Overlapping Generations: the First Jubilee

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Paul Samuelson's (1958) overlapping generations model has turned 50. Seldom has so simple a model been so influential. Its "wow" factor, and the feeling of surprise at its originality and coolness have not paled with the years. The paper, in spite of its ripe age, still elicits wonder.

Starting from the uncontroversial observation that "we live in a world where new generations are always coming along" (all unattributed quotations refer to Samuelson, 1958), Samuelson built a model that violates the credo of the first fundamental welfare theorem with which we still inculcate undergraduates 50 years later. According to Samuelson, all is not necessarily well in the best of market economies: with overlapping generations, even absent the usual suspects such as distortions and market failures, a competitive equilibrium need not be Pareto efficient. Worst of all, this failure of the first welfare theorem in an overlapping generations model occurs in a framework that is, in many ways, more plausible and realistic than the world of agents living synchronous and finite existences in which the theorem is usually proved.

Like Mona Lisa's enigmatic smile, the mysterious welfare properties of the overlapping generations model are, to a significant extent, responsible for its popularity—along with the many economic issues it has illuminated in the last half-century. I take it as my brief in this celebratory paper to provide, after a short exposition of the main results of the overlapping generations model under certainty, an explanation of why the welfare properties of the overlapping generations

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model differ so much from the canonical Arrow–Debreu framework and to review, in a deliberately nonencyclopedic mode, a few striking applications and extensions of Samuelson's deceptively straightforward model.

This paper is not the first attempt at an intellectual history of the overlapping generations model. Solow (2006) sketches the main features of the model in a volume that gathers contributions made by colleagues and friends at Samuelson's 90th birthday celebration in 2005. Interestingly, Solow confesses that he forgot to include the overlapping generations model in his earlier 1983 book on *Paul Samuelson and Modern Economic Theory*. Indeed, it took a while for Samuelson's framework to impose itself on the profession. The Kareken and Wallace (1980) volume and Sargent (1987) textbook played a considerable role in its diffusion, and Geanakoplos (1987) and Farmer (1999, chap. 6) provide superb overviews of its main contributions.

The Model

In this section, I present a streamlined version of the overlapping generations model that at times differs markedly from Samuelson's own rendering. Samuelson's original motivation was, as Solow (2006) notes, to "test Böhm-Bawerk's idea that time preference would be needed to produce a positive rate of interest. (It turned out to be wrong.)" My goal, in an era when the rationale for such objectives belongs to the history of economic thought, is instead to highlight what I take to be the essential features of the model.[1]

Demography: Birth and Death

Imagine the world is comprised of a never-ending succession of generations. The *perpetual* renewal of cohorts (or, under uncertainty, the mere *possibility* that new cohorts might appear), is a crucial element of the overlapping generations model—I will return to this point when I discuss welfare issues.

The arrival of generations is exogenous in Samuelson's overlapping generations model: additional cohorts pop up spontaneously in the economy. Tradition calls this process "birth" and accordingly refers to the "newborn." However, this biological interpretation is only an expositional convenience. The newborn could as well be little green people deposited on our planet by storks or aliens, or immigrants just disembarked on our shores. More radically, souls of all beings may have been planted in the economy, like dormant spies, since time immemorial and

[1] It behooves an economist who received his early training in France to mention here that Allais (1947) developed what amounts to an early version of the overlapping generations model in a 135-page appendix to his book. To explain why it attracted little attention, Malinvaud (1987) opines that it is "rather complex, leading to consideration of many cases and to introduction of long formulas." But *à tout seigneur, tout honneur:* credit must be given where it is due.

could be gradually coming out of the cold as active economic agents (as demonstrated early on by Shell, 1971).

The common denominator of these interpretations is that what matters is *economic birth*: deep down, a "new" agent is not defined by age, nor biological or ethnic characteristics, but by the fact that it is not included in the economic calculus of pre-existing agents. From this vantage point, disowned children who are left by their parents to fend for themselves, or unloved immigrants, are "new" individuals. By contrast, loved children to whom generous ascendants have bequeathed wealth, or immigrants in a society in which they are cherished and helped, are not. They are best thought of as belonging to old bloodlines, to pre-existing families or societies. Even more radically, when borrowing constraints bind, current selves are severed economically from their previous incarnations and constitute "new" individuals.[2] In short, the overlapping generations model is about economic *disconnection* of current and future cohorts.

In combination with the assumption of an unending succession of generations, the hypothesis that generations are comprised of "new" agents implies that the total number of distinct economic agents, together with the number of dated goods, is infinite in the overlapping generations model. By contrast, in the Ramsey–Cass model (which serves as the other workhorse of dynamic macroeconomic theory), no new agent is ever born: every individual is part of a pre-existing family. One should therefore think of the Ramsey model as a limiting case of the overlapping generations model in which the arrival rate of economically new agents has shrunk to zero. I will return to this insight below.

Death (alternatively: kidnapping by storks or aliens, or emigration) is certain. It could be assumed to occur randomly, as when Blanchard (1985) adopts Yaari's (1965) simplifying assumption of age-independent death probabilities, or even with zero probability, as in my model of overlapping infinitely-lived families (Weil, 1989); none of this really matters as the specificity of the overlapping generations model depends, qualitatively, on the arrival of new, disconnected agents rather than on the exact length of lives. How and when consumers vanish is, for the economist who wants to understand why the overlapping generations model is different, of secondary interest.

Samuelson (1958) splits lives into three periods, but he also examines briefly a version with two periods dubbed youth and old age. Most of the literature, following the lead of Cass and Yaari (1966), has adopted the two-period formalization because it has the technical advantage of wiping out intertemporal trade between two consecutive cohorts. When there are two ages of life, I meet my ascendants only once: when I am young (and they are old). This once-only encounter rules out intergenerational exchange because executing an intertempo-

[2] Townsend (1980) and Woodford (1990) pointed out that the overlapping generations model can be reinterpreted as a world of staggered binding borrowing constraints hitting infinitely-lived consumers every other period. In Aiyagari and McGrattan (1998), the average time between the random dates at which the level of assets hits zero serves as a measure of the endogenous average economic lifespan.

ral trade requires meeting twice. The absence of intergenerational trade entailed by the two-period version is convenient because it makes it easy to compute equilibria. Fortunately, the number of periods specified is immaterial for most purposes: more realistic overlapping generations models with an arbitrary number of periods, or in which time flows continuously rather than discretely, yield similar insights although they are much harder to handle.

Finally, it is convenient to assume all agents born at a given date are identical. This limits heterogeneity to the one stemming from the date of birth.

Technology

Following Samuelson (1958), assume that there is only one good in the economy and that it is nonproduced and nonstorable. Samuelson calls this good "chocolate" that agents receive, presumably on a hot day, and must either eat or exchange on the spot with others lest it melts. Call e_1 and e_2 the chocolate endowments received by agents in the first and second periods of their life.

Preferences

To reveal the main properties of the overlapping generations model, it is enough to consider two polar versions of preferences: economies in which consumers care *only* about old-age consumption ("infinitely patient consumers"), and economies in which they *mostly* enjoy eating when young ("almost infinitely impatient consumers"). The reason why I do not go all the way to the extreme of infinitely impatient consumers who only care about current consumption will become clear below. Reality is of course somewhere in between, with the relative weights of the utility of young- versus old-age consumption capturing the degree of impatience, and the concavity of the utility function capturing the desire to smooth consumption across periods. However, I abstract from these details here.

Autarkic Equilibrium

The foregoing assumptions (which streamline Samuelson's original formulation) enable us to conclude right away that consumers must be self-sufficient in equilibrium[3] and must feel happy about it.

There are four reasons there cannot be any trade in equilibrium. Because there are two periods of life, agents belonging to different cohorts meet only once, so that *inter*generational exchange is impossible as discussed above. Because I assumed away within-cohort heterogeneity, there can be no *intra*generational exchange either: should I wish, say, to lend to members of my generation, so would they (because they are just like me), and none of them would borrow from me.

[3] The recursive competitive equilibrium (in which each generation when it is born solves its own two-period maximization problem given the then-prevailing interest rate) coincides under certainty with the Walrasian competitive equilibrium (in which the souls of all agents, born or unborn, meet at the beginning of time and are quoted a sequence of intertemporal prices under which they determine their optimal behavior). As a result, I will not distinguish between the two concepts.

Because the consumption good is not storable, no consumer wants to keep chocolate in a pocket from young to old age (it will melt away). Finally, if the economy or the planet is closed to foreign trade (which I assume), there is no possibility for exchanging goods with foreigners or extraterrestrials.

The interest rate, which determines the terms at which chocolate today trades for chocolate tomorrow, is the market mechanism that eliminates the desire that consumers might have to trade and ensures that markets clear. Its equilibrium level reflects technology (as summarized by the value of the endowments e_1 and e_2 and the preferences of the individuals)—and it is at this point that it starts to matter whether consumers are infinitely patient or impatient.

I will now show that the equilibrium interest rate is either very low or very high (below or above the rate of growth of population) according to whether the economy is peopled with very patient or very impatient consumers. In the former case, the competitive equilibrium is not Pareto-optimal. In the latter, it is. The very fact that it might not be is what elicits wonder: how can it be that the first welfare theorem fails to hold when the interest rate is low?

Low interest rate economies have been dubbed "Samuelsonian" by Gale (1973) because they exhibit the most fascinating features of overlapping generations models. By contrast, high interest rate economies are called "classical," as their welfare properties are standard. Unsurprisingly, I will spend more time discussing the former than the latter.

Samuelsonian Economies

Suppose our agents, who receive endowments in both periods of life, only care about old-age consumption. To mitigate the mismatch between the pattern of endowments and tastes, they will try to exchange the e_1 units of chocolate they get when young but don't enjoy against some extra valuable goods when old. The difficulty, which we discussed above, is that there is no one with whom to execute this exchange.

For agents to be happy with this situation—and remember, this is one of the requirements of a competitive equilibrium—the equilibrium net interest rate must be a punitive –100 percent: faced with such extreme terms of trade between current and future chocolates, our infinitely patient consumer does not wish to deviate from autarky.[4] In this equilibrium, each old consumes e_2 as required in autarky.

But what of the chocolate endowment e_1 of the young? It simply goes to waste, and herein lies the symptom of the Pareto sub-optimality of the competitive equilibrium. In this setting, it is trivial to construct a sequence of intergenerational transfers from young to old that improves the lot of every generation: simply

[4] From microeconomic first principles, the equilibrium interest rate equals the marginal rate of substitution between first- and second-period consumption evaluated at the endowment point, so that $1 + r = u'(e_1)/v'(e_2)$ if the utility function is $u(c_1) + v(c_2)$. In the example I am discussing, the marginal utility of first-period consumption is always nil—that is, $u'(\cdot) = 0$. Hence $1 + r = 0$, and $r = -100$ percent.

confiscate, in perpetuity, a lump-sum amount τ, with $0 < \tau \leq e_1$, from the endowment of the young and transfer it lump-sum to the old. The old then consume, assuming a constant population and thus an equal number of young and old, $e_2 + \tau > e_2$ instead of e_2 in the competitive allocation. If population is growing at the constant rate n, so that the young are $1 + n$ times more numerous than the old, this sequence of perpetual transfer from young to old guarantees a consumption of $e_2 + (1 + n)\tau$ to each older person. Although exogenous population growth is the only source of economic growth I consider here, exogenous growth of the endowments at rate g can easily be added, in which case most of the statements made about n would apply to the rate of growth $n + g$. In every instance, each generation is better off since it only values old-age consumption.

These results are more general than they may seem. Suppose there is instead a linear storage technology for chocolates with *exogenous* net return r, so that one chocolate set aside today mutates into $1 + r$ chocolates tomorrow. The equilibrium interest rate then equals r (the marginal rate of transformation). Chocolates self-destruct, as above, when $r = -1$, melt partially when $r < 0$, and proliferate otherwise. Compared to the equilibrium with nonstorable goods described above, our infinitely impatient consumers have one new possibility: store chocolate under their mattress until old age. Since more consumption is better, the young will use the storage technology to its fullest extent and put e_1 units of goods aside. As a result, their old-age consumption is $e_2 + (1 + r)e_1$ in the competitive equilibrium.

Now compare the possible gains from storing chocolate with the gains that could be achieved by a social planner transferring, starting from some date until infinity, an amount τ, with $0 < \tau \leq 1$, from young to old. The old in the initial time period get something for nothing: no tax when young, but a transfer when old. Subsequent generations do surrender resources to the central planner when young, but they get them back with a vengeance. Left to their own devices, the young can store chocolates at rate r. However, if the rate of population growth n exceeds the interest rate r, intergenerational redistribution provides a superior alternative that yields a larger implicit rate of return n. As long as the interest rate r is below the population growth rate n, the proposed sequence of transfers from young to old is Pareto improving. For each generation, it will be more beneficial to receive a transfer when old from the next younger generation than it would have been to store chocolate. Crowding out private storage by the young is a welfare-improving idea in a Samuelsonian economy.

The *optimum optimorum* is attained in this setting, as long as the interest rate r is fixed by the linear production technology and is below n, when the whole endowment of the young is transferred to the old. More generally, consider what would happen in a world in which the marginal return to storage is decreasing, rather than constant. Then, starting from a competitive situation where there is so much storage that $r < n$, the transfer from young to old should be increased—with the concomitant crowding out of private capital and the ensuing rise in its marginal product—until the interest rate reaches the rate of growth of population. This is

Samuelson's "biological rate of interest" or Phelps's (1961) "golden rule" of capital accumulation.

Notice two essential characteristics of Pareto-improving intergenerational transfers to which I will return when I discuss below why, and not simply how, the first welfare theorem fails when the interest rate r is less than the rate of population growth n. First, Pareto-improving transfers must run from young to old, and not in the opposite direction, because of the fundamental asymmetry of time: there is an initial instant (the big bang, or today), but no last period. As a result, any transfer from old to young, be it implemented in a low or in a high interest rate economy, hurts the first generation of old that it affects. If Eve had been taxed when old to provide transfers to young Cain and Abel, she would have been worse off regardless of the value of the interest rate. Second, young-to-old transfers must be perpetual. If transfers from young to old ceased cold-turkey after a while, one generation—the one that is taxed when young but does not receive anything when old—would complain. This argument can also be generalized to the gradual phasing out of transfers from young to old. Thus, eliminating a pattern of transfers from young to old always involves a Pareto-deterioration.[5]

Classical Economies

High interest rate or "classical" overlapping generations economies are less interesting from the welfare point of view (even though, as I will show below, the usefulness of the overlapping generations model is not limited to the $r < n$ case). Accordingly, I will just sketch their main features.

Suppose that consumers care mainly about consumption in their young age and very little about their old-age consumption. Since old-age consumption is not very valuable, the young would like to borrow against most of their second-period endowment. However, the equilibrium allocation must be autarkic, as discussed above. If consumers are to be happy in this situation (again, this is a requirement for competitive equilibrium), then the equilibrium interest rate must be very high to wipe out the consumers' inclination to borrow. The less that agents care about old-age consumption, the more they want to borrow and the higher the interest rate. This is enough to ensure that the equilibrium interest rate is above the rate of growth of population for a very low utility value of second-period consumption.[6]

The resulting competitive allocation is peculiar: the young consume their endowment in the first period e_1, which they value, while the old consume their endowment e_2, which they value very little. One might tempted to argue that it is

[5] These statements could be wrong in one semi-pathological case: if the marginal rate of return on storage *increased* sharply with the amount stored. Those who are old when the system is eliminated would still be poorer, but they would have stored more goods in anticipation of its curtailment. The resulting beneficial increase in the return on storage could overcome, if it is large enough, the detrimental fall in lifetime wealth. See Weil (2002) for details.

[6] If the marginal utility of second-period consumption goes to zero, the equilibrium interest rate goes to infinity, since the equilibrium interest rate $1 + r = u'(e_1)/v'(e_2)$ tends to positive infinity when $v'(\cdot)$ is small and goes to zero.

Pareto-suboptimal: after all, wouldn't it be better if a social planner confiscated most of the chocolates of the old, since they enjoy them so little, and redistributed them lump-sum to the young who like them so much? Isn't this case exactly symmetrical to the low interest rate economy of the previous subsection? The answer is negative: while generations who take part in both phases of the old-to-young redistribution would obviously be better off (they are giving up chocolates for which they have little appetite when old against chocolates which they crave when young), the initial old (who are taxed in the second period without having received a transfer in the first) are worse off. They are not worse off by much, but they are worse off because they care a little about chocolate, and that is enough to prevent old-to-young redistribution from improving welfare in the Pareto sense. Hence transfers from old to young would improve the lot of all generations but one. A Stalinist social planner, ready to sacrifice the welfare of some currently alive consumers (the initial old) for the benefit of all future generations, would implement old-to-young transfers.

If the old did not enjoy chocolates *at all*—an extreme case I have ruled out—then the symmetry with the low interest rate economy would be reestablished, in the sense that the competitive allocation could be also improved upon in the high interest rate economy. Transfers from old to young (instead of transfers from young to old in the Samuelson case) would be Pareto improving because taxing the initial old would not decrease their welfare. But this is obviously a nongeneric case and its implication ("pay no attention to the initial old") is as extreme as the preferences of the Stalinist planner.

Samuelson, lucid and concise as always, put his finger in a few words on the crucial asymmetry between the existence of an initial period and the absence of a last: "We must give mankind a beginning . . . Must we give mankind an end as well as a beginning? Even the Lord rested after the beginning, so let us tackle one problem at a time and keep births forever constant."

The Strange Welfare Properties

The failure of the first welfare theorem in low interest rate overlapping generations economies is puzzling. Voltaire's Pangloss would like the overlapping generations model, because all seems to be for the best in this best of all worlds: there is perfect competition, there are no externalities and no distortions. So what is going on?

The Wrong Answer

Let us start with a dead end. It is tempting to think that inefficiency stems from the impossibility of conducting trade with previous and future generations. But, as I argued at the outset, the generational interpretation of the overlapping generations model is only an interpretation. We could as well have assumed that all souls are already present at the beginning of time, and meet at that date to trade as they wish in complete Arrow–Debreu markets: we would still encounter inefficiency in low interest rate economies. Thus, as we learn from Shell (1971) the impossibility of trade with ascendants and descendants cannot be the key of the explanation.

The Right Answer

Let us turn to a more subtle but germane argument. The traditional proof of the first welfare theorem fails in overlapping generations models. To understand why, let us leave the world of Samuelson for a short while and examine how one proves that a competitive equilibrium is Pareto-optimal in a static exchange economy.

Imagine that there are two agents, Jane and Paul. They each have preferences over, and endowments of, two perishable goods, apples and oranges. Suppose that, compared to the market outcome, there is (and remember: the point is to prove there is not) a Pareto-improving reallocation of fruits between Jane and Paul. Say it makes Jane strictly better off and leaves Paul's utility unchanged. If Jane is better off, it must be because the proposed reallocation of fruits provides her with a consumption basket that she could not afford at the competitive prices (otherwise, she would have selected it on her own). Hence, in the proposed reallocation and at the competitive prices, Jane spends more on apples and oranges than the value of her fruit endowment. As to Paul, whose welfare is unchanged, he spends as before the value of his endowment (if there had been a way to obtain the same utility more cheaply, Paul would have found it). Thus, in the proposed reallocation, Jane and Paul together spend more on apples and oranges than the combined value of their endowment of apples and oranges. But with two agents and two goods (or more generally a finite number of agents and goods), this implies that aggregate consumption of at least one the fruits must exceed the aggregate endowment of that fruit. Hence the proposed reallocation is infeasible (aggregate demand exceeds aggregate supply for at least one good), and as a result the market equilibrium is Pareto-optimal.

In overlapping generations models, this proof may break down. The key reason is that with an infinite number of households and of dated goods—Shell's (1971) *double infinity*—an allocation that is too expensive at market prices need not be infeasible, so that the first welfare theorem need not hold. In a nutshell, if the value of the resources available to the economy is infinite, spending more on goods is not necessarily synonymous with consuming more than is available.[7]

[7] Here is, for the curious reader, a compact presentation of the strange mathematics of infinity. Index Paul and Jane by h and fruits by i, call their consumption and endowments of the fruits c_{ih} and e_{ih}, and measure the price p_i of the fruits in some numéraire. What we are saying is that a Pareto-improving reallocation must be too expensive in the aggregate: $\Sigma_h(\Sigma_i p_i c_{ih}) > \Sigma_h(\Sigma_i p_i e_{ih})$. With two agents and two goods (more generally a finite number of agents and of goods), we can switch the two summation signs on each side of this inequality. As a result, we can write $\Sigma_h(\Sigma_i p_i c_{ih}) = \Sigma_i p_i(\Sigma_h c_{ih})$, and $\Sigma_h(\Sigma_i p_i e_{ih}) = \Sigma_i p_i(\Sigma_h e_{ih})$. Substituting into the inequality, we get $\Sigma_i p_i(\Sigma_h c_{ih}) > \Sigma_i p_i(\Sigma_h e_{ih})$. For this inequality to hold, there must be at least one good i for which $\Sigma_h c_{ih} > \Sigma_h e_{ih}$ since prices are non-negative. Hence we conclude the proposed reallocation is infeasible. However, in a situation of "double infinity," the

The Gist of the Story

What do we learn from all this?

First, Samuelsonian economies are different because economies with infinite resources are drastically different. With infinite resources, "too expensive" is not synonymous with "infeasible." Indeed, it is possible to improve, in the Pareto sense, on the competitive allocation by redistributing resources from young to old *forever*. Shell's (1971) analogy with Gamow's (1947) static room allocation problem in a hotel with an infinite number of rooms is illuminating (the italics are mine):

> An innkeeper has committed each of the denumerably infinite number of beds on a certain rainy night. A guest asks for a bed when all are occupied, but a bed can be found if the innkeeper requires *each* guest to move down one bed. In our little chocolate game, the imposed allocation can produce one extra chocolate. In the hotel problem, on the other hand, the innkeeper by imposing an allocation will be able to produce a denumerable infinity of extra beds.

In a hotel with a finite number of rooms, it is not possible to produce an extra bed without throwing someone in the street, and the first welfare theorem holds absent market imperfections.

Second, Samuelson's stroke of genius was to construct a model that makes economies in which the first welfare theorem always holds, absent externalities and distortions, look like quite a special case. It is hard to escape the conclusion that the features of the overlapping generations model are the norm, rather than the exception: after all, can we seriously argue, once we understand what "new" means, against the realism of a model that rests entirely on the assumption that "we live in a world where new generations are always coming along?" In that respect, it is not the overlapping generations model, with the wealth of interesting issues it raises and its rich welfare properties, that is a simple toy model, but rather the competing workhorse of modern macroeconomics, the Ramsey–Cass–Koopmans model that assumes that no "new" generation ever comes along as future agents are all part of pre-existing families. Barro's (1974) famous paper on debt neutrality and Weil (1987) make it clear that such a model emerges only if parents love their children (or future immigrants) enough to leave all of them positive bequests. This condi-

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commutation of the summation signs over households and goods that we have performed might not be legitimate because of Fubini's theorem (Rudin, 1987, theorem 8.8), and as a result an allocation that is too expensive is not necessarily infeasible. This happens, in particular, when the value of the aggregate endowment $\Sigma_h(\Sigma_i\, p_i\, e_{ih})$ is infinite. If goods, as in the overlapping generations model, are dated goods, then i in fact indexes time. Normalizing the initial price to 1, p_i is just the present value in the initial period of time i goods, or $(1 + r)^{-i}$ if the interest rate is constant. As a result, the first welfare theorem fails to apply when $r < 0$, that is, in a Samuelsonian economy with an interest rate below the (zero) rate of population growth. This argument was generalized by Okuno and Zilcha (1981) and Balasko and Shell (1980) to economies with time-varying interest rates.

tion is very restrictive, and therefore the Ramsey model, with its long-run interest rate that always exceeds the growth rate, is literally quite extraordinary compared to the overlapping generations model.

How to Cure Inefficiency

To cure inefficiency in low interest rate, Samuelsonian economies, one needs a way to implement perpetual transfers from young to old that will reduce the desire of consumers to transfer goods from youth to old age (remember that it is that desire that drives down the interest rate). There are centralized mechanisms for doing this, and there are market-based solutions.

Centralized Remedies

The most obvious way is of course to set up a straight pay-as-you-go, unfunded social security system that pays retirement benefits to the old from the taxes levied on their young contemporaries. To be Pareto-improving, the system must be perpetual, and its elimination—motivated, for instance, by an unforeseen reversal collapse of population growth n below the interest rate r—will lead to a Pareto-deterioration. When population grows fast ($n > r$), old and young are aligned in support of a pay-as-you-go system. If n falls below r, their interests diverge, and in a democracy it is likely that the old, whose weight is heavier in an aging population, will prevent its elimination. Pay-as-you-go systems are thus a prime illustration of the difficulty of policy design in a democracy: the temptation to create them is large (old and young will vote for them when growth rates are high) but once put in place, they are almost impossible to phase out. Samuelson's overlapping generations model is thus, very naturally, *the* model in which to think about the design and political economy of social security systems.

Another centralized remedy is, following the work of Diamond (1965) who first introduced this approach into the overlapping generations model, public debt. If part of the burden of repaying national debt falls upon future generations comprised of agents who are unconnected to the present cohorts, financing public deficits by issuing national debt raises the net wealth of the currently alive agents (to use the terminology of Barro's 1974 seminal paper), increases their consumption, and crowds out private capital accumulation. This lowers the long-run capital–labor ratio and raises the interest rate. Starting from an economy that, without public debt or unfunded social security, has an interest rate below the growth rate, issuing public debt will be beneficial to all. In point of fact, from a normative viewpoint, public debt should be increased up to the golden rule—the *optimum optimorum* described above where the interest rate equals the growth rate.

At that point, the government (if it spends nothing) can in effect roll over a constant level of public debt per capita from one generation to the next without ever levying any taxes to finance the debt. How is this possible?

Suppose the government borrows a constant $\hat{b} > 0$ from each young cohort and thus owes $(1 + r)\hat{b}$ to each old. Since the young are $1 + n$ times more numerous than the old in a growing economy, the system finances itself without injection of tax revenue provided that $(1 + r)\hat{b} = (1 + n)\hat{b}$—that is, if we are at the golden rule where $r = n$. Note that in such a world, the government has freed itself from the intertemporal budget constraint that ties down the actions of individuals: its debt is equivalent to a Ponzi scheme, a chain letter, or a pyramid game, and it is insolvent in the financial sense (the value of the debt exceeds the present discounted value of future surpluses). This is precisely what enables the government to cure through a dose of public debt the inefficiency intrinsic to low interest rate economies.

The overlapping generations model disciplines the mind into realizing that unfunded social security and public debt operate in a similar fashion. Indeed, it is logically inconsistent to be in favor of pay-as-you-go social security and against public debt—unless, of course, considerations other than intergenerational wealth redistribution are brought to bear on the analysis, such as moral hazard or endogenous labor supply.

Decentralized Cures

Samuelson originally proposed the "social contrivance of money" to cure the inefficiency of low interest rate economies. Suppose the initial old generation prints intrinsically useless "oblongs of paper" into greenbacks or stamp unusable "circles of shells" that they offer in exchange for chocolates to the initial young. Will the young accept paying a positive price for an intrinsically useless asset? The answer is both no and yes: no, if they do not expect to be able to resell it later; yes, if they do and can pass it on like a hot potato for a positive price to the next generation. In other words, multiple equilibria exist.

Let us look more closely at the case when Samuelsonian money is valued. First, observe that when useless greenbacks have a positive price, *the old sell them to the young*, one generation after the other. As a result, the goods that are offered in payment circulate from the young (the buyers) to the old (the sellers). Hence Samuelsonian money when it is valued plays the same role as an unfunded social security system. Second, note that Samuelsonian "money" is a currency with very limited attributes compared to the usual meaning of the word: it does not facilitate transactions as compared to barter, it pays no dividend or interest, and it cannot even be used as a wallpaper as Reichmarks were during the German hyperinflation of the 1920s. In other words, it is only a pure store of value, so that its return can come solely from a capital gain. In the absence of risk, the real price of money (the inverse of the money price of goods) must thus appreciate at the rate of interest: this is the only way money can yield the same rate of return as nonmonetary assets. If the young at time t each buy m_t goods worth of Samuelsonian money, the real value of the money that they hold, and that they will sell to the young born at $t + 1$, will thus increase to $(1 + r)m_t$ by the time they retire. Now what the old will sell, the young must buy. Since there are at any time $1 + n$ young for each old alive, market clearing requires that $(1 + r)m_t = (1 + n)m_{t+1}$.

This equation helps us understand why Samuelsonian money has no place in *high* interest rate economies. Were Samuelsonian money valued when $r > n$, real balances m_t would explode to infinity—which is inconsistent with market-clearing since the resources of the young who buy it are limited at e_1. The intuition is that the real price of a useless asset has to grow at rate $1 + r$. If the speed n at which buyers of this asset (the young) enter the economy falls short of the speed r at which the price appreciates, aggregate demand for the useless asset will eventually fall short of the aggregate supply.[8] If we introduced trading frictions in Samuelson's world, there would still be room for a fully-fledged currency that enables consumers to economize, say, on transaction costs or shopping time. The existence of Samuelsonian money is ruled out by $r > n$, but not the existence of what we usually mean by "money."

Let us return to low interest rate economies and to the expression $(1 + r)m_t = (1 + n)m_{t+1}$ that relates current and future real balances to r and n. We conclude that the golden rule $r = n$ can be reached with real balances constant at the level $\hat{m} = e_1$. The young then devote all their unwanted first-period endowment to the purchase of Samuelsonian money and consume when old the *optimum optimorum* golden rule level $e_2 + (1 + r)\hat{m} = e_2 + (1 + n)e_1$. This allocation is the same as the one that would be reached by optimal social security transfers from young to old or by issuing the right amount of public debt without levying taxes. In fact, the level of public debt that brings the economy to the golden rule, $\hat{b}$, equals $\hat{m}$. In short, Samuelsonian money can cure the inefficiency of low interest rate economies.[9]

Like a pay-as-you-go social security system and like public debt, Samuelsonian money "works" because it is part of a social contract: perpetual intergenerational redistribution from young to old in the case of social security, a long-lived government that does not default on its obligations in the case of public debt, or "a grand consensus on the use of . . . greenbacks as a money of exchange." The message that long-lived informal social contracts, institutions, and common values are of utmost importance for economic outcomes (Caillaud and Cohen, 2000) is not the least of the lessons taught by the overlapping generations model. Like formal laws, they are assets—a characterization I borrow from Kotlikoff, Persson, and Svensson (1988)—that may be vital to reaching efficiency.

[8] For the reasoning to be complete, we should also note that if r is larger than n in the absence of Samuelsonian money, it will be even higher when it is present and valued. The reason is buying money when young and selling it when old involves shifting consumption towards old-age and that an increased interest rate provides the incentive to do so.

[9] I have mentioned here that there exists an equilibrium in which Samuelsonian money is not valued, and I have described the stationary equilibrium with valued currency. There is also a continuum of nonstationary equilibria indexed by initial per capita real balances, which can be anything between 0 and $\hat{m}$. All these equilibria feature declining real balances that converge to zero and interest rates that collapse back towards the value that prevails absent Samuelsonian money. One should not conclude, as was long thought, that indeterminacy occurs because the autarkic steady state is Pareto-suboptimal. The work of Grandmont (1985) establishes in an economy with strong income effects that there is no such causality.

There is another, equivalent interpretation of Samuelsonian money as a rational asset bubble that is relevant to the theory of finance. I have pointed out the greenbacks or stamped shells circulating in the overlapping generations world are intrinsically useless. When an intrinsically useless currency is valued, we are *ipso facto* in a situation where its real price exceeds the present discounted value of the zero dividends that it generates—that is, there is an asset bubble. Tirole (1985) has analyzed exhaustively conditions for the existence of asset bubbles in general equilibrium overlapping generations economies. Rational asset bubbles only occur in low interest rate economies ($r < n$) where generations of new buyers arrive fast enough in the economy. This result should not be a surprise: I am willing to pay for an asset more than its fundamental value (the present discounted value of future dividends) only if I can sell it later to others. A rational asset bubble, like Samuelsonian money, is a hot potato that I only hold for a while—until I find someone to catch it. The arrival of new consumers generates the constant flow of new participants required to keep asset bubbles and similar chain letters going. The age-old postcard game ("send me one dollar in return for this postcard and send a similar postcard requesting one dollar to $1 + r$ of your friends") is feasible and can go on forever if and only if its expansion rate r falls short of the arrival rate n of new participants. On top of that, it is welfare-improving when feasible because it implements the then-needed transfers of goods from new to old participants. A logical corollary of the statement "models in which the first welfare theorem *always* holds are extraordinary" is thus: "economies where asset bubbles are *never* possible are very special."

Beyond Oversaving

I earlier compared the overlapping generations model to the Mona Lisa. One well-known feature of da Vinci's painting is that Mona Lisa's smile is seen differently by each viewer, and that the meaning it seems to conveys depends on the position of the watcher. Similarly, each extension of the overlapping generations approach reveals another aspect of Samuelson's model. I pick four examples of the versatility of the overlapping generations model. There are, of course, many more.

Intergenerational Risk Sharing

I have so far proceeded as Samuelson did in 1958, ignoring uncertainty. Disregarding risk is both convenient and dangerous. It is convenient because the comparison between interest and growth rates yields a sharp delineation of the Samuelsonian and classical cases; it is dangerous, however, because it gives us a false sense of simplicity. In the real world, there is uncertainty, and as a result there are many rates of return: on public debt, on capital, and so on. To aggravate matters, interest rates, like growth rates, fluctuate over time and across events: how are we, then, to assess empirically whether an economy is Samuelsonian or classical? I postpone the answer to this question to the next subsection to address first a further and more serious complication: when there is uncertainty, the competitive equilibrium of the overlapping generations model is never optimal because the unborn cannot take part before their birth in risk-sharing trades with previous generations. This difficulty is specific to uncertainty and to insurance markets. It does not arise under certainty.

To understand what is going on, imagine, for specificity, a version of the overlapping generations model in which the *split* of the constant aggregate endowment between young and old is random and decided by nature a minute before the young are born and youth turns to old age. The young are born too late to be able to share with their elders the risk to which they are exposed. Given that only the split is random, young and old could pool their resources and avoid uncertainty altogether if only they could meet before the realization of uncertainty. This incomplete market participation is enough, even with sequentially complete Arrow–Debreu markets, to prevent the faultless operation of the invisible hand.

This outcome stands in sharp contrast with the certainty case where, as Shell (1971) showed, it is immaterial whether or not the souls of the yet unborn meet at the beginning of time. Under uncertainty, as pointed out by Chattopadhyay and Gottardi (1999), "we cannot find a sequential structure of markets where agents trade only after they are born and which supports the same equilibrium allocations as when agents have unrestricted access to a complete set of markets at the initial date." As a result, even absent any traditional market failure such as distortionary taxes, the equilibrium allocation of an overlapping generations model will always be expected to be Pareto suboptimal: left to their own devices, markets will not properly allocate risk across generations.

Remarkably, this Pareto-suboptimality (and the corrective public intervention it calls for) does not depend on whether the economy is Samuelsonian or classical, on whether there is oversaving or not. Perpetual transfers from young to old that crowd out oversaving are of course beneficial, whether or not there is uncertainty. But even if there is no oversaving (for example, in an economy that we would dub "classical" under certainty), a well-designed intergenerational redistribution scheme can fill in for incomplete participation in insurance markets and can therefore improve welfare. A vast array of papers strives to characterize optimal social security schemes under uncertainty. A good starting point is Ball and Mankiw (2007), who offer an elegant characterization of the policies required to reach the optimal allocation.

This discussion shows that a distinction must be drawn between dynamic inefficiency (oversaving) and Pareto-suboptimality. An economy with capital is dynamically inefficient (or overaccumulates capital) if reducing capital today does not reduce aggregate consumption in any future date or event, and actually increases it in some.[10] Pareto-optimality is a stronger, welfare-based, efficiency

[10] The literature on dynamic inefficiency dates back at least to Malinvaud (1953). A criterion using bond prices for detecting inefficiency under certainty was developed by Cass (1972) and generalized to uncertainty by Zilcha (1990, 1991).

criterion that requires not only the "cake" of aggregate consumption be as large as possible, but also that it be optimally allocated across agents. Hence dynamic inefficiency implies Pareto-suboptimality (because the cake is not as big as it can be), but dynamic efficiency does not in general entail Pareto-efficiency. It does under certainty (if there are no distortions): in that case, Samuelsonian economies with capital are both dynamically inefficient and Pareto-suboptimal. But under uncertainty, the failure of markets to allocate risks properly across generations renders all allocations, even dynamically efficient ones, Pareto-suboptimal. Public policies are thus needed, in Samuelsonian or in classical economies, to achieve the optimal intergenerational risk sharing that would be reached under a Rawlsian veil of ignorance. In Blanchard and Weil (1991), my coauthor and I have studied the existence and beneficial welfare properties of asset pricing bubbles in stochastic overlapping generations economies that are dynamically efficient. This argument shows that the case for intergenerational transfers is not limited in the presence of risk to Samuelsonian economies. The overlapping generations model is a good model to use to think about social security—whether or not dynamic inefficiency is a real-world problem.[11]

Is Dynamic Inefficiency a Real-World Problem?

In a celebrated paper, Abel, Mankiw, Summers, and Zeckhauser (1989) investigate empirically whether actual economies are dynamically efficient. Their interrogation is essential because it could be that the exotic phenomena brought to light by the overlapping generations model (to name but a few: oversaving leading to a low interest rate; Pareto-improving crowding-out of private capital accumulation by pay-as-you-go social security; public debt being beneficially transformed into a Ponzi game; asset bubbles) are only theoretical possibilities that can happily be shelved, after looking at the data, in the library of impractical and useless theories.

The task the authors set for themselves is not an easy one because, as noted above, the presence of risk makes it impossible to talk about "the" interest rate. To circumvent the difficulty of finding out which interest rate to consider and to skirt the added complication stemming from the variation of interest and growth rates over time and events, the authors devise a clever cash-flow–based efficiency criterion. Building on results by Phelps (1961), they show that an economy is "dynamically efficient" (I will explain the quotation marks in a short while) if and only if goods are always (at every date, in every state of nature) flowing out of firms to investors.[12] Measuring the direction of cash flows from national income accounts,

[11] There are aspects of social security programs that have nothing to do with overlapping generations: for instance, the inability to make time-consistent plans for the future and to adequately provide for one's own retirement. Hence, the overlapping generations model is certainly not the *only* model of social security.

[12] The rationale for this criterion is clear in steady state under certainty with constant population growth at rate n. The total return to capital is rK, where K is the aggregate capital stock, while the investment required to keep the capital stock growing at the same rate as population is nK. If the former exceeds the latter, then $r > n$ and the economy is classical.

they conclude that dynamic inefficiency does *not* seem to be a problem for the United States in the 1925–1985 period or for the United Kingdom, France, Germany, Italy, Canada, and Japan between 1960 and 1984.

These results must be interpreted with care for three reasons. First, the authors refer to dynamic inefficiency, but what they characterize is really Pareto-optimality, as evidenced by the utility-based efficiency criterion that they provide.

Second, Abel, Mankiw, Summers, and Zeckhauser (1989, p. 11) themselves note that their empirical conclusion that real-world economies are (Pareto) efficient is valid only "if the economy behaves in the future as it has in the past." Barbie, Hagedorn, and Kaul (2004) point out that the empirical implementation of the Abel et al. (1989) Pareto-optimality criterion, which requires verifying that goods flow out of firms to investors in *every* possible future, is impossible: conventional statistical methods can make statements about what will happen "almost surely" (that is, with probability 1), but they cannot assert for sure what will happen. For instance, flipping only heads in an infinite series of tosses of a fair coin is very unlikely (it will almost surely not happen), but it is not impossible. Fortunately, Barbie, Hagedorn, and Kaul conclude, under assumptions about future paths of the data that are weaker and more plausible than those used by Abel, Mankiw, Summers, and Zeckhauser and using theoretical results due to Zilcha (1991), that the United States economy is dynamically efficient over the period 1890–1999. Pareto-optimality, however, cannot be established.

The third difficulty is subtle. When we declare an economy is dynamically inefficient, we refer to the state the economy would reach in a competitive equilibrium *absent* any intergenerational transfer mechanism. We are in effect asking what the economy would look like in a Lockean "state of nature," that is, before state and market institutions that can transfer goods across generations have been put in place. Would it have low interest rates? Would it exhibit oversaving? In an economy where the marginal return to physical capital is decreasing, would the absence of all the mechanisms that crowd out private saving in physical assets result in interest rates that are below the growth rate? The twentieth century data examined by Abel, Mankiw, Summers, and Zeckhauser (1989) or Barbie, Hagedorn, and Kaul (2004) have little, if anything, to tell us about this fictitious reference point. During the time period they examine, public debt was abundantly used to finance two world wars, unfunded social security systems were deployed in many Western countries, and gold and diamonds and a host of other assets were priced far above their fundamental value. What the data do tell us, however, is that given the many ways our societies have devised to transfer resources from future to current generations, there seems to be no further rationale for crowding out private capital accumulation. The data do not provide, on their own, support for the elimination of policies, such as pay-as-you-go social security, that *already* crowd out private capital accumulation. We just do not know how the economy would look in their absence, whether it would become Samuelsonian or remain classical. The questions raised by Samuelson's model remain relevant.

Keynesian Economics

The overlapping generations model, with the indeterminacy of equilibrium that can occur in one-good exchange economies and the possible inefficiency of competitive equilibria in undistorted environments, turned in the 1980s into the theoretical playground of the attempt to give solid microeconomic foundations to Keynesian economics. What happens today depends on what happens tomorrow, and in the overlapping generations model, no terminal condition ties down this process. As a result, the government becomes a way to anchor expectations, and public policy turns into an equilibrium selection device that molds animal spirits and steers the economy away from inefficient equilibria. This is, for instance, the explicit message of Geanakoplos and Polemarchakis (1986). It is quite revealing that this paper, which conducts its comparative statics in terms of IS–LM diagrams, starts with a sentence almost identical to the one used by Samuelson to introduce the idea of generations (p. 755): "Our point of view is that for some purposes economic activity is better described as a process without end." We are all, indeed, Paul's children.

Behavioral Finance

The overlapping generations model has played a key role in the development of behavioral finance. In one of the founding papers of this literature, DeLong, Shleifer, Summers, and Waldmann (1990) introduce overlapping generations of noise traders who coexist with cohorts of rational traders (see also Shleifer and Summers, 1990). Two characteristics of the overlapping generations model drive the striking result of that paper that noise traders durably affect prices even when there is no fundamental uncertainty on dividends. First, the absence of a last period in the overlapping generations model ensures that asset prices can remain uncertain even in the absence of fundamental risk. Second, the ability of arbitrageurs to buy low and wait to sell high until prices revert to the mean is curtailed by factors (biological or institutional) that limit their economic horizons. The shorter the horizon, the further asset prices diverge from their fundamental values in the presence of noise traders. These mechanisms do not rely on dynamic inefficiency.

Conclusion

Einstein (1924) reminded us, in the introduction to *Relativity: The Special and General Theory*, of the physicist Ludwig Boltzmann's warning that "matters of elegance ought to be left to the tailor and to the cobbler." Although aesthetic pleasure is indeed not the metric of scientific achievement, the beauty radiating from the striking simplicity the overlapping generations model has played a significant role in the paper's impact. What Sassoon (2001) wrote about Mona Lisa applies almost verbatim to the overlapping generations model:

> [M]any of those standing before the Mona Lisa or other famous artefacts are left a little disconcerted. By the cultural conventions of the twentieth century, she is neither beautiful, nor sexy. The painting is not grandiose, or politically inspiring, like Delacroix's *Liberté guidant le peuple*. There is no gore, no violence. It does not tell a story. Just a plain woman, smiling a little.

Solow (2006) marveled at Samuelson's "innocent little device." So should we all until, at the very least, the next jubilee.

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