EQUILIBRIUM SEARCH UNEMPLOYMENT, ENDOGENOUS PARTICIPATION, AND LABOR MARKET FLOWS

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Abstract
The sustainability of welfare states requires high employment and high participation to raise the tax base. To analyze labor supply in a world with market frictions, we propose and solve a macro model of the labor market with unemployment and labor force participation as endogenous and distinct states. In our world, workers' decisions of participating are determined by an entry decision and an exit decision. A calibration of the model improves the usual representations of labor markets, since it quantitatively accounts for the observed flows between employment and nonparticipation. The paper investigates also the effect of payroll taxes and unemployment benefits on participation decisions. Taxes reduce entries and increase exits, whereas unemployment benefits, at a given job-finding rate, raise entries and have ambiguous effects on exits. (JEL: J2, J6)

1. Introduction

Given demographic pressures in Western economies, the sustainability of many welfare-state programs requires high employment and high labor force participation in order to raise the tax base and avoid distortions. However, most economic analysis understands the determinants of labor supply only in a world without market frictions, and workers' participation in the labor market is often described by a neoclassical labor supply function. In the four chapters of the *Handbook of Labor Economics* devoted to labor supply, there are very few references to the role of search frictions.[1] Participation decisions in imperfect labor markets are

1. The most advanced paper in this direction is Blundell and MacCurdy (1999), who discuss intertemporal decisions (p. 1672ff), notably human capital decisions and habit persistence, with no

not yet fully understood. Moreover, from a macroeconomic perspective, we know little about the interactions between workers' participation decisions and firms' incentives to create jobs.

To better understand the functioning of an imperfect labor market with endogenous labor supply, our paper investigates a three-state macro model of the labor market in which the following decisions by agents are endogenous: job creation decisions by firms; job destruction by worker/employer pairs; entry and exit decisions in the labor market by workers; and, in (unreported) extensions, search-effort margins. Our modeling approach is based on the observation that people spend simultaneously a large amount of time in both market and home production, a feature of the data that has already been exploited in the macroeconomic literature.

Recently, the business cycle literature has improved the calibration of various aspects of the data by enriching the time allocation problem on the part of the household so as to explicitly consider the choice between leisure, home production, and market work.[2] But the existing business cycle literature studies home production within frictionless labor markets. Our goal, conversely, is to study the frontier between market and home production in an imperfect labor market.[3] In our world, heterogenous workers face idiosyncratic shocks to home productivity, but market frictions impose a cost to labor market participation. Since we work with a technologically fixed number of hours, our analysis abstracts from the intensive margin of labor supply and concentrates on the extensive margin.

In the paper, we explore in detail the effects of time-consuming search, a labor market friction that has attracted a great deal of attention in the macro literature (Mortensen and Pissarides 1994, 1999; Hall 1999). Our paper shows that job-search costs lead the decisions to participate and stop participating to be dynamic decisions and to differ: labor supply is described by two margins, an entry margin and a quit margin. The two decisions differ all the more when frictions are important, and conversely they coincide when frictions vanish. The gap between the two decisions is due to employed workers hoarding on the job, since quitting involves the loss of irreversible search investment when frictions are positive. Similarly, this employment hoarding effect does not exist in the absence of frictions.[4]

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explicit role for stochastic arrival of job offers. They conclude (p. 1686) by saying that "There remain a number of big issues that we have not touched on in this chapter but that are important for labour market analysis. . . . Another issue relates to the process of job search and job matching."

2. Notably, output volatility, the correlation between hours and productivity, and the correlation between investments in home and market capital (Benhabib, Rogerson, and Wright 1991; Rios-Rull 1993; McGrattan, Rogerson, and Wright 1992; Gomme, Kydland, and Rupert, 2001).

3. An important exception is Nosal, Rogerson, and Wright (1992), who show that, in an indivisible labor model with home production, involuntary unemployment arises in equilibrium without assuming that leisure is an inferior good.

4. To our knowledge, the irreversibility of investments developed in Dixit and Pindyck (1994) has not been transposed to the analysis of labor supply.

The paper, then, explores the positive and normative implications of this setting. From the positive standpoint, we account for a labor market with three states: people spend time in employment, unemployment, and full-time home production. The two labor supply margins also rationalize the recent important work of Jones and Ridell (1999) and Sorrentino (1993, 1995), who emphasize the difficulty of defining the frontier between nonparticipation and unemployment. Notably, they show that there exist agents who report that they would like a job yet do not search, which is one of the main insights of the model. This allows us to define a broader concept of unemployment that takes this population into account. Second, the model can quantitatively account for the large flows between the three labor market states, and we present a calibration that aims to replicate the monthly flows for the United States in the 1990s.

From the normative standpoint, we argue that the existence of two different labor supply margins has some policy implications. We show under what conditions the decentralized unemployment rate is too high and the vacancy/ unemployment ratio inefficiently low in the presence of taxes, even when wages internalize search frictions. The paper also discusses the conditionality of unemployment benefits and examines their entitlement effects—that is, the fact that an increase in unemployment insurance increases the attractiveness of market participation among the noneligible nonemployed (Mortensen 1977; Fredriksson and Holmlund 2001). Our theoretical analysis highlights the existence of a participation hoarding effect, which we define as the additional incentive to hold on to market participation that is induced by conditional eligibility.

Our work is not the first attempt to incorporate endogenous labor market participation features into standard models of search. On the microeconomic side, Seater (1977), Burdett and Mortensen (1978), Burdett (1979), Burdett et al. (1984), and Swaim and Podgursky (1994) have successfully investigated the relations between search frictions and labor supply given a fixed supply of jobs. Our theoretical distinction between inactivity and unemployment, empirically consistent with Flinn and Heckman (1983), is inspired by Burdett and Mortensen (1978). In the macro search literature, Bowden (1980), McKenna (1987), Pissarides (1990, chap. 6), and Sattinger (1995) have introduced a labor demand side and endogenous participation in a way that brings few new insights as compared to the standard (two-state) model of matching. Individuals have a heterogenous value of nonmarket time and decide in a static (though intertemporal) way about their participation in the labor market. It follows that the flows between activity and inactivity are driven by macroeconomic changes (in productivity, in unemployment) and are thus mainly cyclical or conjunctural flows. In contrast, our theory—building on both macroeconomic factors and individual (household) shocks—is able to account for permanent, structural flows between activity and inactivity, even when macro conditions are unchanged. Pries and Rogerson (2002) is another recent attempt to incorporate labor market participation in a macroeconomic framework.

The paper proceeds as follows. Section 2 defines the main properties of the labor supply margins in a partial equilibrium context, when the job-finding rate is exogenously fixed. Section 3 derives the general equilibrium of the model and proves its existence. Section 4 analyzes the two policy dimensions of the paper, namely the role of taxation and unemployment benefits. It highlights also the counterfactual empirical predictions, and it discusses how these can be dealt with in the context of our theory. Section 5 presents a calibration of the baseline model and shows how our framework can rationalize most flows across the three labor market states. We also discuss quantitatively the role of conditional unemployment benefit. Section 6 concludes.

2. Labor Supply with Search Frictions

2.1. Framework

Time is continuous and there is a mass, 1, of risk-neutral individuals who are allocated 1 unit of time. They derive linear utility from home production (leisure) and from market activity. We consider a given skill segment of the labor force in which the marginal productivity is homogenous at a level y. Individuals are paid a wage w (determined later on) and produce x units of utility per unit of time if they are engaged in home production.

Workers wanting to participate in the labor market undertake a time-consuming search. The time allocation problem of the worker is defined as follows: h_w is the number of hours actually worked, h_s is the search intensity necessary to obtain a job, and h_h is the choice of hours spent in leisure/home production. The time constraint is thus

$$1 = h_w + h_s + h_h \text{ with } h_w \in \{0, e\} \text{ and } h_s \in \{0, s\}$$

where e is the inelastic number of hours worked and s is the inelastic number of hours spent looking for a job (we discuss this assumption later on). There is no on-the-job search, so job search and employment are mutually exclusive activities. It follows that in the three states W, U, and H (where W is employment, U is unemployment, and H is full-time home production), the flow utility of agents is given by

$$v^W = (1 - e)x + w$$
$$v^U = (1 - s)x$$
$$v^H = x$$

where x is home productivity and w is the total wage received for the e hours worked. Throughout this section, we assume $1 \geq e \geq s$. It is important to

note that, following Becker (1965), home production or leisure consumption are formally expressed in the same way (raising an individual's utility).[5] Hereafter, we keep the home production interpretation of x, but interpretations in terms of time-varying marginal utility of leisure are possible.

We assume that there is some heterogeneity in the valuation of nonmarket activities. Concretely, home productivity x is heterogeneous and stochastic, and its value changes according to a Poisson process at rate λ.[6] Conditional on the arrival rate of a shock, the value of home productivity takes a value from a continuous distribution $f(x)$ and c.d.f. $F(x)$ defined over the support $x \in [x^{\min}, x^{\max}]$. For a nonemployed individual, the participation decision is whether to spend 0 or s hours in the labor market, whereas the participation decision for an employed worker is whether to work e or 0 hours: our model is an extensive margin model and we ignore hereafter such issues as the intertemporal elasticity of substitution, bargaining over hours, and work sharing. We further assume $x^{\min} \le 0$ to ensure that there will be market participants at equilibrium.

Labeling by W, U, and H the present discounted value of the utility of workers in each state and using W for $W(x)$, W' for $W(x')$, and so forth (for simplicity of exposition), the recursive Bellman equations in the three states read as follows:

$$(r + \lambda)W = v^W + \lambda \int_{x^{\min}}^{x^{\max}} \max(W', U', H') \, dF(x')$$
$$+ \delta[\max(U, H) - W] \tag{1}$$

$$(r + \lambda)U = v^U + \lambda \int_{x^{\min}}^{x^{\max}} \max(U', H') \, dF(x') + p[\max(W, U) - U] \tag{2}$$

$$(r + \lambda)H = v^H + \lambda \int_{x^{\min}}^{x^{\max}} \max(U', H') \, dF(x') \tag{3}$$

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5. The simplest interpretation is that of Becker (1965). Utility is the consumption of bundles, representing a combination of time and money. Here, home production is intensive in time, while market activity is intensive in money. Gronau (1977, p. 1100) states that "[the distinction between home production and leisure], so common in everyday language, disappeared in Becker's more general formulation. The omission is partly due … to the large number of borderline cases (e.g., is playing with a child leisure or work at home?)."

6. Changes in x are thought of as individual and family shocks with large variance and low frequency. The alternative assumption in which x is fixed but heterogeneous and y is time-varying, even in the simplest case in which y takes two values only, gives twice as many margins as in our specification. Our modeling choice is analytically simpler. Further, time variations in the value of nonparticipation (due e.g., to disease, children education, changes in household income, etc.) are sufficiently large for driving a significant part of the transitions between activity and nonparticipation. Pries and Rogerson (2002) have made the opposite modeling choice and hence must keep track of the wage distribution.

where δ is the Poisson parameter of a process of exogenous destruction of the job and p is the Poisson job-finding rate for workers (treated as a parameter in partial equilibrium and endogenized in general equilibrium). The first equation states that the equity value of employment is the sum of the utility flow, the capital gain (or loss) from a home production shock after which workers reoptimize (they decide whether to hold onto the job W', look for another job U', or leave activity H'), and the capital loss of being hit by a destruction shock δ in which case workers decide whether to search for a new job or to resume full-time home production. The second and the third equations have a similar interpretation for the λ shocks. In addition, upon getting a job offer with arrival rate p, unemployed individuals decide whether or not to accept it in considering $\max(U, W)$.

To solve for wages, we need to introduce firms. A firm has either 0 or 1 worker. As long as there are frictions (i.e., when p has finite value), successful matches yield a pure economic rent. As is conventional in the search-matching literature, those rents are split in fixed proportion between firms and workers. Formally, the value of a filled position for the firm depends on x if the wage depends on x. We have

$$(r + \lambda)J(x) = y - w(x) + \delta(V_V - J) + \lambda \int_{x^{\min}}^{x^{\max}} \max(J', V_V)\, dF(x'), \quad (4)$$

where y is the marginal product of the worker and V_V is the value of a job vacancy (treated in partial equilibrium as a parameter). The equity value of a job is the sum of flow profit, the capital loss following exogenous job destruction, and the capital gain after a change in workers' characteristics—possibly leading to job destruction if the worker quits.

Nash bargaining over w follows the usual rule

$$w = \arg \max[W - \max(U, H)]^{\beta}[J - V_V]^{1-\beta}, \quad (5)$$

and it follows that wages split the surplus into shares β and $1 - \beta$. It can be guessed that there are two wage rules, depending on the sign of $U - H$. If $U \geq H$, then workers hit by an exogenous destruction shock (δ) look for another job. If $U < H$, then workers hit by this type of shock exit the labor market and become engaged full-time in home production. The expression for wages is in the Appendix (Section $A.1$, equations (A.4) and (A.5)). They conventionally appear as a weighted average, with weights β and $1 - \beta$, respectively, of the marginal product net of the firm's outside option (in equity value) and a term reflecting the "threat" point of workers (i.e., either U or H).

2.2. *Reservation Strategies and Definitions*

We can now derive the slopes of the value functions W, U, and H with respect to x. With linear utility, the value functions are piecewise linear functions of x, as proved in Appendix A.2. Let us introduce the cutoff points x^v and x^q, defined by

$$U(x^v) = H(x^v), \tag{6}$$

$$W(x^q) = H(x^q). \tag{7}$$

The ordering of the slopes implies the following ordering of intersections: $x^q \geq x^v$. This is always the case in a viable labor market with $W > U$; see Figure 1, which shows these value functions with respect to x. Here $W(x)$ has a kinked point at the cutoff value of home production x^v, corresponding to the change in the outside option of workers.

   We are now in position to clarify a few labor concepts. Above x^q one finds only workers engaged in full-time home production, or nonparticipants. Between the two cutoff points x^v and x^q, one finds two categories of workers. First, some of them are nonparticipants but do not search for a job: this corresponds to a well-identified group of agents in labor statistics: there are indeed persons willing to work but not ready to pay the search cost—nonemployed agents whose home productivity belongs in the interval $[x^v, x^q]$. In Jones and Ridell (1999), those workers are called marginally attached to the labor market. These workers would accept a job if offered one, but they do not wish to pay the search cost. We can thus define a broader concept of unemployment, one that includes unemployed job seekers as well as those who are marginally attached to the labor market; we call this the extended unemployment rate. Second, there are employed workers. The individual history of such workers shows a low value of $x < x^v$, and each has searched for (and found) a job in the past. We call them unattached employed
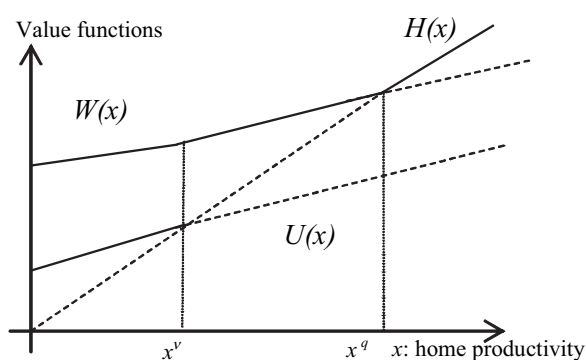

FIGURE 1. Value functions of home productivity x.

workers because they would leave the labor market after a job destruction shock δ. Finally, below x^v, one finds both unemployed job seekers and employed workers. We label the latter attached employed workers because they would be willing to search for a new a job if hit by a job destruction shock δ.

2.2.1. Entry Margin.   The first indifference condition (6) defines an entry margin, a level of home productivity at which the worker is indifferent between full time home production and searching for employment. Formally, the entry margin reads $sx^v = p(W - U)(x^v)$. This states that the forgone value of home production in the job-search activity sx^v must be compensated by an equivalent gain in expected surplus $p(W - U)$, given search frictions. Appendix *A*.3 determines the value of the total surplus and hence of the surplus of workers, leading to

$$\beta \frac{e(x^q - x^v)}{r + \lambda + \delta} = \frac{sx^v}{p} \tag{8}$$

where the term sx^v/p stands for the expected value of foregone home production during search while the left-hand side is the worker's share of the total surplus of the match. Hence this is a free-entry condition into the labor market. At a given p, this equation defines a positive link between x^v and x^q. The higher the quit cutoff point x^q, the higher the surplus on the job and thus the more attractive the labor market is, inducing further entries and a larger x^v.

2.2.2. Quit Margin and Employment Hoarding.   The second indifference condition (7) defines a quit margin, a level of home productivity at which a worker is just indifferent between working in the market and being full time in home production. Using (A.2) in Appendix *A*.1 yields

$$ex^q = w^{na}(x^q) + \lambda\beta\overline{S}, \tag{9}$$

where

$$\overline{S} = \int_{x^{\min}}^{x^q} [J' - V_V + W' - \max(U', H')]\, dF(x') > 0$$

is the average value of a match, net of the firm, and the worker's outside option. Equation (9) states that the forgone value of home production on the job is larger than the wage from market activity by a factor that reflects the future expected surplus of the job given stochastic transitions in x. This is an employment hoarding effect. It is the exact counterpart of the labor hoarding effect for a firm that faces hiring and firing costs and expects higher productivity from labor in the future: the firm thus pays a wage above marginal productivity on a temporary basis in order to save on turnover costs (see e.g., Bertola and Caballero 1994). Note that the employment hoarding effect disappears as the surplus on-the-job $\overline{S}$ goes to zero, which is in particular the case when frictions disappear.

Further, given (A.5) in Appendix *A*.1, we have

$$ex^q = y - rV_V + \lambda \overline{S}. \tag{10}$$

The intuition behind equation (10) is similar to that for equation (9). It states that the marginal worker at the quit margin has home productivity equal to the neoclassical reservation productivity y/e minus a term reflecting the bargaining power of the firm, plus the employment hoarding term. In other words, under free entry of firms ($rV_V = 0$), the sacrifice of home production for the marginally indifferent worker would be above market productivity by a quantity reflecting anticipated future gains of being on the job, given that quitting involves time spent to search. After straightforward calculations of the value of $\overline{S}$ in Appendix *A*.3 (equation (A.6)), we finally obtain

$$x^q = \frac{y}{e} - \frac{rV_V}{e} + \frac{\lambda}{r + \lambda + \delta} \int_{x^v}^{x^q} F(x)dx + \frac{\lambda(e-s)/e}{r + \lambda + \delta + \beta p} \int_{x^{\min}}^{x^v} F(x)dx$$

At a given p, this equation defines a negative link between x^v and x^q: the larger x^v, the smaller the employment hoarding term and thus the less conservative employed workers deciding whether or not to quit.

2.3. *Existence and Properties of Labor Supply in Partial Equilibrium*

In partial equilibrium, both p and V_V are treated as parameters.[7] The two labor supply margins can be usefully analyzed in the space (x^q, x^v); see Figure 2. It shows that there is a unique equilibrium in (x^q, x^v). The proof of this statement is in Appendix *A*.4.

Note also that the quit margin is vertical when $\lambda = 0$, with x^q at a level $(y - rV_V)/e$. This deserves some comment. When $\lambda = 0$ (i.e., when there is no stochastic change in x), we have a "static participation model": people are permanently either in or out of the labor force. The quit cutoff point is still defined as $ex^q = y - rV_V$, but above x^v there are only nonemployed workers, and since $x^q > x^v$, the quit margin is not active. The model has dynamic participation when $\lambda > 0$: the quit margin is activated.

Why does it matter? As an illustration, consider for instance an increase in p, still treated as a parameter here. This affects the entry curve but not the quit curve: the larger p, the easier it is to find a job and thus the larger the incentive to participate in the labor market (higher x^v at a given x^q). Put otherwise, the opportunity cost sx^v/p of searching is lower, raising incentives to participate.

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7. We study x^v and x^q for a given worker in a given firm, whereas rV_V (which is formally derived later on) may be a function of $x^{v'}$ and $x^{q'}$ in *other* firms.
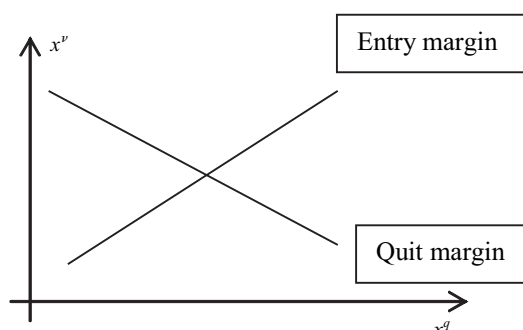
FIGURE 2. Entry and quit margins in partial equilibrium.

The role of frictions on labor supply can thus be understood: in an efficient labor market with large p, workers quit more easily because they can always come back to the labor market. With $\lambda = 0$, the quit point is instead independent of p. In the limit as $p \to +\infty$, the difference $x^q - x^v$ tends to zero and both quantities tend to the neoclassical entry point y/e. In this subsection, we have illustrated the role of dynamic participation decisions in a frictional labor market. The labor supply margins have, in turn, a general equilibrium effect that we now explore.

3. Labor Demand and General Equilibrium

3.1. Labor Demand

The general equilibrium is derived by adding a free-entry condition on firms, which endogenizes V_V and p. In line with the traditional matching literature, an additional vacant position for a firm is established at no fixed cost but at a flow cost c. Thus,

$$rV_V = -c + \chi(J^e - V_V), \qquad (11)$$

where χ is the job-contact intensity for the firm and J^e is the expected value of the job given wage bargaining. The term J^e takes into account the density of workers actively looking for a job in the market. Thanks to the assumption of inelastic search effort s, workers actively looking for a job, who are in the interval $[x^{\min}, x^v]$, are met by firms with identical probabilities. Further, the density of those workers is the conditional density of x in the population.[8] It follows that

$$J^e = \frac{1}{F(x^v)} \int_{x^{\min}}^{x^v} J(x') \, dF(x').$$

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8. This is not assumed here, we proved this result in Garibaldi and Wasmer (2001).

   To obtain the third margin, we assume that there is free entry of firms (i.e., all vacancy opportunities are exhausted), which leads to $V_V = 0$ and hence to $J^e = c/\chi$. The issue is thus to determine the value of J^e, which depends on the expected wage faced by the firm.

   So far we have assumed that $e \geq s$ and have solved for the partial equilibrium properties of the model. To the contrary, we define a fully indivisible labor supply as one in which entering the labor market involves a sacrifice of home production regardless of the employment status, i.e., $e = s$, with $e \leq 1$.[9] At the stage of introducing the labor demand equation, we found it convenient to make this assumption. This insures that $J(x)$ is constant for all newly hired workers (i.e., for $x < x^v$). This assumption implies that, when $V_V = 0$, the job-creation margin is

$$c/\chi = (1 - \beta)\frac{e(x^q - x^v)}{r + \lambda + \delta}. \tag{12}$$

Further, to avoid unimportant constant terms but without implication for the results, $e = 1$. Equation (12) makes clear how the labor supply margins affect the entry decisions of firms. It simply says that the surplus from a job for the firm is equal to the expected search/recruitment costs, determining χ as a function of x^q and x^v.

   The model is then simply closed by the assumption of a matching process between workers and firms. The total number of contacts per unit of time is denoted by $M(u, v)$, where u is the number of unemployed job seekers in the population and v is the number of job vacancies. We denote by $\phi = v/u$ their ratio, traditionally called market tightness. We have, under the usual assumption of constant returns to scale in M, that $\chi = M/v = \chi(\phi)$ with $\chi' < 0$. In addition, the probability of meeting p becomes endogenous also, and can be expressed as a simple function of market tightness ϕ. Formally, p is defined as a function $p = M/u = p(\phi)$ with $p' > 0$; thus p is uniquely obtained from χ by the job-creation margin.

3.2. General Equilibrium

Denote by n the nonparticipation rate, that is, the ratio of the number of inactive workers to the total population (normalized to unity).

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9. That $e > s$ is documented, for instance, in Layard, Nickel, and Jackman (1991, pp. 237–241). On the other hand, we may view $e = s$ as an extreme form of indivisibility of labor. This may actually capture the fact that the decision to enter the labor market involves a new organization of an individual's life, and in an irreversible way (which is precisely what our modeling choice is about). Given this new organization, unemployed workers must (at least temporarily) be immediately available for a job, which reduces the extent to which they can produce domestic goods or services.

DEFINITION 1.   A market equilibrium consists of an n-tuple (x^v, x^q, ϕ, u, n) and two wage rules (one for attached, one for unattached workers) satisfying: the entry margin for workers; the quit margin for workers; the job-creation margin for firms; the steady-state condition for unemployment flows; and, the steady-state condition for inactivity flows.

Derivation of the general equilibrium involves three equations solving for three endogenous variables: x^q, x^v, ϕ. Then comes the derivation of the stocks (unemployment and nonparticipation) from steady-state conditions on flows. The three equations are:

$$\frac{c}{\chi(\phi)} = (1 - \beta)\frac{x^q - x^v}{r + \lambda + \delta} \tag{JC}$$

$$\frac{x^v}{p(\phi)} = \beta\frac{x^q - x^v}{r + \lambda + \delta} \tag{Entry}$$

$$x^q = y + \frac{\lambda}{r + \lambda + \delta}\int_{x^v}^{x^q} F(z)\, dz. \tag{Quit}$$

Equation (JC) was obtained from the labor demand equation (12). Equations (Entry) and (Quit) are simply derived from the definitions of entry and quit in equations (8) and (10) when $e = s = 1$ and $V_V = 0$.

PROPOSITION 1.   *A sufficient condition for existence and uniqueness is $y > 0$.*

The proof is given in Appendix *A*.5.

3.3. Stocks

In this section we derive the equilibrium stock of workers in different states. Given the steady-state assumption, one may obtain the unemployment rate, which is defined as the ratio of the number of unemployed to the active population (employed plus unemployed):

$$u_r = \frac{\delta + q}{p + \delta + q} \tag{13}$$

with $q = \lambda[1 - F(x^q)]$. The steady-state stocks of the other states are too complicated to report here, but they can be calculated (see proofs in Appendix *A*.6).

In the general case, equilibrium unemployment is determined by a whole new set of parameters that are linked to inactivity and nonmarket production through the quantity q appearing in equation (13). Those parameters are absent from the classical two-state analysis of the labor market. Second, in steady state, the effect of the quit rate is exactly the same as an increase in the job-destruction

rate: it increases the inflows into unemployment, because the number of people leaving a job for inactivity will be matched by an equivalent number of workers entering activity through unemployment. Equation (13) has also the implication that unemployment is affected by q through an indirect effect affecting p: the quit rate is anticipated by firms along the job creation margin, and a higher q reduces vacancy posting at a given x^v and so leads to a lower job-finding rate p, raising unemployment.

4. Further Issues

We now explore several additional questions raised by the model: (1) the efficiency of participation margins in a decentralized equilibrium; (2) the distortive role of taxes on the allocation of time; (3) the role of unemployment benefits in attracting and keeping workers in the labor force; (4) the role of heterogeneity of market productivity in wage and employment differences across groups; and (5) the existence of a search-effort margin that extends our model. Most of these extensions will be used in the last section of the paper, which is devoted to a quantitative exercise and a calibration of U.S. flows.

4.1. Welfare and Efficiency

As in Section 3, we assume throughout this section that $e = s = 1$. Let us first consider the central planner's problem. The central planner is maximizing the sum of market and nonmarket production. The general program is given as

$$\max_{N_U, x^v, x^q} \Omega(z) = y(1 - n - N_U) - c\phi N_U + \mathcal{H}$$

under constraints (A.13) and (A.14) in Appendix *A*.7, where $\mathcal{H}$ is total home production, n is the number of nonparticipants and N_U is the mass of unemployed workers (total population is normalized to unity). After some intermediate steps detailed in Appendix *A*.7, one can show that the optimal values of x^q, x^v, and ϕ (the latter obtained from optimal N_U) are jointly determined by the following expressions:

$$\frac{c}{\chi(\phi)} = (1 - \eta)\frac{x^q - x^v}{\lambda + \delta} \qquad \text{(JC*)}$$

$$\frac{x^v}{p(\phi)} = \eta\frac{x^q - x^v}{\lambda + \delta} \qquad \text{(Entry*)}$$

$$x^q = y + \frac{\lambda}{\lambda + \delta}\int_{x^v}^{x^q} F(x)\,dx \qquad \text{(Quit*)}$$

where $-\eta$ is the elasticity of χ with respect to ϕ.[10] Comparing these results with the decentralized equilibrium described by equations (JC), (Entry), and (Quit), we immediately obtain the Hosios condition $\eta = \beta$. In this case, the decentralized equilibrium is efficient: it reaches a labor market allocation that is identical to the social planner's allocation with optimal taxation. This result might have been expected, since it is a synthesis of the efficiency results obtained in Pissarides (2000, chapters 6 and 8) with either endogenous destruction but fixed participation or exogenous destruction but endogenous participation (though static, with $\lambda = 0$). In what follows, we assume that the Hosios condition is satisfied. To save on space, all analytical proofs of next sections are in a technical Appendix, available on request, or in Garibaldi and Wasmer (2003).

4.2. Taxation and Welfare

Things are different when taxes affect wage earnings. Let us introduce a proportional tax on wages at rate t. In this case, all participation margins are distorted. The labor cost can be shown to be $w^a = \beta(y + c\phi)$ and $w^{na}(x) = \beta y + x(1 - \beta)/(1 - t)$, and the reduced form of the model now reads:

$$\frac{c(1-t)}{\chi(\phi)} = (1 - \beta)\frac{x^q - x^v}{r + \lambda + \delta} \qquad \text{(JC}(t))$$

$$\frac{x^v}{p(\phi)} = \beta \frac{x^q - x^v}{r + \lambda + \delta} \qquad \text{(Entry}(t))$$

$$x^q = y(1 - t) + \frac{\lambda}{r + \lambda + \delta}\int_{x^v}^{x^q} F(x)dx. \qquad \text{(Quit}(t))$$

Inspection of these three equations immediately shows that payroll taxes influence the equilibrium.

PROPOSITION 2.   *When the quit margin is active* $(\lambda > 0)$, *a marginal increase in payroll taxation reduces the two cutoff points and labor market tightness to an inefficiently low level:* $\partial\phi/\partial t < 0$, $\partial x^q/\partial t < 0$, *and* $\partial x^q/\partial t < 0$. *It also raises the unemployment rate and reduces the employment rate compared to socially optimum levels.*

The intuition is that taxation of wages reduces the payoff from labor market activity and thus reduces the incentives to participate (effect on the two participation margins, x^v and x^q). It also raises labor costs compared to market and home productivity and discourages job creation (effect on ϕ). The clarification we thus

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10.   Formally, $\eta = \eta(\phi) = -\phi\chi'(\phi)/\chi(\phi)$.

bring here is that taxes distort the economy along two dimensions—though the literature tends to consider one (or the other) at a time—and that these distortions are, moreover, fairly independent of each other.

In fact, in the standard and simplest matching model with exogenous job destruction (Pissarides 1987), payroll taxes do increase equilibrium unemployment if one interprets the unemployment income as a non taxable home production. Yet, in such models the size of the labor force is fixed, and the only endogenous variable that may respond to changes in taxation is market tightness. As Pissarides (1998) has shown, such an effect is present as long as unemployed income is not taxed. But in our economy both the entry and the quit margins would also be affected.[11]

To see the relative independence of the two distortions, consider a special case of our model when $\lambda = 0$, i.e., when home production is constant over time and the quit margin is latent. In this special case, our previous three equations simplify to

$$\frac{c}{\chi(\phi)} = \frac{y(1 - \beta) - c\beta\phi}{r + \delta} \tag{14}$$

$$x^v = \frac{c(1 - t)\phi\beta}{1 - \beta} \tag{15}$$

$$x^q = y(1 - t). \tag{16}$$

In this special economy, employed workers are all attached, since workers with x above the entry cutoff point x^v never participate. The quit margin is thus latent. In this special case, all wages are independent of home production and labor costs are equal to $w = \beta(y + c\phi)$. As a result, firms create the appropriate number of jobs, so that vacancy and unemployment are at the efficient level. Yet, the overall returns to market participation are distorted and fewer people enter the labor market. This is a pure labor supply effect. As shown in equation (15), the entry margin is affected only by taxation through t, since ϕ is unchanged when $\lambda = 0$. In the general case of $\lambda > 0$, a lower ϕ brings an additional distortion of x^v.

We can finally adduce two further results. First, in the general case of our model with $\lambda \geq 0$, taxes tends to reduce the employment hoarding effect by

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11. Taxation with home production was also studied by Holmlund (2002), but in his paper, the effect on the quit margin was not considered. Other papers such as Sandmo (1990), Frediksen et al. (1995), Sorensen (1997), and Kolm (2000) have studied taxation with home production, but these latter models do not focus on job search and are mainly concerned with tax differentials between market and home production. There exists also an extensive empirical literature on the effects of taxes on labor costs. The results of that literature are mixed. See, for example, Tyrväinen (1995) or Gruber (1997).

decreasing the distance between x^q and x^v.[12] Since taxes increase the relative value of home production, a larger tax rate clearly reduces the dynamic incentive to hold on to a the job. The second result concerns a reverse causality: the larger is the difference between x^q and x^v, the larger is the distortive effect of taxation on market tightness.[13] Using the results of Section 2.3 (in particular that the gap between x^v and x^q is larger when search frictions increase), this implies that taxation reduces job creation quantitatively all the more, the more frictions there are in the economy. Conversely, the adverse marginal effect of taxation on unemployment disappears as frictions vanish.

4.3. Unemployment Benefits in Partial Equilibrium

We now discuss the effect of the level of and eligibility for unemployment benefits. It is often put forward that they have an insurance role at the cost of reducing search efforts of the unemployed, with additional adverse effects on wages. Overall, unemployment is increased by a lower labor demand. Despite such disincentive effects on the insured workers, the existing literature has also emphasized a positive link between unemployment benefits and market participation, since an increase in unemployment insurance reinforces the attractiveness of market participation among the noneligible, nonemployed in general and in particular among people who are out of the labor force. This is called the entitlement effect.[14] In this section, we briefly discuss the implications of an extension of our model to unemployment benefits when p is fixed, with a focus on the entitlement effect.

Let us assume here that a benefit b is available to workers under two conditions: they have a significant job-search activity and have been previously employed (i.e., they do not come from nonparticipation; see Fredriksson and Holmlund (2001) for a similar assumption).[15] Unemployed workers coming from full-time home production then differ from those who were previously employed. We refer to the latter as covered and to the former as uncovered, with present discounted value of unemployment denoted by U^c and U^u, respectively. The x^v cutoff point is thus doubled, and we need to define x^v and x^{vc} such that $U^c(x^{vc}) = H(x^{vc})$ and $U^u(x^{vu}) = H(x^{vu})$. Let $\tilde{b} = \lambda/(r+\lambda) \int_{x^{vu}}^{x^{vc}} F(x)\,dx$.

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12. Equation JC(t) shows that $x^q - x^v$ decreases both because of a lower ϕ (Proposition 2) and because of the denominator (a lower $1 - t$).

13. Indeed, $\partial\phi/\partial t$ can be shown to be proportional to $-\lambda \int_{x^v}^{x^q} z f(z)\,dz$.

14. This was first pointed out by Mortensen (1977), mentioned by Atkinson and Micklewright (1991) in an influential survey, and has recently received more attention. Notably, Fredriksson and Holmlund (2001) studied such effects in their analysis of optimal sequencing of unemployment benefits. Related papers include Cahuc and Lehmann (2000) and Lehmann and Vanderlinden (2002).

15. We ignore issues of imperfect monitoring from the public service providing the benefits, since s is assumed to be exogenous.

Straightforward calculations lead to

$$\frac{c}{\chi(\phi)} = (1 - \beta)\frac{x^q - x^{vc}}{r + \lambda + \delta} \tag{JC$'$}$$

$$x^q = y + \frac{\lambda}{r + \lambda + \delta}\int_{x^{vc}}^{x^q} F(x)\,dx + \tilde{b} \tag{Quit$'$}$$

$$\frac{x^{vu}}{p} = \beta\frac{x^q - x^{vc}}{r + \lambda + \delta} + \frac{\tilde{b} + b}{r + \lambda + p} \tag{Entry$'$}$$

$$\frac{x^{vc}}{p} = \beta\frac{x^q - x^{vc}}{r + \lambda + \delta} + \frac{\tilde{b} + b}{p}. \tag{Entry$^{c\prime}$}$$

Note that $\tilde{b}$ is a crucial additional term. It reflects the gain in surplus for workers due to the existence of unemployment benefits. We have that $\tilde{b} > 0$ and $\tilde{b} \to 0$ when $b = 0$. One can further show that U^u and U^c do not depend on x, and that the following inequalities hold: $U^c > U^u$ and $x^{vc} > x^{vu}$. That is, the covered unemployed are better off than uncovered unemployed, and the decision of covered unemployed to return to full-time home production is reached for higher values of home productivity than for the uncovered. Thus, unemployment benefits attract and retain more active job seekers. Finally, from the perspective of the firm, there are now two different types of job seekers: the new entrant ones with $x < x^v$, and the laid-off unemployed workers with $x < x^{vc}$; see Figure 3.

The novelty of our analysis compared to the literature is the existence of a participation hoarding effect, which is an additional hoarding effect, different from the employment hoarding effect described in Section 2.2.2.

DEFINITION 2.   The participation hoarding effect is the additional incentive to participate to the labor market that is induced by conditional eligibility to benefits.
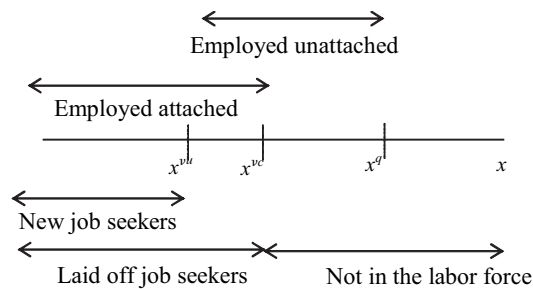

FIGURE 3. Employed workers, unemployed workers (covered and uncovered), and nonparticipants as a function of home productivity.

This effect is accounted for by the term $\tilde{b}$, that is, the loss of eligibility in case of a withdrawal from market activity.

Eligible unemployed individuals and employed workers—in order to keep eligibility—hold on to market participation in anticipation of future changes in the value of home production. Note that all three cutoff points are affected: $\tilde{b}$ directly raises x^{vc} and x^q linearly with slope 1, but by $p/(r + \lambda + p)$ it also affects (intertemporally) the cutoff point x^{vu}. The participation hoarding effect exists only when $b > 0$ and when b is strictly conditional on a previous employment spell. A higher b makes $\tilde{b}$ larger; i.e., it becomes more costly to quit, because doing so entails a loss of eligibility. The increase in x^q can be shown to be exactly equal to $\tilde{b}$. Hence, this effect parallels the employment hoarding effect in (Quit), which was the additional term $\lambda/(r + \lambda) \int_{x^v}^{x^q} F(x)\, dx$ compared to the neoclassical labor supply rule. One can also formally establish the partial equilibrium comparative statics of the model over the three labor supply cutoff points x^q, x^{vc}, and x^v, holding fixed market tightness ϕ.

PROPOSITION 3.   *At fixed $p(\phi)$, the effect of benefits is such that: $\partial x^{vc}/\partial b > 0$; $\partial x^v/\partial b > 0$; and, if δ is sufficiently low, $\partial x^q/\partial b < 0$.*

The proof is omitted here (see Garibaldi and Wasmer (2003) for details). This proposition suggests that the effect of an increase in b on x^{vc} and x^v is the same as the standard eligibility effect of benefits, which induces an increase in the entry cutoff points of both eligible and noneligible unemployed. The effect of an increase in b on the quit cutoff point is now more complicated. In the quit margin there are now both the employment hoarding effect and the participation hoarding effect. While the increase in b reduces the employment hoarding effect, causing a potential reduction in x^q, it also increases the participation hoarding effect, since with larger benefits workers lose more from a voluntary quit into inactivity. The overall effect is thus ambiguous and depends on the size of δ. For low values of δ, the employment hoarding effect prevails and a larger b reduces the quit margin. For sufficiently large values of δ the second effect dominates, since larger δ reduces the size of the employment hoarding effect.

To sum up, in the four-state model the presence of the quit margin mitigates the entitlement effect—but only at low values of δ. The general equilibrium results of benefits are more complex and are explored in the calibration exercise (Section 5).

4.4. Counterfactual Predictions

The model rationalizes five of the six flows in the labor market and allows for the previous series of tractable extensions. However, it seems to fail in two dimensions. First, it does not account for the sixth flow between nonparticipation and

employment; second, it generates a seemingly counterfactual prediction on wages. In this section we discuss how our model can be consistent with both phenomena.

4.4.1. Wages.   In our model, the wage of unattached workers is higher than the wage of the attached workers. To see this, consider the general equilibrium value of the wages $w_a = \beta y + \beta c\phi$ and $w_{na}(x) = \beta y + (1 - \beta)x$. Equation (Entry) implies that if $x > x^v$ then $(1 - \beta)x > \beta c\phi$. This result is theoretically sound, since unattached workers have a higher threat point, and it may capture the intuition that the reservation wage of workers with a stronger preference for leisure is higher, ceteris paribus. However, this result does not fit with the intuition that, in a cross-section of workers, higher attachment is positively associated with wages and participation. To reconcile the two intuitions, one can extend the model and introduce heterogeneity in market productivity: a higher market productivity jointly raise wages and labor market attachment. A similar argument could be made if the heterogeneity across groups includes home productivity parameters such as λ. These claims are more fully documented in the calibration exercise of Section 5.

4.4.2. Flows from N to E.   Our model accounts for five out of six flows between employment, unemployment, and nonparticipation (see Figure 4). Let us first establish some notation. We use lowercase letters (e.g., $e_a n$) to denote the flows rates between those stocks, while uppercase letters (e.g., EN) denote the total number of transitions.

We do not properly account for worker flows from N to E, even though (as detailed in next section) a significant number of workers do actually move directly from nonemployment to employment. Such workers thus make no transition to unemployment, which precisely was in our theoretical definition the state in which workers do actively seek a job.
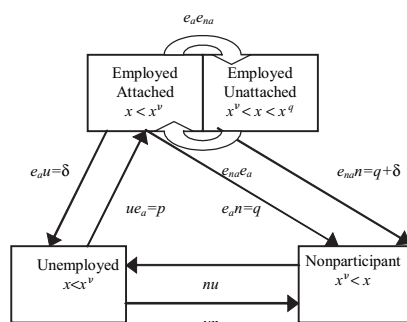

FIGURE 4. Labor market flows between three labor market states plus internal flows.

Several authors, Petrongolo and Pissarides (2001) in particular, have nonetheless argued that the direct flows from inactivity to employment are due to misclassification problems, known technically as a "time aggregation bias." Any person now holding a job must have made some minimal effort (going to an interview, negotiating the wage or working conditions), which cannot be detected by labor force surveys. The working hypothesis is that *EN* flows in the data are a pure misclassification problem, due to undetected inframonthly transitions. The time aggregation bias is the basic route we follow. There are additional rationalizations.

An interpretation similar to the time aggregation bias is that search effort is actually continuous, whereas we took it to be inelastically set to either 0 or *s*. Thus, some workers with low search effort are misclassified as nonparticipants when their search effort is strictly positive, yet below the detection point of statisticians.[16] These individuals get jobs despite low search effort, and the transition is recorded as part of the *NE* flows. The last interpretation, which we have not yet allowed, is that "jobs bump into people," even in the absence of search effort, meaning that truly inactive workers also obtain job offers. An extension of the model to endogenous search, derived in our technical Appendix, encompasses these interpretations.

5. A Quantitative Analysis

5.1. The Stylized Facts

Let us first start with a description of the facts we wish to illustrate. Following Abraham and Shimer (2001) and Faraglia (2003), we use the gross monthly flows of workers between the three ILO (International Labor Office) market states *E*, *U*, and *N*. Appendix *A*.8 provides details of a correction procedure to account for various biases. Table 1 lists the sample averages for the different flows and stocks and different age categories. It shows that there are large flows to and from inactivity, even when we take away the extremes of the age distribution (as we do when considering the 25–54 sample instead of the 15–64 sample). It is notably the case that exits from employment to unemployment are less frequent than exits from employment to inactivity. The other flows have standard values. Table 1 also indicates that there are important direct flows from inactivity to employment. To be consistent with the time-aggregation interpretation developed in Subsection 4.4.2, any *NE* transition may mask two inframonthly transitions *NU* and *UE*. Our calibration strategy will account for this correction.

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16. See Frijters and Van Der Klaauw (2003) for a recent empirical paper on the intensity of search and changes in transitions from *U* to *N*; included there is a discussion of the impact of personal characteristics on the arrival rate of offers.

TABLE 1. Average monthly flows in the U.S. labor market.

	Flows[a]					
	eu	*en*	*ue*	*un*	*nu*	*ne*
15–64 Population						
Transitions	1.02	1.62	25.90	16.59	3.46	4.43
Flow rates	0.74	1.18	0.90	0.58	0.81	1.04
Stocks [b]	*E/P*	*U/P*	*N/P*	*U/L*		
	72.90	3.50	23.60	4.58		
25–54 Population						
Transitions	0.83	1.01	25.61	13.28	4.61	3.38
Flow rates	0.68	0.82	0.76	0.40	0.71	0.52
Stocks	*E/P*	*U/P*	*N/P*	*U/L*		
	81.58	3.00	15.42	3.55		

Notes: [a] The first (second) letter refers to the source (destination) population; e.g., *eu* is the flow from employment to unemployment. [b] E is employment, N is out of the labor force, U is unemployment, and L is the labor force. Values listed are averages from 1995:10 through 2001:12 with the Abowd and Zellner (1985, Table 5) correction. Source: Author calculations based on Gross CPS data provided by Robert Shimer and Elisa Faraglia.

5.2. *Calibration: Baseline Model*

As displayed in Table 1, there are six flows to consider, and the model endogenizes five of them. Consistent with the model and the foregoing discussion on inframonthly transitions, the calibration will be based on the modified rates: $(nu)^{TA} = nu + ne$ and $(ue)^{TA} = ue + ne$, where the superscript *TA* refers to the correction for the time aggregation bias. This implies that workers flowing from N to E are assumed to have made two transitions, from N to U and from U to E.

The calibration exercise determines the performance of the model in accounting for labor market flows once the stocks, determined in steady state from these flows, are calibrated to replicate the U.S. quantities for the 25–54 population in the second half of the 1990s. In particular, we want to replicate an unemployment rate of 3.5% and a nonparticipation rate of 15.4%. The target for market tightness is set at 0.5, a reference value for most of the matching literature. The calibration code searches the parameter space for values of y, δ, c, and x_0.[17]

Table 2 reports the assumptions (upper part) and the outcome of calibration (lower part). We find that 8% of the total working age population is between x^v and x^q, among which 5.6% are employed unattached. This means that 2.4%

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17.   The total number of contacts is $x_0 \psi^{-\eta} v$ (i.e., x_0 is a scale parameter and $-\eta$ is the elasticity of $\chi(\psi)$, the finding rate of workers by firms). The pure monthly discount rate r is 0.005, and $\beta = \eta = 0.5$ so as to satisfy the Hosios efficiency condition with standard values. The distribution of home productivity is exponential with parameter $\Lambda = 0.5$. The arrival rate of the idiosyncratic shock λ is set to 0.06.

TABLE 2. Calibration to the U.S. labor market.

Parameters	Notation	Value	U.S. economy
Fixed parameters			
Matching elasticity	η	0.5	
Average home production[a]	Λ^{-1}	2.00	
Unemployed income	b	0.00	
Discount rate	r	0.005	
Idiosyncratic shock rate	λ	0.06	
Workers' surplus share	β	0.5	
Code determined parameters			
Separation rate	δ	0.01	
Productivity	y	3.93	
Matching function constant	x_o	0.71	
Search costs	c	6.30	
Equilibrium values			
Entry margin	x^v	3.15	
	$F(x^v)$	0.79	
Quit margin	x^q	4.08	
	$F(x^q)$	0.87	
Market tightness	ϕ	0.50	
Calibrated statistics			
Unemployment rate	u	3.55	3.55
Nonparticipation rate	n	15.40	15.42
Implied statistics			
Share household GDP		0.30	0.33
eu Flow rate	eu	0.83	0.68
en Flow rate	en	0.67	0.82
$(ue)^{TA}$ Flow rate	$(ue)^{TA}$	1.50	1.47
un Flow rate	un	0.04	0.40
$(nu)^{TA}$ Flow rate	$(nu)^{TA}$	0.18	1.23
Extended unemployment		5.41	4.97
Attached employed	E_a	76.31	
Nonattached employed	E_{na}	5.29	
Employment hoarding[b]		0.14	
Attached wage	w_a	3.54	
Nonattached wage	w_{na}	3.77	
Diagnostic statistics			
Absolute fit	R^{abs}	0.62	1
Relative fit	R^{dev}	0.56	1

Notes: [a] Distribution is exponential with parameter $\Lambda = 0.50$. [b] as a fraction of market productivity. Source: Author calculations.

of the population corresponds to the marginally attached defined by Jones and Ridell (1999) with reference to the Canadian labor market.[18] As Table 2 shows,

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18. For the United States, Sorrentino (1993, 1995) has established several definitions of unemployment ranging from 1 (the most conservative) to 7 (the broadest one) on the basis of answers of respondents to survey questions about their willingness to have a job, the desired number of hours, and the duration of their current unemployed spell. The ILO definition corresponds to Definition 4; Definition 6 includes workers who report wanting a job but not searching for one. Using the estimates of Sorrentino (1993), extended unemployment in the United States (including marginally attached workers) is 40% larger than given by the conventional definition. This implies that extended unemployment in the United States is about 5% of the working-age population.

extended unemployment in our calibration is 5.41%, less than half a percentage point larger than the corresponding U.S. estimates. Household production is approximately 30% of market production (GNP), a statistic that appears to be in line with existing estimates on the size of the informal sector (Eisner 1988 finds 33%). Table 2 presents also a quantitative measure of the employment hoarding effect, and it shows that the incentive to hold on to job a accounts for 14% of market productivity.

In order to assess the goodness of fit of our calibration exercise, we rely on two quantitative indicators, that measure the distance between our calibration and the U.S. economy. The first indicator is R^{abs}; it is expressed as

$$R^{abs} = 1 - \frac{\sum_{i=1}^{5} \left| flow_i^{US} - flow_i^{Model} \right|}{\sum_{i=1}^{5} flow_i^{US}}$$

where $flow_i^{US}$ and $flow_i^{Model}$ refer to one of the five labor flows calculated for the U.S. economy and for the artificial economy. The indicator R^{abs} indicates the average fit of our with respect to the U.S. statistics, where the fit is measured in absolute value. A perfect match would yield a value of R^{abs} equal to 1 while a value of 0 would indicate an average deviation of an order of magnitude. The indicator R^{rel} is constructed in a similar way but with the distance calculated in percentage terms.

Table 2 shows that the model economy calibrated to U.S. stocks can match between 56% and 62% of the flows, depending on which of the two indicators is being used. Nevertheless, as is clear from Table 2, the actual degree of resemblance of the various statistics varies across flows. In particular, our model economy matches very well three of the five target flows (eu, ue^{TA}, and en) but fall short of accounting for the flows un and nu. Further, our economy implies that non attached workers enjoy a higher wage than attached workers. In the next section, we assess whether a more accurate accounting of the structure of the unemployment benefits as well as relaxing the representative agent assumption can improve various dimensions of the calibration.

5.3. Calibration: Extensions

In this section we present two extensions of our baseline calibration. The first extension deals with market productivity differences in the population and the second with unemployment benefits. The results are displayed in Table 3.

In the first simulation we assume that there are two types of agents in the population: individuals with high market productivity and individuals with low

TABLE 3. Calibration to the U.S. labor market: Extensions.

| Parameters | Notation | Heterogeneity[a] | | | Four states | U.S. |
		l	a	h		
Productivity	y	3.53	3.93	4.33	4.13	
Matching elasticity	η		0.5		0.4	
Discount rate	r		0.01		0.01	
Idiosyncratic shock rate	λ		0.058		0.06	
Workers' surplus share	β		0.5		0.4	
Average home production	Λ^{-1}		2.0		2.0	
Time in market activity	e		0.90		0.85	
Matching function constant	x_o		0.71		0.74	
Separation rate	δ		0.01		0.01	
Search costs	c		6.30		6.03	
Eligible unemployed income	b		0.00		1.149	
Calibrated statistics						
Unemployment rate	u	4.10	3.61	3.12	3.55	3.55
Nonparticipation rate	n	19.02	15.76	12.50	15.40	15.42
Market tightness	ϕ	0.44	0.50	0.56	0.50	
Implied statistics						
eu Flow rate	eu	0.78	0.83	0.87	0.78	0.68
en Flow rate	en	0.78	0.68	0.57	0.69	0.82
$(ue)^{TA}$ Flow rate	ue^{TA}	1.56	1.6	1.44	1.47	1.47
un Flow rate	un	0.05	0.04	0.03	0.04	0.40
$(nu)^{TA}$ Flow rate	nu^{TA}	0.27	0.20	0.13	0.19	1.23
Share of covered unemployed		0.00	0.00	0.00	0.52	0.33
Replacement rate		0.00	0.00	0.00	0.32	0.2
Attached employed	E_a	71.92	75.95	79.98	77.24	
Nonattached employment	E_{na}	5.74	5.27	4.79	4.36	
Employment hoarding[b]		0.14	0.14	0.14	0.11	
Participation hoarding[b]		0.00	0.00	0.00	0.02	
Average wage	$\overline{w}$	3.17	3.55	3.93	3.56	
Diagnostic statistics						
Absolute fit	R^{abs}		0.63		0.65	1
Relative fit	R^{dev}		0.57		0.59	1

Notes: [a]l, a, and h refer, respectively, to the low, average, and high segment of the economy. [b]As a fraction of market productivity. Source: Author calculations.

market productivity. The former have productivity equal to y^h while the latter have productivity equal to y^l, with $y^h > y^l$. For simplicity, we assume that both populations have identical size. We also assume that the aggregate economy comprises the high-productivity and low-productivity segments. We start from the parameters of the three-state model and then set y^l and y^h so that the average values of endogenous stock variables match the U.S. economy. We thereby uncover the underlying properties that are the result of an aggregate composition effect.

Table 3 highlights several important implications. First, high-productivity individuals are high-wage individuals. Second, low-wage individuals have larger flows to and from inactivity. Observe that the en flow for the high productivity group is just 0.1% while it is ten times larger for low-productivity workers. The same is true of other flows to and from inactivity. This suggests that low-wage

individuals (and hence low productivity) account for most transitions to and from inactivity. Third, the table shows that a larger share of unattached workers is made up of low productivity workers.

The second extension presented in Table 3 provides a better description of the unemployment benefit system, one that is in line with the model presented here. The calibration is based on the aggregate statistics used in Table 2. The parameters are similar to those used in the baseline calibration, with the only notable exception being the Hosios condition, which is now satisfied for a value of $\beta = \eta = 0.4$. The results are as follows. First, our aggregate indicators of fits increase to 59% and 65% percent respectively, suggesting that improving the specification of the unemployment benefit system helps explain a larger share of the flows. Second, the calibration of the unemployment benefits is fairly accurate, since it implies a replacement rate of 30% and a coverage of 50%, statistics that are in line with the U.S. market. Finally, Table 3 presents also a quantitative measure of the participation hoarding effect and indicates that it amounts to (almost) 10% of the level of unemployment benefits.

6. Conclusions

Our model allows for a rather precise description of the labor market. It includes several categories of individuals: attached employed workers, unattached employed workers, unemployed workers, marginally attached nonemployed workers and true nonparticipants. All this is delivered within a tractable model of endogenous job creation and the solution is characterized using only three equations, solving for two reservation values for workers and the job creation rate. Five of the six usual labor market flows are accounted for in the benchmark model; the sixth requires specific assumptions about flows from inactivity to employment. Policy implications are explored: the roles of taxation and unemployment benefits are different and more complex than with inelastic labor supply or static participation, and this is due to the emergence of the quit margin.

An additional possible extension concerns the dynamic implications of the model. Is it possible that accounting for participation improves the quality of macroeconomic models of the business cycle? The answer is a priori ambiguous,[19] and extending our model to investigate its dynamic properties is therefore

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19.   Veracierto (2002) argues that including leisure-work choices into RBC models generates counterfactual implications—notably, with too low volatility of employment fluctuations. Shimer (2003a, 2003b) and Hall (2003) have also argued that the standard search model was predicting too high volatility of wages and too low volatility of employment and vacancies. In our setup, a positive productivity shock would reduce workers' incentives to quit and thus additionally raise employment. Further, as we noticed in Section 3.3, a lower quit rate further raises vacancy creation as firms anticipate longer periods of profits. The bottom line is that our model may generate a higher employment response to productivity shocks. All these interesting points were made clear to us by a referee.

important. Faraglia (2003) has made some progress in this direction, but with indications that there are difficulties associated with accounting for the dynamics of distribution of workers across states.

Beyond dynamic issues, extensions of this work include policy simulations of the impact of "workfare" policies and subsidies toward activity, a better accounting of firms' heterogeneity, and the introduction of several classes of workers. This paper is a first step in the direction of an accurate calibration of frictional labor markets.

Appendix

A.1. Wage Determination

The proof involves computing the average surplus of workers, firms, and the match. We define

$$\overline{S}_w = \int_{x^{\min}}^{x^{\max}} \max(W', U', H') - \max(U', H') dF(x')$$

and

$$\overline{S}_f = \int_{x^{\min}}^{x^{\max}} \max(J' - J^V, 0) dF(x').$$

Let $\overline{S} = \overline{S}_f + \overline{S}_w$. Thanks to (5), we have $\overline{S}_f = (1 - \beta)\overline{S}$ and $\overline{S}_w = \beta\overline{S}$. Note first that, given $v^H > v^U$, we have that, for all x such that $H(x) > U(x)$, necessarily $\max(W - U, 0) = W - U$. This is easily seen from the Bellman equations (2) and (3). Taking differences of the Bellman equations $(1)-(3)$, we obtain: if $H \leq U$,

$$(r + \lambda + \delta + p)(W - U)(x) = w(x) + (s - e)x + \lambda\beta\overline{S} \qquad (A.1)$$

if $H \leq U$,

$$(r + \lambda + \delta)(W - H)(x) = w(x) - ex + \lambda\beta\overline{S} \qquad (A.2)$$

and

$$(r + \lambda + \delta)(J - J^V)(x) = y - w(x) + \lambda(1 - \beta)\overline{S} - rJ^V. \qquad (A.3)$$

Using (5) and simplifying for discount factors $(r + \delta + \lambda)$, terms in $\overline{S}$ cancel out and the expression for wages comes easily: we obtain

$$w^a(x) = \beta(y - rV_V) + (1 - \beta)[(e - s)x + p(W - U)(x)] \qquad (A.4)$$

$$w^{na}(x) = \beta(y - rV_V) + (1 - \beta)ex \qquad (A.5)$$

where a refers to attached workers $(U \geq H)$ and na refers to unattached workers $(H \geq U)$.

A.2. *Slopes of Value Functions and Reservation Strategies*

Recall from Section *A*.1 that for all x such that $H > U$, we also have $W > U$. It is thus sufficient to prove that asset values are increasing, linear (or piecewise linear), continuous, and that $U(x)$ is less steep than $H(x)$. Hence for all $x < x^{\nu}$, where x^{ν} is defined as $U(x^{\nu}) = H(x^{\nu})$, we necessarily have $W > U > H$. Now, if $x^{\min} > x^{\nu} > x^{\max}$ (which we will assume), then $x^{q} > x^{\nu}$.

Let's prove this. Using the wages derived in (*A*.1), we can rewrite the four Bellman equations of employed (attached and unattached), unemployed, and not in the labor force. Taking the derivative with respect to x, and denoting by a_W^a, a_W^{na}, a_U, and a_H the slopes, we obtain a four-by-four system linking the four slopes. Taking two-by-two differences, we find that $a_W^a - a_U$ is proportional to $-(e - s)$ and thus negative, $a_W^{na} - a_H$ is proportional to $-e$ and thus negative and $a_U - a_H$ is the sum of $-s/(r + \lambda)$ and a term proportional to $a_W^a - a_U$, and thus is negative as well. Overall, $a_H \geq a_U \geq a_W^a$ and $a_H \geq a_W^{na} \geq a_W^a$. Given $W(x) > U(x) > H(x)$ for $x < x^{\nu}$ and $a_H \geq a_U$, the intersection of $W(x)$ with $H(x)$ denoted by x^q is thus necessarily to the right of the intersection of $U(x)$ and $H(x)$. Thus $x^q \geq x^{\nu}$. This is represented in Figure 1. One can also prove that $a_W^{na} \geq a_W^a$.

A.3. *Determining the Surplus $S(x)$ and $\overline{S}$*

Let us first define $S(x) = J(x) + V_V + W(x) - \max(U(x), H(x))$. Observe that $\frac{\partial S}{\partial x} = \frac{-e}{r + \lambda + \delta}$ for $x^q \geq x \geq x^{\nu}$ whereas $\partial S/\partial x = (-e + s)/(r + \lambda + \delta + \beta p)$ for $x^{\nu} \geq x \geq x^{\min}$. Given that $S(x^q) = 0$ and that $S(x)$ is continuous in x^{ν} with a discontinuity in slopes, we have

$$S(x) = \frac{e(x^q - x)}{r + \lambda + \delta} \text{ for } x^q \geq x \geq x^{\nu}.$$

We can also determine the value of $\overline{S}$ defined in *A*.1. After an integration by parts, we obtain

$$\overline{S} = \frac{e - s}{r + \lambda + \delta + \beta p} \int_{x^{\min}}^{x^{\nu}} F(x)dx + \frac{e}{r + \lambda + \delta} \int_{x^{\nu}}^{x^q} F(x)dx. \qquad (A.6)$$

A.4. *Existence and Uniqueness in Partial Equilibrium*

The proof for uniqueness of x^{ν}, x^q for a given p is simple to obtain. First, the expression of the quit margin in equation (10) is downward sloping, while the

expression for the entry margin in equation (8) is upward sloping. It is also easy to see that the intersections with the horizontal axis ($x^v = x^{\min}$) are such that the intercept of the entry margin is below $x^{\min}$ while the intercept of the quit margin is given implicitly by

$$x^q = \frac{y}{e} - \frac{r}{e}V^V + \frac{\lambda}{r + \lambda + \delta} \int_{x^{\min}}^{x^q} F(x)dx.$$

A sufficient condition for uniqueness is that the latter is above $x^{\min}$, which is the case when y is sufficiently large.

A.5. Existence and Uniqueness of the General Equilibrium

Proof of Proposition 1. The existence (and uniqueness) of the equilibrium described by (Entry), (Quit), and (JC) can be shown by eliminating x^v from those equations, and than noting that (Entry) and (JC) jointly imply

$$x^v = \frac{\beta}{1 - \beta} \, c\phi. \tag{A.7}$$

This states, in a reduced form, that higher shares of the surplus and better labor market prospects induce further entry of workers into the labor market. Using (A.7), yields two relations between ϕ and x^q that have opposite slopes. One can thus express the equilibrium in the space $[\phi, x^q]$. The modified JC curve is positively sloped and states that more stable workers (higher x^q) raise job creation. The modified (Quit) equation is downward sloping and states that, in a tighter labor market, the surplus of a job is lower and the capital loss of quitting is lower, reducing x^q. Comparing the intercepts of the two modified (JC) and (Quit) equations, we can show existence and uniqueness. The intercept of the (JC) curve is $x^q = 0$, and the intercept of the quit margin curve is defined by $q_0 = y + \lambda/(r + \lambda + \delta) \int_0^{q_0} dF(x) \geq y > 0$ as long as $y > 0$. ☐

A.6. Stocks and Flows

We use upper letters for the stocks of workers, where E_a, E_{na}, N_U, and $N = n$ denote (respectively) the employed attached, employed unattached, unemployed, and nonparticipants (in full-time home production). One can write the evolution

of the stocks of workers in the four categories by:

$$\frac{dE_a}{dt} = -(e_a n + e_a u + e_a e_{na})E_a + u e_a N_U + e_a e_{na} E_{na} \qquad (A.8)$$

$$\frac{dE_{na}}{dt} = -(e_{na} n + e_{na} u + e_{na} e_a)E_{na} + e_a e_{na} E_{na} \qquad (A.9)$$

$$\frac{dN_U}{dt} = -(u e_a + un)N_U + e_a u E_a + nu N \qquad (A.10)$$

$$\frac{dN}{dt} = -nu N + e_{na} n E_{na} + un N_U + e_a n E_a. \qquad (A.11)$$

Intermediate calculations available in our technical appendix lead to equation (13) and

$$N\lambda F(\theta^v) = (\lambda - \lambda F(x^v) + p)N_U - \delta\left(\frac{\delta + q + \lambda F(x^v)}{\delta + \lambda} E\right).$$

A.7. Welfare Analysis

The proof of the social planner problem is as follows. The social planner maximizes, over N_U, x^v, and x^q

$$\Omega = y(1 - n - N_U) - c\phi N_U + \mathcal{H} \qquad (A.12)$$

where $\mathcal{H}$ is home production of inactive workers, n are the non participants, and N_U is the mass of unemployed workers (total population is normalized to unity) implying that $\phi N_U = v$ the vacancy rate. Observe that $u_r = N_U/(E + N_U)$. Denoting by $f^H(x)$ the density of inactive workers, home production is

$$\mathcal{H} = n \int_{x^v}^{+\infty} x f^H(x) d\theta.$$

We can prove that f^H is proportional to f; then, $\max_{u,x^v,x^q} \Omega$ subject to the constraints

$$p N_u - (\delta + q)(1 - N_U - n) = 0 \qquad (A.13)$$

$$(1 - N_U - n)\left[\frac{\delta}{\delta + \lambda}(\delta + q) - \frac{\lambda}{\lambda + \delta}\lambda F(x^v)\right]$$
$$- N_U(\lambda + p) + \lambda F(x^v) = 0 \qquad (A.14)$$

immediately leads to (Entry*), (Quit*), and (JC*).

A.8. Flow Data

All flows tabulated from labor force surveys suffer from serious misclassification problems. There are two major problems with the unadjusted gross flows data derived from the CPS (Current Population Survey). First, imperfect matching of labor force data leaves approximately 15% of the eligible observations with labor force status missing in one month or the other. Second, the measurement of changes in labor-force status may be biased because of random respondent, interviewer, or coding errors even when these classification errors do not generate bias in measurement of the levels. Abowd and Zellner (1985) have proposed a procedure that resolves these issues. We apply their adjustment under the assumption that these biases are time invariant, as in Abraham and Shimer (2001). We only consider the post June 1995 period (since there are missing data between June and September 1995) for two groups of workers: the 15–64 population (hereafter referred to as "total") and the 25–54 population ("prime-age"). When the flows are deflated by the origin population, they are called transition rates; when deflated by the total (or prime-age) population, they are called "flow rates".

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