Asymmetric (S,s) pricing: implications for monetary policy
Zakaria Babutsidze

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This paper presents a model of asymmetric $(S,s)$ pricing. We investigate implications of such a behavior for the effectiveness of the monetary policy. We discuss two types of asymmetric responses to monetary interventions. One is the symmetry in the responses to positive and negative monetary shocks. The other is the variance in responses to monetary shocks during booms and recessions. The conclusion is that first type of asymmetry can be attributed to the asymmetry in adjustment bands, while the second kind of asymmetry is a result of firm heterogeneity, and asymmetry of $(S,s)$ bands does not contribute to it.

**Keywords:** $(S,s)$ Pricing, Monetary policy, Heterogeneity, Asymmetry.

Pricing behavior of individual firms has implications for the aggregate price and output movements. The propagation of money supply shocks crucially depends on pricing patterns. If firms in every moment in time charge the optimal price and there are no imperfections on financial markets, it is easy to show that money supply shocks have no real effects (Akerlof and Yellen, 1985). If financial markets are imperfect, for example there are information asymmetries (Greenwald and Stiglitz, 2001) or the credit market is characterized by financial accelerator (Bernanke et al., 1999), money neutrality disappears. In this paper we abstract from the possibility of financial market imperfections and concentrate on the possibility

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of individual price deviations from optimum. We believe that in real life prices are rarely at the optimum. There are two reasons for this. One is that there exists costs to price adjustment. The other one is that firms do not reconsider their prices as regularly as it would be necessary for keeping them at optimum permanently.

This paper does not go into the discussion of which of these arguments is more plausible. Instead we assume a specific type of firm pricing behavior which has been empirically well documented and try to understand the implications of this behavior for the effectiveness of monetary policy during equilibrium as well as during different phases of business cycle. More precisely we assess the effectiveness of monetary policy during booms and recessions.

We adopt the framework of (S,s) pricing (Caplin and Spulber, 1987) which introduces the inaction interval around the optimal price. As long as price is within the interval it is optimal for the seller not to adjust the price. In this type of models money has been found to be neutral (Caplin and Spulber, 1987). However, this finding is not robust to asymmetry of inaction bands above and below optimal price. Asymmetry creates some room for monetary policy. In asymmetric setup money is not neutral. We build on empirical finding pointing to the possibility of asymmetry in adjustment bands around the optimal price and analyze a simple model. We do not model neither the fine-grained micro behavior nor non-market interaction among firms. We simply assume asymmetry of price adjustment bands.\(^2\)

\(^2\) Although the present work does not concentrate on the derivation of the optimality of asymmetric bands, here we provide further possible explanations and a sketch of possible modeling technique. As we argued before, menu costs and adjustment costs are not exactly the same. So, adjustment costs can be different for movements of price in different directions. For example there are some psychological factors that can be at work making adjustment costs different (Greenslade and Parker, 2012). Then optimality of asymmetric bands can be derived from the usual monopolist profit maximization problem (Babutsidze, 2006). In principle, the asymmetric adjustment cost is not the only way to get asymmetric bands of adjustment. Similar results can be obtained by assuming the asymmetric profit function. Namely, profit function that is steeper before optimal price and flatter after it. This assumption makes not adjustment, but rather deviation costs asymmetric. To see this define deviation costs as

\[
C_{dev} = \frac{1}{2} \frac{\partial^2 \pi}{\partial p^2} (p - p^*)^2.
\]

Then if a profit function is flatter when \( p > p^* \) for the same absolute value of deviation

\[
C_{dev}^{p<p^*} > C_{dev}^{p>p^*}.
\]

Thus, even with symmetric adjustment costs firm’s pricing behavior will feature a longer right tail and a shorter left one.
hit by an idiosyncratic shock at any point in time that might push its price outside the inaction interval and induce it to adjust its price to the optimal level. Besides, government can conduct active monetary policy that would equally affect all the firms. Firms’ responses to these policies are analyzed in order to assess the effectiveness of the monetary interventions.

Under the assumption of infinitesimally small idiosyncratic shocks, or alternatively very wide inaction bands, the model can be solved analytically. We derive a long-run density function of price level distributions in absence of monetary shocks. This is interpreted as equilibrium distribution. The effects of monetary policy in equilibrium can be also assessed analytically. However, there are two interesting departures from equilibrium that are worthy of analysis. One is a moderate size of adjustment bands. This is because wide adjustment bands imply excessively large adjustment costs that are not in line with empirical findings. The other departure is related to the cyclical nature of the economy. Any external aggregate shock that hits the economy may knock it out of the equilibrium state. As price adjustments are not instantaneous, it takes a while until the system converges back to the ergodic price distribution. We try to assess the powers of monetary policy during this transitional dynamics.

These two exercises cannot be performed using analytic tools. This is where Agent Based Modeling (AMB) comes in handy. ABM is a flexible framework that does not require analytical tractability, which simplifies the task in the present case. It is a bottom-up modeling framework, which means that modeler can specify behavior of individual agents at the microscopic level and explore its implications for macroscopic outcomes. Merits of ABM are extensively discussed in few of the articles in this special issue (e.g. Fagiolo and Roventini, 2012; Napoletano and Gaffard, 2012). Using computational tools we set up an ABM equivalent of the model and explore the behavior of the system in simulated environments. Using ABM methodology we analyze the effects of the monetary intervention in presence of non-trivial idiosyncratic shocks and during booms and recessions.

There are two major findings. One is that in presence of sufficiently large shocks the model is able to reproduce significant asymmetry in output’s reaction to positive and negative macroeco-
nomic shocks. Asymmetry of adjustment bands plays the crucial role in this. The second major finding is that model is characterized by asymmetry in responses to similar shocks across different phases of business cycle. However, the asymmetry on micro level is not necessary for this. The difference in responses to similar shocks across booms and recessions seems to be the result of simple existence of inaction interval, rather than its asymmetry.

The rest of the work is organized as follows. Section 1 reviews related strands of literature. Section 2 lays out the model. Section 3 presents the results. Section 4 concludes.

1. Related literature

Our work is closely related to two large strands of literature. One strand is concerned with the pricing behavior of firms and implications of this behavior for macroeconomics. The second one discusses the empirical findings about asymmetries on micro and macro levels.

From wide range of models concerning firms’ pricing behavior most closely related to the work presented in this paper are sticky price models. During the last few decades sticky price models have proved to be of great importance. The empirical findings illustrate that prices are not flexible enough to always be at the optimum. The evidence of price stickiness is found in many markets. For example, Stigler and Kindahl (1970) and Carlton (1986) find evidence of price stickiness for various industrial goods, Cecchetti (1986) for magazine prices.3

Sticky price models can be divided into two parts: in one class of models firms follow a time-dependent policy of price adjustment; in the other one they follow a state dependent policy. Time-dependent pricing models assume that a firm’s decisions of revising and modifying the existing price are constrained by some time limits. For example, in Fisher (1977) and Taylor (1980) models of staggered pricing firms are allowed to set their prices every other period. In Calvo (1983) the information about the changes in market conjuncture arrives randomly in time. So, decisions about

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the price changes also follow a random process. These models imply forward-looking price setting and result into the phenomenon called New Keynesian Philips Curve, which differs from the classical Philips curve that is constructed using backward-looking pricing. Time dependent price setting models feature money non-neutrality and some empirical support has been found for them (Gali and Gertler, 1999; Fabiani et al., 2006). However, they have also raised some criticism because they do not match wide range of macroeconomic regularities (see for example Fuhrer and Moore, 1995; Mankiw, 2001) and prompted researchers to propose alternative models (such as one due Mankiw and Reis, 2002).

State-dependent pricing models are more intuitive. The baseline logic here is that firms change prices depending on the state of economy. In this setup firms may change the price every period or leave it unchanged for a number of periods. The best representation of state-dependence is (S,s) pricing (Caplin and Spulber, 1987; Caplin and Leahy, 1991). The (S,s) rule was first introduced by Arrow et al. (1951) for inventory management purposes. Later, Barro (1977) and Sheshinski and Weiss (1977; 1983) also applied it to pricing models. In these models, due to the existence of adjustment costs, the zone of inaction is created around the optimal price for the firm. As long as the price is inside of the band, it is optimal not to adjust it. When the price crosses any of the inaction bands the adjustment to optimal price is observed. More recently (S,s) pricing models have been used to gain insights into the effects of monopolistic competition (see for example Caplin and Leahy, 1997). They have also been used successfully in multi-sector general equilibrium models (e.g. Damjanovic and Nolan, 2007).

All these pricing models allow for agent heterogeneity despite the fact that the strategies and the incentives of all of them are usually assumed to be identical. Heterogeneity comes with the different prices of the producers that are due to the frictions to the price adjustment. If there were no frictions, all the prices would coincide and the behavior of the aggregate variables would be the same as the individual ones (scale adjusted).

Recent years have seen a development of mixed models, or so called generalized state dependent pricing models (Devereux and Siu, 2007; Woodford, 2009; Costain and Nakov, 2011). In this paper we present a model with asymmetric (S,s) bands which
belongs to this later class of general models. We take the asymmetry of inaction bands as given, based on a well-documented empirical findings (e.g. Tobin, 1972; Ball and Mankiw, 1994).

The literature on empirics about asymmetries can be divided in two parts. One part documents asymmetries on micro level, the other—on macro level. The fact that prices do not change very often is a well documented fact (Klenow and Malin, 2011; Greenslade and Parker, 2012). The present work is based on a more fine-grained finding which is that individual prices are more rigid downwards than upwards, but if they decline, they decline by a higher magnitude relative to price increases. This means that firms' adjustment policies are asymmetric on microeconomic level. There are two types of asymmetries observed on aggregate level also. One is that the aggregate output has low and high response regimes to the monetary policy (Lo and Piger, 2005; Peersman and Smets, 2001). Namely, the output responds to a somewhat lesser extent to positive monetary shocks during the recession than during the normal periods and even lesser than during the booms. Second, the output response is smaller in magnitude when we have positive money supply shocks rather than when we have negative ones (Cover, 1992).

The asymmetry of microeconomic adjustment policies has been documented long ago. In the 70’s, economists were talking about the downward rigidity of prices (Tobin, 1972). More recent research also shows the overwhelming evidence on more frequent price increases than decreases. For example, Borenstein et al. (1997) find the microeconomic asymmetry on gasoline and agricultural products’ markets, Jackson (1997) finds it on bank deposits. To this Chen et al. (2004) add the documentation of the asymmetry in price changes in American supermarket chains.

The asymmetry in the frequency and the magnitude of adjustment is better documented for European countries. Loupias and Ricart (2004) investigate the pricing behavior of over 1600 French manufacturing firms and find that positive price changes are more frequent than negative ones. They also find that the magnitude of up- and downward price changes are different: they report an average of 3% for price upgrades in contrast with an average of -5% for price downgrades. Their findings are supported by another study of French manufacturing firms’ behavior by Baudry et al.
(2004), who find no evidence of nominal downward rigidity but support the asymmetry in magnitude of changes, although less pronounced (+4% versus -5%). Similar relation between frequency of price change and its magnitude has also been found recently for the UK (Bunn and Ellis, 2012).

A similar picture emerges in other European countries. In Belgium, Aucremanne and Dhyne (2005) find no differences in the frequency, but in the magnitude of price changes: +6.8% versus -8.7%. For Spain, Alvarez and Hernando (2004) find that the ratio of price increases to price decreases is 1.6. With regard to the asymmetry in the magnitude of price changes they report +8.2% for price increases versus 10.3% of price decreases. For Portugal, Dias et al. (2004) find no difference in magnitude of changes but a huge contrast in the frequency of price changes in different directions; they report the ratio of positive to negative price changes equal to 2.34.4

Lach and Tsiddon (1992) also find the asymmetry in magnitudes of price deviations for Israel. They examine disaggregated price data of foodstuffs in Israel during 1978-1984. Their main conclusion is that the asymmetry is more pronounced during high inflation periods, more precisely when the annual inflation goes above 130%.

Of course, these findings are not left without attention. Ball and Mankiw (1994) incorporate the difference in frequency into their model. They do this by introducing the positive drift in inflation process justifying this with some kind of Harrod-Balassa-Samuelson effect due to the faster economic integration and the development of countries. This introduces the asymmetry in price distribution. Although Ball and Mankiw’s (1994) model is able to feature more frequent price upgrades than downgrades, still the magnitudes of changes on the firms level are equal. Thus, anticipated positive drift in inflation explains only half of the story.

Tsiddon (1991) presents a simple menu cost model for high inflationary environment. He introduces the costs for adjustment that are proportional to the deviation from the optimal price and

4. Further evidence on asymmetry for all EU15 countries is provided by Lunnemann and Matha (2004).
derives the optimal pricing policy for the representative firm. The author distinguishes between price stickiness and downward rigidity and concludes that the model features the latter. The model exhibits an asymmetry in the following way. According to the optimal pricing policy, during the low inflation periods firms adjust their prices more frequently than during the high inflation periods. This is due to the fact that high inflation increases the uncertainty in future optimal price movements and the optimality is achieved by waiting. A similar result is obtained by Hansen (1999) who derives the dependence of the "first passage time" function on the degree of uncertainty. So, in a sense, Tsiddon’s (1991) model features the difference in the magnitudes of the price adjustment as well as the difference in the frequency of price adjustment.

Although the inflation trend assumed in these models is an intuitive device for introducing asymmetry, as it aggravates the effect of a positive shock and mitigates the effect of a negative one, it is not well matched with the empirical findings. For example, Peltzman (2000) shows that asymmetry is very pronounced in the United States in the period 1982-1996, when the positive drift in inflation was measured to be less than 2%. DeLong and Summers (1988) find an asymmetry during the Great Depression period when the price trend was deflationary. All this points to the fact that trend inflation can not explain even the different frequency of price up- and downgrades. Some other factors seem to be in work.

The overwhelming majority of sticky price models (e.g. Tsiddon, 1991; Ball and Mankiw, 1994) take the inaction bands lying on an equal distance from the optimal price. If we take the adjustment cost to be a menu cost type, the symmetry is justified: there is no reason why the menu costs can be different for changing the prices in different directions. But the problem is that the adjustment cost is a much wider notion than the menu cost. There are many other factors that can be regarded as the ingredients of the cost of changing price. For example, the psychological factor as seeing the

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5. There are also the examples of the other kinds of asymmetry in price adjustment derived in different setups. See for example Danziger (1988) where asymmetry is due to the discounting of future profits in inflationary environment. There every price spends most of the time being below the optimal one.

product’s price raising with large jumps can result in loss of consumers and decreasing profits. This can further propagate to firm’s large negative jump in purchases of inputs offending the suppliers. Large discrete downward jumps are better justified: this will probably result in “stealing” the buyers from competitors and also hoping to bargain a good discount with a supplier on a larger order due to the increased output. Recent empirical support for this view is due Greenslade and Parker (2012) who analyze large sample of UK firms.

The importance of these considerations is outlined in Bowman (2002). The author presents a model of sticky prices without any menu costs. In this model for firms it is optimal not to change prices in response to nominal shocks because doing so increases their profits by expanding the customer base. Then the non-neutrality of the money is obtained without any kind of menu costs. Some other kinds of cost seem to deter firms from adjusting prices.

Also, as documented by Kwapil et al. (2005), firm’s decisions about price upgrades and downgrades depend on different factors. Research on Austrian manufacturing firms shows that changes in wage and intermediate goods’ costs are two of the most important factors for price increases, while changes in competitors’ prices and technological improvements are the main driving factors for price reductions. Furthermore, Loupias and Ricart (2004) conclude that menu costs are absolutely not important for price changes of manufacturing products. Then, from this point of view, there is absolutely no reason why the costs of price changes in different directions have to be the same.

The literature on asymmetries on macro level concentrates on two major asymmetries. The first one is the asymmetry in responses of output to the expansionary and contractionary shocks of the same size. This is a well documented empirical finding for developed economies. For example, Cover (1992) exploits the quarterly data spanning 1951:1-1987:4 and finds a very high degree of asymmetry. He uses three model specifications for the identification of the asymmetry: the one proposed by Barro and Rush (1980), modified specification of Mishkin (1982) and his own. Asymmetry is pronounced in all three models. In Barro-Rush model 73% of a negative monetary shock is passed to output, while the same indicator for positive shocks is only 1% and it is not significant. In the modified Mishkin model the same indicator is 66% versus 6% (the
latter again not significant). In Cover’s original model 96% of negative monetary shock is passed to output, while, although not significant, the passthrough from positive shocks has the wrong sign. From these considerations one can conclude that positive monetary shocks do not have any effect on output and they basically pass to prices while negative shocks are passed to output to a larger extent. The more recent study of Ravn and Sola (2004) confirms the basic conclusions of Cover (1992) about the existence of asymmetry, but in their case the asymmetry is less pronounced.7

The second type of macro asymmetry is in reaction of output to monetary shocks during different phases of business cycle. Lo and Piger (2005) employ a Markov regime-switching model to investigate the asymmetry in output movements after monetary shocks to different directions. Their finding is that there is a very well pronounced time variation in output responses that can be explained by the time varying transition probability model. Basically, they find that the variation can be explained by inclusion in the model of a simple dummy variable indicating whether the economy is in a recession or in a boom. This confirms the authors’ hypothesis that output reaction has two regimes: “low response” and “high response.” In particular, policy actions taken during recessions seem to have larger effects on output than those taken during expansions.

Similar two-regime character of output responses has been found for number of economies. For example, Garcia and Schaller (2002) found asymmetry in US output response a bit earlier than Lo and Piger (2005). Peersman and Smets (2001) find the same type of asymmetry for the whole set of European countries. Furthermore, Kaufmann (2002) and Kwapil et al. (2005) document two regimes of output reaction for Austria.

All in all there is an asymmetry on macro as well as on micro levels. However, the link between micro- and macroeconomic asymmetries is complicated. In fact, microeconomic asymmetry in price adjustment can totally cancel out at the aggregate level, or macroeconomic asymmetry can be introduced by aggregation of

7. The asymmetry to positive and negative monetary shock responses is also found in other parts of the world. Karras (1996) finds asymmetry in 18 European countries. Chu and Ratti (1997) find asymmetry in the Japanese economy.
the firms with absolutely symmetric microeconomic pricing properties. A simple model presented by Caballero (1992) is an excellent demonstration of this point. Caballero (1992) demonstrates the link between micro and macro asymmetries has to be analyzed very carefully. There is no distinct link identified between these two phenomena. The motivation of the present work is to contribute to this line of research with aspiration of gaining further insight into the functionality of monetary policy. In the next section we provide the baseline model of the present paper.

2. The model

2.1. Setup of the model

We model Chamberlinian monopolistic competition following Dixit and Stiglitz (1977). The economy consists of a continuum of monopolistically competitive firms indexed on [0;1] interval that produce close (but not perfect) substitutes. This form is chosen because in a perfect competition setup a positive deviation from the optimal price results in large losses due to the loss of the entire market share. This is because, in the case of perfect competition, the profit function of the firm is not continuous in own price: it has a discrete jump immediately after the optimal price (Akerlof and Yellen, 1985). This makes competitive environment useless for the purposes of this paper.

Consider a monopolistic firm that faces downward sloping demand of a form

\[ Y = \left( \frac{P}{\bar{P}} \right)^{-\eta} \frac{M}{\bar{P}}, \]

where \( P \) is the own price of firm’s product, \( M \) is the money supply per firm, \( \bar{P} \) is the aggregate price. The positivity of monopolistic markup gives the condition \( \eta > 1 \). The firm operates at a constant real marginal costs \( C = \beta Y^\alpha \), where \( \beta \) can be interpreted as the real wage per unit of effort (in equilibrium it is constant), \( \alpha \) is the inverse of productivity parameter. Then, the monopolistic profit maximization problem is

\[ \max_{P} \pi = PY - PC \]

(2)
with respect to the demand on $Y$. Assuming symmetry, that the prices of all the goods are equal, the problem results in $P = \bar{P}$ and gives $P = GM$, where $G$ is constant and is equal to:

$$G = \left(\frac{\eta - 1}{\beta(\eta\alpha - 1)}\right)^{\frac{1}{1-\alpha}}.$$  \hfill (8)

Notice that in this (no adjustment costs) setup the output of a single firm, and as a consequence of the whole economy, is constant at a value $G$.

Taking the natural logarithms of the price-money supply relationship, denoting the logarithms by lower case letters, we get

$$p^* = g + m.$$  \hfill (3)

Then, it is apparent that $dp^* = dm$. Thus, the idiosyncratic, mean-zero shocks in money supply would call for no aggregate price changes.

Let’s introduce a variable $x$ that is the deviation of firm’s actual price from its optimal one, defined as $x = p - p^*$. Note that unlike other papers (e.g. Hansen, 1999) the negative value of $x$ means that the actual price is lower and the positive value—that the actual price is higher than the desired price. We make this assumption because of simpler tractability of results of the density function of $x$ derived in the next sub-section.

We also assume that there is a fixed cost of adjustment that is not necessarily equal for up- and downgrading the price. And there is a cost of being apart from the optimal price. Following Hansen (1999) we assume that this cost is incurred at every moment when $p \neq p^*$ and can be measured as accumulated flow costs. Note that due to the concavity of the profit function, the cost of being at non-optimum is the second order. Then an entrepreneur makes a decision by comparing the two costs. As long as the deviation cost is sufficiently lower prices do not change. This behavior creates the zone of inaction that is \textit{not necessarily symmetric} around the optimal price.

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8. Note that the solution puts stricter requirement on $\eta$. It requires $\eta > 1/\alpha$ for the positivity of $G$. 

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2.2. Deriving the long-run density

In this framework we can derive the long-run density of price deviations. Define \( f(x) \) as the long-run, time-invariant density function of price deviations. This function can also be interpreted as the likelihood of having a price deviation equal to \( x \) at any particular moment. For the derivation of the density function we assume that Brownian motion in money supply has very simple properties: it is a mean zero process and at every instant \( dt \) it can change \( x \) by \( dx \) with equal probabilities going up and down. This means that if we are now at \( x \) after one period \((dt)\) we will be at \( x + dx \) with probability 0.5 and at \( x - dx \) with probability 0.5. Then,

\[
f(x) = \frac{1}{2} f(x+dx) + \frac{1}{2} f(x-dx),
\]

as being today at \( x \) means being either at \( x - dx \) or at \( x + dx \) a moment ago. This is a very convenient property. We can rewrite (4) as

\[
(f(x+dx) - f(x)) - (f(x) - f(x-dx)) = 0.
\]

Then, division by \( dx \) gives

\[
\frac{f(x+dx) - f(x)}{dx} - \frac{f(x) - f(x-dx)}{dx} = 0.
\]

Notice that as \( dx \to 0 \) two parts of left hand side of expression (5) converge to derivatives of \( f(x) \) and then whole left hand side is something like the change in the derivative from point \( x+dx \) to point \( x \)\(^9\). Then the whole expression (5) is equivalent to the second derivative of \( f(x) \) being zero

\[
\frac{d^2 f(x)}{dx^2} = 0.
\]

Now, as \( f(x) \) is a density function, we know that

\[
\int_{-a}^{b} f(x)dx = 1,
\]

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\(^9\) In real life this would mean to assume that inaction bands on both sides of the optimal price are wide in comparison to the average size of an idiosyncratic shock. This assumption is necessary for deriving analytical results and is relaxed in coming sections when we employ ABM techniques.
Now, as \( f(x) \) is a density function, we know that
\[
\int_{-a}^{b} f(x) \, dx = 1,
\]
where \(-a\) and \(b\) are optimal bands of price adjustment. Thus, price deviation \((x)\) is distributed between \(-a\) and \(b\). We also have two boundary conditions \(f(-a) = f(b) = 0\), by assumption that prices are adjusted immediately as they reach any of the boundaries, thus none of them, in principle, are reached. Then we can split the integral (6) into two parts
\[
\int_{-a}^{0} f(x) \, dx + \int_{0}^{b} f(x) \, dx = 1.
\]

From the second derivative of \( f(x) \) being zero we know that both of these parts are linear. From the boundary conditions we know their crossing points with \(x\) axis are \(x = -a\) and \(x = b\). Also, note that \(f(b)\) has to reach maximum at \(x = 0\), because has the highest probability equal to
\[
\frac{1}{2} (f(-dx) + f(b-dx)) + \frac{1}{2} (f(dx) + f(-a+dx)) = f(0)
\]

This is the probability of being either at \(-dx\) or at \(b - dx\) and getting a positive shock plus the probability of being either at \(dx\) or at \(-a + dx\) and getting a negative shock. Then, two straight lines have to cross at \(x = 0\), otherwise the density function will not be continuous.

All these conditions together imply that \(f(x)\) has a triangular shape with the base \(a + b\) and the height \(2/(a + b)\) (and it reaches maximum at \(x = 0\)). This gives us the solution to the problem
\[
f(x) = \begin{cases} 
\frac{2}{a+b} \left(1+\frac{x}{a}\right) & \text{if } x < 0 \\
\frac{2}{a+b} \left(1-\frac{x}{b}\right) & \text{if } x \geq 0
\end{cases}
\]
Thus, the resulting density function looks like the one shown on Figure 1.\textsuperscript{10}

The shape of the resulting density function has an interesting implication. The figure is drawn for the case when $a < b$ which seems to be a realistic scenario given the empirical findings summarized in section 1 of this paper. This implies difference in an intensive margin (Klenow and Kryvtsov 2008). From the figure we can infer that near the upgrading band (near $-a$) there are relatively more firms than near the downgrading band (near $b$). This demonstrates the difference in an extensive margin. This result emphasizes the obscurity of the link between micro- and macro-asymmetry: although price downgrades are higher in magnitude there are fewer firms who want to reduce their prices as a result of a shock. Consequently, it is not obvious that the positive shock in price deviations\textsuperscript{11} will induce the aggregate price level to reduce with higher magnitude than the rise caused by the negative shock of the same magnitude. In fact, there is a chance that these two factors completely cancel out each other and we get the same result as Caballero (1992).

The long-run density (9) has few interesting characteristics. The share of firms that hold price under their optimal price is $\frac{a}{a + b}$. Consequently the share of firms holding the price over the optimal one is $\frac{b}{a + b}$. In fact this average price deviation can be calculated as

$$\bar{x} = \frac{a}{a + b} \int_{-a}^{0} 2 \frac{1 + \frac{x}{a}}{a + b} x \, dx + \frac{b}{a + b} \int_{0}^{b} 2 \frac{1 - \frac{x}{b}}{a + b} x \, dx,$$

which results into

$$\bar{x} = \frac{b^3 - a^3}{3(a + b)^2}. \tag{11}$$

This is interesting as it implies that in case of asymmetry (when $a < b$) the average price deviation will be positive. In other words

\textsuperscript{10} Notice that the original assumption of discretization of a continuous process, mainly that $x$ can go to only two states, either $x + dx$ or $x - dx$ is not crucial for the form of the density function. If one assumes many different type of idiosyncratic shock distribution it is easy to show that the same shape results. A crucial assumption for the shape is that the distribution is symmetric and centered around zero, which is maintained throughout the whole paper.

\textsuperscript{11} As shown in the next section a positive shock in price deviations is equivalent to a negative monetary shock.
an “average” firm will be holding the price above the optimal level. This happens without assuming any inflationary expectations.

3. Response to monetary policy

For the analysis of the responses of the system to monetary policy we setup an ABM equivalent to the model described in previous section. There are two reasons for this. One is that we want to depart from the unrealistic assumption of infinitesimally small idiosyncratic shocks. Recall that this was a necessary assumption for derivation of the long-run density. If idiosyncratic shocks are not of a negligible size compared to the adjustment bands, the price deviation density will depart from the one described by equation (4). In this case larger share of firms will hold prices close (or equal to) the optimal price.

The second reason for using ABM is that we want to analyze the implications of the model for the effectiveness of monetary policy during turbulent periods. We want to check how system responds to monetary shocks during booms and recessions. Recall one of the empirical findings regarding macro asymmetry has been that expansionary monetary policy is more effective during recessions than during booms. We want to check the implications of our model in this respect.

3.1. Methodology

In this sub-section we provide essential details of the simulation methodology. Of course, we can not work with the continuum of firms any longer. As we work with price deviations we have to transform the results in terms of price and output responses. Let \( x_0 \) be an initial price deviation for a single firm \( x_0 = p_0 - p^*_0 \). Then money supply shock of a magnitude \( \varepsilon \) is also an optimal price shock of the same magnitude \( p^*_1 = p^*_0 + \varepsilon \). This gives \( x_1 = p_1 - p^*_0 - \varepsilon \). From these identities we get \( x_1 = p_1 - p^*_0 - \varepsilon \). Then it is apparent that a positive shock in money supply transforms into a negative shock in price deviations and vice versa. Intuitively, the immediate rise in optimal price for the firm means that its relative price has lowered. Finally, one can express the evolution of the price of a single firm as

\[
p_1 - p_0 = \varepsilon + x_1 - x_0
\] (12)
We track the evolution of every single price in the economy. Then, the evolution of the aggregate price is derived by simply averaging all the prices in the economy.

For output changes, we proceed with demand functions. Taking natural logarithms of the original demand function and totally differentiating gives

$$ dy = \eta(d\bar{p} - dp) + (dm - d\bar{p}) $$

From here it is obvious that the output changes for every single firm depend on the parameter \( \eta \). But on the aggregate level, note that by definition

$$ \sum dp = \sum dp, \text{ as } n d\bar{p} = n \sum \frac{dp}{n}. $$

So, the first summand in (13) disappears on the aggregate level and we are left with

$$ dm = d\bar{p} + d\bar{y} $$

where \( \bar{y} \) is a log of aggregate output. So, on the aggregate level the role of price elasticity of demand disappears. Then, to simplify calculations, for aggregate output we proceed with the rearrangement of (14), as we know \( dm \) and also \( d\bar{p} \).

Results of the model depend on the size of the policy and idiosyncratic shock compared to firms’ inaction band. Therefore, we fix the size of the inaction band and calibrate monetary policy and idiosyncratic shocks in corresponding units. We normalize the size of the inaction band \( a + b = 100 \). In this case an idiosyncratic shock of size \( w \), can be interpreted as the shock of \( w\% \) of the inaction band. The same is true for monetary policy—its size will be measured as a corresponding percent of an inaction band. Then, the asymmetry of the pricing policy can be described by parameter \( a \). If \( a = 50 \), there is no asymmetry in firms’ pricing strategy. If \( a < 50 \) firms tolerate larger price deviations above the optimal price compared to the deviations below it. If \( a > 50 \) situation is reversed.

We assume the idiosyncratic shocks are normally distributed with zero mean and variance that is measured in units comparable to the size of the inaction band. Variance being equal to \( w \), means shock are drawn from \( N(0, \sigma) \), which corresponds to the shock variance being equal to the \( \sigma^2 \% \) of the inaction band. If \( w \) is small enough, we have demonstrated that the time invariant price deviation distribution density is given by (9). However, when shock
variance increases the long run distribution departs from the one derived analytically. Larger mass of firm’s will be adjusting each period to optimal price and as a consequence larger mass will be concentrate at \( x = 0 \). In order to permit the system to converge to the time invariant distribution before starting a policy experiment we initialize the system with a uniform distribution of \( x \) over the interval \([-a;b]\) and let the system run without any aggregate shock for 3000 periods\(^{12}\). Once the system has settled to the time invariant distribution we conduct a policy experiment—we introduce a monetary shock of certain size and analyze the system’s response to it.

We study the economy populated by 1000 firms. For reporting each result we conduct 150 Monte-Carlo simulation and report the average values across all 150 runs. In all cases standard deviations are extremely small, therefore they are not reported on graphs below.

### 3.2. Results

A major contribution of the paper to the literature is that we can discuss the implications of the extent of the asymmetry of the adjustment bands. Recall that we have normalized \( a + b = 100 \). Then parameter \( a \) completely characterizes the adjustment band asymmetry. Asymmetry of inaction interval \( (a) \) is one of the major parameters in our investigation. This is because the results of a recent study by Álvarez et al. (2007) that has assembled the evidence from european countries suggests variation in levels of asymmetry across countries.

Figure 2 presents the results of agent-based model that demonstrates the effect of the asymmetry on the effectiveness of the monetary policy. On the bottom axis the parameter \( a \) is plotted, while on the vertical axes we have plotted the share of the monetary shock passed to output. The value 0.3 on the vertical axis should be interpreted as 30% of the shock being passed to output while 70% being absorbed by the prices. The graph is reproduced

\(^{12}\) Numerous simulations show that in case of sufficiently low variance long-run equilibrium is indeed the one given by (9). As a consequence, the results reported in this paper are not dependent on initial conditions unlike, for example, Caplin and Spulber (1987) where initial distribution is crucial for basic results of the model.
by setting the variance of the idiosyncratic shock to unity, which is a low enough level for the time invariant price deviation distribution to be well described by the analytical one presented by equation (9). We are usually interested by the left half of the graph because that half implies $a < b$ which is a realistic case based on the empirical findings reviewed in this paper.

Figure 2 presents three sequences for three different sizes of the expansionary monetary policy: for 20, 50 and 80% of the inaction band. As one can clearly see the asymmetry plays virtually no role if the magnitude of the monetary policy is small. With increasing size of the monetary intervention role of asymmetry becomes prominent. For instance with policy size of 80 greater asymmetry (going to the left on the graph) implies higher efficiency of the policy. This is intuitive as larger asymmetry leaves fewer firms at the right edge of the $x$ distribution which will adjust prices when policy is implemented. Fewer firms adjusting prices induces larger share of monetary shock being passed to output.

![Figure 2. The effect of the adjustment band asymmetry](image)

Another important effect that has been demonstrated by figure is the impact of the policy size on its passage to output. This effect is better demonstrated by the left panel of figure 3. Similar to the figure, in this figure $\sigma^2 = 1$. The model predicts that the size of the monetary intervention negatively affects its efficiency. The logic behind this result is that larger monetary shock knocks more firms out of the inaction bands, induces them to adjust to the optimal price and as a result drives up the inflation instead of affecting real economy.
Although in all the figures we present in this paper we are discussing expansionary monetary policy, we can in fact draw conclusions also about the effects of the contractionary policies. This is due to the symmetry of the results. If $a = 50$ we do not have asymmetry in inaction bands and the size of the results of positive and negative monetary interventions are equal. However, in case of asymmetry for any given $a$ we can construct a scenario to derive the corresponding results for the contractionary policy. Consider an arbitrary $a$. We know that $b = 100 - a$. Therefore, the contractionary monetary policy for inaction band asymmetry being described by $a$, is exactly equal to the result generated by $100 - a$. This means that in figure 2 the effect of the positive and negative monetary policy are given by mirroring at $a = 50$. For example, when $a = 20$, 25% of positive monetary shock of size 80 is passes to output, as documented by the graph. However, contractionary monetary policy is sterile, which is seen by observing the passage to output being equal to zero at $a = 80$ (which is a mirror to $a = 20$).

The implications of the model in this regard are easily seen on the left panel of figure 3. In this figure we plot three series, each corresponding to different values of asymmetry. Two of them correspond to $a = 20$ and $a = 80$ which are the mirror cases comparable to each other. The discrepancy between these two series implies differential response to positive and negative monetary shocks. As we know that reality calls for $a < 50$, and figure presents results for the expansionary monetary policy, it is intuitive to view results of $a = 20$ as response to expansionary monetary policy and that of $a = 80$ as contractionary monetary policy.
As one can clearly see the two series depart from each other as policy size grows. The results predict that if there exists an asymmetry in adjustment bands (of sort that $a < b$), for large enough monetary intervention, positive expansionary monetary policies are more effective than contractionary monetary policies. This is in line with the empirical findings surveyed above.

Another result concerns the analytical long-run price deviation distribution derived above and its implications. Recall that for the derivation of function (9) we had to assume infinitesimally small idiosyncratic shocks, which in case of numerical simulations means $\sigma^2 \to 0$. As $\sigma$ is measured as the constant share of the inaction band size, this effectively means infinitely large inaction interval. This is, clearly not realistic. Agent-based simulations present us a chance to explore the effect of relaxing this assumption and exploring the effects on the monetary policy.

![Figure 4. Output response during boom and recession](image)

The right panel of the figure 3 presents results where we vary the value of $\sigma^2$. As we have anticipated in the text above, larger variance would imply larger mass of firms leaving adjustment bands and reseting themselves to the mode of the distribution at $x = 0$. This would effectively mean that at any point in time greater number of firms holding optimal prices and monetary policy being less effective. As results presented in figure 3 show, this is indeed the case: for any size of asymmetry effectiveness of monetary policy is strictly decreasing in idiosyncratic shock variance. This result stresses the importance of the size of the inaction bands when taking the decision on the size of the monetary policy.
3.2.1. Monetary policy during booms and recessions

Here we present results of our model regarding the asymmetric response to aggregate monetary shocks during different phases of business cycle. The current model is a kind of hybrid of sticky and flexible price models. Everything depends on the distribution of price deviations and the direction of the monetary shock. For example, if economy is in a boom, that is, it has been hit with several positive shocks, the distribution of price deviations shifts to the left border of inaction interval. And any further positive monetary shock induces a large number of firms to raise their prices. The model gets closer to flexible price models and the output response is dampened. But this is only for positive monetary shocks. If, in this situation, the economy is hit by a negative monetary shock the distribution will shift to the right and basically no firm will adjust prices. Then, the model gets closer to sticky price models and the whole shock is passed to the output. So, the regime of output responses crucially depends on the direction of the aggregate shock.

Figure 4 presents five series. One of them, termed “equilibrium” is the series generated the same way as all the series up to now. The other four series represent responses to expansionary monetary policy during the different phases of a business cycle. Cycles in our computational environment are generated artificially by shocking the economy in several consecutive times. More precisely, early boom and early recession is generated by introducing policy of size +1 and -1 respectively, while late boom and recession are generated by introducing shocks of the same size for 20 consecutive time periods. After we bring the system to the state of boom or recession we exercise expansionary monetary policy and calculate the response that is presented on the figure.

The results are close to linear and conform to our conjectures. Expansionary monetary policy is becoming increasingly ineffective as we progress further into the boom and it becomes increasingly effective as we go deeper into the recession. It is worth mentioning that this statement is valid only in the case of positive monetary shocks. For negative ones, the situation is the mirror image. In case of contractionary policy, it is absorbed by prices in recessions but passed to the output in booms. But, the point is that this particular kind of heterogeneity of agents is able to produce
some type of asymmetry. Stemming from the theoretical considerations above, these results can be derived from any (S,s) pricing model. The asymmetry of the bands is not required for this result. It is purely due to the shifts of the price deviation density to one of the edges of the distribution. So, asymmetry on the micro level is not the cause of the aggregate output having two regime property, but rather this is due to (S,s) pricing behavior itself. Thus, this kind of aggregate asymmetry is the direct consequence of heterogeneity of agents, no matter whether their micro policies are symmetric or asymmetric.

4. Conclusion

Individual prices change rarely, and there is a staggering in the adjustment since the price changes across the firms differ in time. This behavior is due to some costs involved in the price adjustment process: costs of gathering information about the market conjuncture, costs of loosing the market share, etc. So, the adjustment cost is a wider notion than “menu cost”; the latter is one of the components of the former. Due to the fact that some ingredients of price adjustment costs are asymmetric for price changes in different directions, the adjustment costs, as a whole, are also different for price upgrades and downgrades.

In the current paper we presented the model where individual firms follow asymmetric (S,s) pricing behavior. This is due to the asymmetry in the adjustment costs mentioned in the previous paragraph. We investigate few important questions such as asymmetry in responses to expansionary and contrationaty monetary policies and variance of the effectiveness of the policy during different phases of the business cycle. We also investigate the role of the asymmetry in adjustment bands in these processes.

The basic results were derived by numerically simulating the model. However, for the small idiosyncratic shocks the time invariant price deviation distribution had been analytically derived. This distribution does not depend on the initial conditions of the model. One more specific character of the current paper is that, unlike the most similar papers, we did not use simple binomial random walk for the description of shock process. Rather we used more elaborate shock process that allows for the variance in the
size of the idiosyncratic shocks. This is important as it highlights the importance of firms located in the interior of the adjustment bands. This contrasts to the models with binomial shocks (e.g. Caballero, 1992), that but emphasis on firms located on margins of the inaction interval. We have also explored the effects of the changing variance in shock process.

We have explored at the implications of the asymmetric (S,s) pricing behavior of firms for two kinds of stylized facts about the asymmetry in the aggregate output dynamics. The first is the asymmetric response of output to positive and negative monetary shocks. Here the finding is that in the case of sufficiently high shocks, the model is able to produce significant asymmetry on the aggregate level between responses. The second type of asymmetry is that the aggregate output has low and high response regimes with respect to monetary shocks, depending on whether the economy is in boom or in recession. Although the model is able to produce this kind of effect for positive shocks, the main conclusion is that this is not due to the asymmetry on the micro level. Instead, firm heterogeneity itself creates the asymmetry on aggregate level.

References


Asymmetric (S,s) pricing: Implications for monetary policy


