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Income Inequality and the Progressivity of Taxes in a Coalition Formation Model

Zsófia L. Bárány*

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Abstract: In this paper, I relate the degree of progressivity of the income tax scheme to the prevailing income inequality in the society. I find that, consistent with the data, more unequal societies implement more progressive income tax systems. I build a model of political coalition formation, where different income groups have to agree on a tax scheme to finance the public good. I show that, the greater income inequality is, i.e. the further away the rich are from the rest of the population, the less able they are to credibly commit to participating in a coalition. Therefore, as income inequality rises, the rich are increasingly excluded from the design of the income tax scheme. Consequently, the rich bear a larger fraction of the public good, and the tax system becomes more progressive.

Keywords: Income inequality, coalition formation, income taxes, progressivity

JEL-Classification: D72, C71, H23, H24.

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1 Introduction

The widespread progressivity of income taxes is a puzzling phenomenon for economists. The normative literature is inconclusive on the optimality of progressive income taxes. The results depend on the equity-efficiency trade-off, and hence are very sensitive to the social welfare function and the elasticity of labour supply. In the positive literature, self-interested citizens, politicians or parties are the central element. When modeled as a classical Downsian competition, additional restrictions on policies or preferences have to be implemented, otherwise the typical problem of voting cycles arises in the multi-dimensional setting. Even in models with policy-motivated politicians the conclusions about the progressivity of the tax scheme is ambiguous. The question, however, has not been analyzed in the context of endogenous coalition formation, which is a natural framework to think about redistributive issues. In this paper, I relate the *degree* of progressivity of the income tax scheme to the prevailing income inequality in the society. I find, that as in the data, more unequal societies implement more progressive income tax systems.

This paper contributes to the discussion on progressivity from both an empirical and a theoretical perspective. In order to understand how uniform the progressivity of income taxes is in advanced economies, I calculate the progressivity index of income taxes for 17 OECD countries. I find that there is a substantial variation in the index of progressivity, and that more unequal countries have more progressive income tax schemes in place. From a theoretical perspective, I present a model of political coalition formation in which the tax scheme is determined. The society has to decide on how to share the burden of financing a given level of public good. I show that in such a model, as inequality increases, the representative of the rich group becomes less able to participate in any coalition, and the equilibrium tax scheme shifts the tax burden towards the rich, thus increasing the progressivity of taxes.

In this paper, I analyze an economy where the citizens have to decide on how to raise funds to provide a given level of public good. The society has to select a tax scheme that raises a fixed amount of revenue, and the different income groups have conflicting interests on which groups to tax more heavily. In the elections, each income group is represented by a politician, whose interests coincide with his group's. The representatives have to decide whether to run alone, or to form a coalition with another representative. Each representative or coalition chooses a tax scheme to offer, and citizens then vote on the candidate or party that offers the tax scheme that maximises their utility. As in the citizen-candidate model of Besley and Coate (1997), single representatives can only offer their ideal policy, as they cannot credibly commit to the implementation of any other platform. On the other hand, coalitions offer a commitment mechanism for parties: due to the internal conflict of party members, the party can credibly commit to any policy that is in the Pareto set of its members. I identify the stable coalitions and the equilibrium winning platforms.

I find that when income inequality is low, then a coalition of the poor and the rich wins and implements a tax system that puts a large fraction of the tax burden on the middle income group. When income differences are small, then the middle income group's preferred policy is the median policy.

This implies that in the absence of coalitions, the middle income group wins and implements its ideal policy. Thus, the rich and the poor can credibly commit to cooperate and choose a platform that is better for both than the middle income group's ideal policy. As inequality increases, the policy role of tax rates on higher levels of income increases: more revenue can be raised by a marginal increase in the tax rates. This implies that there is more room to trade-off tax rates on middle and high incomes. This gives a lot of power to the poor, as the preferences of the middle income and the rich are very different. If inequality is moderately high, then the poor are not powerful enough to stop the middle income group and the rich from forming a coalition. This coalition implements policies which feature high taxes on low levels of income and moderate taxes on higher levels of income. When inequality is very high, the poor can prevent the rich and the middle income group from forming a stable coalition, and hence implement a highly progressive tax scheme.

I calculate the pre- and post-tax Gini coefficients, the progressivity index of the personal income tax, and the average tax rate for 17 OECD countries. I proxy the distribution of income by the employment share and average earnings of the main occupation groups based on labour force surveys. I find that low inequality countries have a less progressive income tax scheme. Hence, contrary to common belief, the progressivity index is relatively low in the Nordic countries, and it is relatively high for Southern European countries. However, the average tax rate, which also contributes to redistribution, is higher in countries with lower inequality. The overall redistributive effect of the personal income tax scheme also increases as inequality increases. Therefore the model I present here is in line with the data in predicting that as inequality increases in a society, the implemented tax scheme becomes more progressive.

2 Related literature

One strand of models of voting on the progressivity of income taxes is in the Downsian tradition: parties or politicians only care about holding office, and can perfectly commit to implementing any policy platform. Snyder and Kramer (1988) were one of the first to address the progressivity of income taxes from a political economy perspective. Under the restriction that parties can only offer policies that are ideal for some citizens, and citizens optimally allocate their time between taxable and non-taxable activities, they show that marginal rate progressivity emerges due to the desire of middle-income citizens to reduce their own tax burden. Cukierman and Meltzer (1991) analyze the question in cases when a Condorcet winner exists, over quadratic tax schemes, but only succeed in showing the prevalence of progressive taxes under very strong restrictions. Marhuenda and Ortuño-Ortín (1995) relax the requirement of the existence of a Condorcet winner, and show that a marginal rate progressive tax always defeats a marginal rate regressive tax, if the median income is below the mean income. Hindriks (2001) shows under similar conditions, that for any tax scheme there exists a less progressive one, which is supported by a majority of voters, thus demonstrating that the demand for progressivity cannot be derived from the standard Downsian framework. These voting cycles arise, because to analyze the pro-

gressivity of the tax scheme, the policy space has to be at least two-dimensional. In a multi-dimensional policy space pure strategy Nash-equilibrium of the standard two-party game generally does not exist. Carbonell-Nicolau and Ok (2007) identify mixed strategy equilibria and find that in an unconstrained policy space there is an equilibrium which is not marginal rate progressive. Carbonell-Nicolau (2009) circumvents the problem of voting cycles by allowing politicians to reveal their policy platforms gradually in more than one period, and shows that the tax scheme benefits the most populous groups, and puts the burden of taxation on groups with fewer voters. Therefore in log-normal income distributions the income tax scheme is not progressive, as the tax burden is on the rich *and* the poor.

This paper is closer to the non-Downsian strand of the literature which assumes that politicians have some preferences over the policy to be implemented. Roemer (1999) introduces a new equilibrium concept (Party Unanimity Nash Equilibrium, PUNE), one which is based on the idea that parties have internal conflicts: some members only care about winning the election, whereas others care about the policy that will be implemented. In such a setup, he shows that from the set of quadratic tax functions in a two-party election, both parties propose a progressive tax scheme. This paper is similar to mine in the sense that the existence of parties and the internal conflict allows the parties to offer policy platforms that a single candidate (either office- or policy-motivated) would not be able to offer. However, in my model the candidates only care about the policy that is implemented, and party formation is endogenous.

Carbonell-Nicolau and Klor (2003) analyze a similar setup to the one presented here: there is an exogenous set of parties, who have preferences over after-tax inequality. Each party decides whether to enter the election with a candidate or not. Voters vote sincerely in order to minimise their expected tax payment. In such a setup they characterise the conditions under which a strong Nash equilibrium exists, and show that these equilibria feature increasing marginal rates. Their setup is similar to the one of this paper in the sense that the citizens vote on representatives, and that the representatives have preferences over the implemented policy. However, in my model, the representatives are citizens as well and they have preferences over their own after-tax income, and not over the after-tax income inequality as in the paper of Carbonell-Nicolau and Klor (2003). Moreover, in my model, candidates are allowed to form coalitions and this way improve their chances of implementing a platform that increases their utility.

Most of positive literature has omitted the equity-efficiency trade-off, which is at the heart of the normative literature. This trade-off arises, as progressive taxes provide a mechanism for the state to redistribute income from the rich to the poor, however, high marginal tax rates have efficiency costs. Since the seminal paper by Mirrlees (1971), this literature has developed significantly, but as Saez (2001) notes, its implications for policy are still very limited.¹

Donder and Hindriks (2003) is a notable exception in the positive literature in the sense that they

¹Mirrlees (1971) shows that the tax rates have to be non-negative and below full taxation. The most well-known result is that when the income distribution is bounded, the top marginal tax rate should be zero, Sadka (1976) and Seade (1977) show this result. Seade (1977) showed that the marginal rate at the bottom should also be zero if everyone in the society works.

explicitly consider labour supply choices. They show through simulation results that under less demanding equilibrium concepts than majority winner, progressive tax schemes are more likely to arise, and they are the only possibility if the distribution of abilities is sufficiently concentrated at the middle.

From an empirical perspective only a few papers have quantified the progressivity of the income tax schemes across countries. Kakwani (1977), who introduced the progressivity index that I use in this paper, calculates the redistributive effect and the progressivity of the tax scheme in the US, Canada, the UK and Australia. Suits (1977), introduces a similar measure of progressivity and calculates the change in the progressivity of different US taxes. A more recent study by Wagstaff et al (1999) calculates and decomposes the redistributive effect of income tax schemes in 12 OECD countries.

3 Tax progressivity in 17 OECD countries

3.1 Measuring progressivity

It is generally accepted that the progressivity of taxes at a given income level depends on the elasticity of the tax function with respect to income. If the elasticity is equal to unity, then the tax is exactly proportional at that income level, if this elasticity exceeds 1, then the tax is progressive, while if it is below 1, then it is regressive at that income level. Based on this definition, one can look at tax systems and determine the parts of the income distribution where the tax system is regressive, progressive and proportional. However, this is a rather tedious exercise to compare the progressivity of tax systems across countries.

To measure the progressivity of taxes, I use an index proposed by Kakwani (1977).² This measure is based on the above definition of progressivity, but characterises an entire tax system with a single index. This index is essentially the difference between the inequality of income and the inequality of tax payments. Figure 1 shows this measure.

The cumulative density function of income, $F(x)$ is plotted on the horizontal axis, where x is the income level. The vertical axis represents two other cumulative density functions. The one closer to the 45 degree line is the accumulated fraction of total income of those who have income less or equal to x , denoted by $F_1(x)$. This is the Lorenz curve of income and the Gini coefficient, G is twice the area between the 45 degree line and $F_1(x)$. The line further from the 45 degree line is the accumulated fraction of the total tax burden of those who have income less than or equal to x , this is denoted by $F_1(T(x))$. This curve is the *concentration curve* of taxes. The concentration index of taxes, C is defined as twice the area between the 45 degree line and $F_1[T(x)]$. Kakwani's measure of progressivity is $P = C - G$, which is equivalent to twice the integral of $(F_1(x) - F_1(T(x)))$, which is twice the shaded area on Figure 1.

If this measure is positive (as in Figure 1), then the tax system is said to be progressive, conversely a

²This is quite similar, although not the same as the one developed in Suits (1977). For a comparison of the two see Formby, Seaks, and Smith (1981).

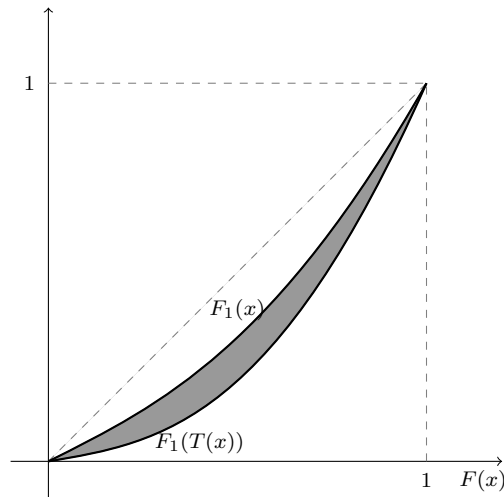


Figure 1: Index of progressivity

negative measure implies a regressive system, while a zero value implies a proportional system. However, since this measure captures the progressivity of a system in a single number, the same value P can be assigned to quite different tax systems, just as different income distributions can have the same Gini coefficient.

Consider for example a perfectly proportional tax system, where those who earn k percent of total income bear exactly k percent of the total tax burden, and hence the $F_1(T(x))$ curve and the $F_1(x)$ curve are perfectly aligned. This implies a progressivity index equal to zero, $P = 0$. Note however, that a progressivity index of zero can be reached in other ways, for example by $F_1(T(x))$ below $F_1(x)$ for low values of x and above it for higher values of x . This system is only neutral on average, on some parts of the income distribution it is progressive, whereas on other parts it is regressive.

This is a drawback that is bound to arise with any measure that captures progressivity in a single index, since progressivity depends on the entire income distribution. However, when considering tax systems already in place, this drawback is not so severe, since the shape of the curve $F_1(x) - F_1(T(x))$ is similar in all countries considered.³ As the shape is in general similar, similar values of P truly reflect similar degree of progressivity.

An alternative and widespread measure of tax progressivity is the difference between the pre-tax (G) and the post-tax (G^*) Gini coefficients, which was introduced by Musgrave and Thin (1948). Kakwani (1977) shows, that while this difference measures the *redistributive effect* of a tax system, it captures not only the progressivity of the system (P), but also the effects of the average tax rate ($\bar{\tau}$).⁴ The decomposition is the following:

³See graphs in Section C.1 of the Appendix.

⁴The decomposition by Kakwani assumes that individuals with equal income pay equal tax, i.e. that there exists a tax function, $T(x)$ which takes the same value for everyone with income x . Aronson, Johnson, and Lambert (1994) show that the difference between the pre- and the post-tax Gini coefficients depends on other factors as well, if there is unequal treatment of equals, for example by treating incomes differently depending on the source of the income. Here, since I do not have such detailed data on incomes, I omit this analysis.

$$RE = G - G^* = \frac{\bar{\tau}}{1 - \bar{\tau}} P.$$

This shows, that the redistributive effect is increasing both in the average tax rate and in the progressivity of the tax scheme. For example, by doubling all tax rates, the progressivity of the system does not change, but the average tax rate doubles, implying that the Musgrave-Thin measure increases as well.

In what follows, I present both Kakwani's measure P of progressivity and Musgrave-Thin's measure of the redistributive effect of tax systems for 17 OECD countries, and a similar pattern emerges for both measures.

3.2 Data sources

To calculate the progressivity index across countries I need data on the income distribution and the tax scheme of these countries.

I use the harmonised European Union Labour Force Survey (ELFS) from 2005, supplemented by earnings data from the Eurostat Structure of Earnings Survey 2006 (SES) to create the income distribution for 17 OECD countries.⁵ I proxy the distribution of incomes with a discrete categorization of the workforce into occupation groups. Since finer than first-digit occupational data is not available for all countries, I use first-digit occupation groups from the International Standard Classification of Occupations (1988 - ISO-88(COM)), which divides the workforce into nine occupations. The ELFS contains information on the number of employees and self-employed individuals for these nine main occupation groups, however, data on earnings is not available in this survey due to anonymity requirements.

Table 1: Relative earnings and self-employment across occupations

| occupation category | relative earnings | share in workforce | share of self-employed |
|--|-------------------|--------------------|------------------------|
| Legislators, senior officials & managers | 1.87 | 0.09 | 0.37 |
| Professionals | 1.41 | 0.15 | 0.11 |
| Technicians & associate professionals | 1.08 | 0.16 | 0.09 |
| Clerks | 0.83 | 0.11 | 0.02 |
| Craft & related trades workers | 0.85 | 0.13 | 0.15 |
| Plant & machine operators & assemblers | 0.82 | 0.08 | 0.07 |
| Service, shop & market sales workers | 0.69 | 0.15 | 0.06 |
| Skilled agricultural & fishery workers | 0.67 | 0.04 | 0.52 |
| Elementary occupations | 0.64 | 0.09 | 0.04 |

Notes: The first column contains the occupation categories. The second column contains the cross-country average of relative earnings of each occupation category relative to the average earnings within that country, for the 17 countries based on data from the SES 2006 data. The third column contains the share of the workforce working in that specific occupation across 16 countries from the EU LFS 2005 data (excluding Italy, as data is not available in the 2005 survey). The fourth column contains the average share of self-employed across 16 countries. Including Italy using the 2003 and 2008 surveys does not significantly change the values reported here.

Earnings information is taken from the SES, which, like most earnings surveys only records employees, since earnings data for self-employed individuals is generally not reliable. As can be seen in Table

⁵Austria, Belgium, Germany, Denmark, Spain, Finland, France, Greece, Iceland, Ireland, Italy, Luxembourg, Netherlands, Norway, Portugal, Sweden, United Kingdom. I use the ELFS 2005 and 2008 for Italy, as Italy is missing from the 2005 survey.

1, in some occupations a significant part of the workforce is self-employed, hence excluding them when analyzing the distribution of earnings would lead to significant discrepancies. Therefore, I include the self-employed when constructing the income distributions. Since data on the earnings of self-employed is not available, I assume that their average earnings are the same as the average earnings of employees within each occupation category.⁶ For some countries the average earnings in certain occupation groups are missing, I interpolate these values from the average earnings in the other occupations groups.⁷

The data on tax structures is taken from the OECD's Taxing Wages 2006 publication, where each country's tax code is described in detail. In all calculations I only take into account personal income taxes at all levels of government. I calculate the income tax function for all countries in my sample. Most tax schemes provide tax brakes or tax credits after dependents, varying with the number of earners in the household. For simplicity, I treat everyone in my sample as a single earner without any dependents.

3.3 Progressivity indices

Table 2 contains the Gini coefficient, the progressivity index, the redistributive effect and the average tax rate calculated in the above described way. It is important to note that all countries in my sample implement a progressive income tax scheme. Note that Denmark, Norway, and Sweden, which are typically regarded as countries with very progressive tax systems, all have relatively *low* progressivity indices (0.04, 0.06, 0.07). Their progressivity indices are so low, that even with relatively high average tax rates, the redistributive effect of the personal income tax is below average. On the other hand, Greece, Portugal, the Netherlands and Ireland have very high progressivity indices (0.38, 0.34, 0.29, 0.25), and relatively low average tax rates (0.10, 0.13, 0.12, 0.12), resulting in a high redistributive effect.

Figure 2 presents the correlation between income inequality and the progressivity of taxes and the redistributive effect. The left panel of this Figure shows the progressivity index of the personal income tax, P , which is the integral of the difference between the cumulative share of total income and the cumulative share of total taxes with respect to $F(x)$ plotted against the Gini index. The right panel in the same figure shows the redistributive effect of the personal income tax, RE , again plotted against the Gini coefficient. Note that both graphs show a clear positive correlation: countries with a higher Gini coefficient tend to have more progressive tax systems, that achieve more redistribution. Since $P = C - G$, if C , the concentration index of taxes was unrelated to inequality, then one would expect a negative relationship between P and G . However, the linear trend line shows a positive correlation that is significant at 1.3%: higher income inequality and more progressivity seem to go together. The right panel shows the change in the Gini coefficient due to the income tax. The positive correlation implies

⁶Several studies show (for example Pissarides and Weber (1989), Clotfelter (1983), Slemrod (1985), Feinstein (1991), Andreoni, Erard, and Feinstein (1998), Slemrod (2007), Feldman and Slemrod (2007), Kleven, Knudsen, Kreiner, Pedersen, and Saez (2011), Hurst, Li, and Pugsley (2011)), that on average, self-employed individuals have lower earnings than employees, and that they consume a significantly higher fraction of their earnings than employees. This can be explained either by self-selection and preferences (i.e. self-employed enjoy the freedom of setting their own hours more and have a lower savings rate) or by tax evasion, whereas the self-employed earn the same amount, but declare a lower fraction of it, which explains the discrepancy both in earnings, and in saving rates. Here I take the stand, that self-employed earn the same amount, just declare a lower fraction of their earnings.

⁷Robustness checks on interpolation methods shows that the results are not sensitive to the different methods.

Table 2: Gini coefficients, progressivity and redistributive effect

| country | Gini | Progressivity | Redistributive effect | Average tax rate |
|-------------|------|---------------|-----------------------|------------------|
| Austria | 0.21 | 0.17 | 0.05 | 0.21 |
| Belgium | 0.20 | 0.10 | 0.04 | 0.28 |
| Denmark | 0.12 | 0.04 | 0.02 | 0.38 |
| Finland | 0.17 | 0.11 | 0.04 | 0.27 |
| France | 0.17 | 0.10 | 0.02 | 0.16 |
| Germany | 0.18 | 0.13 | 0.04 | 0.22 |
| Greece | 0.20 | 0.38 | 0.04 | 0.10 |
| Ireland | 0.17 | 0.25 | 0.04 | 0.12 |
| Iceland | 0.15 | 0.07 | 0.02 | 0.24 |
| Italy | 0.23 | 0.15 | 0.04 | 0.19 |
| Luxembourg | 0.24 | 0.22 | 0.04 | 0.16 |
| Netherlands | 0.15 | 0.29 | 0.04 | 0.12 |
| Norway | 0.12 | 0.06 | 0.02 | 0.22 |
| Portugal | 0.33 | 0.34 | 0.05 | 0.13 |
| Spain | 0.20 | 0.16 | 0.03 | 0.15 |
| Sweden | 0.12 | 0.07 | 0.02 | 0.26 |
| UK | 0.23 | 0.11 | 0.03 | 0.20 |
| average | 0.19 | 0.16 | 0.03 | 0.20 |

Notes: The first column contains the countries, the second column contains the Gini coefficients, the third contains the progressivity indices, the fourth column contains the redistributive effect, and the fifth column the average personal income tax rates. Author's own calculations from ELFS 2003, 2005 and 2008, SES 2006, OECD Taxing Wages 2006.

that the income tax has a larger redistributive effect in more unequal societies. However, the coefficient of correlation is significantly smaller than one, implying that countries where gross earnings are more unequal, the net earnings are more unequal as well, even though to a smaller extent.

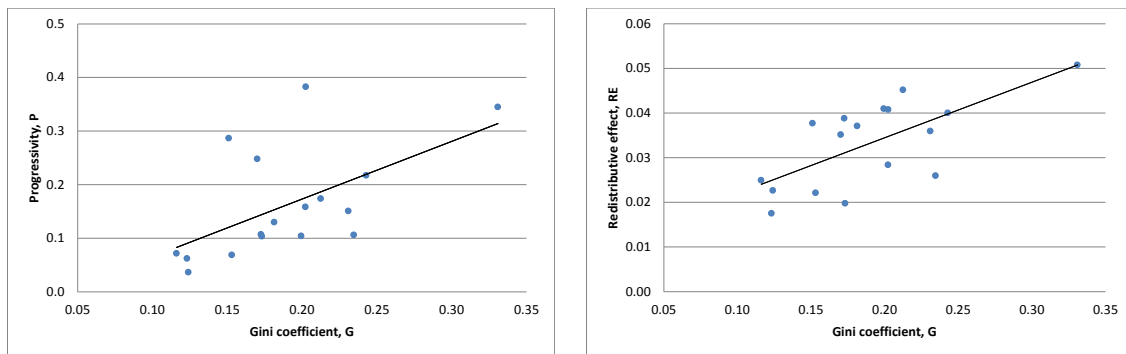


Figure 2: Progressivity of the income tax, redistributive effect and Gini coefficients

Notes: For the progressivity index the linear trendline is given by $P = 1.0763G - 0.042$, where the coefficient on G is significant at 1.3%. The redistributive effect's trendline $RE = 0.1244G + 0.0096$, where the coefficient on G is significant at 0.1%. The progressivity indices and the redistributive effects are the author's own calculations as described in the text. Data sources: OECD Taxing wages 2006, ELFS 2005 (2003 and 2008 for Italy), SES 2006.

In the next section I present a theoretical explanation for why more unequal countries have a more progressive income tax scheme. The basic idea is that the more unequal the society is, the further are the rich from the rest of the population. The further are the rich, the less likely it is that they participate in setting the tax scheme, and hence the more likely it is that a significant fraction of the tax burden will fall on them.

4 Model

The society consists of three groups, who have to decide on how to finance the public good, g . They set three tax rates, made to resemble the most commonly observed bracket-type tax structure, with the marginal rates given for each bracket. The trade-off is clear: those with lower income aim to have high marginal rates at the top, whereas those with high income aim to have high marginal rates at the bottom.

The coalition formation model is based on the model developed by Levy (2004). Each of the three groups is represented by one candidate, who decides whether to run alone, and offer his ideal policy, or to run in coalition with another candidate, in which case they offer a policy from their Pareto set. Each citizen votes on the candidate or party that offers the policy that is best for him. The equilibrium of the game is a partition of the candidates into parties and the policy platform that each offers.

4.1 Admissible policies and preferences

Citizens differ in their level of income: people are either poor, middle income, or rich, with the following income levels: $y^P < y^M < y^R$. The size of group $i \in \{P, M, R\}$ is α^i , and the total population is normalised to one, $\sum_i \alpha^i = 1$.

A public good, g has to be financed by taxes. The tax structure is the following: everyone pays τ^P fraction of their income below y^P , τ^M fraction of income between y^P and y^M , and τ^R fraction of income above y^M . The balanced budget condition is:

$$\tau^P y^P + (\alpha^M + \alpha^R) \tau^M (y^M - y^P) + \alpha^R \tau^R (y^R - y^M) = g \quad (1)$$

Let $\Delta(g) \subseteq \mathbb{R}^3$ denote the set of *admissible policies*, which consists of tax triples $\theta = \{\tau^P, \tau^M, \tau^R\}$, for which the balanced budget condition is met and all tax rates are between 0 and 1. Taking g as exogenous leaves two free variables: any two tax rates uniquely determine the third.

Given a tax structure, θ , the utility of the poor, middle income and rich individuals are the following:

$$U^P(\theta) = H(g) + (1 - \tau^P)y^P, \quad (2)$$

$$U^M(\theta) = H(g) + (1 - \tau^P)y^P + (1 - \tau^M)(y^M - y^P), \quad (3)$$

$$U^R(\theta) = H(g) + (1 - \tau^P)y^P + (1 - \tau^M)(y^M - y^P) + (1 - \tau^R)(y^R - y^M). \quad (4)$$

Citizens derive utility $H(g)$ from the public good, g . For simplicity I assume that the utility is linear in disposable income. This simplification does not affect the results qualitatively, as the shape of the utility function only has a quantitative effect on how the different groups value the different taxes and the trade-offs between them.

4.2 Ideal policies and indifference curves

Within an income group agents have identical preferences. The *ideal policy* (or the set of ideal policies) of a group is the policy platform that maximises their utility from the admissible set. In general each group wants to reduce their own tax payment. This implies that the poor want as low τ^P as possible, the middle want τ^P and τ^M to be low, while the rich would prefer all tax rates to be as low as possible. However, since the public good has to be financed from taxes, there is a trade-off between the tax rates, and the different income groups value this trade-off differently. The poor only care about the level of τ^P , and their utility increases as τ^P decreases. Hence their ideal policy or set of ideal policies are those where τ^P is minimal within $\Delta(g)$. The middle income care about τ^M and τ^P , hence their ideal policy is where τ^R is the highest possible. If expropriating all income above y^M does not cover the public good, the middle income prefer to increase τ^P , as one unit of revenue is financed by all groups, whereas increasing τ^M would only increase the burden on the rich and themselves. Finally, the rich would like to share as much of the financing of the public good as possible with the other members of the society, hence they favour to first increase τ^P as much as possible, and then τ^M .

Figure 3 shows the ideal policy of each group and their indifference curves.⁸ Note that each group has linear indifference curves, as utility only depends on disposable income, which is a linear function of the tax rates.

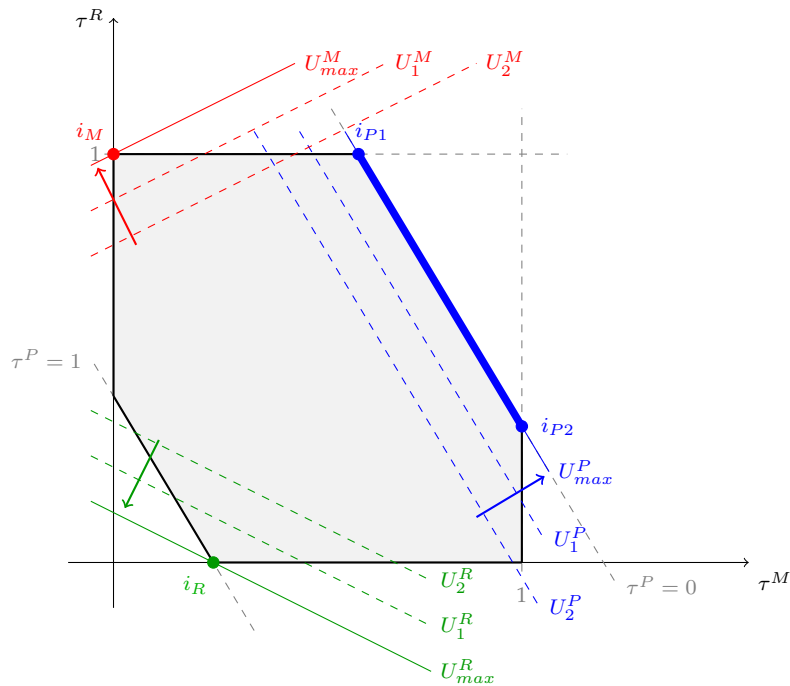


Figure 3: Ideal policies and indifference curves

The horizontal axis represents the middle tax rate, τ^M , whereas on the vertical axis is τ^R , the top tax

⁸See Section C.2 of the Appendix for the analytical solution of the ideal policies.

rate. As mentioned earlier, the third tax rate, τ^P is implicitly defined as one that allows the budget to be balanced. The grey dashed lines represent the boundaries on the minimum and maximum possible rates ($\tau^i = 0$ and $\tau^i = 1$). The set of admissible policies, $\Delta(g)$, is the shaded hexagon bounded by the horizontal ($\tau^R = 0$) and vertical ($\tau^M = 0$) axis, the $\tau^R = 1$ and $\tau^M = 1$ line and finally by the $\tau^P = 0$ and $\tau^P = 1$ lines.⁹

The set of ideal policies of the poor are represented by the thick blue line, at the top-right part of $\Delta(g)$. The poor want to finance as little as possible of the public good from τ^P , hence their ideal policy is where τ^P is minimal. In this graph the ideal policies of the poor are the platforms where $\tau^P = 0$. If the required amount of tax would be higher, the $\tau^P = 0$ line would shift out, gradually reducing the measure of ideal policies of the poor, up to the point of leaving the poor with one ideal policy, where the other two tax rates reach their maximum level, $\tau^M = \tau^R = 1$.

The indifference curves of the poor are depicted by the dashed blue lines parallel to the $\tau^P = 0$ line. Utility is increasing along the blue arrow, and maximum utility is achieved on the solid blue line. As the poor only care about the level of τ^P , their indifference curves are sets of points for which τ^P is constant. It is straightforward that these indifference curves are downward sloping in the $\{\tau^M, \tau^R\}$ space: if τ^M increases, a lower τ^R is enough to balance the budget.¹⁰

The ideal policy of the middle income group is represented by the red dot at the top left corner of the admissible policies. The middle income prefer to finance the public good primarily by τ^R , then by τ^P and want as low τ^M as possible. In this graph this platform is where $\tau^R = 1$, $\tau^M = 0$ and τ^P is such that the tax requirement is met. For lower levels of public good, the $\tau^P = 0$ line shifts down, eventually until the ideal policy of the middle income would consist of $\tau^M = \tau^P = 0$ and $\tau^R \leq 1$ to meet the tax requirement.¹¹ On the other hand, with the level of public good increasing, the $\tau^P = 1$ line would shift out, leading to the ideal policy of the middle moving along the $\tau^R = 1$ line, with positive τ^M .

The dashed red lines represent the middle income group's indifference curves, with utility increasing along the red arrow, and maximum utility reached at the solid red line. An increase in τ^M reduces the utility of middle income individuals by more than how much it improves the budget. To be kept indifferent they need a decrease in τ^P , which would leave the budget in deficit unless τ^R increases as well. This implies that their indifference curves are upward sloping.

Finally, the ideal policy of the rich is represented by the green dot in the bottom left part of the admissible policies. The rich prefer to finance as much as possible of the public good by τ^P , and as little as possible by τ^R . If the tax requirement is higher, the $\tau^P = 1$ line shifts out, thus moving the ideal policy of the rich along the $\tau^R = 0$ line.

The indifference curves of the rich are represented by the dashed green lines, with utility increasing along the green arrow, and the highest utility achieved at the solid green line. To understand why the indifference curves are downward sloping, consider an upward sloping line in the $\{\tau^M, \tau^R\}$ space.

⁹ $\Delta(g)$ is not necessarily a hexagon: as the $\tau^P = 0$ line shifts out, the top corner of the square will not be cut off, while as the $\tau^P = 1$ line shifts down, the bottom corner will not be cut off, thus leaving $\Delta(g)$ a pentagon or quadrangle.

¹⁰See Section C.2 of the Appendix for the analytical expression of the slope of the indifference curves.

¹¹In this case the ideal policy of the middle income would be an ideal policy for the poor as well.

An increase in τ^M or τ^R hurts rich individuals more, than by how much it improves the balance of the budget. Hence, the magnitude of reduction they require in τ^P would leave the budget unbalanced. Therefore their indifference curves are downward sloping. It is easy to see, that their indifference curves are less steep than those of the poor. Rich individuals are hurt more by an increase in τ^R , than the poor, hence a decrease in τ^M has to be financed by an increase in both τ^P and τ^R . (As opposed to the poor, who can be compensated by solely increasing τ^R .)

4.3 Policy selection

The society consists of three very distinct groups of citizens, who are divided on the issue of how to finance the public good. Through a political process the citizens choose a policy to be implemented from the set $\Delta(g)$. I adapt the political model introduced by Levy (2004).

The most important assumption is that parties or single candidates can only offer *credible policies*, policies that are in the Pareto set of their members. Parties are the union of one or more representatives. Therefore, if a representative of the poor, middle income or rich group runs on his own, his only option is to offer his (and his group's) ideal policy. Underlying this assumption is the idea that politicians cannot credibly commit to implementing a policy. The citizens understand that once elected, the politician has the freedom to implement whatever he wishes, which leads to the implementation of his own ideal policy. In such a setup parties serve as a commitment device. Since members of parties have potentially opposing interests, once they offer a policy from their Pareto set, they cannot agree on implementing something else, as this would make some member of the party worse off.

I assume that each group has one representative, who I denote by P , M and R , and each representative's ideal policy coincides with his group's. Consider a partition on the set of politicians.¹² For instance $PM|R$ represents the case where the representative of the poor and the middle income form a party, and compete in the election with a joint platform against the representative of the rich. On the other hand, for example, $P|M|R$ represents the case where all representatives run as individual candidates.

For now, assume that the partition of politicians into parties is given. Each party decides whether to run in the election or not, and if running, decides which policy to offer from its Pareto set. Running in the election entails a small cost, $\varepsilon > 0$. Citizens then vote on the policy that maximises their utility. The election is won by the party or candidate who receives the highest vote share, and this party then implements the policy they offered.¹³

Given a partition, a set of policy platforms is an equilibrium, if taking the other parties' actions as given, no party has an incentive to alter its action. A party can alter its action by switching to another platform, by withdrawing from or by joining the electoral competition. The party has an incentive to

¹²I do not consider the possibility of a full coalition, i.e. when all candidates join in one party.

¹³In case of a tie, all parties that tied have equal probabilities of winning. If none of the parties decides to run in the election, a status quo is implemented. I assume, as Levy (2005) that the status quo is worse for everyone than any other outcome. This ensures that in equilibrium, at least one party runs and some platform is chosen.

do this, if this action improves the utility of all of its members. Given the partition of representatives into parties, the set of equilibrium winning platforms can be found, which are the platforms that are implemented given a set of equilibrium strategies.¹⁴

Finally, the last step is to identify the *stable political outcomes*. Stability is defined in a recursive way as in Ray (1997). Representatives start from some coalition structure and are allowed to break this structure up into finer ones. Deviations can be done by one or more representatives jointly. Credible threats are deviations to finer partitions which are stable themselves, since deviators take into account future deviations. In this setup, since there is either one two-member coalition, or everyone is running as a single representative (which is stable by nature), the only deviation to consider is a member leaving the coalition and thus reducing the game to three single representatives. Representatives, when considering splitting from the coalition, take into account the outcome of the single representative election. I identify the stable partitions together with their equilibrium winning platforms.

5 Equilibrium

In this section I present the equilibrium outcome of the coalition formation game. The results suggest that when inequality is low, the poor and the rich representative form a coalition and win the election with a platform that has a high middle tax rate and low tax rates on the bottom and top parts of income. At intermediate inequality levels all or some of the two-representative coalitions can be stable. Finally, at high levels of inequality, the poor run alone and win the elections with a highly progressive tax scheme.

First, I present the equilibrium outcomes in the absence of coalitions, as this is the outside option if a coalition member decides to split from his party. Then I present the equilibrium partitions and strategies for the full model.

5.1 Equilibrium without coalitions

Consider the case when the representatives cannot form coalitions. In such a setup, the only question is who will compete in the election and with which platform. As Figure 3 shows, the representative of the middle income group and of the rich each have one ideal point, i_M and i_R . The representative of the poor can offer any point from the $i_P = [i_{P1}, i_{P2}]$ set, as all these points give him equal utility.

Figure 4 depicts the different regimes. The colour red is assigned to the middle income group: the red dot indicates their ideal policy, i_M , while the dashed red line is one of their indifference curves. The colour green is assigned to the rich, while blue indicates the poor. The equilibrium winning strategy (or set of strategies) is indicated in black. It is important to note, that the poor always weakly prefer the policy of the middle income group to that of the rich, since i_R contains the highest possible τ^P , and

¹⁴Given a set of platforms, in general, there is only one platform that receives the highest vote share in pure strategies. I focus on pure strategy equilibria when they exist.

hence the worst payoff for them.

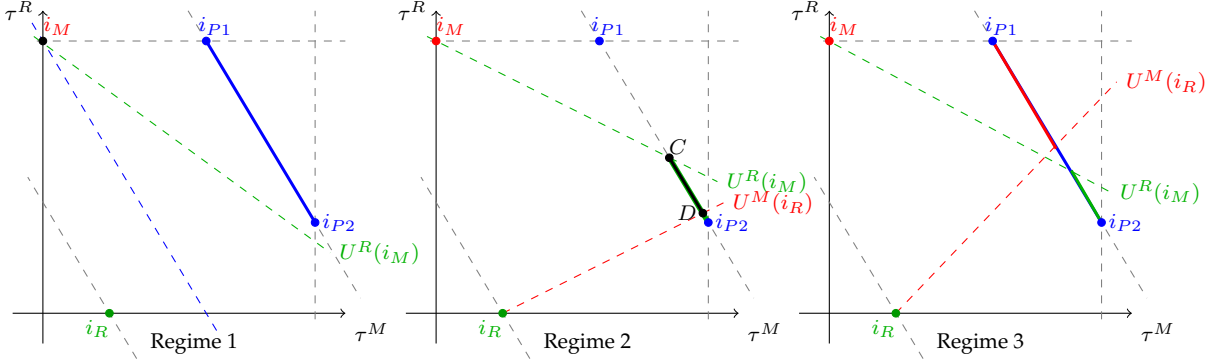


Figure 4: Winning party and platform

In Regime 1, the rich prefer the ideal policy of the middle income group to the entire set of ideal policies of the poor. All three candidates entering the electoral competition is not an equilibrium, as the representative of the group with the highest population share wins, and either one of the other representatives have an incentive to drop out, as that does not change the outcome and saves the cost ε . Similarly, two candidates running does not constitute an equilibrium, since the losing candidate has an incentive to withdraw from the competition. The only possible equilibria are those where only one candidate is running, and this can only be the middle income group's representative. If any other candidate is running on his own, the middle representative can enter and win the election, since the third group (the rich or the poor) will vote for him in the election. The only equilibrium strategy in this case is: $\{\emptyset, i_M, \emptyset\}$, and the equilibrium winning platform is i_M . This is represented by the black dot at the ideal policy of the middle income group.

In Regime 2 and in Regime 3, the rich prefer some policies of the poor to the the ideal policy of the middle (the policies below $U^R(i_M)$), and the middle prefer some policies of the poor to the ideal policy of the rich (the policies above $U^M(i_R)$).

In Regime 2 these two sets have an intersection: the black segment of i_P contains those ideal policies of the poor, which are preferred by the rich to the middle income group's ideal policy, and by the middle income to the rich group's ideal policy. Similarly to the previous case, more than one candidate running cannot constitute an equilibrium. In these cases the equilibrium strategy set has P running uncontested with a policy from the black line segment and winning the election: $\{\theta_P \in (C, D), \emptyset, \emptyset\}$. The equilibrium winning platform can be any policy from the set (C, D) , and in expectation it will be: $E = (C + D)/2$.

In Regime 3, none of the ideal policies of the poor are preferred both by the rich to the middle income's ideal policy and by the middle income to the rich group's ideal policy: $i_P \cap U^R(i_M)^+ \cap U^M(i_R)^+ = \emptyset$. In these cases, there is no equilibrium where one representative runs uncontested and

wins the election, since none of the ideal policies is a Condorcet winner.¹⁵ The equilibrium in this case features mixed strategies, and depends on which group is the largest in the society. In what follows I present the results; for details of mixing probabilities and complete characterization of the equilibria see Section C.3 of the Appendix.

If the rich constitute the largest part of the society ($\alpha^R > \alpha^M, \alpha^P$), then P mixes between running with i_{P1} and not running, M mixes between running and not running, while R runs with i_R . The expected equilibrium winning platform, E in this case is close to i_R , as this is implemented if both or neither of the other representatives enters the electoral competition.

When the middle income is the largest group, then P plays i_{P2} and M and R are mixing between running and not. In this case the expected platform E is close to i_{P2} , as if R does not enter, then i_{P2} is implemented.

Finally if P is the largest group, then all representatives mix: P between i_{P1} and i_{P2} , M and R between running and not.¹⁶ The expected equilibrium winning platform in this case is an interior point of i_P , as in most cases one of the ideal points of the poor is implemented.

5.1.1 Inequality and indifference curves

To understand what determines whether the economy is in Regime 1, Regime 2 or Regime 3, the forces that shape the indifference curves and the admissible policies have to be analyzed. The differences in income levels, the shares of the different income groups and the level of public good are the factors that have to be considered. The differences in income levels are crucial, since these govern how efficient the three tax rates are in raising revenue.

As $y^R - y^M$ decreases, τ^R loses its role as a policy tool: an increase in τ^R changes the revenue collected by less, and hence allows a smaller reduction in the burden on middle and low incomes. As the role of τ^R is reduced the conflict between M and P becomes sharper. A reduction in τ^P cannot be offset so easily by increasing τ^R , and implies a larger increase in τ^M . The set of ideal policies of the poor, i_P is reduced, and in particular all policies will feature high τ^M rates. At the same time, i_M will feature a high τ^P . In addition, a low $y^R - y^M$ implies that the effect of τ^R on the utility of the rich is relatively small. Given this, the rich will prefer the ideal policy of the middle income group. So as $y^R - y^M$ decreases, it is more likely that the economy is in Regime 1, where the median policy is i_M .

A small difference between y^M and y^P erodes the role of τ^M as a policy tool. This leads to an increased conflict of interest between the poor and the rich: they can only achieve their goals at the expense of one and other. The set of ideal policies of the poor is small in this case as well. However, with low $y^M - y^P$, all policies in i_P feature relatively high τ^R rates. This in turn makes the rich prefer

¹⁵To see this consider first the poor representative running alone with a policy from i_P below $U^M(i_R)$, from the green part of i_P . The rich group has an incentive to enter and win the election with policy i_P . If P runs with a policy from i_P and above $U^R(i_M)$, from the red part of i_P , then the middle income group can enter and win the election with i_M . If P runs with a policy from the blue part of i_P , then both M and R can enter and win the election. If M runs alone, then P can beat him for example by i_{P2} , while R can be beaten by M or by P if running with i_{P1} .

¹⁶When P is mixing between the extremes, it could be possible that it is better to mix between some interior points. However, numerical tests show that P achieves the highest payoff when mixing between i_{P1} and i_{P2} .

the ideal policy of the middle income group, as they prefer increasing τ^P to increasing τ^M . Again, as $y^M - y^P$ decreases, the likelihood of the economy being in Regime 1 increases.

Finally consider the case of a very low y^P . In this case, there is not much to gain from increasing τ^P , and it is the rich and the middle income whose goals are in sharp contrast. In such a case, P can offer a policy from i_P , which both the rich and the middle income prefer to each others ideal policy. So as y^P decreases, it is more likely that the economy is in a situation depicted in Regime 3.

When none of these measures is small, a situation arises when there is no Condorcet winner among the ideal policies. The ideal policy of M is not better than all the ideal policies of P in the eyes of the rich, as i_P contains some policies with low enough τ^R . However, these policies have too high τ^M , so the middle income prefer i^R , which is characterised by a lower τ^M and a high τ^P . On the other hand, the middle prefer some ideal policies of the poor to i_R , since there are some with high τ^R and low τ^P . Compared to these policies, the rich prefer the ideal policy of the middle income. In such cases, the economy is in Regime 3.

To summarise, Regime 1 depicts economies where $y^R - y^M$ or $y^M - y^P$ is small, Regime 2 shows economies where y^P is small or $y^R - y^M$ or $y^M - y^P$ is higher, and Regime 3 shows a situation, when all these measures are intermediate.

The total amount of revenue to be collected, g , also affects the incidence of the different cases. For a very low g , it is relatively easier to finance the public good, both the $\tau^P = 0$ and the $\tau^P = 1$ lines shift down. This implies, that Regime 1 arises for fewer combinations of parameters: P can always offer a platform with a low τ^R and τ^P , which the rich prefer to i_M . Regime 3 is also less likely, as i_P can contain platforms with moderate τ^M and τ^R , something that both the rich and the middle income prefer to the other's ideal policy. On the other hand, as g increases, the $\tau^P = 0$ and the $\tau^P = 1$ lines both shift out, thus reducing the ideal policies of the poor. This implies that it will be less likely that P can offer a policy that the rich prefer to i_M , hence making Regime 1 more likely.

5.2 Equilibrium with coalitions

The definition of a stable political outcome immediately implies that a representative who would win in the absence of parties cannot be a member of a party in a stable political outcome. If a winning representative were a member of a party, he could break up this coalition, thus returning to the individual candidate election, run and win. This would guarantee him the highest possible payoff, making a partition with him in a coalition unstable with any platform.

In Regime 1, when the middle income group's ideal policy is a Condorcet winner, the only real coalition that can be part of a stable political outcome consists of the representative of the poor and the rich. Therefore, the only partitions that can be stable are $\{PR|M\}$ and $\{P|M|R\}$.

A party consisting of P and R can offer anything from their Pareto set, which is depicted by the black line, on the border of $\Delta(g)$ between i_{P2} and i_R . Any point below the dashed green line is better for the rich than i_M , while any point to the right of the dashed blue line is better for the poor than

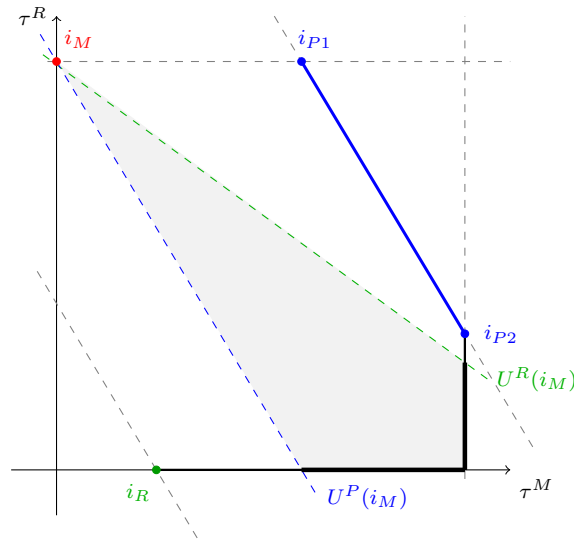


Figure 5: Poor-rich coalition

i_M . The shaded area contains the platforms that are better for both the rich and the poor than i_M , and hence any platform from this area would receive the votes of both groups. The platforms that can be *winning equilibrium platforms* are indicated by the thick black line, and are at the intersection of the shaded area and the Pareto set of R and P . Since the indifference curves of the rich are less steep than the indifference curves of the poor, this set is never empty.

Thus the party of the rich and the poor can win by advocating policies which are characterised by heavy taxation on incomes between y^P and y^M , and low taxes on income below y^P and above y^M . Against competition from the middle income, these policies attract the votes of the groups it represents. The party is also stable as neither the rich nor the poor want to break it. When the economy is in Regime 1, when the middle income are not sufficiently different from either the rich or the poor, a coalition comprising the rich and the poor will be stable. This is the case, as when the middle income group is very similar to either the rich or the poor, then their ideal policy constitutes a median policy, which gives the group too much power, thus disabling them from credibly committing to any coalition. However, since the middle income group is still different from the other two groups, the others can agree on at least one aspect: to put a disproportionate burden on middle income levels, this way achieving relatively low tax rates at both extremes.

In Regime 2, the only possibly stable coalition is between M and R , since P wins the single candidate election. Whether M and R can form a stable coalition and offer a pure strategy winning equilibrium platform depends on whether there is a platform that beats the set of ideal policies of the poor for both their groups.

Figure 6 shows the two possible cases. The left panel, Regime 2a, the platforms from the shaded area are preferred both by the rich and by the middle income group to the *entire set* of ideal policies of

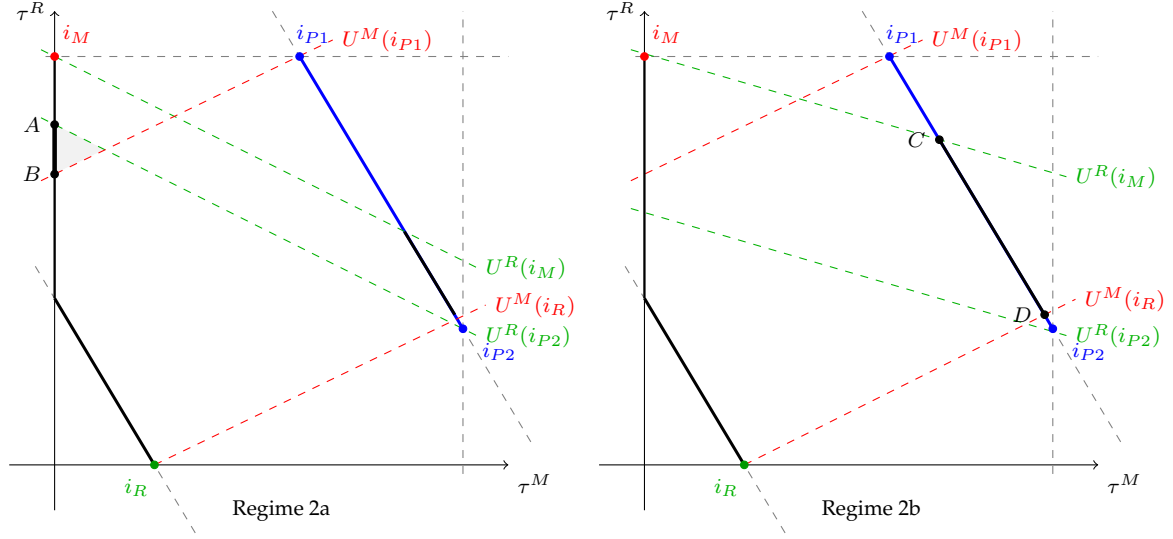


Figure 6: Middle-rich coalition

the poor. Any platform from this area is preferred by all members of the rich and the middle income group to any point from i_P . The Pareto set of the middle income group and the rich are the black line running along the border of $\Delta(g)$ between i_M and i_R . The MR coalition can run uncontested with a platform from $[A, B]$. This guarantees the stability of the coalition, as neither the rich, nor the middle income want to split, and both groups vote for the MR coalition regardless of the policy P would run with. In such scenarios, the MR coalition will offer policies that tax the lowest part of the income very heavily, and have low to medium tax rates on higher parts of income. The expected equilibrium winning platform is $E = (A + B)/2$.

In Regime 2b, on the right panel of 6 there are no platforms in the set of admissible policies that both the rich and the middle income prefer to all of the poor's ideal policies. This implies that the MR coalition cannot offer anything in pure strategies, since P could then offer something from i_P that would be preferred either by R or by M to the coalition's platform, and win. In this case the coalition members would be better off by not running in the election, thus reducing the game to the single representative competition and having P win with $p \in [C, D]$. This implies that in such cases the only stable political outcome is the partition $\{P|M|R\}$ with strategies $\{p \in [C, D], \emptyset, \emptyset\}$, that is P runs uncontested and wins the election with a platform from $[C, D]$.¹⁷ The expected equilibrium winning platform, $E = (C + D)/2$, has very low tax rates on low income levels, and medium to high rates on higher income levels.

Whether the economy is in Regime 2a or in Regime 2b depends on whether M and R can find a platform that is better for both of them than any policy in the ideal set of P . This crucially depends on how sharp the conflict of interest is between the two groups. When y^P is relatively high, the conflict of the two groups is less pronounced. They can agree on increasing τ^P , and when y^P is relatively high,

¹⁷There are cases in which the MR coalition could be stable with a mixed strategy. However, the stability of a coalition with mixed strategies is questionable, therefore I omit the discussion here.

this action sufficiently reduces the amount of public good that has to be financed from τ^M and τ^R , mitigating the intensity of conflict. On the other hand, if y^P is low, even if they agree on taxing it very heavily, this does not allow a satisfactory reduction in *both* τ^M and τ^R . Another important factor is the size of $y^R - y^M$. If this difference is high, then i_M and i_{P1} are close, which makes finding a policy better than i_{P1} for M a hard task. Hence, when y^P is relatively high and $y^R - y^M$ is not very high, M and R can find a platform that is better for both groups than any policy from i_P .

In Regime 3, the model does not uniquely predict the winning party. Since the outside option is always a mixed strategy equilibrium, the expected payoff from breaking the coalition is in general a weighted average of some of the ideal policies. Let E denote the expected equilibrium winning platform of the single candidate game, which is an internal point of $\Delta(g)$.

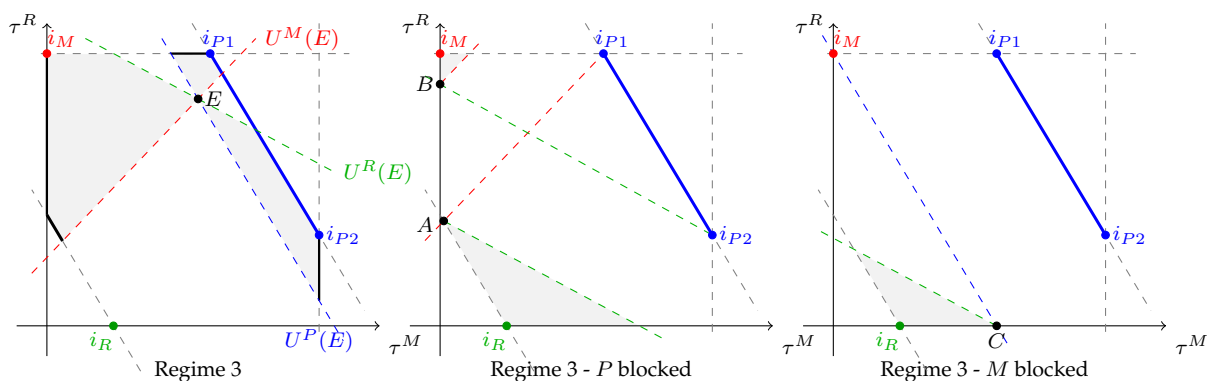


Figure 7: Any coalition

In the left panel of Figure 7 the shaded areas show the platforms that M and R prefer to E (the area to the left of E), that M and P prefer to E (the area above) and that P and R prefer to E (the area to the right). It is easy to see, that due to the slope of the indifference curves, whenever E is strictly inside $\Delta(g)$, any two-representative party has a segment in their Pareto set, which is better than E for both party members.¹⁸

The final thing to consider is whether the third representative is able to offer a platform which captures the votes of one of the coalition member's group, this way winning against the coalition. If this is the case, then the other coalition member does not have an incentive to enter the coalition, because the expected policy E is better for him than the ideal policy of the third representative. This happens, if the policies preferred to E and the policies preferred to the third representative's ideal policy by both coalition members do not have an intersection with the Pareto set of the coalition.

The MP coalition always has an equilibrium winning platform, that cannot be beaten by i_R , since that is worse for both M and P than their entire Pareto set.

¹⁸In the case that $\alpha^M > \alpha^R, \alpha^P$, as $\varepsilon \rightarrow 0$ the implemented platform tends to i_{P2} , which is in the border. In such a case P cannot be a member of a stable coalition.

The middle panel in Figure 7 shows the areas that are blocked by P in case of an MR coalition. Consider a case when the expected policy E is in the top left shaded area. If MR were to run with a policy they both prefer to policy E , this would have to be from $[i_m, B]$. However, this could not be a winning platform, since P could run with i_{P2} and win, since the rich and poor both vote for P . In this case M would not have an incentive to join coalition, since E is better for M than i_{P2} . Hence, if the expected platform of the game in the absence of coalitions is in the top left shaded area, then P can *block* the MR coalition from winning the election. Similarly, if E is in the bottom left shaded area, then P can capture the votes of the middle income by running with i_{P1} , and R would not want to participate in the coalition.

The right panel in Figure 7 shows the area blocked by M if P and R were to form a coalition. Imagine that E is in the shaded area. PR would have to run with a policy from $[i_R, C]$ to be better off than E . In this case, if M enters the electoral competition with i_M , the poor would vote for him, and he would win the election. In this case, the PR coalition is better off by withdrawing from the coalition.

In Regime 3 in general, more than one partition can be stable. In these cases, the prediction of the model on the equilibrium winning platforms are in expected terms, which usually gives an internal point as the expected equilibrium winning platform. It is hard to evaluate the progressivity implied by these expected platforms, as it is in fact an expected progressivity index, one that is never realised. The evaluation of these indices would be difficult, however, none of the 17 countries falls into Regime 3.

6 Predictions and data

In this section I summarise the model's predictions about the relation between inequality and the implied progressivity between regimes as well as within regimes. I also show the progressivity indices the model predicts for the sample countries and compare these predictions to the data.

6.1 Progressivity in the model

When the income difference between the middle income and either the poor or the rich is low to moderate, then the taxes are set either by a coalition of the rich and the poor, or by the middle income group. The coalition sets relatively high tax rates on middle income levels, and low rates on both extremes. This implies a tax system that is not very progressive. It is progressive moving from low to middle incomes, but for higher income levels it is actually regressive. On the other hand, if the middle income group sets the tax scheme, then the rates show the opposite pattern: high rates on top income levels, low rates on middle income levels, and low to medium rates on the lowest part of income. This tax scheme shows more progressivity, as the marginal rates on high incomes are the largest. This implies that for low income inequality countries, if the PR coalition sets the tax rates, then the system will show little progressivity, whereas if M is setting the tax scheme, then the system will be quite progressive.

If the PR coalition is setting the tax scheme within these regimes, for a higher $y^R - y^M$, the system

becomes more progressive, while for a higher $y^M - y^P$ the system becomes less progressive. This is because when the rich are relatively richer, the objectives of the poor and the middle income are more aligned. Hence, even when the poor are in coalition with the rich, the rich have to bear a larger burden of the public good. On the other hand, when $y^M - y^P$ is higher, then a vast part of the public good can be financed by taxing middle incomes, which makes the system less progressive.

If M is setting the tax scheme, then in terms of $y^R - y^M$ a similar pattern emerges. If $y^R - y^M$ is high, then most of the public good is financed by taxes levied on R . However, the progressivity of the tax system does not depend too much on the difference in income levels between the middle income and the poor, since the taxes on middle incomes are kept low anyway.

To summarise, within Regime 1, progressivity increases as inequality increases between the rich and the middle income. Overall progressivity and its dependence on the inequality between the middle income and the poor, depends on who is setting the tax rates.

On the other extreme, when the rich are very rich, or the poor are very poor, Regime 2b is implemented, and it is the poor who regulate the tax system. The system set by the poor is characterised by no taxation for low income individuals. All the public good is financed by the middle income and the rich. These systems are very progressive, as the low income individuals do not (or hardly) contribute to the public good. Within these regimes, as $y^M - y^P$ increases, the progressivity of the implemented regime decreases. This is due to the fact that when $y^M - y^P$ is higher, it is relatively easy to finance the public good by taxing middle incomes.

Regime 2a is implemented for relatively low levels of $y^M - y^P$ and intermediate levels of y^R , and in these cases either the MR coalition or P sets the tax scheme. If P is setting the regime, then the same holds as for Regime 2b. When MR is setting the tax scheme, they put high taxes on low incomes, low taxes on the middle income levels, and intermediate taxes on high income levels. These regimes typically are not progressive, and progressivity is increasing as $y^M - y^P$ is decreasing, since a higher fraction of the public good has to be financed by τ^R .

Finally, the predictions of the model in Regime 3 are less precise and harder to interpret. Without coalitions, the equilibrium is in mixed strategies, whereas with coalitions, there is always more than one stable coalition. However, as presented in the next section, for the countries in my sample, none fall in the region where Regime 3 would be implemented.

6.2 Model predictions and data

In this section I present the model's predictions about the progressivity index in the 17 OECD countries. In general, the model does quite well: more unequal countries implement more progressive tax schemes. Even though the model over-predicts the level of progressivity, it gives a good prediction of the magnitude of increase in progressivity with income inequality.

There are a few difficulties in translating the data into the parameters of the model. First of all, I have data on nine occupation categories, which I have to transform into three groups, the poor, the

middle income and the rich. Second, I have to fix the amount of revenue that needs to be collected.

I divide the nine income categories into three income groups in two ways for each country in my sample. First, I group the three richest occupations into the rich group, the three middle income occupations into the middle income group and the three lowest paid occupations into the poor group (Panel A). The second is a more sophisticated division: country-by-country I cut the nine groups into three categories depending on the distance in income between the different occupations (Panel B).¹⁹

Taking the level of public good to be provided from the data raises several issues. One problem is what part of government expenditure to take as the public good, g . Taking the entire government expenditure is problematic, as not all of it is financed from personal income taxes. Also, the public good has to be measured in terms of economy-wide personal income (or average income): $g = \eta \bar{y}$. From this point of view, it is not clear what to take as \bar{y} , which in the model is economy-wide income. Since the earnings data is only on earnings from employment and self-employment, the best measure would be to take total personal income or labour earnings, however, this data is not available. Another way of getting η from the data is through the average tax rate. In the model, the required amount of public good pins down the average tax rate: $\bar{\tau} = \frac{g}{\bar{y}} = \eta$, and since I have data on the average tax rate for each country, I use these as a proxy for η .

Figure 8 plots the predicted progressivity indices against the actual progressivity indices for the 17 countries. The top row contains the first type of division (Panel A), with three occupations in each income group, while the second row has the country specific division (Panel B), based on the difference in income levels. The left column shows the predictions when the cross country average of the country-specific average income tax rate is used as a proxy for η , whereas the right panel shows the predictions using for each country their own average tax rate as a proxy for η . In all cases I calculated the progressivity index of the implied equilibrium winning platform for all 17 countries, using their actual income and population shares. The full circles show the progressivity implied by the single candidate equilibrium, while if a stable coalition exists, I indicate the progressivity implied by their equilibrium winning platform as well, with empty circles.

The model predicts the existence of stable equilibrium platforms for only a few countries, where income inequality is low. The stable equilibrium platforms that emerge in these cases are mostly PR platforms in Regime 1 or MR in Regime 2a. This can be seen in Figure 8, as the empty circles are in most cases *below* the full circles, in line with the observation that PR coalitions implement a less progressive tax scheme than the one that representative M would implement, and that MR coalitions implement a less progressive system than the one that representative P would implement.

Note that in all cases the model over-predicts the progressivity indices obtained from the data. The reason for this is that the model predicts that in the majority of cases it is the representative of the poor, who wins the election (Regime 2b). In this case, the implemented policy is one that is better for the rich than the ideal policy of the middle income group, and is better for the middle than the ideal policy of the

¹⁹See Section C.4 of the Appendix for division.

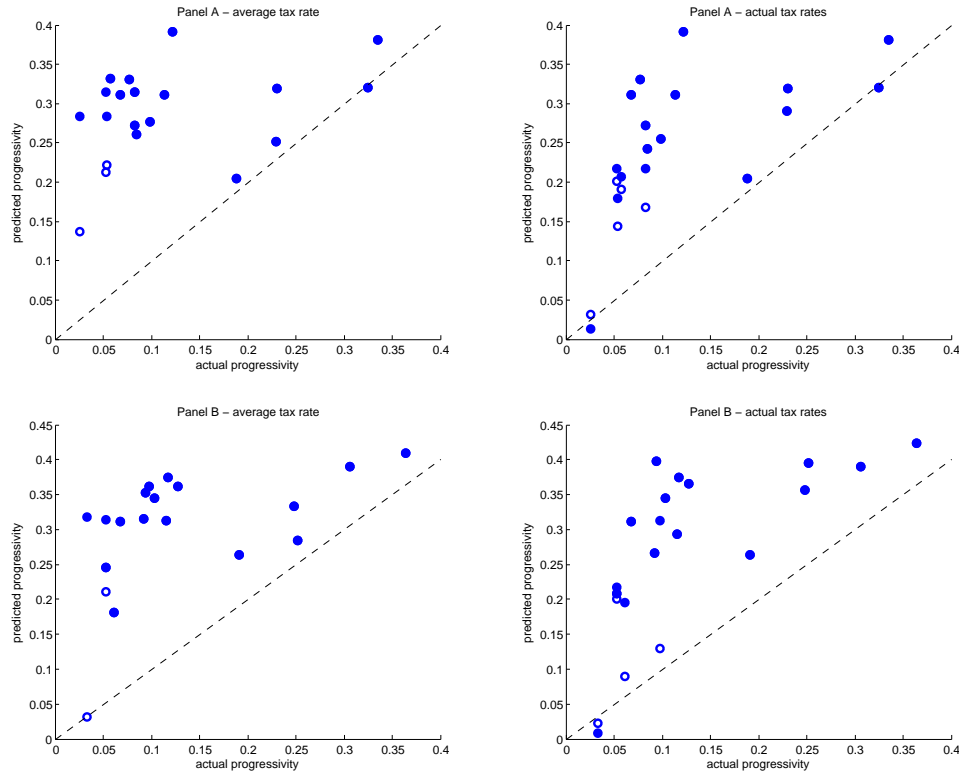


Figure 8: Actual and predicted progressivity indices

Notes: Actual and predicted progressivity index for the 17 OECD countries with different divisions of occupation groups into income categories (Panel A and Panel B) and different levels of public good (actual average tax rate for each country and their cross country average). The actual progressivity indices are calculated using the same division of occupation groups into income categories.

rich. These policies implemented by the poor are always progressive, but the degree of progressivity depends on the income of the other two groups. As inequality increases, the implemented policy taxes the rich more heavily, and hence is more progressive.

The degree of over-prediction of the model is smaller if the actual tax rates are used. This is the case, since countries with lower inequality tend to have higher average tax rates. A higher average tax rate in the data implies a higher public good provision in the model. This in turn implies, that Regime 2b will be implemented in fewer cases, thus leading to lower progressivity, especially for the low inequality countries, where the over-prediction of the model is the highest. Hence, using the actual average tax rates rather than their cross-country average improves the fit of the model.

7 Conclusion

In this paper I present a model of political coalition formation, where a society has to decide how to share the burden of providing the public good. I show that in such a model, more unequal societies implement a more progressive system. This is due to two factors: the more unequal a society is, the

more power the poor have and the less likely it is that the rich can be in a winning coalition. Thus depending on income inequality different regimes are in place, which implement different type of tax schemes. Second, within a regime, more inequality increases the progressivity of the system. If inequality is higher because the rich are further away from the middle and low income individuals, then it is relatively easy to tax the rich.

I test the predictions of the model by applying it to the income distribution of 17 OECD countries. By comparing the model predictions to the data on the progressivity of the tax system, I find that even though the model over-predicts the degree of progressivity, it predicts, in line with real world tax schemes, that more unequal societies implement more progressive tax schemes. The predicting power of the model is improved if the division of occupation groups into the three income categories is country-specific and if country level average tax rates are used as a proxy for g/\bar{y} .

The main dispersion in the model's prediction on the progressivity of income tax systems is driven by within regime variation, as I find that in a majority of the countries the representative of the poor sets the tax scheme. Where stable coalitions emerge, in low inequality countries, the fit of the model is significantly improved, by reducing the predicted progressivity.

These findings imply that the predictive power of the model could be improved by obtaining better data on the distribution of income. My proxy for the distribution of earnings is very coarse: I combine two labour force surveys, the ELFS and the SES, to obtain the share and relative earnings of the nine main occupation groups. The relative average earnings of the occupation groups might be too coarse to capture the full dispersion income dispersion in reality, and might over-predict the power of poorest groups. An alternative would be to use household surveys for all countries, this way having a better proxy for the distribution of income. Parallel to improving the data on the distribution of income, the income distribution could also be refined in the model. Allowing for more than three income groups would potentially reduce the power of the poorest group, and open up the possibility to more diverse coalition formation, which seems to be the key in matching the progressivity indices.

Dealing with data from household surveys would also allow a more extensive analysis of the progressivity of the tax scheme of these countries. Since household surveys contain data on earnings from various sources, number of dependants and consumption, an analysis could be conducted on the progressivity of the different type of taxes (personal income tax, capital tax, consumption tax, social security contributions) as well as of the tax scheme as a whole. Looking at the progressivity of the tax scheme as a whole would allow a better proxy for the required revenue from the tax scheme, for example government spending net of changes in government debt. On the other hand, while the personal income tax has a clear redistributive role, this is not true for social security contributions and consumption taxes, which might be determined through another process.

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A Progressivity curves

Figure 9 plots the *progressivity curves*, the difference between the Lorenz curve of income and the concentration curve of taxes ($F_1(x) - F_1(T(x))$) against $F(x)$ for the income tax. Notice, that the progressivity curves follow similar patterns across countries. All of the progressivity curves in all countries are upwards sloping for lowest deciles of the income distribution, start declining between the 6th, 7th or 8th decile, but remain positive. This implies, that the non-invertibility of the P index is not a major problem when looking at the progressivity of existing tax systems. The fact that the progressivity curves are upward sloping for the lowest deciles means that the income tax shows some degree of progressivity at least at the very bottom of the distribution.

B Progressivity indices across countries

Table 3: Progressivity and Gini coefficients

| country | tax | tax and employee social security | tax and total social security | Gini |
|-------------|--------|----------------------------------|-------------------------------|--------|
| Denmark | 0.0168 | 0.0120 | 0.0110 | 0.1139 |
| Sweden | 0.0362 | 0.0245 | 0.0133 | 0.1176 |
| Norway | 0.0307 | 0.0227 | 0.0157 | 0.1234 |
| Netherlands | 0.1427 | 0.0276 | 0.0099 | 0.1528 |
| Iceland | 0.0364 | 0.0305 | 0.0253 | 0.1620 |
| Ireland | 0.1227 | 0.1057 | 0.0630 | 0.1714 |
| Finland | 0.0545 | 0.0442 | 0.0260 | 0.1725 |
| France | 0.0510 | 0.0255 | 0.0108 | 0.1772 |
| Germany | 0.0653 | 0.0248 | 0.0094 | 0.1829 |
| Belgium | 0.0519 | 0.0340 | 0.0160 | 0.1983 |
| Spain | 0.0784 | 0.0512 | 0.0116 | 0.2050 |
| Greece | 0.1951 | 0.0739 | 0.0357 | 0.2059 |
| Austria | 0.0866 | 0.0341 | 0.0107 | 0.2100 |
| UK | 0.0524 | 0.0317 | 0.0294 | 0.2353 |
| Luxembourg | 0.1043 | 0.0553 | 0.0369 | 0.2370 |
| Italy | 0.0803 | 0.0548 | 0.0256 | 0.2473 |
| Portugal | 0.1735 | 0.0921 | 0.0459 | 0.3076 |

C Ideal policies and indifference curves

The ideal policy of a group can be found by a constrained maximization: each group maximizes their utility ((?)) subject to the balanced budget condition ((?)). This maximization leads to the following ideal policies:

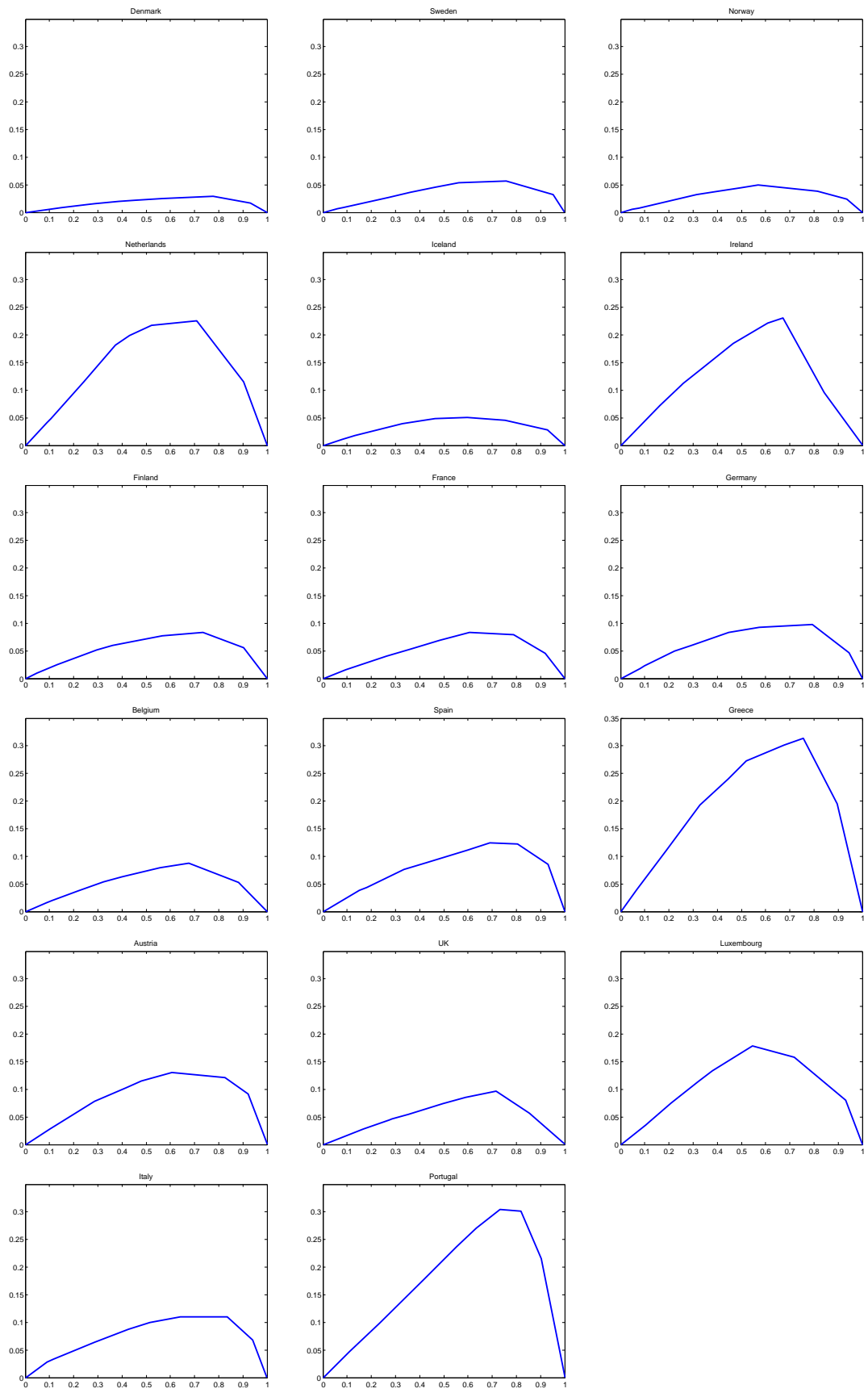


Figure 9: Progressivity across countries

$$\begin{aligned}
i_{P1} &= \left\{ \max\left(0, \frac{g - \alpha^R(y^R - y^M) - (\alpha^R + \alpha^M)(y^M - y^P)}{y^P}\right), \min\left(1, \max\left(0, \frac{g - \alpha^R(y^R - y^M)}{(\alpha^R + \alpha^M)(y^M - y^P)}\right)\right), \min\left(1, \frac{g}{\alpha^R(y^R - y^M)}\right) \right\} \\
i_{P2} &= \left\{ \max\left(0, \frac{g - \alpha^R(y^R - y^M) - (\alpha^R + \alpha^M)(y^M - y^P)}{y^P}\right), \min\left(1, \frac{g}{(\alpha^R + \alpha^M)(y^M - y^P)}\right), \min\left(1, \max\left(0, \frac{g - (\alpha^R + \alpha^M)(y^M - y^P)}{\alpha^R(y^R - y^M)}\right)\right) \right\} \\
i_M &= \left\{ \min\left(1, \max\left(0, \frac{g - \alpha^R(y^R - y^M)}{y^P}\right)\right), \max\left(0, \frac{g - \alpha^R(y^R - y^M) - y^P}{(\alpha^M + \alpha^R)(y^M - y^P)}\right), \min\left(1, \frac{g}{\alpha^R(y^R - y^M)}\right) \right\} \\
i_R &= \left\{ \min\left(1, \frac{g}{y^P}\right), \min\left(1, \max\left(0, \frac{g - y^P}{(\alpha^M + \alpha^R)(y^M - y^P)}\right)\right), \max\left(0, \frac{g - y^P - (\alpha^M + \alpha^R)(y^M - y^P)}{\alpha^R(y^R - y^M)}\right) \right\}
\end{aligned}$$

Each group is indifferent over a hyperplane, which is defined by $U_a^i = T \in \mathbb{R}^3 \text{ s.t. } U^i(T) = a$. However, the relevant part of these hyperplanes is their intersection with the set of admissible policies, $\Delta(g)$. The indifference set is reduced to a line segment within $\Delta(g)$. Figure ?? shows these indifference lines. The slope of the indifference lines can be then found by solving $U_a^i = T \in \mathbb{R}^3 \text{ s.t. } U^i(T) = a \wedge T \in \Delta(g)$. This yields the following slopes:

$$\begin{aligned}
P : \quad \frac{d\tau^R}{d\tau^M} &= -\frac{(\alpha^M + \alpha^R)(y^M - y^P)}{\alpha^R(y^R - y^M)} \\
M : \quad \frac{d\tau^R}{d\tau^M} &= \frac{\alpha^P(y^M - y^P)}{\alpha^R(y^R - y^M)} \\
R : \quad \frac{d\tau^R}{d\tau^M} &= -\frac{\alpha^P(y^M - y^P)}{(1 - \alpha^R)(y^R - y^M)}
\end{aligned}$$

D Equilibria without coalitions

Lemma 1. *In the absence of coalitions the following equilibria exist:*

1. If $U^R(i_M) \geq U^R(i_{P2})$, then the representative of the middle income group runs alone and wins with the platform i_M .
2. If $\exists p \in [i_{P1}, i_{P2}]$ such that $U^M(p) \geq U^M(i_R)$ and $U^R(p) \geq U^R(i_M)$, the representative of the poor runs alone with such a platform p from the set of his ideal policies and wins.
3. If $U^R(i_M) < U^R(i_{P2})$, but $[i_{P1}, i_{P2}] \cap \{p | U^M(p) \geq U^M(i_R) \vee U^R(p) \geq U^R(i_M)\} = \emptyset$,
 - $\alpha^R > \alpha^M, \alpha^P$: R runs, M mixes between running (with probability δ^M), P mixes between i_{P1} (with probability δ^P) and not running. The mixing probabilities are:

$$\begin{aligned}
\delta^M &= \frac{U^P(i_P) - U^P(i_R) - \varepsilon}{U^P(i_P) + U^P(i_M) - 2U^P(i_R)} \\
\delta^P &= \frac{U^M(i_M) - U^M(i_R) - \varepsilon}{U^M(i_M) + U^M(i_{P1}) - 2U^M(i_R)}
\end{aligned}$$

- $\alpha^M > \alpha^R, \alpha^P$: P runs with i_{P2} and M mixes between running (with probability δ^M), and R mixes between running (with probability δ^R) and not running. The mixing probabilities are:

$$\begin{aligned}
\delta^M &= \frac{U^R(i_R) - U^R(i_{P2}) - \varepsilon}{U^R(i_R) - U^R(i_M)} \\
\delta^R &= \frac{\varepsilon}{U^M(i_M) - U^M(i_R)}
\end{aligned}$$

- $\alpha^P > \alpha^R, \alpha^M$: P mixing between i_{P1} (with probability δ^P) and i_{P2} , M and R mixing between running (with probability δ^M and δ^R) and not. The mixing probabilities are:

$$\begin{aligned}\delta^R &= \frac{(U^P(i_P) - U^P(i_M)) \sqrt{\frac{U^P(i_P) - U^P(i_R)}{U^P(i_P) - U^P(i_M)} \frac{U^R(i_R) - U^R(i_{P2})}{U^M(i_R) - U^M(i_{P2})} \frac{U^M(i_M) - U^M(i_{P1})}{U^R(i_M) - U^R(i_{P1})}}}{(U^P(i_P) - U^P(i_M)) \sqrt{\frac{U^P(i_P) - U^P(i_R)}{U^P(i_P) - U^P(i_M)} \frac{U^R(i_R) - U^R(i_{P2})}{U^M(i_R) - U^M(i_{P2})} \frac{U^M(i_M) - U^M(i_{P1})}{U^R(i_M) - U^R(i_{P1})}} + U^P(i_P) - U^P(i_R)} \\ \delta^M &= \frac{\sqrt{\frac{U^P(i_P) - U^P(i_R)}{U^P(i_P) - U^P(i_M)} \frac{U^R(i_R) - U^R(i_{P2})}{U^M(i_R) - U^M(i_{P2})} \frac{U^M(i_M) - U^M(i_{P1})}{U^R(i_M) - U^R(i_{P1})}}}{1 + \sqrt{\frac{U^P(i_P) - U^P(i_R)}{U^P(i_P) - U^P(i_M)} \frac{U^R(i_R) - U^R(i_{P2})}{U^M(i_R) - U^M(i_{P2})} \frac{U^M(i_M) - U^M(i_{P1})}{U^R(i_M) - U^R(i_{P1})}}}} \\ \delta^P &= \frac{\delta^R (U^M(i_R) - U^M(i_{P2})) + \varepsilon}{(U^M(i_R) - U^M(i_{P2})) \delta^R + (U^M(i_M) + U^M(i_{P1})) (1 - \delta^R)}\end{aligned}$$

Proof. 1. See in text.

2. See in text.

3. When $U^R(i_M) \leq U^R(i_{P2})$ and $[i_{P1}, i_{P2}] \cap \{p | U^M(p) \geq U^M(i_R) \vee U^R(p) \geq U^R(i_M)\} = \emptyset$, then as explained in the text, there aren't any pure strategy equilibria. Dividing the cases based on which group is the largest and going through each case yields the identification of the equilibria as described in the Lemma.

(a) $\alpha^R > \alpha^M, \alpha^P$

The only equilibria is P mixing between running with i_{P1} probability δ_P and not running with probability $1 - \delta^P$, M mixes between running with probability δ^M and not running with probability $1 - \delta^M$, and R running for sure.

The mixing probabilities are:

$$\begin{aligned}\delta^M &= \frac{U^P(i_P) - U^P(i_R) - \varepsilon}{U^P(i_P) + U^P(i_M) - 2U^P(i_R)} \\ \delta^P &= \frac{U^M(i_M) - U^M(i_R) - \varepsilon}{U^M(i_M) + U^M(i_{P1}) - 2U^M(i_R)}\end{aligned}$$

The equilibrium expected payoffs are:

$$\begin{aligned}E(U^P) &= \delta^M U^P(i_R) + (1 - \delta^M) U^P(i_P) \\ E(U^M) &= \delta^P U^M(i_R) + (1 - \delta^P) U^M(i_M) \\ E(U^R) &= \delta^M (1 - \delta^P) U^R(i_M) + (1 - \delta^M) \delta^P U^R(i_{P1}) + (1 - \delta^P - \delta^M + 2\delta^M \delta^P) U^R(i_R)\end{aligned}$$

The expected equilibrium winning platform, E in this case is:

$$E = (\delta^P \delta^M + (1 - \delta^P)(1 - \delta^M)) i_R + \delta^M (1 - \delta^P) i_M + \delta^P (1 - \delta^M) i_{P1}$$

This expected platform will be quite close to i_R , as i_R has at least 1/2 weight.

(b) $\alpha^M > \alpha^R, \alpha^P$

The only equilibrium is P playing i_{P2} and $M(R)$ playing $i_M(i_R)$ with probability $\delta^M(\delta^R)$ and not running with probability $1 - \delta^M(1 - \delta^R)$. The mixing probabilities are:

$$\begin{aligned}\delta^M &= \frac{U^R(i_R) - U^R(i_{P2}) - \varepsilon}{U^R(i_R) - U^R(i_M)} \\ \delta^R &= \frac{\varepsilon}{U^M(i_M) - U^M(i_R)}\end{aligned}$$

The equilibrium expected payoffs are:

$$\begin{aligned}E(U^P) &= \delta^R U^P(i_M) + (1 - \delta^R) U^P(i_P) - \varepsilon \\ E(U^M) &= \delta^R U^M(i_R) + (1 - \delta^R) U^M(i_{P2}) \\ E(U^R) &= U^R(i_{P2})\end{aligned}$$

The expected equilibrium winning platform, E in this case is:

$$E = \delta^M \delta^R i_M + (1 - \delta^M) \delta^R i_R + (1 - \delta^R) i_{P2}$$

As $\varepsilon \rightarrow 0$, $\delta^R \rightarrow 0$ and the implemented platform is i_{P2} . Hence it is likely that only an MR coalition is feasible.

(c) $\alpha^P > \alpha^M, \alpha^R$

Only equilibrium: P mixing between i_{P1} and i_{P2} , M and R mixing between running and not. The mixing probabilities satisfy:

$$\begin{aligned}\delta^R(U^P(i_P) - U^P(i_R)) - \delta^M(U^P(i^P) - U^P(i_M)) &= \delta^M \delta^R (U^P(i_M) - U^P(i_R)) \\ \delta^P(1 - \delta^R)(U^M(i_M) - U^M(i_{P1})) &= \delta^R(1 - \delta^P)(U^M(i_R) - U^M(i_{P2})) + \varepsilon \\ \delta^P \delta^M (U^R(i_M) - U^R(i_{P1})) + \varepsilon &= (1 - \delta^P)(1 - \delta^M)(U^R(i_R) - U^R(i_{P2}))\end{aligned}$$

The solution of these three equations yields the probabilities in the Lemma. The expected payoffs are:

$$\begin{aligned}E(U^P) &= \delta^M(1 - \delta^R)U^P(i_M) + (1 - \delta^M + \delta^M \delta^R)U^P(i_P) \\ E(U^M) &= \delta^R \delta^P U^M(i_{P1}) + (1 - \delta^R) \delta^P U^M(i_M) + (1 - \delta^P)U^M(i_{P2}) \\ E(U^R) &= \delta^M(1 - \delta^P)U^R(i_{P2}) + (1 - \delta^M)(1 - \delta^P)U^R(i_R) + \delta^P U^R(i_{P1})\end{aligned}$$

The expected equilibrium winning platform is:

$$E = \delta^P(\delta^R + (1 - \delta^R)(1 - \delta^M))i_{P1} + (1 - \delta^P)(\delta^M + (1 - \delta^R)(1 - \delta^M))i_{P2} + (1 - \delta^P)(1 - \delta^M)\delta^R i_R + \delta^P \delta^M(1 - \delta^R)i_M$$

This platform is close to an internal point of i_P . As P wins the election in most of the cases.

□

| country | 1 | 2 | 3 | 4 | 7 | 8 | 5 | 6 | 9 |
|----------------|---------|---------|---------|------|---------|------|---------|---------|---------|
| Austria | 2.12(R) | 1.43(R) | 1.15(R) | 0.92 | 0.77 | 0.80 | 0.63(P) | 0.62(P) | 0.61(P) |
| Belgium | 1.75(R) | 1.37(R) | 0.91 | 0.81 | 0.72 | 0.76 | 0.67(P) | 0.64(P) | 0.61(P) |
| Germany | 1.96(R) | 1.45(R) | 1.04 | 0.95 | 0.83 | 0.81 | 0.64(P) | 0.62(P) | 0.62(P) |
| Denmark | 1.59(R) | 1.25(R) | 1.04 | 0.87 | 0.98 | 0.95 | 0.73(P) | 0.83 | 0.78(P) |
| Spain | 2.25(R) | 1.48(R) | 1.14(R) | 0.82 | 0.84 | 0.85 | 0.71(P) | 0.78 | 0.64(P) |
| Finland | 1.84(R) | 1.27(R) | 0.98 | 0.81 | 0.89 | 0.88 | 0.71(P) | 0.66(P) | 0.70(P) |
| France | 1.82(R) | 1.46(R) | 1.07 | 0.80 | 0.81 | 0.83 | 0.74(P) | 0.73(P) | 0.67(P) |
| Greece | 1.89(R) | 1.47(R) | 0.96 | 0.80 | 0.93 | 0.81 | 0.70(P) | 0.66(P) | 0.63(P) |
| Ireland | 1.38(R) | 1.47(R) | 1.07 | 0.79 | 0.92 | 0.77 | 0.65(P) | 0.69(P) | 0.68(P) |
| Iceland | 1.88(R) | 1.22(R) | 1.07 | 0.84 | 0.95 | 0.86 | 0.76 | 0.72(P) | 0.69(P) |
| Italy | 2.46(R) | 1.62(R) | 1.09 | 0.94 | 0.73(P) | 0.79 | 0.75 | 0.60(P) | 0.45(P) |
| Luxembourg | 2.08(R) | 1.39(R) | 1.18(R) | 0.83 | 0.69(P) | 0.72 | 0.61(P) | 0.49(P) | 0.37(P) |
| Netherlands | 1.49(R) | 1.28(R) | 1.04 | 0.83 | 0.97 | 0.93 | 0.70(P) | 0.63(P) | 0.61(P) |
| Norway | 1.59(R) | 1.25(R) | 1.13(R) | 0.86 | 0.92 | 0.92 | 0.76(P) | 0.80(P) | 0.74(P) |
| Portugal | 2.79(R) | 1.96(R) | 1.32(R) | 0.83 | 0.58(P) | 0.67 | 0.59(P) | 0.52(P) | 0.57(P) |
| Sweden | 1.71(R) | 1.22(R) | 1.07 | 0.84 | 0.91 | 0.90 | 0.81 | 0.76(P) | 0.74(P) |
| United Kingdom | 1.68(R) | 1.58(R) | 1.10 | 0.73 | 0.87 | 0.72 | 0.56(P) | 0.63(P) | 0.58(P) |

Table 4: Relative earnings of occupation groups within countries and categorization B

Notes: The values indicate the average earnings within that occupation compared to the average earning in the country. The occupations are: 1 - Legislators, senior officials & managers; 2 - Professionals; 3 - Technicians & associate professionals; 4 - Clerks; 7 - Craft & related trades workers; 8 - Plant & machine operators & assemblers; 5 - Service, shop & market sales workers; 6 - Skilled agricultural & fishery workers; 9 - Elementary occupations. An (R) behind the value indicates that this occupation belongs to the rich group in Panel B, a (P) implies that it belongs to the poor, nothing implies that it belongs to the middle income group. Values calculated from SES 2006, averages calculated with weights from the ELFS 2005 (2003 and 2008 for Italy).