Increasing Returns and Animal Spirits

Philippe Weil

Increasing Returns and Animal Spirits

By PHILIPPE WEIL*

External increasing returns have recently been advanced by Paul Romer (1986) as providing, in the tradition of Kenneth Arrow (1962), a foundation upon which to build a competitive, general equilibrium theory of endogenous long-term economic growth, and a framework within which to re-assess the long-standing empirical and theoretical debate about the long-run convergence of growth rates.[1]

As is well known from the literature on static external effects, in particular in international trade theory, increasing returns which are external to the firm often lead to the existence of multiple competitive equilibria.[2] The purpose of this note is to simply point out that the same multiplicity, intrinsic of course to the presence of non-convexities, characterizes the models of growth with external increasing returns which have been studied recently, and, more importantly, to show that the many competitive equilibria which often arise in those models can be interpreted as "animal spirits" equilibria—that is, equilibria indexed by the consumers' optimism or pessimism.

This multiplicity of competitive equilibrium paths can be viewed as providing a new foundation upon which to build a theory of self-fulfilling prophecies, one which relies solely on the existence of external effects and does not rest, as previous models did,[3] on the overlapping generation structure or on backward-bending offer curves. It may also be envisaged as suggesting that a general belief in material progress (or decline) may be sufficient, under conditions to be made precise below, to generate economic growth (or contraction). In a way reminiscent of recent reinterpretations of Keynesian economics by Russell Cooper and Andrew John (1988), Nobuhiro Kiyotaki (1985), or Olivier Blanchard and Kiyotaki (1987), the failure of an economy to develop may thus be interpreted, according to this approach, as stemming from a coordination failure, rather than being the necessary consequence of fundamental factors such as tastes or factor endowments—a result which suggests the possibility of developing a Keynesian theory of growth and development.

The theoretical fact, illustrated in this paper, that belief in economic growth may be self-fulfilling should not and cannot be dismissed simply on the teleological ground that the nonuniqueness of competitive equilibria in models with external increasing returns leads to an empirical hara-kiri—as real effects of psychological phenomena are the norm, rather than the exception, in rational expectations models.

The paper is organized as follows. Section I introduces the model. Section II presents the multiplicity results. Section III discusses welfare. The conclusion outlines directions for future research.

I. The Basic Framework

To keep the results as transparent as possible, consider a simple two-period[4] economy inhabited by (a continuum of) identical individuals. A representative consumer derives utility

$$U = u(c_1, c_2) \tag{1}$$

*Department of Economics, Harvard University, Cambridge, MA 02138. I thank Costas Azariadis, Nobuhiro Kiyotaki, Paul Krugman, and two anonymous referees for their useful comments and suggestions.

[1] See, for instance, William Baumol (1986) and Bradford DeLong (1988).

[2] See, for instance, Elhanan Helpman and Paul Krugman (1985).

[3] For example, Costas Azariadis and Roger Guesnerie (1986), Jean-Michel Grandmont (1985).

[4] Romer (1986, Section IV) studies a similar two-period model.

from his consumption (c_1, c_2) when young and old. The standard concavity, continuity, and differentiability conditions are imposed on $u(\cdot,\cdot)$, and the following additional assumptions are made:

ASSUMPTION 1: For all $x > 0$, $u_1(0, x) = \infty$.

ASSUMPTION 2: For all $x > 0$, $u_2(0, x) < \infty$.

Each individual receives an endowment (e_1, e_2) when young and old of the single storable consumption good, with $e_1 > 0$ and $e_2 \geq 0$. Storage is, from a *private* point of view, a constant-returns-to-scale activity,[5] with the private gross rate of return on storage, R, being taken as given by each individual (or firm). A representative consumer therefore maximizes U in (1) subject to the budget constraints

$$c_1 + k = e_1 \tag{2}$$

$$c_2 = e_2 + Rk \tag{3}$$

$$c_1, c_2, k \geq 0, \tag{4}$$

where k denotes the amount of storage. The first-order condition for a consumption optimum is simply

$$-u_1(c_1, c_2) + Ru_2(c_1, c_2) \leq 0 \quad = 0 \text{ if } k > 0. \tag{5}$$

It is assumed that the gross rate of return on storage is an increasing function $R(\cdot)$ of the *aggregate* amount K of storage

$$R = R(K), \quad R(\cdot) > 0, \quad R'(\cdot) > 0. \tag{6}$$

In other terms, the more society stores, the more productive the storage technology becomes.

As a consequence, storage is, from a *social* point of view, an increasing returns to scale activity since, letting $F(K, k) \equiv R(K)k$ denote (as in Romer) per capita output from storage, we find that for any $\lambda > 1$,

$$F(\lambda K, \lambda k) > F(K, \lambda k) = \lambda R(K)k = \lambda F(K, k), \tag{7}$$

as $R'(\cdot)$ is positive by assumption. These increasing returns are external to the firms (or the consumers): private agents, because they are atomistic, neglect the effect of their own savings decision on the economywide productivity of the storage technology.

It is useful to think, as in Romer, of storage as investment in knowledge: today's endowment of the consumption good can be used interchangeably as consumption or as an input—knowledge—into a process $F(\cdot,\cdot)$ which produces more consumption tomorrow with knowledge. According to this interpretation, investment in knowledge is subject to an externality, presumably because of an imperfect protection intellectual property.

II. Competitive Equilibria

Because of the technological externality at the core of this model, one should not expect competitive equilibria to be in general unique. Rather, very much as the recent models of Peter Diamond and Drew Fudenberg (1987) or Peter Howitt and Paul McAfee (1988) (which are based the existence of labor market transaction externalities), the overlapping generation economy with threshold externalities analyzed by Costas Azariadis and Alan Drazen (1988), the monopolistic competition environment studied by Kiyotaki, or the Keynesian economy recently studied by John Bryant (1987), the simple framework developed *supra* may give rise, under assumptions which will be made precise below, to the existence of "optimistic" or "pessimistic" rational expectations equilibria which are reminiscent, but in a different context, of the recent literature on Keynesian coordination failures (see Cooper and John).

The intuition behind that multiplicity of competitive equilibria is straightforward. If agents are optimistic in the sense that they

[5] Relaxing this assumption to allow for private decreasing returns to storage would not qualitatively affect the results.

expect a high rate of return on storage tomorrow, then, *provided this lead them to store more*, storage will indeed be highly productive tomorrow—thus validating today's optimistic expectation. Pessimism about storage prospects will, in an analogous fashion, be self-fulfilling.

Formally, normalizing the size of population to 1, it must be the case that in equilibrium

$$k = K, \tag{8}$$

since all agents are identical.[6] From equations (2) to (6) and (8), it is easily seen that equilibrium storage solves the following inequality:

$$z(k) \leq 0 \quad = 0 \text{ if } k > 0, \tag{9}$$

where

$$z(k) \equiv -u_1[e_1 - k, e_2 + R(k)k] + R(k)u_2[e_1 - k, e_2 + R(k)k]. \tag{10}$$

To delineate the conditions under which there might be multiple competitive equilibria, it is first useful to notice that, as suggested above, a negative interest elasticity of savings is inconsistent with multiplicity:

PROPOSITION 1: *An interior competitive equilibrium is unique (when it exists) if the interest elasticity of savings, ξ, is negative.*

PROOF:

An *interior* competitive equilibrium is a capital stock $\hat{k}$, $0 < \hat{k} < e_1$, such that $z(\hat{k}) = 0$. From equation (10), $z'(\hat{k}) = \Delta[1 - \xi\eta]$, where $\Delta \equiv u_{11} - 2Ru_{21} + R^2u_{22}$ is negative by the second-order condition, $\xi \equiv -[(u_2 + Rku_{22} - ku_{12})/\Delta][R/k]$ is the interest elasticity of savings, and $\eta \equiv R'(k)k/R > 0$ denotes the elasticity of the rate of return on storage with respect to capital. Therefore $z'(k) < 0$ throughout the interval $(0, e_1)$ if $\xi < 0$. Hence, if there exists a $\hat{k}$ such that $z(\hat{k}) = 0$, it is unique. □

Proposition 1 establishes that the multiplicity of competitive equilibria to be exhibited *infra* does not rely on strong income effects—as it does in the overlapping generation sunspot literature exemplified by the Azariadis-Guesnerie model. Instead, a negative response of savings to interest rates is sufficient in our framework to rule out multiplicity. The intuition behind this result is obvious. For optimism (resp. pessimism) to be self-fulfilling in our model, the prospect of high (resp. low) rates of return on investment must lead consumers to save more (resp. less). In other, but equivalent terms, strategic complementarity between agents is required (but obviously not sufficient) for the existence of multiple equilibria: each agent must follow the crowd and go along with what he expects to be the others' investment decision.

A sufficient condition for multiple interior competitive equilibria to come in pairs is given by

PROPOSITION 2: *Under assumptions* 1 *and* 2, *there is an even number of interior competitive equilibria if* $R(0) < u_1[e_1, e_2]/u_2[e_1, e_2]$.

PROOF:

Under assumptions 1 and 2, $z(e_1) = -\infty$, while, under the condition of the proposition and the properties of $R(\cdot)$, $z(0) < 0$. Since $z(\cdot)$ is continuous over the interval $(0, e_1)$, there must therefore be an even number l (possibly zero)[7] of solutions $k_1, \ldots, k_l$ to the equation $z(k) = 0$, that is, an even number of interior competitive equilibria. □

The condition of Proposition 2 imposes a form of strategic complementarity stronger than is already implicit in a nonnegative interest elasticity of savings. It requires that

[6] Pareto-improving collusive behavior by consumers or firms is ruled out in this model, as it in Romer's.

[7] Obviously, no equilibrium exists if the conditions of Propositions 1 and 2 are both satisfied.

faced with the gross rate of return of storage $R(0)$ competitive agents choose not to store but instead remain at the corner $k=0$;[8] under this condition, *a consumer stores only if he/she expects other agents to undertake positive storage*.

To show that Proposition 2, which relies on weak assumptions, is not vacuous, it suffices to consider the following simple and non-pathological example:

EXAMPLE: Suppose that $u(c_1, c_2) = \ln(c_1) + 0.95\ln(c_2)$, $e_1 = e_2 = 100$ and $R(K) = 1 + 0.035K$. The condition of the proposition is satisfied and there are two interior perfect foresight equilibria: $\hat{k}_1 = 4.76$ and $\hat{k}_2 = 15.38$, which can be dubbed, respectively, pessimistic and optimistic equilibrium.

Two comments on Proposition 2 are in order. First, the satisfaction of the condition of the proposition does not guarantee, as its proof makes clear(l may be zero), the existence of interior competitive equilibria—even for a nonnegative interest elasticity of savings. In particular, it is easy to prove the intuitive result that there will be no interior competitive equilibrium if the external effect, as measured by η, is too small and the condition of the proposition is satisfied. In that case, the corner no-storage equilibrium will be the unique competitive equilibrium. Not surprisingly, a large enough externality is therefore required to guarantee the existence of a positive and even number of interior competitive equilibria.

Second, the sufficient condition of Proposition 2 is overly strong for economies in which interior competitive equilibria do exist. It could easily be weakened by specializing the model, but there is little point in doing so, as the objective of this paper—pointing out that a theory of self-fulfilling prophecies can easily be conducted, in models with external increasing returns, on foundations which do not rely on backward-bending offer curves or on an overlapping generation structure—is attained by the foregoing analysis.

III. Welfare

Propositions 1 and 2 have established that, in the presence of enough strategic complementarity between agents, interior competitive equilibria will come, when they exist, in pairs. In this section, I show that, as in the coordination failure literature, these multiple equilibria can be Pareto-ranked. Optimistic competitive equilibria (those with a high expected and realized level of investment) Pareto-dominate pessimistic ones—which is precisely the ranking dictated by intuition.

It is first useful to remember that because of the technological externality at the core of this economy, all interior competitive equilibria are of course suboptimal. For any competitive equilibrium capital stock $\hat{k}$, the private return to storage falls short of its social benefit by an amount $\hat{k}R'(\hat{k}) > 0$.

Whether interior competitive equilibria can be Pareto-ranked is a more important issue, which is addressed in the following

PROPOSITION 3:[9] *If $\hat{k}_i$ and $\hat{k}_j$ are two competitive equilibrium capital stocks and $\hat{k}_j > \hat{k}_i > 0$, then utility is higher in the $\hat{k}_j$ equilibrium—that is, optimistic (high investment) equilibria Pareto-dominate pessimistic (low investment) competitive outcomes.*

PROOF:

Since $R'(\cdot) > 0$, the affordable consumption set at $R(\hat{k}_j)$ includes the equilibrium consumption basket at $R(\hat{k}_i)$. By the strict quasi-concavity of $u(\cdot,\cdot)$, the equilibrium consumption basket at $R(\hat{k}_j)$, $(c_1, c_2) = (e_1 - \hat{k}_j, e_2 + R(\hat{k}_j)\hat{k}_j)$, must therefore be strictly preferred to the equilibrium consumption basket at $R(\hat{k}_i)$, and the $\hat{k}_j$ equilibrium Pareto-dominates the $\hat{k}_i$ competitive outcome. □

[8] No capital accumulation ($k = 0$) is therefore, under the condition of Proposition 1, an additional *corner* competitive equilibrium.

[9] I thank an anonymous referee for suggesting this more elegant formulation of my original proposition.

Proposition 3, as is manifest from its proof, is very general as it can deal with cases of multiplicity not covered by the sufficient (but not necessary) condition of Proposition 2.[10] To prove Proposition 3, and thus to be able to Pareto-rank multiple equilibria, one only needs two ingredients: a positive externality ($R' > 0$), and the strict quasi-concavity of preferences. Multiple competitive equilibria, whenever they occur, can therefore always be Pareto-ranked in the simple economy analyzed here:[11] more knowledge is preferable to less. Pareto-ranking is, of course, not always possible: with heterogeneous agents, for instance, borrowers would be made worse off by higher interest rates.

IV. Conclusion

This paper has shown that, given enough strategic complementarity between agents, multiple competitive equilibria, indexed by consumers' optimism or pessimism, may arise in a simple dynamic model by external increasing returns. In contrast with some previous literature, the existence of multiple animal spirits, self-fulfilling competitive equilibria does not rely on backward-bending offer curves or the overlapping generation structure. Moreover, these multiple equilibria can always be Pareto-ranked, with optimistic, high-knowledge equilibria dominating pessimistic, low-investment equilibria.

The conclusion that multiple animal spirits competitive equilibria are likely to arise in a model of external increasing returns would survive the generalization of the framework used in this paper to many-period environments—be they overlapping generation or representative agent, infinite horizon structures.[12] The reason is of course that what essentially matters for multiplicity in this model is the presence of an externality, and not demographic details.

From a broader point of view, this paper illustrates a coming together of the literatures on Keynesian economics and growth theory, precisely at a time where development economics is returning to the center stage of macroeconomics. The integration of the largely static issues which have preoccupied students of Keynes with the dynamic perspective of growth theory should provide fertile ground for new research.

[10] Proposition 3 can also easily be extended to include $k = 0$, when it is a corner equilibrium.

[11] Cooper and John also succeed, but in a different context, to Pareto-rank the multiple competitive equilibria which emerge from their model.

[12] Imbedding the model of this paper into an overlapping generation structure is immediate, as there would be no intergenerational trade in equilibrium in this economy; sunspots and cycles would then emerge under the conditions derived *supra*. With immortal agents, multiplicity propositions would, however, become substantially more complex; for a given initial knowledge stock, for instance, there could be, depending on the details of the model, two equilibrium trajectories, one leading to a high, the other to a low, long-run capital stock.

REFERENCES

Arrow, Kenneth, "The Economic Implications of Learning by Doing," *Review of Economic Studies*, June 1962, *29*, 155–73.

Azariadis, Costas and Drazen, Alan, "Threshold Externalities in Economic Development," mimeo., 1988.

_____ **and Guesnerie, Roger,** "Sunspots and Cycles," *Review of Economic Studies*, October 1986, *58*, 725–37.

Baumol, William, "Productivity Growth, Convergence, and Welfare: What the Long-Run Data Show," *American Economic Review*, December 1986, *76*, 1072–85.

Blanchard, Oliver and Kiyotaki, Nobuhiro, "Monopolistic Competition and the Effects of Aggregate Demand," *American Economic Review*, September 1987, *77*, 647–66.

Bryant, John, "The Paradox of Thrift, Liquidity Preference and Animal Spirits," *Econometrica*, September 1987, *55*, 1231–36.

Cooper, Russell and John, Andrew, "Coordinating Coordination Failures in Keynesian Models," *Quarterly Journal of Economics*, August 1988, *103*, 441–64.

DeLong, Bradford, "Productivity Growth, Convergence, and Welfare: Comment," *American Economic Review*, December 1988, *78*, 1138–54.

Diamond, Peter and Fudenberg, Drew, "Rational Expectations Business Cycles in Search Equilibrium," mimeo., MIT, 1987.

Grandmont, Jean-Michel, "On Endogenous Competitive Business Cycles," *Econometrica*, September 1985, *53*, 995–1045.

Helpman, Elhanan and Krugman, Paul, *Market Structure and Foreign Trade*, Cambridge, MA: MIT Press, 1985.

Howitt, Peter and McAfee, Paul, "Animal Spirits," mimeo., MIT and the University of Ontario, Canada, 1988.

Kiyotaki, Nobuhiro, "Implications of Multiple Equilibria Under Imperfect Competition," mimeo., Harvard University, 1985.

Romer, Paul, "Increasing Returns and Long-Run Growth," *Journal of Political Economy*, October 1986, *94*, 1002–37.