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Dynamic Efficiency, the Riskless Rate, and Debt Ponzi Games under Uncertainty

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Abstract

In a dynamically efficient economy, can a government roll its debt forever and avoid the need to raise taxes? In a series of examples of economies with zero growth, this paper shows that such Ponzi games may be infeasible even when the average rate of return on bonds is negative, and may be feasible even when the average rate of return on bonds is positive. The paper then reveals the structure which underlies these examples.

The average realized real rate of return on government debt for major OECD countries over the last 30 years has been smaller than the growth rate. Does this imply that governments can play a Ponzi debt game, rolling over their debt without ever increasing taxes?

If only economies were both nonstochastic and in steady state, the answer to this question would be a simple one. All interest rates would be the same, and if the interest rate were less than the growth rate, the economy would be dynamically inefficient. In this case, the government could issue debt and roll it over forever, never increasing taxes, and covering interest payments by new debt issues. Debt would grow at the interest rate, but the ratio of debt to GNP would eventually tend to zero. Such a policy would crowd out the excessive capital accumulation characteristic of dynamically inefficient economies and, as we also know, it would in general be Pareto improving.[1]

Actual economies however are stochastic. And in actual economies, there are many different interest rates, some which are on average above the growth rate, some which are below. It has been shown in particular that stochastic economies may be dynamically efficient while having an average riskless real rate below the growth rate.[2] But this brings us back, with an additional twist, to our initial question. In dynamically efficient economies (i.e., in economies that are not plagued by capital overaccumulation), can governments issue and roll over riskless debt, and thus play a Ponzi debt game? This is the issue we take up in this paper.

Our paper is constructed using the Socratic method. This means two things. First, that our objective in this paper is *not* to derive new theoretical results on dynamic efficiency and Ponzi games. Instead, we explicitly and unabashedly rely on existing theoretical results (most often developed in contexts quite different from ours) to study an issue— the feasibility of Ponzi debt games in dynamically efficient economies with low safe interest rates— that often leaves macroeconomists somewhat baffled. Second, that we use an unorthodox exposition style. Instead of proceeding from the general to the particular, we rely on a series of simple, but increasingly richer, examples to clear up common misconceptions on Ponzi games in stochastic economies.

[1] These results require various conditions, both technical and substantive, to be satisfied. One such substantive condition is that the interest rate be equal to the social marginal product of capital, an assumption which fails for example if there are external returns to capital. For an analysis of dynamic efficiency and Ponzi games in an endogenous growth model with external returns to capital, see Saint-Paul (1992).

[2] See Abel, Mankiw, Summers and Zeckhauser (1989).

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We use these examples to move from the specific to the general, and to reveal the theoretical structure that we need to understand when and why Ponzi games can exist in dynamically efficient economies under uncertainty.

Sections 1 to 4 of the paper present our four examples. All share the following features. *First*, all characterize economies which are subject to technological shocks and thus to uncertainty, so that assets with different risk characteristics have different rates of return. *Second*, all characterize overlapping generation economies with capital accumulation, thus economies for which capital overaccumulation—dynamic inefficiency— is not ruled out a priori. But, *third*, in each case, we choose, relying on the criterion derived by Zilcha (1991), underlying taste and technology parameters such the economy is actually dynamically efficient. In each of these economies, we then ask what would happen to the debt-to-GNP ratio if the government were to issue debt and roll it over time, issuing new debt to pay interest on the existing debt. And each of the four examples gives us a very different answer.

In our first example, the average riskless rate is negative; nevertheless, the expected value of the debt-to-GNP ratio under a rollover strategy explodes, and thus the government cannot play a debt Ponzi game. In the second, the average riskless rate is again negative, and, furthermore, the expected value of the debt-to-GNP ratio under rollover goes to zero. But the fact that a Ponzi game appears viable in expected value does not imply that it is feasible. Indeed, in this example, with strictly positive probability, debt rollover leads to an arbitrarily large value of the debt-to-GNP ratio and is thus again infeasible. By then, the reader may suspect that dynamic efficiency rules out Ponzi games, no matter what the behavior of the average riskless rate may be. But the last two examples show this guess to be wrong. In our third example, the riskless rate is always negative and under a Ponzi debt game, the debt-to-GNP ratio goes to zero with certainty, so that rollover is indeed feasible. Our fourth example offers a nearly perfect counterpoint to the first. In that example the average riskless rate is positive; yet by issuing and rolling over two-period bonds, the government can still play a Ponzi debt game...

We spend section 5 of the paper to reveal the structure behind the results, to explain why Ponzi games are feasible in the last two examples but not in the first two, and to acknowledge our many debts to the literature up to 1990, when this paper was first written. The literature on debt Ponzi games has made much progress since then (in the direction pointed out by this paper!): the last section of the paper reviews these contributions.

We do not want to give the answers away in the introduction. For the

busy reader, they are restated in the conclusion.

1 A first example: a Diamond model with logarithmic preferences

Our first example is a straightforward extension of the Diamond (1965) model to uncertainty.[3] Consider an overlapping generation economy in which two-period consumers inelastically supply one unit of labor when young and retire when old, and in which population is constant and normalized to one.[4] Assume that consumers have time- and state-additive logarithmic preferences. An individual representative of the generation born at time t therefore maximizes

$$(1-\beta)\ln C_{1,t} + \beta \,\mathrm{E}_t \ln C_{2,t+1} \tag{1.1}$$

subject to

$$\begin{aligned} C_{1t} + K_{t+1} &= W_t, & (1.2)\\ C_{2,t+1} &= R_{t+1}K_{t+1}, & (1.3) \end{aligned}$$

where $C_{1,t}$ and $C_{2,t+1}$ denote first and second period consumptions of an individual born at t, W_t and R_t the wage and capital rental rates at t, K_t the capital stock at t, and the discount factor $\beta \in (0,1)$ measures subjective time preference.

Output is produced according to a Cobb-Douglas technology. Output at time t is given by:

$$Y_t = \Upsilon_t K_t^{\alpha}, \tag{1.4}$$

where K_t denotes capital per worker at time t, and $\alpha \in (0,1)$ the constant share of capital in output. The logarithm of the productivity shock Υ is assumed to be independently and identically normally distributed, with mean zero and variance σ^2. Capital fully depreciates in production.

[3] This model is by now standard. See for example Blanchard and Fischer (1989).

[4] We shall throughout consider economies with no growth. Introducing nonstochastic population or productivity growth would be straightforward, and only complicate notation.

1.1 Equilibrium capital accumulation and dynamic efficiency

Solving for consumption from utility maximization, and replacing wages and rental rates by their values from profit maximization, gives:

$$K_{t+1} = (1-\alpha)\beta\Upsilon_t K_t^\alpha, \tag{1.5}$$
$$C_{1,t} = (1-\beta)(1-\alpha)\Upsilon_t K_t^\alpha, \tag{1.6}$$
$$C_{2,t} = \alpha\Upsilon_t K_t^\alpha. \tag{1.7}$$

Note—this will be relevant later—that, at any time t, the consumptions of the young and the old are perfectly correlated. Our interest for the moment is however in the behavior of capital. Denote, hereafter, the logarithm of an uppercase variable by its lowercase counterpart. We then have

$$k_{t+1} = \ln[(1-\alpha)\beta] + \alpha k_t + \upsilon_t. \tag{1.8}$$

And the behavior of output is given by

$$y_t = \alpha \ln[(1-\alpha)\beta] + \alpha y_{t-1} + \upsilon_t. \tag{1.9}$$

Capital accumulation leads to serial correlation of capital and output in response to white noise shocks.

We can now ask under what parameter values this economy is dynamically efficient. The natural extension of the aggregative Cass (1972) dynamic efficiency criterion under certainty is that the economy is dynamically efficient if there does not exist another feasible sequence of capital which provides at least as much *aggregate* consumption at all dates and in all states, and strictly higher aggregate consumption in at least one date or state. Zilcha (1991) has derived a necessary and sufficient condition for the efficiency of *stationary* economies such as the one we consider. In this economy with constant population, the condition is that unconditional expectation of the *logarithm* of the gross marginal product of capital, $\mathrm{E}\, r_t$, be nonnegative. Here,

$$\mathrm{E}\, r_t = \ln\alpha + (\alpha-1)\,\mathrm{E}\, k_t = \ln\left[\frac{\alpha}{(1-\alpha)\beta}\right], \tag{1.10}$$

so that the economy is dynamically efficient if and only if

$$\theta \equiv \frac{\alpha}{(1-\alpha)\beta} - 1 \geq 0. \tag{1.11}$$

In the rest of this section, we assume that condition (1.11) is satisfied and that the economy is dynamically efficient.[5] We now turn to the determination of the riskless rate.

1.2 The riskless rate

The equilibrium riskfree rate of return R^f_{t+1} on a safe one-period bond (which pays one unit of the consumption good in every state at date $t+1$) satisfies the first-order condition for utility maximization:

$$(1-\beta)C_{1,t}^{-1} = \beta R^f_{t+1}\,\mathrm{E}_t\, C_{2,t+1}^{-1}, \tag{1.12}$$

Evaluating the equilibrium risk free rate at a zero net supply of bonds, and thus replacing first- and second-period consumption in (1.12) by their values from equations (1.6) and (1.7), we get

$$R^f_{t+1} = \frac{1}{\mathrm{E}_t\,\Upsilon_{t+1}^{-1}}\alpha K_{t+1}^{\alpha-1}, \tag{1.13}$$

which, using our distributional assumptions about υ, implies that

$$r^f_{t+1} = \ln\alpha + (\alpha-1)k_{t+1} - \sigma^2/2. \tag{1.14}$$

Using (1.5), (1.14) and (1.11), the unconditional mean and variance of the logarithm of the riskfree rate are thus given by

$$\mathrm{E}\, r^f_{t+1} = \ln(1+\theta) - \sigma^2/2, \tag{1.15}$$

$$\mathrm{Var}\, r^f_{t+1} = \frac{1-\alpha}{1+\alpha}\sigma^2, \tag{1.16}$$

so that, finally,

$$\mathrm{E}\, R^f_{t+1} = (1+\theta)\,\mathrm{e}^{-\alpha\sigma^2/(1+\alpha)}\,. \tag{1.17}$$

Were there no uncertainty, the net riskless rate would be equal to θ, and thus would be positive under dynamic efficiency. But if the variance of the technological shocks is large enough, the average gross riskless rate under uncertainty may be less than one, and the net rate may be negative.

[5] Note that the condition for dynamic efficiency of Abel et al. (1989)—which, being a sufficient condition, is often not conclusive in particular models—is satisfied here. Gross profits, $R_tK_t = \alpha\Upsilon_t K_t^\alpha$, exceed gross investment, $K_{t+1} = \beta(1-\alpha)\Upsilon_t K_t^\alpha$, at all dates and in all states if and only if $\alpha \geq \beta(1-\alpha)$, i.e., if and only if $\theta \geq 0$.

1.3 Debt Ponzi games

Assume that the underlying parameters are such that the economy is dynamically efficient and that the average net riskless rate is negative. Does the average negative riskless rate imply that the government can roll its debt forever?

Our strategy in assessing whether or not Ponzi debt games are feasible in each of our examples will be to characterize the behavior of debt using the interest rates corresponding to a zero net supply of debt, thus corresponding to the case where consumption and capital dynamics are given by (1.5) to (1.7). If we can show that Ponzi games are not feasible under such interest rates, this will a fortiori be true were we to do the same exercise at the interest rates corresponding to a positive supply of debt (an exercise however considerably more difficult analytically): under our assumption on the utility function which implies that saving is a constant fraction of labor income, Ponzi games crowd out capital accumulation (see Weil (1987)) and thus raise, *ceteris paribus*, all interest rates. If instead we can show that Ponzi games are feasible under such interest rates, that the debt-to-GNP ratio implodes over time under rollover, then this will remain true if the government issues a small enough amount of debt at time 0.

Let B_0 be the amount of one-period riskless bonds issued at time 0. Then, under a rollover strategy, the debt-to-GNP ratio at time t follows

$$\begin{aligned} B_t/Y_t &= (R_t^f \dots R_1^f)B_0/Y_t, && (1.18)\\ &= [R_t^f/(Y_t/Y_{t-1})] \dots [R_1^f/(Y_1/Y_0)](B_0/Y_0). && (1.19) \end{aligned}$$

Using the characterization of output and the riskless rate given in equations (1.9) and (1.14), this implies:

$$b_t - y_t = (b_0 - y_0) + t[\ln(1+\theta) - \sigma^2/2] - \sum_{s=1}^{t} \upsilon_s. \qquad (1.20)$$

The logarithm of the debt-to-GNP ratio follows a random walk with drift, with innovations equal to the technological shocks. This in turn implies that the expected value of the debt-to-GNP ratio follows:

$$\mathrm{E}[B_t/Y_t] = [B_0/Y_0](1+\theta)^t. \qquad (1.21)$$

Thus, the behavior of the expected debt-to-GNP ratio is simple: it grows

at rate θ, irrespective of the value of the average riskless rate.[6] This implies that a Ponzi game is not feasible: in expectation, debt becomes larger than saving, which is proportional to GNP—an impossibility.

What is at work here is Jensen's inequality. As is clear from equation (1.14), the riskless rate from t to $t+1$ is known at time t, but varies stochastically through time. What matters for the behavior of debt is the expectation of the product of the riskless rates, not the product of the expected riskless rates. This is why a negative average riskless rate is consistent with an exploding expected debt-to-GNP ratio.[7]

This first example gives a clear warning. An economy may have an average negative riskless rate (or, if we were to allow for growth, an average riskless rate below the average growth rate) but this does not imply that the government can rollover debt forever; in this example, under the condition that the economy is dynamically efficient, the expected debt-to-GNP ratio grows at a rate which is necessarily positive, regardless of the riskless rate. One may ask however what would happen if people were more risk averse than implied by logarithmic utility. Wouldn't this further decrease the average riskless rate and reintroduce room for a Ponzi game? We take the issue up in the next example.

2 A second example: Allowing for more risk aversion

In this example, we maintain the assumption that consumers still have logarithmic intertemporal preferences, but no longer restrict their coefficient of relative risk aversion to equal unity. The rationale for adopting this specification is that, while the assumption of a unit elasticity of intertemporal substitution is necessary to preserve the simplicity of the model, the ability to choose the degree of risk aversion allows us to examine the comparative dynamic effects of increased risk aversion on equilibrium returns and on the asymptotic rate of growth of debt Ponzi games.

[6] The behavior of debt itself—as opposed to the debt-to-GNP ratio—is more complex. If the average riskless rate is negative, expected debt initially decreases. Asymptotically however, it also grows at rate θ.

[7] A similar point has been made in Galli and Giavazzi (1992) in the context of a Ramsey economy.

Thus, following Selden (1978) and Weil (1990), we assume that consumers now maximize

$$(1-\beta)\ln C_{1,t} + \beta \ln[\mathrm{E}_t\, C_{2,t+1}^{1-\gamma}]^{1/(1-\gamma)}, \tag{2.1}$$

where $\gamma > 0$ ($\gamma \neq 1$) is the coefficient of relative risk aversion.[8]

2.1 Equilibrium capital accumulation and dynamic efficiency

Because of the assumption of logarithmic intertemporal preferences, consumers choose to save a constant fraction β of their lifetime wealth regardless of the coefficient of relative risk aversion: attitudes towards risk are irrelevant for consumption and savings decisions when income and substitution effects cancel each other out. As a consequence, the equilibrium consumption allocation and capital accumulation process are the same as in the previous section, and the condition for dynamic efficiency is still, as in (1.11), $\theta \geq 0$.

2.2 The riskless rate

While the value of γ does not matter for capital accumulation when intertemporal preferences are logarithmic, attitudes towards risk are of course a crucial determinant of the implicit riskless interest rate. Adapting the argument in the previous section, the equilibrium gross return on a safe claim on consumption at $t+1$ must satisfy

$$\frac{1-\beta}{C_{1,t}} = \beta R_{t+1}^{f,\gamma} \frac{\mathrm{E}_t\, C_{2,t+1}^{-\gamma}}{\mathrm{E}_t\, C_{2,t+1}^{1-\gamma}}, \tag{2.2}$$

where the notation $R_{t+1}^{f,\gamma}$ is adopted to highlight the dependence of the riskless rate on γ.

Evaluating this expression along the no debt path, one finds that:

$$R_{t+1}^{f,\gamma} = \frac{\mathrm{E}_t\, \Upsilon_{t+1}^{1-\gamma}}{\mathrm{E}_t\, \Upsilon_{t+1}^{-\gamma}} \alpha K_{t+1}^{\alpha-1}. \tag{2.3}$$

[8] Note that $[\mathrm{E}_t\, C_{2,t+1}^{1-\gamma}]^{1/(1-\gamma)}$ is simply the certainty equivalent of second period consumption for an individual with constant relative risk aversion γ. One can also verify, using L'Hospital's rule, that the preferences represented in (2.1) reduce to the time- and state- additive logarithmic form in (1.1) when $\gamma \to 1$.

Under the lognormality assumption, it follows that:

$$R_{t+1}^{f,\gamma} = R_{t+1}^{f,1}\, \mathrm{e}^{(1-\gamma)\sigma^2}, \tag{2.4}$$

where $R_{t+1}^{f,1}$ is the riskless rate for the unit risk aversion case computed (in logarithmic form) in (1.14).

As a consequence,

$$\begin{aligned} \mathrm{E}\, R_{t+1}^{f,\gamma} &= \mathrm{E}\, R_{t+1}^{f,1}\, \mathrm{e}^{(1-\gamma)\sigma^2} \\ &= (1+\theta)\, \mathrm{e}^{[-(\alpha/(1+\alpha))]\sigma^2}\, \mathrm{e}^{[(1-\gamma)]\sigma^2}\,. \end{aligned} \tag{2.5}$$

The effect of increased risk aversion is thus to decrease proportionately the riskless rate by the same factor in all states and at all dates. For a given variance of productivity shocks, the average net riskless rate can be very negative if agents are very risk averse.

2.3 Debt Ponzi games

Following the same logic as in the previous section, we derive the behavior of the debt-to-GNP ratio under rollover, using the processes for the riskless rate and for output characterized above. We get that the logarithm of the debt-to-GNP ratio at time t under Ponzi finance follows:

$$b_t - y_t = (b_0 - y_0) + t[\ln(1+\theta) + (1-2\gamma)\sigma^2/2] - \sum_{s=1}^{t} \upsilon_s. \tag{2.6}$$

As before, the debt-to-GNP ratio follows a random walk with drift. However, for a given variance of shocks, the drift may now be arbitrarily large and negative. Equation (2.6) in turn implies:

$$\mathrm{E}[B_t/Y_t] = [B_0/Y_0](1+\theta)^t\, \mathrm{e}^{(1-\gamma)\sigma^2 t}\,. \tag{2.7}$$

The unconditional expected debt-to-GNP ratio thus grows (or declines...) at the constant rate $(1+\theta)\, \mathrm{e}^{(1-\gamma)\sigma^2}$. Provided that agents are sufficiently risk averse, dynamic efficiency ($\theta \geq 0$) need not imply an exploding expected debt-to-GNP ratio under rollover. So is this Ponzi game feasible? The answer is no. It is not enough that the *expected value* of the ratio go to zero. What is required is that the Ponzi game be feasible in all states.[9] Equation (2.6) shows that the logarithm of the debt-to-GNP ratio

[9] This is indeed the main point raised by Bohn (1995).

follows a random walk with drift. It follows that the debt to GNP ratio will exceed any finite limit, such as the ratio of saving to GNP, with probability 1 if the expected debt-to-GNP ratio rises, and with positive probability if the expected debt to GNP ratio decreases. Thus, with strictly positive probability, even if the expected debt to GNP ratio goes to zero, the Ponzi game will prove infeasible.

This second example shows that, in an economy which is dynamically efficient, the average riskless rate may be negative, the debt-to-GNP ratio may go to zero in expected value under rollover, and yet there is a strictly positive probability that the Ponzi game cannot be played forever. The proximate cause of the result is that, in this example, the debt-to-GNP ratio follows a random walk with drift: no matter how small the initial amount of debt issued, there is a positive probability that the (nonstationary) debt-to-GNP ratio eventually exceeds some given bound, and in particular the wage income of the young—an event inconsistent with equilibrium. This naturally raises another question. Could the government issue bonds with different risk characteristics so that, under some conditions, the debt to GNP ratio would instead be stationary around a mean and the probability that it reaches some critical value be made arbitrarily small or even zero? We defer the question to Section 6 below; there we shall show that the answer is no.

3 A third example: An economy with stochastic storage

Assume that, as in our first example, consumers have time- and state-additive logarithmic preferences. Assume, however, that the technology is different. People born at time t receive a nonstochastic first period endowment $W > 0$, and have access to a stochastic constant returns to scale storage technology with random gross rate of return $\tilde{R}_{t+1}$.[10] The logarithm of $\tilde{R}_t$ is assumed to be identically and independently distributed with mean μ and variance σ^2.

Using the Zilcha criterion, this economy is dynamically efficient if and only if $\mathrm{E} \ln \tilde{R}_t = \mu \geq 0$.[11]

[10] This model is also standard. It was used by Koda (1984) to study the existence of fiat currency equilibria, and was more recently analyzed by Gale (1990) in the context of public debt. A similar model was also used by Summers (1984).

[11] Formally, our example, which has constant returns to storage does not satisfy the concavity conditions required for the Zilcha criterion to apply. An alternative formalization,

Under those assumptions, the consumption of the young, the consumption of the old and the capital stock are given by:

$$C_{1t} = (1-\beta)W, \tag{3.1}$$
$$C_{2t} = \beta\tilde{R}_t W, \tag{3.2}$$
$$K_t = \beta W. \tag{3.3}$$

Because the endowment is nonstochastic, the consumption of the young and capital accumulation are also nonstochastic. Because storage is stochastic, the consumption of the old is stochastic, and independently and identically distributed over time. In contrast to the previous examples, the consumption of young and old are not perfectly correlated, a point to which we shall return below.

Having characterized consumption, we can derive the implicit equilibrium rate of return on a one-period bond. From the first order conditions of the consumer, it is given by:

$$R^f = \frac{1}{\mathrm{E}\,\tilde{R}^{-1}} = \mathrm{e}^{\mu-\sigma^2/2}. \tag{3.4}$$

Thus the rate of return on one-period bonds is also nonstochastic. It may well be negative if the variance of $\tilde{R}$ is sufficiently large.[12] Since both the first-period endowment and the riskless rate are constant, a necessary and sufficient condition for the feasibility of issuing at least a small quantity of debt and rolling it over is simply

$$R^f < 1. \tag{3.5}$$

Thus, in sharp contrast to our previous examples, in this dynamically efficient economy, debt Ponzi games may actually be feasible. And the riskless rate plays a crucial role in determining the feasibility of Ponzi finance. Ponzi finance is feasible if and only if the net riskless rate is negative.[13] This suggests that, after all, the riskless rate may be an important variable in assessing whether Ponzi finance is feasible or not. The last example shows that this guess would also be wrong.

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which satisfies those conditions, and has the same implications for the existence of debt rollover is one where depreciation is stochastic so that output is produced according to $Y_t = {K_t}^{\alpha} - \delta K_t$ where δ is a random variable.

[12] Or, if we had used the preferences of section 2, if consumers are sufficiently risk averse.

[13] This result is closely related to Koda's (1984) characterization in this model of the existence of monetary equilibria with constant money supply.

4 A fourth example: Serially correlated returns in storage

We consider the same storage economy, but now allow for serially correlated returns on storage.[14] The reason for doing this will be clear later. More specifically, we assume that the rate of return on storage follows a geometric MA(1) process:

$$R_{t+1} = \epsilon_t^{\rho} \epsilon_{t+1}, \tag{4.1}$$

where the logarithm of ϵ is identically and independently distributed with mean μ and variance σ^2, and with $\rho \geq -1$.

From Zilcha's criterion, taking the logarithm and the unconditional expectation of both sides of (4.1), this economy is dynamically efficient if and only if

$$\mathrm{E} \ln R_t = (1+\rho)\, \mathrm{E} \ln \epsilon \geq 0, \tag{4.2}$$

or, equivalently, if and only if $\mu \geq 0$.

Consider now the following debt Ponzi game. At time 0, the government issues debt in the form of two-period bonds, titles to one unit of good two periods later. At time 1, the government buys back what are now one-period bonds, and pays with the proceeds of new issues of two-period bonds, and so on. To characterize the behavior of debt under this scheme, we first determine the equilibrium prices of one- and two-period bonds.

The price at time t of a two-period bond, issued at $t-1$ and maturing at $t+1$, is simply, from the first-order condition of the consumers

$$\begin{aligned} p_{1t} &= \frac{\beta}{1-\beta}\, \mathrm{E}_t \left\{ \frac{C_{1,t}}{C_{2,t+1}} \right\} \\ &= \mathrm{E}_t\, R_{t+1}^{-1} \\ &= \epsilon_t^{-\rho}\, \mathrm{E}\, \epsilon^{-1}, \end{aligned} \tag{4.3}$$

Note, for use below, that the average one-period gross riskless rate is given by $\mathrm{E}(1/p_{1t})$.[15] The average net riskless rate is thus nonnegative if and only if $(\mathrm{E}\, \epsilon^{\rho})/(\mathrm{E}\, \epsilon^{-1}) \geq 1$, or equivalently, $\mu - (1-\rho)\sigma^2/2 \geq 0$.

Notice, also for later use, that, if ρ is positive, the price of a one-period bond is high when the current rate of return on storage is low[16]—because a

[14] This example is a direct application of Gale (1990).

[15] By arbitrage, the rates of return on one-period bonds, and on old two-period bonds one year to maturity must be equal.

[16] Given ϵ_{t-1} , a high ϵ_t means a low p_{1t} and a high R_t.

low return on storage today creates the expectation of a low future return.

The price at time t of a newly issued two-period bond is

$$\begin{aligned} p_{2t} &= \left(\frac{\beta}{1-\beta}\right)^2 \mathrm{E}_t \left\{ \frac{C_{1,t}}{C_{2,t+1}} \frac{C_{1,t+1}}{C_{2,t+2}} \right\} \\ &= \mathrm{E}_t (R_{t+1} R_{t+2})^{-1} \\ &= \epsilon_t^{-\rho} \, \mathrm{E}\, \epsilon^{-(1+\rho)} \, \mathrm{E}\, \epsilon^{-1}. \end{aligned} \tag{4.4}$$

Now consider the dynamic behavior of debt (or—trivially—the ratio of debt to first period endowment, as the endowment, W, is constant). At time t, debt satisfies:

$$p_{2t} B_t = p_{1t} B_{t-1}, \tag{4.5}$$

so that

$$B_t = \frac{1}{\mathrm{E}\, \epsilon^{-(1+\rho)}} B_{t-1}. \tag{4.6}$$

Thus, the rate of growth of the debt is deterministic, and of the same sign as $\mu - (1+\rho)\sigma^2$.

Are there parameters such that i) the economy is dynamically efficient, ii) the average one-period net riskless rate is nonnegative, (iii) debt decreases or stays constant every period? From our derivations above, it is clear that the answer is yes. The conditions on μ, ρ and σ^2 are that $\rho > 0, (1+\rho)\sigma^2 \geq \mu \geq (1-\rho)\sigma^2$. These conditions are satisfied for example by $\rho = \mu = \sigma = 1$.

Thus, Ponzi finance using noncontingent two-period bonds is feasible, although the economy is dynamically efficient and the average one-period net riskless rate is positive.

5 Dynamic efficiency and Pareto-optimality

Why do results differ between the first two and the last two examples? Under uncertainty, overlapping generation models differ from Ramsey economies in two ways. The first is that people have finite economic horizons. The second is that markets are incomplete. In all four examples, the condition that dynamic efficiency holds rules out the capital overaccumulation which may arise from the first feature. The difference between the two sets of examples comes from the second feature. In the first two examples, incomplete markets do not matter, and thus the equilibrium is Pareto optimal. In the last two,

they do, and debt is Pareto improving. We consider these propositions in more detail.

In all four examples, the overlapping generation structure implies that the young cannot enter insurance contracts with the old so as to share risk in the first period of their life. But in the first two, the consumption of the young and the consumption of the old are perfectly correlated, so that there is no room anyway for further risk sharing. Indeed, it is easy to check that the decentralized allocation in our first two examples maximizes the following social welfare function

$$\mathrm{E}_t[\sum_{s=0}^{\infty}(1+\theta)^{-s}((1-\beta)\ln C_{1,t+s}+\beta\,\mathrm{E}_{t+s}\ln C_{2,t+s+1})] \tag{5.1}$$

where θ is defined as earlier and is therefore positive under dynamic efficiency. In line with previous literature,[17] this means that the market outcome is Pareto optimal: there is no way to improve the welfare of current generations without hurting some future generation. Thus the government cannot play Ponzi games, no matter what the average riskless rate or the expected value of the debt-to-GNP ratio.

This is not the case in the last two examples. In the third example, technological uncertainty falls only on the old at time t, while the consumption of the young remains constant. This suggests room for debt to provide such insurance, and this indeed the case. This has been emphasized by Gale (1990) who has shown that in the context of that example, the government can not only issue and rollover debt, but further issue new debt every period so as to maintain a constant debt-to-endowment ratio. Not only is such a policy feasible but it is Pareto improving, so long as the amount of debt does not lead to a positive riskless rate. This can be seen easily. Carrying a constant amount of debt at a negative riskless rate is equivalent to transferring a constant amount, τ from the young to the old. It is easy to check that the condition for such a transfer to increase expected utility is that the riskless rate be negative.[18] Put another way, the reason why the government is able to play a Ponzi game is that bonds provide insurance and require a low rate of return. (This conclusion is similar to that in Bertocchi (1991), who focuses on the equivalent phenomenon of bubbles.) The role for noncontingent debt

[17]See, for instance, Peled (1982), Aiyagari and Peled (1991), or Gottardi (1996).

[18]The proof is immediate and holds for a general utility function. Expected utility when an intergenerational transfer of size τ takes place is $U = \mathrm{E}\,u[W - K(\tau) - \tau, \tilde{R}K(\tau) + \tau]$. Straightforward algebra shows that $dU/d\tau > 0$ as long as $R^f < 1$.

to provide insurance may however be quite limited (or not present at all). This explains the role of the debt in the last example. When returns to storage are positively correlated, a low realization of the rate of return on storage is associated with a lower riskfree rate next period and thus a high price of one-period bonds. Thus, issuing two period bonds and buying them back as one-period bonds provides insurance to the old. The price they get for their bonds is high when the returns to storage are low. This suggests that there may be an optimal maturity for bonds, a line that has been explored by Gale. Indeed, this suggests the issuance of explicitly contingent bonds to provide the needed insurance. And under those conditions, the government may well be able to issue and rollover debt.

Should one conclude that Pareto suboptimality is a sufficient condition for the feasibility of debt Ponzi games, for some form of debt? Surely not. Competitive economies may be Pareto suboptimal for reasons that have nothing to do with inadequate intergenerational transfers. To see this, consider for example a simple modification of our third example. Assume that preferences and the storage technology facing each individual are the same as in that example, but that uncertainty is idiosyncratic so that, while the return on individual storage is uncertain, aggregate storage is not. Assume further that individual realizations are private information, preventing private contracts that pool away this individual uncertainty from being written. The lack of *intra*-generational sharing makes the equilibrium obviously Pareto suboptimal. Can the government play a Ponzi game? As aggregate variables are nonstochastic, we can, without loss of generality, look at riskless debt games. The riskless rate is the same as it was in our example, so that the condition for Ponzi games to be feasible is again that the average net riskless rate be negative, namely that $\mu - \sigma^2/2 \leq 0$. If this condition is not satisfied, if for example, storage is sufficiently productive, there is no scope for Ponzi finance. [19]

Should one conclude that Ponzi games, when feasible, are Pareto-improving? In the four examples considered in this paper the answer is yes, but this is not true generally, for reasons that have nothing to do with uncertainty *per se* but rather with second-best theory. An example is given by Saint-Paul (1992), who studies an endogenously growing overlapping gen-

[19]Cass, Okuno and Zilcha (1980) have shown, under certainty, that the often-asserted connection between the Pareto suboptimality of "nonmonetary" equilibria and the existence of "monetary" equilibria does not generally hold: their results about fiat currency translate almost directly to debt Ponzi games, and are valid, *a fortiori* under uncertainty.

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erations economy with a distortion coming from external increasing returns to capital. In his model, Ponzi games are feasible whenever the private marginal product of capital falls short of the growth rate (which is itself always lower than the social rate of return on capital). But a debt Ponzi game is never Pareto improving: since it crowds out capital accumulation, it lowers the growth rate, so that eventually the welfare of some future generations is adversely affected.

6 Subsequent literature

The literature has made much progress since this paper was first written in 1990. Authors looking for existence conditions for fiat-currency equilibria (another name for debt Ponzi games) have confirmed the essential themes of this paper: the irrelevance of the average riskless rate, and the crucial role of debt Ponzi games in providing intergenerational insurance. Some papers have gone beyond the territory explored here, and have provided the necessary and sufficient condition for the existence of monetary equilibria (or debt Ponzi games) we were unable to derive ten years ago.

Manuelli (1990) shows, in a stochastic overlapping generations *exchange* economy (in which the very issue of dynamic productive efficiency that preoccupied us is moot) that the sign of the expected riskfree interest rate puts no restriction of the existence of fiat currency equilibria. In addition, monetary equilibria, when they exist, often do not look like simple deterministic transfers from the young to the old, but instead provide partial insurance between generations. Manuelli (1990) constructs an example in which it is indeed "this 'insurance' aspect that explains the value of money" (p. 278). All this is of course in direct line, albeit in an exchange economy, with the message of our four Socratic examples.

Also in a stochastic exchange economy, Chattopadhyay and Gottardi (1999) provide a necessary and sufficient for conditional Pareto optimality.

Closer to our framework, Wang (1993) analyzes the existence and uniqueness of stationary equilibria in overlapping generations economies with stochastic *production*. He does not deal however with dynamic efficiency, optimality, or Ponzi games. In a stochastic Diamond (1965) economy, Bertocchi (1994) carefully analyzes *safe* debt Ponzi games in the dynamically *in*efficient case. Using a log-linear stochastic economy with capital similar to our first example, she shows that safe debt might be unable to remove inefficiency, be-

cause of inadequate risk-sharing between generations. Furthermore, Bertocchi and Kehagias (1995) develop a stochastic version of Pontryagin's maximum principle for Markov controls which enables them to derive a complete characterization of Pareto optimality for stochastic economies with capital based on stochastic multipliers. They emphasize, as we did, that a "careful distinction between dynamic efficiency and Pareto optimality is crucial for understanding phenomena such as Ponzi games and speculative bubbles... [E]ven when dynamic efficiency obtains, Ponzi games and bubbles are sustainable if they can provide insurance and thus improve, in the Pareto sense, the allocation of aggregate risk" (pp. 321–322).

This is also the message of Barbie, Hagedorn and Kaul (2000), who generalize the results of Chattopadhyay and Gottardi (1999) to an economy with production. They provide a single necessary and sufficient condition for conditional Pareto optimality with dynamically complete markets. This criterion—which alas eluded us when we wrote our paper— involves checking that a properly weighted sum of inverse contingent prices does *not* converge over any possible history of the economy. Chattopadhyay and Gottardi (1999) also show that whenever the competitive equilibrium is Pareto suboptimal, Pareto optimality can always be restored by a scheme that closely resembles a Ponzi game, and rolls over an *insurance* contract in exchange for contributions. The lesson that "sophisticated" Ponzi games improve intergenerational risk sharing is indeed the one taught by our example 4.

Last but not least, Barbie, Hagedorn and Kaul (2001) use data from the maturity structure of U.S. government debt to argue empirically that the U.S. is dynamically efficient (this was the gist of the Abel et al. (1989) paper) but also conditionally Pareto suboptimal. Barbie et al. (2001) conclude that there is, therefore, room for a "dynamic fiscal policy" to insure against macroeconomic risks. This policy involves running (sophisticated) debt Ponzi games in a dynamically efficient economy under uncertainty...

7 Conclusion

Arguments as to whether governments can rollover debt are often cast in terms of a comparison of the average growth rate and average riskless rate. In a series of examples, we have shown that this may be misleading. The average riskless rate may be less than the growth rate while Ponzi games are infeasible, or it may be greater than the growth rate, while Ponzi games are

feasible.

Turning to the structure underlying those examples, we have shown that, even in economies in which equilibrium is dynamically efficient, Pareto suboptimality may also lead to the feasibility of Ponzi schemes.

In thinking about the implications of Pareto suboptimality, we have focused in this paper on the suboptimality which comes naturally from the incompleteness of markets under uncertainty in overlapping generation models. But there are many other reasons why actual economies may not be Pareto optima. Missing markets may be missing for reasons ranging from asymmetric information to transaction costs, leading for a potential role of public debt, and opening the possibility of Ponzi games.[20] Distortions, from externalities to taxation, may also create wedges between risk adjusted interest rates and the social marginal product of capital.[21] Thus, Ponzi games may be feasible. And if they are, they may—but need not—be Pareto improving.

Colophon

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[20]For instance, Woodford (1986), Scheinkman and Weiss (1986) and Kocherlakota (1992) provide examples of economies with infinitely-lived agents in an unbacked fiat currency—equivalent to a debt Ponzi game—may substitute for an exogenously closed market.

[21]The paper by Saint-Paul (1992) we have already referred to earlier explores that route.

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