_R^1 \frac{\partial w_S(z')}{\partial Z} \frac{\partial Z}{\partial n_s(z)} n_S(z') dz' \right. \right. \\ \left. \left. - w_s(z) \frac{\partial N_S}{\partial n_s(z)} - \lambda G(R) T \frac{\partial N_S}{\partial n_s(z)} \right) dt + \frac{\partial \bar{\Pi}}{\partial Z'} [(1-\lambda dt) \frac{\partial Z}{\partial n_s(z)} + \lambda \frac{\partial N_S}{\partial n_s(z)} \int_R^1 g(z') z' h dz' dt] \right\}$$

and, after multiplication by  $(1+rdt)$ , we have:

$$(1+rdt) \frac{\partial \bar{\Pi}}{\partial Z} \frac{\partial Z}{\partial n_s(z)} = \left( F_Z \frac{\partial Z}{\partial n_s(z)} - \frac{\partial w_J}{\partial Z} \frac{\partial Z}{\partial n_s(z)} N_J - \int_R^1 \frac{\partial w_S(z')}{\partial Z} \frac{\partial Z}{\partial n_s(z)} n_S(z') dz' \right. \\ \left. - w_s(z) \frac{\partial N_S}{\partial n_s(z)} - \lambda G(R) T \frac{\partial N_S}{\partial n_s(z)} \right) dt + \frac{\partial \bar{\Pi}}{\partial Z'} [(1-\lambda dt) \frac{\partial Z}{\partial n_s(z)} + \lambda \frac{\partial N_S}{\partial n_s(z)} \int_R^1 g(z') z' h dz' dt]$$

we obtain in a steady-state:

$$(r+\lambda) dt \frac{\partial \bar{\Pi}}{\partial Z} \frac{\partial Z}{\partial n_s(z)} = \left( F_Z \frac{\partial Z}{\partial n_s(z)} - \frac{\partial w_J}{\partial Z} \frac{\partial Z}{\partial n_s(z)} N_J - \int_R^1 \frac{\partial w_S(z')}{\partial Z} \frac{\partial Z}{\partial n_s(z)} n_S(z') dz' \right. \\ \left. - w_s(z) \frac{\partial N_S}{\partial n_s(z)} - \lambda G(R) T \frac{\partial N_S}{\partial n_s(z)} \right) dt + \frac{\partial \bar{\Pi}}{\partial Z} \frac{\partial N_S}{\partial n_s(z)} \lambda \int_R^1 g(z') z' h dz' dt$$

Dividing by  $dt$ :

$$(r+\lambda) \frac{\partial \bar{\Pi}}{\partial Z} \frac{\partial Z}{\partial n_s(z)} = F_Z \frac{\partial Z}{\partial n_s(z)} - \frac{\partial w_J}{\partial Z} \frac{\partial Z}{\partial n_s(z)} N_J - \int_R^1 \frac{\partial w_S(z')}{\partial Z} \frac{\partial Z}{\partial n_s(z)} n_S(z') dz' \\ - w_s(z) \frac{\partial N_S}{\partial n_s(z)} - \lambda G(R) T \frac{\partial N_S}{\partial n_s(z)} + \frac{\partial \bar{\Pi}}{\partial Z} \frac{\partial N_S}{\partial n_s(z)} \lambda \int_R^1 g(z') z' h dz'$$

Terms in  $\frac{\partial N_S}{\partial n_s(z)}$  can be removed, in noticing that  $\frac{\partial Z}{\partial n_s(z)} = zh \frac{\partial N_S}{\partial n_s(z)}$ :

$$(r+\lambda) \frac{\partial \bar{\Pi}}{\partial Z} zh = F_Z zh - \frac{\partial w_J}{\partial Z} zh N_J - \int_R^1 \frac{\partial w_S(z')}{\partial Z} zh n_S(z') dz' - w_s(z) \\ - \lambda G(R) T + \frac{\partial \bar{\Pi}}{\partial Z} \lambda \int_R^1 g(z') z' h dz'$$

or

$$(r+\lambda) \frac{\partial \bar{\Pi}}{\partial Z} zh = F_Z zh - \frac{\partial w_J}{\partial Z} zh N_J - \int_R^1 \frac{\partial w_S(z')}{\partial Z} zh n_S(z') dz' - w_s(z) \\ - \lambda G(R) T + \lambda \int_R^1 \frac{\partial \bar{\Pi}}{\partial Z} z' h dG(z')$$

Use the definition of  $\Pi_z = \frac{\partial \bar{\Pi}}{\partial Z} \frac{\partial Z}{\partial n_s(z)}$ :

$$(r + \lambda)\Pi_z = F_Z zh - \frac{\partial w_J}{\partial Z} zh N_J - \int_R^1 \frac{\partial w_S(z')}{\partial Z} zh n_S(z') dz' - w_s(z) - \lambda G(R)T + \lambda \int_R^1 \Pi_{z'} dG(z')$$

Add up  $(r + \lambda)T$  on both sides:

$$(r + \lambda)(\Pi_z + T) = F_Z zh - \frac{\partial w_J}{\partial Z} zh N_J - \int_R^1 \frac{\partial w_S(z')}{\partial Z} zh n_S(z') dz' - w_s(z) + rT + \lambda G(R)T + \lambda[1 - G(R)]T - \lambda G(R)T + \lambda \int_R^1 \Pi_{z'} dG(z')$$

Simple manipulation leads to :

$$(r + \lambda)(\Pi_z + T) = F_Z zh - \frac{\partial w_J}{\partial Z} zh N_J - \int_R^1 \frac{\partial w_S(z')}{\partial Z} zh n_S(z') dz' - w_s(z) + rT + \lambda[1 - G(R)]T + \lambda \int_R^1 \Pi_{z'} dG(z')$$

Insert  $\lambda[1 - G(R)]T$  into the integral and one obtains equation (63) that is precisely equation (15) in the text:

$$(r + \lambda)(\Pi_z + T) = F_Z zh - \frac{\partial w_J}{\partial Z} zh N_J - \int_R^1 \frac{\partial w_S(z')}{\partial Z} zh n_S(z') dz' - w_s(z) + rT + \lambda \int_R^1 (\Pi_{z'} + T) dG(z') \quad (63)$$



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