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Macroeconomic Dynamics in a Model of Goods, Labor and Credit Market Frictions*

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Abstract

Building a model with three imperfect markets - goods, labor and credit - representing a product's life-cycle, we find that goods market frictions drastically change the qualitative and quantitative dynamics of labor market variables. The calibrated model leads to a significant reduction in the gap with the data, both in terms of persistence and volatility while search models of the labor market fail in one of the two dimensions. Two factors related to goods market frictions generate these results: i) the expected dynamics of consumers' search for goods, itself depending on the income redistributed by firms and the entry of new products; and ii) the expected dynamics of prices, which alter future profit flows.

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1 Introduction

In this paper, we build a model with three imperfect markets - goods, labor and credit - and find that goods market frictions drastically change the qualitative and quantitative dynamics of the labor market, bridging the gap with the data both in terms of persistence and volatility. Since its inception the Real Business Cycle literature has faced the same challenge, emphasized in King and Rebelo (1999) and Cogley and Nason (1995): that of the propagation of technological shocks. In the standard RBC model, it is necessary to assume large innovations in order to obtain realistic business cycle fluctuations. Moreover, the standard model cannot generate the amount of autocorrelation in the growth rate of output that we see in the data. This twin failing in the lack of both amplification and persistence is even more severe for search models of unemployment.

Our modeling approach attempts to improve reduce the gap between data and calibrated statistics as follows. We abstract each friction as a process matching two sides of a market, expanding the dynamic view of entry and exit of, respectively, jobs, lending relationships and goods. In particular, we rely on the observation that the allocation of final goods to end consumers is a costly process: in the US, the retail sector, for instance, represents more than 5 percent of GDP, suggesting that a frictionless view of the goods market would be an oversimplification. The relative supply and demand measures a degree of market tightness: the familiar vacancy to unemployment ratio for the labor market; the ratio of prospecting consumers and products on the goods market; and the ratio of investment projects to banks on the credit market.

Our model aims at understanding the respective role of each market friction for the dynamic propagation of shocks. Since we are agnostic about the respective role of frictions in each market, our model will let each frictions play a role and the calibration will determine their qualitative and quantitative importance. We find that imperfect goods markets, working through the forward looking nature of job creation, are the key ingredient changing the qualitative and quantitative responses of the model to productivity shocks. In particular, the dynamics of the goods market generate persistence in the growth of the incentives to hire workers, which translates into responses of labor market tightness to productivity shocks that are hump-shaped, or highly persistent. During the first stages of an economic expansion, more firms enter the goods market relative to the change in the effective demand from consumers.

As these firms generate more revenue, they will distribute more income to consumers who will raise their effort to reach their desired consumption level. However, in the immediate aftermath of the shock, the entry of firms causes an increase in congestion in the goods markets, from the point of view of firms, and a decline in the negotiated price at which the goods are eventually sold. From the perspective of a firm deciding to hire a worker, this moderates the incentive to create a vacancy at the beginning of an expansion as it is less likely the additional production will find an outlet, and if it does, it sells at a lower price.

The goods market eases, in the sense of there being relatively more demand from consumers than products competing for customers, only after a few periods. This decrease in congestion, which also leads to firms obtaining a better price, actually increases the incentive to recruit workers. Overall, these mechanisms combine to generate amplification, and persistence, visible by the rise in labor market tightness for several periods after the initial shock. Propagation arises from the fact that the economic value of hiring a worker is tied through interesting intertemporal linkages to congestion and prices on goods markets. These mechanisms are absent from the standard labor search model and a large class of extensions.

It should be noted that there has also been a revival of the interest in the impact of demand shocks in goods markets and their implications for the identification and propagation of technology shocks within the RBC paradigm. Bai et al. (2011), in independent work, model the process of matching between consumers and firms as a matching model, with a focus on the respective role of technology shocks and shocks to the demand side of the market. Our model will generate dynamics of propagation of income that rely exclusively on productivity shocks, not on consumer or demand shocks, in contrast with Bai et al. (2011).

This paper is organized as follows. In Section 2, we briefly review the literature and discuss the evidence motivating our modeling of the goods market. In Section 3, we develop the model. In Section 4, we calibrate the model to quarterly data, using evidence on goods market flows, and investigate the sources of propagation in detail. Section 5 concludes.

2 A brief overview of the literature

Earlier research into propagation in models of the business cycle focused on the labor market, either increasing the elasticity of labor supply, e.g. models of indivisible

labor in Hansen (1985) and Rogerson (1988), or introducing of a market friction in the form of wage rigidity.¹ The importance of the latter for amplifying the response of the demand for labor to changes in productivity has received renewed attention in search models of equilibrium unemployment as a means of addressing the lack of volatility in job vacancies and unemployment.²

The role of credit markets in amplifying exogenous shocks to economies and the existence of a *financial accelerator* has been emphasized in papers such as Bernanke and Gertler (1989), Kiyotaki and Moore (1997). We take into account the potential importance of frictional credit markets by introducing a financial accelerator of the type explored in Wasmer and Weil (2004) and Petrosky-Nadeau and Wasmer (2010). Both papers have introduced frictions in the labor and credit market and studied, respectively, the static and dynamic properties of the financial accelerator.

Our main novelty here is to develop a model in which the introduction of goods market imperfections generates additional insights in a modern theoretical setup. It has of course been recognized for long that non-clearing or departures from Walrasian equilibria in goods markets can generate additional unemployment. Several waves of research have attempted to put this intuition into models (see the survey in Bénassy, 1993). This previous literature has mostly been centered around the idea of price rigidities leading to excess supply (or demand) of goods, in turn generating inefficient outcomes in the labor market. In our paper, goods market imperfections will propagate shocks without solely relying on price rigidity. Finally, a large fraction of the search literature has an explicit focus on frictions in the good market, as in Diamond (1971, 1982) for instance.

Frictions in the market for delivery goods from producers to end consumers are important. As an indication, the BEA statistics report that in 2009, wholesale trade amounts to 5.5% of GDP, while retail trade amounts to 5.8%, suggestive of a costly allocation of final goods to consumers. The presence of frictions in producers accessing a distribution network to reach consumers, or consumers acquiring information about a producer, can explain why new firms face lower demand than comparable older firms (Foster et al., 2008). These frictions lead to time and costs for both side of the goods market in searching before acquiring or beginning to consume a good for the

¹See, for example, Taylor (1980), Christiano, Eichenbaum, and Evans (2005).

²This deficiency of the canonical model was shown in Cole and Rogerson (1999) and Shimer (2005). See also Hall (2005). Other mechanisms were suggested in the literature, such as introducing on-the-job search (Mortensen and Nagypál, 2007). Fujita and Ramey (2007) also focus on the lack of persistence in the growth rates of labor market variables in this class of models.

first time.

Beyond, new data on goods market flows build on a literature measuring gross and net creation and destruction flows in the labor and credit markets. Following the seminal contributions for labor markets of Davis and Haltiwanger (1990, 1992), Del’Arricia and Garibaldi (2005) have measured creation and destruction in the US loans market, while, most recently, Broda and Weinstein (2010) have carefully documented the magnitude of flows of entry and exits of goods, as well as the procyclical features of net product creation flows. These empirical works point to the directions theory should take. Their insights have

inspired our work, in particular the details of the modelization and its interpretation. The authors built up and used a unique dataset with 700 000 products with bar codes purchased by 55 000 households. The covered sectors amount to 40 percent of all expenditures on goods in the CPI. Their findings, relevant to our approach, are as follows: they find large flows of entries and exits of “products,” actually four times more entry and exit in product markets than is found in labor markets, and a large share of product turnover happens within firms. Further, net product creation is strongly procyclical and primarily driven by creation rather than destruction. Over their 9 year sample period, 1994-2003, the product entry rate, defined as the number of new product codes divided by the stock is 0.78 quarterly. The product exit rate, defined as the number of disappearing product codes over the stock, is 0.72. Third, net creation of products is strongly pro-cyclical, and driven by creation as destruction is weakly counter-cyclical. This suggests that high demand leads to the introduction of new goods, reminiscent of the implementation cycles in Shleifer (1986).

Our model will follow closely the empirical results: throughout the paper, we will follow the birth of a “product line”, its development and finally its death due in part to technological obsolescence, in part due to changes in consumer tastes, and will calibrate parts of the model based on the creation and destruction statistics above.

3 An economy with goods, labor and credit market frictions

We consider the case of a firm evaluating marginal investment projects. These projects first need to obtain financing on the credit market. A financed project is then managed so as to maximize the value to the firm and the creditor, and needs to hire a worker

to produce a good. In that we will follow the structure in Wasmer and Weil (2004) and Petrosky-Nadeau and Wasmer (2010). However, the good cannot be sold until a consumer has been found. Once a match in the goods market is formed, consumers and producers bargain over the price. The creditor (hereafter named the bank) has the monopoly over the ability to allocate resources from one period to the other. There is no money in this economy, contrary to Berentsen et al. (2011) who study jointly a search-money framework with labor market frictions with a focus on the impact of interest rates on unemployment. In particular, we assume that consumers do not have the ability to transfer wealth from one period to the other. However, this assumption is not binding as, with search frictions in the goods market, savings will be shown to be a dominated strategy.

3.1 Financing investment projects

Time is discrete. An investment project is initially in need of a financial partner (hereafter called a “banker”). This financing will cover the cost of recruiting a worker and the wage bill when the firm has not found a demand for its product. Prospecting on the credit market costs κ_E units of effort per period of time. With probability p_t it finds a banker, and with complementary probability it remains in this stage (denoted by c like *credit*). We denote by J_c the asset value of the investment project in this stage. At the time of the meeting between banker and project, both sides agree on the terms of a financial contract whereby the resulting costs of the project are financed by the bank when the project’s cash-flow is negative (in stages 2 and 3) and pays the banker when the cash-flow is positive (in stage 4).

Now matched with a banker, the project enters the second stage, where it prospects on the labor market in order to hire a worker. It must pay a per-period cost γ to maintain an active job vacancy. With probability q_t the firm is successful in hiring a worker, with complementary probability it remains in this stage (denoted by l like *labor*). We denote by J_l the asset value of a project in this stage. The firm offers a wage w_t to the worker as long as the firm is active.

In the third stage, now endowed with a worker, the firm could start producing y_t units of output from this particular project and attempt to sell it on the goods market, but it has no customers. Meeting with a consumer comes with probability λ_t , and production can be sold the following period. By assuming that production involves an operating cost Ω over and above the wage, and that the good cannot be

stored, the firm chooses not to produce in this stage (denote by g for *goods* market). The value of this stage is denoted J_g . Note that the bank is still financing the firm by transferring the amount of cash necessary to pay the worker. With probability s , projects are hit by an exogenous destruction shock at the end of the period in this stage and the next.

In the fourth and final stage, the firm is now matched with a consumer and its output x_t is sold at price \mathcal{P}_t . We assume that x_t follows a stationary process. Later on, the process is specified to be an AR(1), but this assumption is unessential at this stage. With revenue $\mathcal{P}_t x_t$, the firm pays the worker w_t , the operating cost Ω , an amount ρ_t to the bank, and enjoys the difference. We denote this stage by π , standing for profit, and by J_π its associated asset value. In addition, the consumer may stop consuming the particular good produced by this project with probability τ , in which case the project returns to the previous stage g to search for another consumer.

Finally, as in Pissarides (2000), all profit opportunities are exhausted by new entrants such that the value of the entry stages are always driven to zero. In the case of the credit market, this implies that $J_{c,t} \equiv 0$ at all times, which is also the continuation value following the destruction shock s .

Given these assumptions, the Bellman equations of the investment project, which faces a discount rate r and assuming that transitions from the credit to the labor market stages occur within a single period, are:

$$J_{c,t} = 0 = -\kappa_I + p_t J_{l,t} \tag{1}$$

$$J_{l,t} = -\gamma + \gamma + \frac{1}{1+r} \mathbb{E}_t [q_t J_{g,t+1} + (1 - q_t) J_{l,t+1}] \tag{2}$$

$$J_{g,t} = -w_t + w_t + \left(\frac{1-s}{1+r} \right) \mathbb{E}_t [\lambda_t J_{\pi,t+1} + (1 - \lambda_t) J_{g,t+1}] \tag{3}$$

$$J_{\pi,t} = \mathcal{P}_t x_t - w_t - \rho_t - \Omega_t + \left(\frac{1-s}{1+r} \right) \mathbb{E}_t [(1 - \tau) J_{\pi,t+1} + \tau J_{g,t+1}] \tag{4}$$

The bank's lifetime closely follows that of the investment project, with values denoted by B_j , $j = c, l, g$ or π for each of the stages. In stage c , it prospects on the credit market to find a viable project to finance, which occurs with probability \hat{p}_t , and pays a per period screening cost κ_B . Free entry on this side of the credit market implies that $B_{c,t} = 0$ at all times. In stage l , the bank pays the cost of a vacancy γ and waits for the hiring to be realized. In stage g , the bank now pays the wage cost w_t and waits for the firm to be matched with a consumer. In stage π , the bank cashes

in the repayment ϱ_t .

The corresponding Bellman equations for the banker are

$$B_{c,t} = 0 = -\kappa_B + \hat{p}_t B_{l,t} \quad (5)$$

$$B_{l,t} = -\gamma + \frac{1}{1+r} \mathbb{E}_t [q_t B_{g,t+1} + (1-q_t) B_{l,t+1}] \quad (6)$$

$$B_{g,t} = -w_t + \left(\frac{1-s}{1+r} \right) \mathbb{E}_t [\lambda_t B_{\pi,t+1} + (1-\lambda_t) B_{g,t+1}] \quad (7)$$

$$B_{\pi,t} = \rho_t + \left(\frac{1-s}{1+r} \right) \mathbb{E}_t [(1-\tau) B_{\pi,t+1} + \tau B_{g,t+1}] \quad (8)$$

Going forward, we will be interested in the joint values of a creditor and investment project, which we refer to as a “firm.” Let the value of a firm for each of the stages be denoted by $S_{j,t} = J_{j,t} + B_{j,t}$, with $j = c, l, g, \pi$. The Bellman equations for the value of a firm in each stage can be obtained by summing the corresponding equations for projects and banks, that is (1) to (4) and (5) to (8). We have, after rearrangement:

$$S_{c,t} = 0 \Leftrightarrow \frac{\kappa_B}{\hat{p}_t} + \frac{\kappa_E}{p_t} = S_{l,t} \quad (9)$$

$$S_{l,t} = -\gamma + \frac{1}{1+r} \mathbb{E}_t [q_t S_{g,t+1} + (1-q_t) S_{l,t+1}] \quad (10)$$

$$S_{g,t} = -w_t + \left(\frac{1-s}{1+r} \right) \mathbb{E}_t [\lambda_t S_{\pi,t+1} + (1-\lambda_t) S_{g,t+1}] \quad (11)$$

$$S_{\pi,t} = \mathcal{P}_t x_t - w_t - \Omega + \left(\frac{1-s}{1+r} \right) \mathbb{E}_t [(1-\tau) S_{\pi,t+1} + \tau S_{g,t+1}] \quad (12)$$

Equation (9) states that the value of a firm in the hiring stage is equal to the sum of capitalized search costs paid by each side in the previous credit market stage. This is driven to zero in the absence of credit market frictions. The formulation the labor market stage in equation (10) describes the value of a job vacancy as a flow cost γ and an expected gain from hiring a worker, valued at S_g . As we will discuss in detail, the presence of a frictional goods markets fundamentally alters the dynamics of S_g compared to the standard framework through the dynamics of the goods market meeting rate, λ_t , and the price, \mathcal{P}_t .

3.2 Search and matching on the goods markets

3.2.1 Matching in the goods market

Consumers may spend a disposable income Y^d , which we define below, on either an essential goods (serving as a numeraire), c_0 , or a preferred manufactured good, c_1 . Consuming the later first requires searching on the goods market. When a consumer is matched with a manufacturing firm, it purchases the production, x_t , at a unit price \mathcal{P}_t . The remaining income is spent on the essential good, which is supplied as a transfer of resource across individuals.³

At any point in time in this economy there are matched and unmatched consumers. Normalizing the mass of consumers to 1, we denote these shares by $\mathcal{C}_{1,t}$ and $\mathcal{C}_{0,t}$, respectively. In equilibrium, these will be the fractions of disposable household income allocated to either category of goods. Unmatched consumers $\mathcal{C}_{0,t}$, exert an average search effort, \bar{e}_t , to find unmatched goods, $\mathcal{N}_{g,t}$, through a process summarized by a constant returns to scale function $M_G(\bar{e}_t\mathcal{C}_{0,t}, \mathcal{N}_{g,t})$. $\bar{e}_t\mathcal{C}_{0,t}$ can be thought of being the effective demand for new goods. The meeting rates between consumers and firms are given by:

$$\begin{aligned} \frac{M_G(\bar{e}_t\mathcal{C}_{0,t}, \mathcal{N}_{g,t})}{\mathcal{N}_{g,t}} &= \lambda(\xi_t) \quad \text{with} \quad \lambda'(\xi_t) > 0 \\ \frac{M_G(\bar{e}_t\mathcal{C}_{0,t}, \mathcal{N}_{g,t})}{\bar{e}_t\mathcal{C}_{0,t}} &= \tilde{\lambda}(\xi_t) \quad \text{with} \quad \tilde{\lambda}'(\xi_t) < 0 \end{aligned}$$

where $\xi_t = \frac{\bar{e}_t\mathcal{C}_{0,t}}{\mathcal{N}_{g,t}}$ is the natural concept for tightness in the goods market (from the point of view of consumers) and $\lambda(\xi_t) = \xi_t\tilde{\lambda}(\xi_t)$. That is, $\tilde{\lambda}_t$, the probability that an unmatched consumer finds a suitable firm from which to buy goods, is decreasing in goods market tightness. Conversely, the greater ξ_t , the greater the demand from consumers relative to the goods awaiting to find consumers, and the shorter the duration of search for producers. This creates an important feedback from the goods market to the labor market as the returns to hiring a worker are greater when it is easiest to find customers.

³To avoid any further complication, we assume that the numeraire is produced by a technology without labor: it is therefore the fruit of a Lucas tree with no search friction here. The extension of our framework along the lines of Berentsen et al. (2011) with money-search would permit richer interpretations.

3.2.2 Disposable income

The total net profits in this economy, Π_t , are the sum of profit flows to projects and banks. This corresponds to:

$$\Pi_t = (\mathcal{P}_t x_t - \Omega) \mathcal{N}_{\pi,t} - w_t \mathcal{N}_t - \gamma \mathcal{N}_{l,t} - \kappa \mathcal{B}_{c,t}$$

where $\mathcal{N}_t = \mathcal{N}_{\pi,t} + \mathcal{N}_{g,t}$ is the sum of the number of firms matched with a consumer $\mathcal{N}_{\pi,t}$, and of the number of firms in stage g $\mathcal{N}_{g,t}$ (that is, matched with a bank and a worker but not with a consumer). The number of firms prospecting for workers in stage l is $\mathcal{N}_{l,t}$ and the number of banks screening projects in stage c is $\mathcal{B}_{c,t}$. In the equation above, the first term is the revenue generated by firms in stage 4, net of operating costs. The second term represents wage payments in the economy. The remaining terms represent the negative cash-flows of the bank during the first stages due to search costs in labor and credit markets.

These profits net of search costs are pooled and distributed lump sum to workers. The mass 1 of workers, the unemployed and employed, therefore receive per person and per period Π_t as a cash transfer. Further, resources are pooled across categories of workers, as in Merz (1995) and Andolfatto (1996). The average disposable income of a representative consumer is $\Pi_t + \mathcal{N}_t w_t$ plus an additional lump sum transfer T distributed to all consumers, matched and unmatched.⁴ The timing of the transmission of income from firms to consumer within a period corresponds to the simplest possible assumption, and also the most natural.⁵

3.2.3 Value functions for consumers

Individuals want to consume manufactured goods but may not buy them before prospecting on the goods market. Let us denote by $D_{0,t}$ and $D_{1,t}$ the values for a consumer of being unmatched and matched, respectively. The generic utility of consuming both goods is denoted by $v(c_1, c_0)$, where c_1 and c_0 are the consumption of the manufactured good (subject to search frictions) and the essential goods (not subject to search frictions). Unmatched consumers search for a good at an effort cost

⁴This lump sum transfer has no consequence on the price level, determined later on and simply ensures the balancing of the budget of a matched consumer. For unmatched consumers, this simply raises the consumption of the numeraire.

⁵Alternative specifications, where the disposable income in t would be generated through profits and sales from period $t - 1$ would affect the dynamics and add up a source of lags, which we avoid here by our specification: the dynamic path we obtained is thus not slowed down artificially.

$\sigma(e)$, with $\sigma'(e) > 0$, $\sigma''(e) \geq 0$ and $\sigma(0) = 0$, and perceive their search effort as influencing their effective finding rate, $e_t \tilde{\lambda}_t$. Consequently, we have:

$$D_{0,t} = v(0, c_{0,t}) - \sigma(e_t) + \frac{1}{1+r} \mathbb{E}_t \left[e_t \tilde{\lambda}_t D_{1,t+1} + (1 - \tilde{\lambda}_t e_t) D_{0,t+1} \right] \quad (13)$$

$$D_{1,t} = v(c_{1,t}, c_{0,t}) + \left(\frac{1-s}{1+r} \right) \mathbb{E}_t [\tau D_{0,t+1} + (1-\tau) D_{1,t+1}] + \frac{s}{1+r} \mathbb{E}_t D_{0,t+1} \quad (14)$$

Assuming the manufactured good has greater marginal utility, matched consumers will always spend up to $\mathcal{P}_t x_t$ on $c_{1,t}$ and then expend what is left, $Y_t^d - \mathcal{P}_t x_t$, on $c_{0,t}$. In other words, the utility in the first equation (unmatched consumer) is $v(0, Y_t^d)$ and the utility in the second equation (matched consumer) will be $v(x_t, Y_t^d - \mathcal{P}_t x_t)$. In the current version, we assume a marginal utility for c_1 of $\Phi > 0$ and that the essential good provides a basic level of utility independent of the quantity consumed (we think of food and utilities, for example). We also discuss in appendix why the assumption of the absence of savings by consumers is not binding.

3.2.4 Optimal search effort

The optimal individual search effort is simply given by a condition equating the marginal cost of effort to the discounted, expected benefit yielded by that marginal unit of effort:

$$\sigma'(e_t^*) = \frac{\tilde{\lambda}_t}{1+r} \mathbb{E}_t [(D_{1,t+1} - D_{0,t+1})] \quad (15)$$

and it follows that all consumers exert the same effort :

$$e_t^* = \bar{e}_t$$

Equation (15) implies that consumer search effort is increasing in the expected capital gain from consuming the manufactured good. Combining the first order condition above and the derivation of value equations for consumers, we can see that the level of effort will depend on the gains from consumption next period, that is, from the difference in utility between being matched and unmatched. Both disposable income and the dynamics of the price \mathcal{P} , play a determining role in this respect as :

$$\sigma'(\bar{e}_t) = \frac{\tilde{\lambda}_t}{1+r} \mathbb{E}_t \sum_{i=0} \psi^i \left[\left(\frac{\Phi}{\mathcal{P}_{t+1+i}} - 1 \right) Y_{t+1+i}^d + \tilde{\sigma}(e_{t+1+i}) \right]$$

where $\psi \equiv (1 - \tau) \frac{1-s}{1+r}$ is a discount factor, Φ is the marginal utility from consuming c_1 and $\tilde{\sigma}(e_t) \equiv \sigma(e_t) - e_t \sigma'(e_t)$.

3.2.5 Determining the dynamics of the goods surplus and price

Consistent with the search literature, we postulate that the price \mathcal{P}_t is bargained between a consumer and a firm. The total surplus to the consumption relationship is $G_t = (S_{\pi,t} - S_{g,t}) + (D_{1,t} - D_{0,t})$. The price for the good is determined as $\mathcal{P}_t = \underset{\mathcal{P}_t}{\operatorname{argmax}} (S_{\pi,t} - S_{g,t})^{1-\delta} (D_{1,t} - D_{0,t})^\delta$, where $\delta \in (0, 1)$ is the share of the goods surplus G_t going to the consumer. This results in the sharing rule

$$(1 - \delta) (D_{1,t} - D_{0,t}) = \delta (S_{\pi,t} - S_{g,t}) \quad (16)$$

Introducing the notation for the elasticity of the effort cost function, $\eta_\sigma > 0$, we then obtain the negotiated price rule as:

$$\mathcal{P}_t x_t = (1 - \delta) [\Phi x_t + (1 - \eta_\sigma) \sigma(e)] + \delta \Omega + (1 - \delta) \lambda_t \frac{1 - s}{1 + r} \mathbb{E}_t [\delta G_{t+1}]$$

which emphasizes the forward looking aspect of price determination: today's price is increasing in the expectation of tomorrow's surplus on the goods market.⁶ Finally, we obtain which first states the that price is increasing in the marginal utility Φ . The cost of consumer search effort puts $\sigma(\bar{e}_t)$ downward pressure on the price if the elasticity $\eta_\sigma > 1$. Finally, the negotiated price is increasing in goods market tightness ξ_t : the greater the effective demand on the consumer side relative to the supply of unmatched goods \mathcal{N}_g , the greater the price and hence profits for firms.

3.3 Matching in other markets

3.3.1 Matching in the labor market and wage determination

We assume that matching in the labor market is governed by a function $M_L(\mathcal{N}_{l,t}, u_t)$, where u_t is the rate of unemployment and the total number of unemployed workers since the labor force is normalized to 1. $\mathcal{N}_{l,t}$, already defined as the number of firms in stage l , is also the number of "vacancies." The function is assumed to be constant returns to scale, hence the rate at which firms fill vacancies is a function of the ratio $\mathcal{N}_{l,t}/u_t = \theta_t$, a measure of the tightness of the labor market. This rate, $q(\theta_t)$, is given

⁶The details of the derivation for this and subsequent equations are provided in the appendix.

by

$$q(\theta_t) = \frac{M_L(\mathcal{N}_{l,t}, u_t)}{\mathcal{N}_{l,t}} \text{ with } q'(\theta_t) < 0.$$

Conversely, the rate at which the unemployed find a job is

$$\frac{M_L(\mathcal{N}_{l,t}, u_t)}{u_t} = \theta_t q(\theta_t) = f(\theta_t) \text{ with } f'(\theta_t) < 0.$$

Once employed, workers earn a wage w_t , which we assume, for simplicity, takes the functional form

$$w_t = \chi_w (\mathcal{P}_t x_t)^{\eta_w} \tag{17}$$

where η_w can be interpreted as the elasticity of wages to the marginal product of labor $\mathcal{P}_t y_t$. In the spirit of search models, one may want to have a wage schedule as the outcome of Nash-bargaining between the firm and the worker. We decided to avoid the complications implied by Nash-bargaining in this context in order, focusing on the role played by the elasticity of wages to productivity for propagation, thus leaving aside the question of bargaining in this context for future work.⁷

3.3.2 Matching in the credit market and bargaining on loan repayment

The matching rates p_t and \widehat{p}_t are made mutually consistent by the existence of a matching function $M_C(\mathcal{B}_{c,t}, \mathcal{N}_{c,t})$, where $\mathcal{B}_{c,t}$ (already defined) and $\mathcal{N}_{c,t}$ are, respectively, the number of bankers and projects in stage c . This function is assumed to have constant returns to scale. Hence, denoting by ϕ_t the ratio $\mathcal{N}_{c,t}/\mathcal{B}_{c,t}$, which is a reflection tightness of the credit market from the point of view of projects, we have

$$p_t = \frac{M_C(\mathcal{B}_{c,t}, \mathcal{N}_{c,t})}{\mathcal{N}_c} = p(\phi_t) \text{ with } p'(\phi_t) < 0. \tag{18}$$

$$\widehat{p}_t = \phi_t p(\phi_t) \text{ with } \widehat{p}'(\phi_t) > 0. \tag{19}$$

The division of rents from implementing a project, $S_{l,t}$, are determined by bargaining about ϱ upon meeting. Calling $\beta \in (0, 1)$ the bargaining power of the bank,

⁷Some complications with bargaining are as follows. First, given that firms pays the worker in two different stages (when it does not produce and when it does), this would imply not one but two wage schedules, with analytical complications but for a small quantitative difference since the surplus value of the firm in each stage are very close and exactly equal when the discount rate is small compared to the rate at which it finds a consumer. Hence, a similar wage rule in the two stages is a quantitatively good assumption. Second, given the number of parties, several complexities arise in which we would need to make assumptions on timing and bargaining structure. We ignore these here by choosing a rather simple wage determination rule.

the Nash-bargaining condition

$$(1 - \beta)B_{l,t} = \beta J_{l,t} \quad (20)$$

states that with $\beta = 1$ the bank receives all the surplus. Note that the rule for ϱ is determined at the time of the meeting but paid a few periods after the negotiation, when the firms becomes profitable. We assume that there is no commitment problem (as in Wasmer and Weil 2004) so that any new realization of aggregate productivity will not undo the financial contract and there is no renegotiation.

Combining (1), (5) and (20), as well as the definition of \widehat{p} in (19), we can obtain the equilibrium value of ϕ_t denoted by ϕ^* with

$$\phi^* = \frac{\kappa_B}{\kappa_E} \frac{1 - \beta}{\beta} \forall t \quad (21)$$

Free-entry of both banks and projects on credit markets implies a credit market tightness that is constant over time, even out of the steady-state. Going forward, all the information pertaining to the credit market is contained in the total transaction costs paid by both firms and banks in stage c :

$$K(\phi^*) \equiv \frac{\kappa_B}{\phi^* p(\phi^*)} + \frac{\kappa_E}{p(\phi^*)} \quad (22)$$

3.4 Stocks of consumers, employment and unemployment

Having stipulated the transition rates for all agents in the economy, we can now write the laws of motion for the stocks of consumers, firms and, consequently, employment. Potential consumers \mathcal{C}_0 become consumers the period after meeting a producer, and a fraction $0 < \tau < 1$ of current consumers separate from their product only to return to the pool of potential consumers the following period. The stocks of consumers in the goods market therefore evolve according to:

$$\mathcal{C}_{0,t+1} = (1 - \tilde{\lambda}_t)\mathcal{C}_{0,t} + [s + (1 - s)\tau]\mathcal{C}_{1,t} \quad (23)$$

$$\mathcal{C}_{1,t+1} = (1 - s)(1 - \tau)\mathcal{C}_{1,t} + \tilde{\lambda}_t\mathcal{C}_{0,t} \quad (24)$$

There is an inflow $q(\theta_t)\mathcal{N}_{l,t}$ into the stock of firms searching on the goods market every period, where $\mathcal{N}_{l,t}$ is the number of vacancies at time t . To this, $(1 - s)\tau N_{\pi,t}$

firms separated from consumers lead the stocks \mathcal{N}_g and \mathcal{N}_π to evolve according to:

$$\mathcal{N}_{g,t+1} = (1-s)(1-\lambda_t)\mathcal{N}_{g,t} + (1-s)\tau N_{\pi,t} + q(\theta_t)\mathcal{N}_{l,t} \quad (25)$$

$$\mathcal{N}_{\pi,t+1} = (1-s)(1-\tau)\mathcal{N}_{\pi,t} + (1-s)\lambda_t\mathcal{N}_{g,t} \quad (26)$$

Finally, the dynamics of aggregate unemployment and employment are then given by

$$u_{t+1} = s(1-u_t) + (1-f(\theta_t))u_t \quad (27)$$

$$1-u_t = \mathcal{N}_{g,t} + \mathcal{N}_{\pi,t} \quad (28)$$

3.5 Looking into the sources of propagation

The central equation relating labor market tightness and the expected value of hiring a worker, equation (10), lies at the heart of propagation in this class of models. In combination with (9) and calling $o_t(r) \equiv r \left(\frac{1/q(\theta_t)-1}{1+r} \right)$ a term vanishing as the discount rate goes to zero, this is:

$$K(\phi^*)(1+o_t(r)) + \frac{\gamma}{q(\theta_t)} = \frac{1}{1+r} \mathbb{E}_t S_{g,t+1} \quad (29)$$

which equates the average cost of creating a job - the left-hand side, equal to the financial costs properly discounted, $K(\phi)$, and the expected costs of search on the labor market, $\gamma/q(\theta_t)$ - to the discounted expected value of a worker to the firm in the goods market stage (the right-hand side). A few words of comparison with the canonical search model are warranted here. First, the costs of financial intermediation enter the left hand side of the equation and place a lower bound on the value of a “vacancy” to a firm. Absent credit market frictions the average cost of creation depends on the flow cost of a vacancy γ and congestion on the labor market. Second, the expected value on the right hand side corresponds to the ability to produce and sell a good once a consumer has been located. Under frictionless goods markets, the right hand side is simply the value of the profit stage. Thus the current model nest the canonical search model when $K(\phi^*)$ tends to zero and the goods market friction is removed.

A log-linear approximation around the deterministic steady state of this job creation condition yields

$$\widehat{\theta}_t = \frac{1}{\eta_L} \frac{S_g}{S_g - K(\phi)} \mathbb{E}_t \widehat{S}_{g,t+1} \quad (30)$$

where η_L is the elasticity of the job filling rate with respect to labor market tightness and “hatted” variables indicate proportional deviations from the steady state. Over and above the amplification of changes in S_g from frictions in the labor market, measured as the inverse of the elasticity of the labor matching function, frictions in credit markets create an amplifying factor of $\frac{S_g}{S_g - K(\phi)}$. This financial accelerator is decreasing in the firm’s surplus to hiring a worker, $S_g - K(\phi)$, and its full potential is explored in detail in Petrosky-Nadeau and Wasmer (2010).

Goods market frictions fundamentally change the dynamics of S_g along two dimensions: 1) the expected likelihood of reaching the profit stage in the period after hiring the worker, λ ; 2) the expected profit flow, which is now dependent on the expectation of what price the goods will fetch on the market, \mathcal{P} . In order to see this more clearly, recall that the values of the goods market stage derived earlier is a function F of goods market frictions λ_t , of the initial payoff of the firm $\pi_{0,t}$ and the profits stage S_π :

$$S_{g,t} = F(\pi_{0,t}, \lambda_t) = \pi_{0,t} + \left(\frac{1-s}{1+r} \right) \mathbb{E}_t [\lambda_t S_{\pi,t+1} + (1-\lambda_t) S_{g,t+1}]$$

with $\pi_{0,t} = -w_t$. It can be observed that when $\lambda_t \equiv 1$ and $\pi_{0,t} = x_t - w_t$, $F(\pi_{0,t}, \lambda_t)$ converges to the Mortensen-Pissarides world in which the value of the goods market stage is $S_{g,t}^{MP} = x_t - w_t + \left(\frac{1-s}{1+r} \right) \mathbb{E}_t S_{g,t+1}^{MP}$. From the recursive nature of $S_{g,t}^{MP}$, all that matters for the dynamics of labor market tightness is the expected path of the net profit flow $x - w_t$. By comparison, congestion in the goods market through λ_t affect the duration of the costly stage g before reach the profit stage π , with the later being affect by the negotiated price \mathcal{P} .

4 Quantitative results

We begin by detailing our calibration strategy. Next, we present the quantitative results for the full model and discuss in detail the sources of propagation. This section also presents some robustness results with respect to parameters of the goods market and compares the role of the different market frictions.

Table 1: Targeted moments: goods, labor and credit markets

Unemployment rate	u	10%
Wage rate	w/\mathcal{P}	0.75
Average recruiting cost over wage bill	$[\gamma\mathcal{N}_i/q(\theta)]/(w\mathcal{N})$	3%
Unmatched goods	$\mathcal{N}_g/\mathcal{N}$	19%
Consumer matching rate	$\tilde{\lambda}$	0.75
Consumer search effort	$\mathcal{C}_0\sigma(\bar{e})/w\mathcal{N}$	0.1
Share of essential good in consumption	$(\mathcal{C}_0ExpC_{0,0} + \mathcal{C}_1ExpC_{1,0})/Y^d$	15%
Mark-up over marginal cost	$\mathcal{P}/(\mathcal{N}_g w + \mathcal{N}_\pi(w + \Omega)) - 1$	15%
Share of financial sector in GDP	Σ	3%

4.1 Calibration strategy

We consider the basic unit of time to be a quarter and calibrate the model accordingly. The risk free rate r is set to 1%, corresponding to an annualized return close to the historical average on 3-month Treasury bills. The labor and goods market parameters are determined by matching a set of first moments, presented in Table 1 and discussed below. For all parameters the match with the moments is exact, with the exception of the bargaining weight δ for which we do not have a target, and the autocorrelation coefficients. We instead estimate the values of the AR(1) parameters and the consumer bargaining weight δ by maximizing the likelihood of the rational expectations solution to a linear approximation of the model on quarterly data for labor market tightness over the period 1977:1 to 2004:3. This estimation procedure yields parameter estimates for technology presented in Table 2. The typical productivity shock is a 1% deviation.⁸ The estimated consumer bargaining weight is $\delta = 0.34$.

We target an average rate of unemployment of 10%, midway between the values in Cole and Rogerson (1999) and Shimer (2005). In addition, we require recruiting costs to represent 3% of the wage bill in steady state, consistent with the evidence reported in Silva and Toledo (2007). Based on the evidence in Davis et al. (2006), we set the exogenous job separation rate to $s = 0.05$. The elasticity of the labor matching function, $M_L(\mathcal{N}_l, u) = \chi_L \mathcal{N}_l^{1-\eta_L} u^{\eta_L}$, is set to $\eta_L = 0.5$, in the mid-range of

⁸This is small enough to make sure that the saving motive is limited even if we had assumed a concave utility function. Indeed, since the consumers pool the unemployment risk, individual consumers do not face unemployment shocks, and the only income fluctuations arise from these very small perturbations in productivity. Hence, a concave utility would then be well approximated locally by a linear utility.

Table 2: Baseline parameter values

Labor market			Goods market		
job separation rate	s	0.05	goods exit rate	τ	0.01
matching elasticity	η_L	0.5	matching level param.	χ_G	0.52
wage elasticity	η_w	0.5	matching elasticity	η_G	0.5
matching level param.	χ_L	0.58	consumer barg. weight	δ	0.34
wage level param.	χ_w	0.57	cost function - level	χ_σ	0.55
vacancy cost	γ	0.01	cost function - elasticity	η_σ	6.22
			Marginal utility of c_1	Φ	0.64
Credit market			Technology		
bank's barg. weight	β	0.27	labor productivity	x	1
matching elasticity	η_C	0.5	persistence param.	ρ_x	0.975
matching level param.	χ_C	1.9	standard deviation	σ_x	0.01
search costs	$\kappa = c$	0.05	risk free rate	r	0.01

values reported in the survey by Petrongolo and Pissarides (2001). The elasticity of the wage to the marginal product is set to 0.5, close to the value suggested by Gertler and Trigari (2009). We also target the wage to be 75% of the marginal revenue of a sale \mathcal{P} , the reminding 25% being shared between the firm and the bank, through repayment ρ .

Only firms matched with a consumer sell their goods in this economy, thus workers associated with projects that are not selling their goods are, in a sense, un-utilized capacity. We thus target a capacity utilization rate of 81%, similar to the calibration in Bai et al. (2011). These authors target a capacity utilization rate in the consumption sector of 81% based on the Federal Reserve's Statistical Release of Industrial Production and Capacity Utilization. Finally, the cost parameter Ω is adjusted to match a 15% price mark-up over marginal cost.

With respect to consumer search, we target an average search duration of a little over 5 weeks before finding and deciding on a new consumption good, implying $\tilde{\lambda} = 0.75$. Given our other calibration targets, the steady state rate of product entry, defined as $\tilde{\lambda}\mathcal{C}_0/\mathcal{C}_1$, is 0.25 on an annualized basis. This is consistent with the product entry rate, weighted by expenditure shares, found by Broda and Weinstein (2010). The goods market matching function is assumed to take the form $M_G(\bar{e}\mathcal{C}_0, \mathcal{N}_g) = \chi_G (\bar{e}\mathcal{C}_0)^{1-\eta_G} \mathcal{N}_g^{\eta_G}$ and we assume an elasticity of 0.5, performing a series of sensitivity tests below. To calibrate the effort placed into the search for consumption goods, we

rely on the BLS' time use survey which reports that households spend on average half an hour a day purchasing goods and services (0.4 hours for men, 0.6 for women). Of course, this is not necessarily time spent searching and comparing goods before making a choice. Nor does it include travel related to these activities. Assuming an individual works on average 5 hours a day, spread over a week, the cost of time searching in the goods market corresponds to approximately 10% of wage income. That is, we target $\mathcal{C}_0\sigma(\bar{e})/w\mathcal{N} \simeq 0.1$. The product exit rate is given by $s + (1 - s)\tau$, which Broda and Weinstein found to be 0.24 at an annual frequency (weighted by expenditure shares). This implies quarterly goods separation rate of $\tau = 0.011$. Finally, we target an expenditure share in the essential good based on the Household Consumption Expenditure survey's average annual expenditure on food consumed at home, plus utilities, over the period 1984 to 2009. This amounts to 15% of total annual expenditures. In the model, this share is defined as $(\mathcal{C}_0ExpC_{0,0} + \mathcal{C}_1ExpC_{1,0})/Y^d$, where $ExpC_{1,0} = Y^d - \mathcal{P}$ is the expenditure on the essential good of a matched consumer, and $ExpC_{0,0} = Y^d$ the expenditure of an unmatched consumer. These expenditures are weighted by the fraction of unmatched and matched consumer.

The calibration of the credit market requires choosing parameters of the credit matching function, assumed to be of the form $M_c(\mathcal{B}_c, \mathcal{N}_c) = \chi_C \mathcal{N}_c^{1-\eta_C} \mathcal{B}_c^{\eta_C}$, the costs of prospecting on credit markets and the bargaining weight β , and follows Petrosky-Nadeau and Wasmer (2010). We assume symmetry in prospecting costs $\kappa = c$, and the remaining parameters, χ_C , η_C and β , are adjusted to accommodate a targeted share of corporate lending in GDP equal to 3%, using national account data and calculations described in the appendix. In the model this share is calculated from

$$\Sigma = \frac{\mathcal{B}_\pi \varrho - \mathcal{B}_g w - \mathcal{B}_l \gamma - \mathcal{B}_c \kappa}{Y^d}$$

4.2 Results

In the benchmark economy with frictions in the three markets, as reported in Figure 1, labor market tightness and the value of filled vacancy S_g reach their peak 11 periods after the realization of the shock to technology. The evolution of labor market tightness is driven by the expected value of hiring a worker, as displayed in the top right panel of Figure 1. The inverted U-shape for the response of labor market tightness is not a frequent property in most economic models, yet is a feature of the

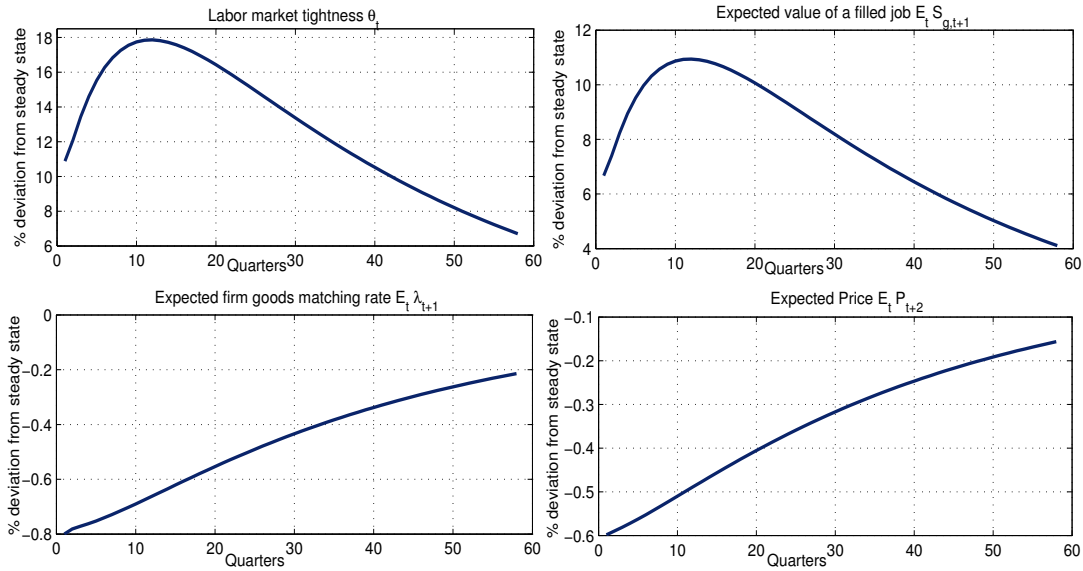


Figure 1: Goods market frictions - inspecting the mechanism. IRFs to a positive productivity shock

data (see Fujita and Ramey, 2007) our model achieves through mechanisms described below.

In the bottom panels of Figure 1, we study the various channels at work, in particular the intertemporal linkages between the labor and goods markets. Firms expect next period to face a drop in the likelihood of selling their goods following recruiting a worker. The reason being that there will be an inflow of vacancies today, leading to more supply of goods tomorrow due to more firms matched with a worker and able to look for consumers. On the other hand, search effort by consumers will also rise, contributing to an increase in the probability of selling for firms, but this effect is dominated by the first one and overall, λ_t will decrease. As productivity reverts to its trend, conditions on goods market improve over time for firms as the inflow of new goods on the market slows relative to the increases in effective demand by searching consumers both in terms of market congestion and the price at which firms sell their product. The evolution of the goods market thus creates increasing incentives to hire workers even as productivity and the profit flow are returning to trend.

Table 4 reports the business cycles characteristics of labor market variables, along with those for aggregate consumption and output. The baseline calibrated model comes close to replicating the amount of volatility of labor market tightness seen in

Table 3: Unconditional second moments - goods market variables

Goods market		a	b			a	b
Tightness	ξ	1.37	-0.69	Consumer search effort	e	0.65	0.91
Firms hazard rate	λ	0.69	-0.69	Unmatched consumers	\mathcal{C}_0	0.53	-0.64
Consumer hazard rate	$\tilde{\lambda}$	0.69	0.69	Unmatched firms	\mathcal{N}_g	1.71	0.70
Price of goods c_1	\mathcal{P}	0.49	-0.79				

a: standard deviation relative to output; b: contemporaneous correlation with output. All moments are HP filtered

the data, generating a standard deviation relative to that of output of 11.10. The same is true for the volatilities of vacancies and unemployment, and the contemporaneous correlations of each variable with the cyclical component of aggregate output is consistent with the data. However, the main improvement over the existing literature lies in the last rows of Table 4. They report the autocorrelation of output and labor market tightness growth rates at the first three lags. This measure indicates that goods market frictions generate a substantial amount of persistence in the dynamics of the labor market: it is positive, around 0.17, largely above the corresponding measures in the models without goods market frictions (last column, -0.01), yet still below the data (0.61).

The next table summarizes the business cycle dynamics of the goods market by presenting a series of second moments for goods market variables, in terms of H.-P. filtered standard deviations relative to output and contemporaneous correlations with output. Focusing first on congestion in the goods market, tightness $\xi = \bar{e}\mathcal{C}_0/\mathcal{N}_g$ is countercyclical, consumers match more quickly with goods in a boom while, as we mentioned, the matching rate of firms λ is countercyclical. The number of unmatched firms, or goods on the market searching for consumers, is greater during an expansion, capturing a notion as in Shleifer (1986) of booms being periods when more projects are implemented. Consumers search effort is pro-cyclical while the fraction of unmatched consumers is counter-cyclical.

Summarizing, this model with search frictions in the goods market features a lagged response of the demand side of the goods market to productivity shocks, both directly through the income distributed to consumers, and indirectly through consumer effort and prices. As compared to Bai et al. (2011), we don't rely on consumer demand shocks: the endogenous response of the consumer side of the market amplifies and propagates productivity shocks.

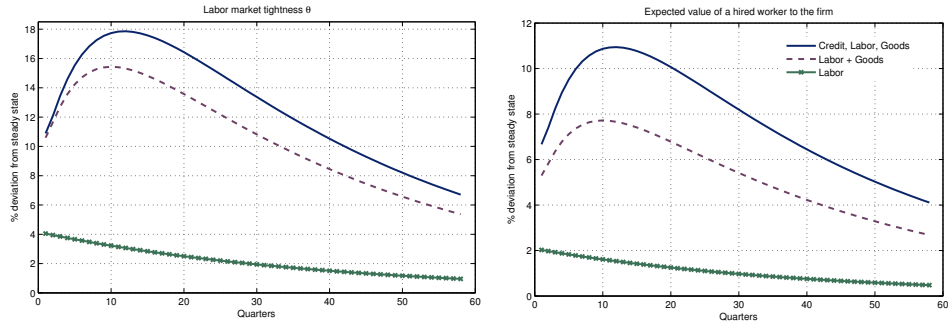


Figure 2: Comparing frictions: IFS to a positive productivity shock

4.3 The respective role of different market frictions

In order to assess the role of each frictions, we will now compare the previous results with two alternatives: one in which we have frictions in the goods and labor markets (perfect financial markets, $K = 0$) and one in which we only have frictions in the labor market (a Mortensen-Pissarides economy). These alternative economies were calibrated in order to match the same statistics for unemployment.⁹

Figure 2 illustrates the response of these different calibrated economies by plotting the impulse response of labor market tightness to a positive productivity shock. Compared to the benchmark economy, there is still a peak in the impulse response function when there are frictions both in the goods and in the labor market, but at a lower level. Indeed, in this calibration with goods market frictions, the absence of financial market imperfections has a relatively moderate impact.¹⁰

There is instead a very classical response of the economy with only labor market frictions, with little amplification and persistence. This is because, for most wage rules, labor market tightness simply follows the path of the process for productivity. This shows a contrario that search frictions in the goods market, by generating a snowball effect between entry of firms and redistribution to consumers, is responsible for the increasing part of the inverted U-shape curve of labor market tightness discussed above.

Table 4 reports the corresponding business cycle moments from these models and

⁹An alternative is to preserve the parameter values obtained in the baseline calibration the model, successively removing the different market frictions. The business cycle moments of the resulting economies would be difficult to compare.

¹⁰This result is in contrast with Petrosky-Nadeau and Wasmer (2010) and Petrosky-Nadeau (2010) where, in the absence of good market frictions, the role of financial market imperfections was much more important and lead to a greater financial multiplier. In that sense, financial and goods market frictions are substitutes in producing volatility.

Table 4: Second moments - comparing frictions

	US data		Goods, labor and credit		Goods & labor		Labor only	
	a	b	a	b	a	b	a	b
Vacancies	8.83	0.89	7.51	0.94	7.18	0.92	3.09	0.88
Unemployment	6.82	-0.88	5.00	-0.81	4.72	-0.81	0.16	-0.67
Labor tightness	15.41	0.90	11.10	0.99	10.50	0.99	3.20	0.88
Wage	0.40	0.44	0.29	0.61	0.29	0.66	0.40	0.88
Consumption	0.59	0.80	0.68	0.89	0.65	0.89	0.66	0.75
$\sigma(GDP)$	1.40		1.13		1.13		1.18	
Persistence:	GDP	θ	GDP	θ	GDP	θ	GDP	θ
$corr(\Delta z_t, \Delta z_{t-1})$	0.24	0.61	0.06	0.17	0.05	0.14	0.26	-0.01
$corr(\Delta z_t, \Delta z_{t-2})$	0.19	0.40	0.26	0.18	0.24	0.14	0.21	-0.01
$corr(\Delta z_t, \Delta z_{t-3})$	0.05	0.30	0.12	0.15	0.11	0.11	0.18	0

Notes: H.-P. filtered (a) standard deviation relative to GDP; (b) contemporaneous correlation with GDP. Data sources: B.E.A., B.L.S. and Conference Board, 1977:Q1 to 2007:Q4.

summarizes the empirical shortcomings of the canonical search model of unemployment. The first concern is the well known lack of amplification of productivity shocks: labor market tightness is nearly 15 time more volatile than GDP over the business cycle whereas the model generates of relative volatility of 3. The second concerns persistence, measured by autocorrelation in growth rates. Labor market tightness is very persistent in the data, much more so than GDP, whereas the canonical model generates no persistence: θ follows exactly the shock process. Table 4 also reveals that goods market frictions contribute most to improving the qualitative and quantitative dynamics of labor market variables.

4.4 Robustness to alternative parametrizations

We now verify whether the elasticity of the goods matching function η_G and the consumer's bargaining weight δ affect the responses of the variables that are key for propagation through the goods market. Figure 3 plots the impulse responses to the same technological innovation of labor market tightness, S_g and its determinants when we increase the bargaining weight δ from 0.34 to 0.5, or when we reduce the elasticity η_G from 0.5 to 0.25. Table 5 reports the filtered second moments for each scenario.

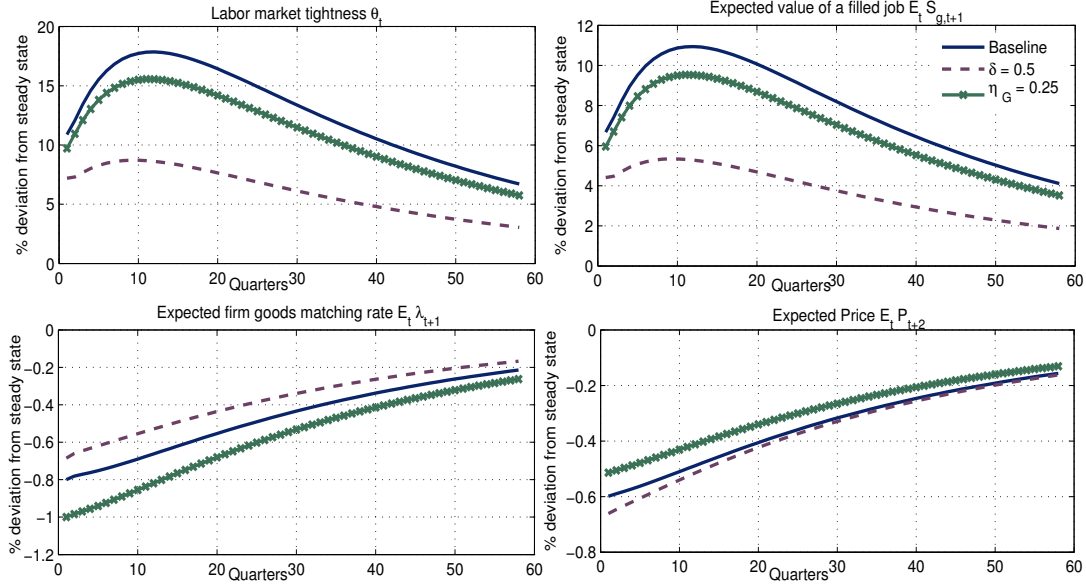


Figure 3: Impulse responses to a positive productivity shock- Baseline and Sensitivity to goods market parameters η_G and δ

Increasing the share of the goods surplus accruing to the consumer implies a stronger downward response of the price following the shock and, although the goods matching rate for firms λ still drops at first, its return to steady state is very progressive. The result is much more muted response of the value of hiring a worker, with a modest “hump”. The persistence of labor market tightness is thus only a third of what it was under the baseline parameter values, and the relative volatility of θ decreases from 11.10 to 7.04. On the other hand, reducing the elasticity of the goods matching function has only a minor impact on the quantitative results: the second moments in Table 5 are mostly the same as in the baseline parameterization.

We perform a final sensitivity analysis in this Section in which we set the elasticities of the labor and goods matching function both to 0.25, retain the estimated value for the bargaining weight of 0.34. While this has little impact on the relative volatility of labor market tightness, there is a much stronger response of employment and output to the same changes in market tightness θ . As the job finding rate varies more over the business cycle, there is a significant increase in the relative volatility of unemployment reinforcing the feedback between labor and goods markets and, in terms of persistence, the model is now much closer to the autocorrelation in the growth rate of θ measured in the data.

Table 5: Business cycle moments and sensitivity to goods market parameters

	Baseline		Consumer		Goods Match.		$\eta_L = \eta_G = 0.25,$	
	$\delta = 0.34, \eta_G = 0.5$		barg. $\delta = 0.5$		$\eta_G = 0.25$		$\delta = 0.34$	
	a	b	a	b	a	b	a	b
Vacancies	7.51	0.94	5.04	0.91	7.04	0.95	5.83	0.73
Unemployment	5.00	-0.81	3.17	-0.76	4.68	-0.79	7.47	-0.90
Labor tightness	11.10	0.99	7.04	0.99	10.41	0.99	11.08	0.99
Wage	0.29	0.61	0.31	0.65	0.31	0.73	0.24	0.53
Consumption	0.68	0.89	0.65	0.81	0.71	0.92	0.81	0.97
$\sigma(GDP)$	1.13		1.03		1.08		1.47	
Persistence:	GDP	θ	GDP	θ	GDP	θ	GDP	θ
$corr(\Delta z_t, \Delta z_{t-1})$	0.06	0.17	0.003	0.06	0.07	0.18	0.31	0.33
$corr(\Delta z_t, \Delta z_{t-2})$	0.26	0.18	0.14	0.07	0.16	0.17	0.44	0.29
$corr(\Delta z_t, \Delta z_{t-3})$	0.12	0.15	0.04	0.06	0.09	0.14	0.32	0.24

Notes: H.-P. filtered (a) standard deviation relative to GDP; (b) contemporaneous correlation with GDP.

5 Conclusion

This paper investigates the significance of goods market frictions for the dynamics of the labor market in particular, and the macroeconomy in general. The model with search frictions in the goods market features a lagged response of the demand side of the goods market to productivity shocks, acting through the income distributed to consumers. This affects consumer search effort for goods and negotiated prices leading to rich intertemporal linkages between labor and goods markets.

We find that the feedback from goods to labor markets significantly affects the qualitative and quantitative features of labor market dynamics. During the first stages of an economic expansion, more firms enter the goods market and more profits are generated. The additional redistributed revenues push consumers to raise their search effort in order to achieve a higher desired level of consumption. Simultaneously, the inflows of new firms causes an increase in congestion in the goods markets, from the point of view of firms, and a decline in the negotiated price at which the goods are eventually sold, dampening the incentive to create a vacancy at the beginning of an expansion.

Overall, we account for a rise in labor market tightness for several periods after the initial shock through improved conditions in the goods market for firms in the

later stages of an expansion. Our model features the propagation of supply shocks through the demand side of the economy, contrary to the standard labor search model and a large class of its extensions.

In contrast, the steady-state properties of the model are rather different. In a companion paper, Wasmer (2009) had explored these properties and found instead, little amplification from the good market. The reason is that in the steady-state the demand and the supply of goods always match and cannot drift away from each other, since all the income generated by firms is distributed to consumers without the interesting dynamics studied here.

Finally, our model of a frictional economy is a natural framework for introducing additional sources of shocks, in particular demand shocks and money as in Berentsen et al. (2011). This is left to future work.

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Appendix

A Data used in calibration

In order to calculate the financial sector's share of aggregate value added, we rely on the BEA's industry value added tables available at http://www.bea.gov/industry/io_histannual.htm. Because of i) changes in industry labels in 1987; and ii) a change of classifications in 1998, we report the exact series used for each time period:

- 1947 to 1987: we sum the value added of “banking” (code 60), “Credit agencies other than banks” (code 61) and “Security and commodity brokers” (code 62).
- 1987 to 1998: we sum the value added of “Depository institutions” (code 60), “Non-depository institutions” (code 61) and “Security and commodity brokers” (code 62).
- 1998 to 2009: we sum the value added of “Federal Reserve banks, credit intermediation, and related activities” (code 521 and 522), “Securities, commodity contracts, and investments” (code 523) and “Funds, trusts, and other financial vehicles” (code 525).

The share Σ is obtained by dividing by the corresponding year's aggregate GDP from the same value added tables.

Data on consumption expenditure is obtained from BLS series CXUTE000201 on Total average annual expenditures, series CXUFH000201 for food consumed at home and series CXUUT000101 for Utilities, fuels, and public services, covering the years 1984-2009

B Determinations of price \mathcal{P}_t

Utility

We begin by assuming a quasi-linear form for consumer utility:

$$U(c_{1,t}, c_{0,t}) = v(c_{1,t}) + c_{0,t}$$

Given these preferences, we can determine the negotiated price for the good c_1 as the outcome of Nash bargaining over the consumption surplus $G_t = (S_{\pi,t} - S_{g,t}) + (D_{1,t} - D_{0,t})$. $\delta \in (0, 1)$ will be the consumer's bargaining weight. As a first step, we derive expressions for the surplus to the consumption relationship for each side of the market. We have a budget constraint per period (since we assume no savings):

$$\mathcal{P}_t c_{1,t} + c_{0,t} = Y_t^d$$

Consumer surplus

When matched, the consumer consumes what is available, that is

$$c_{1,t} = x_t$$

Recall the Bellman equations for unmatched and matched consumers:

$$\begin{aligned} D_{0,t} &= c_{0,t} - \sigma(e_t) + \frac{1}{1+r} \mathbb{E}_t \left[e_t \tilde{\lambda}_t D_{1,t+1} + (1 - \tilde{\lambda}_t e_t) D_{0,t+1} \right] \\ &= Y_t^d - \sigma(e_t) + \frac{1}{1+r} \mathbb{E}_t \left[e_t \tilde{\lambda}_t D_{1,t+1} + (1 - \tilde{\lambda}_t e_t) D_{0,t+1} \right] \\ D_{1,t} &= v(c_{1,t}) + Y_t^d - \mathcal{P}_t c_{1,t} + \left(\frac{1-s}{1+r} \right) \mathbb{E}_t [\tau D_{0,t+1} + (1-\tau) D_{1,t+1}] + \frac{s}{1+r} \mathbb{E}_t D_{0,t+1} \\ &= v(x_t) + Y_t^d - \mathcal{P}_t x_t \end{aligned}$$

Then the surplus to a match for a consumer is

$$\begin{aligned} D_{1,t} - D_{0,t} &= v(x_t) - \mathcal{P}_t x_t + \sigma(e_t) + \tau \frac{1-s}{1+r} \mathbb{E}_t [D_{0,t+1} - D_{1,t+1}] \\ &\quad + \frac{1-s}{1+r} \mathbb{E}_t D_{1,t+1} + \frac{s}{1+r} \mathbb{E}_t D_{0,t+1} \\ &\quad - \tilde{\lambda}_t e_t \frac{1}{1+r} \mathbb{E}_t [D_{1,t+1} - D_{0,t+1}] - \frac{1}{1+r} \mathbb{E}_t D_{0,t+1} \end{aligned}$$

From the optimal choice of effort, we had that $\tilde{\lambda}_t e_t \frac{1}{1+r} \mathbb{E}_t [D_{1,t+1} - D_{0,t+1}] = e_t \sigma'(e_t)$.

Hence,

$$\begin{aligned} D_{1,t} - D_{0,t} &= v(x_t) - \mathcal{P}_t x_t + \sigma(e_t) - e_t \sigma'(e_t) + \tau \frac{1-s}{1+r} \mathbb{E}_t [D_{0,t+1} - D_{1,t+1}] \\ &\quad + \frac{1-s}{1+r} \mathbb{E}_t [D_{1,t+1} - D_{0,t+1}] \end{aligned}$$

$$D_{1,t} - D_{0,t} = v(x_t) - \mathcal{P}_t x_t + \sigma(e_t) - e_t \sigma'(e_t) + (1 - \tau) \frac{1-s}{1+r} \mathbb{E}_t [D_{1,t+1} - D_{0,t+1}]$$

Firm surplus

Turning to the surplus of a consumption match for a firm, we have

$$\begin{aligned} S_{\pi,t} - S_{g,t} &= \mathcal{P}_t x_t - \Omega - \tau \frac{1-s}{1+r} \mathbb{E}_t [S_{\pi,t+1} - S_{g,t+1}] + \frac{1-s}{1+r} \mathbb{E}_t S_{\pi,t+1} \\ &\quad - \lambda_t \frac{1-s}{1+r} \mathbb{E}_t [S_{\pi,t+1} - S_{g,t+1}] - \frac{1-s}{1+r} \mathbb{E}_t S_{g,t+1} \end{aligned}$$

$$S_{\pi,t} - S_{g,t} = \mathcal{P}_t x_t - \Omega + (1 - \tau - \lambda_t) \frac{1-s}{1+r} \mathbb{E}_t [S_{\pi,t+1} - S_{g,t+1}]$$

Sharing rule

$\mathcal{P}_t = \underset{\mathcal{P}_t}{\operatorname{argmax}} (S_{\pi,t} - S_{g,t})^{1-\delta} (D_{1,t} - D_{0,t})^\delta$. Since $\partial (S_{\pi,t} - S_{g,t}) / \partial \mathcal{P}_t = x_t$ and $\partial (D_{1,t} - D_{0,t}) / \partial \mathcal{P}_t = -x_t$, the sharing rule is

$$(1 - \delta) (D_{1,t} - D_{0,t}) = \delta (S_{\pi,t} - S_{g,t})$$

Goods surplus

$$\begin{aligned} G_t &= (S_{\pi,t} - S_{g,t}) + (D_{1,t} - D_{0,t}) \\ &= v(x_t) - \Omega + (1 - \tau - \lambda_t) \frac{1-s}{1+r} \mathbb{E}_t [S_{\pi,t+1} - S_{g,t+1}] \\ &\quad + \sigma(e_t) - e_t \sigma'(e_t) + (1 - \tau) \frac{1-s}{1+r} \mathbb{E}_t [D_{0,t+1} - D_{1,t+1}] \end{aligned}$$

$$\begin{aligned} G_t &= v(x_t) - \Omega + \sigma(e_t) - e_t \sigma'(e_t) \\ &\quad + (1 - \tau) \frac{1-s}{1+r} \mathbb{E}_t G_{t+1} - \lambda_t \frac{1-s}{1+r} \mathbb{E}_t [S_{\pi,t+1} - S_{g,t+1}] \end{aligned}$$

Under regular assumptions on the effort cost function $\sigma(e)$, it will be the case that $e\sigma'(e) = \eta_\sigma\sigma(e)$ where $\eta_\sigma > 0$ is the elasticity of the effort cost function.

$$G_t = v(x_t) + (1 - \eta_\sigma)\sigma(e) - \Omega + [(1 - \tau) - (1 - \delta)\lambda_t] \frac{1 - s}{1 + r} \mathbb{E}_t G_{t+1}$$

Negotiated price \mathcal{P}

In order to solve for the price, we equate the firm's surplus $S_{\pi,t} - S_{g,t} = (1 - \delta)G_t$ to the previous expression for the goods surplus

$$(1 - \delta)G_t = (1 - \delta) [v(x_t) + (1 - \eta_\sigma)\sigma(e) - \Omega] + (1 - \delta) [(1 - \tau) - (1 - \delta)\lambda_t] \frac{1 - s}{1 + r} \mathbb{E}_t G_{t+1}$$

$$\begin{aligned} (1 - \delta) [v(x_t) + (1 - \eta_\sigma)\sigma(e) - \Omega] + (1 - \delta) [(1 - \tau) - (1 - \delta)\lambda_t] \frac{1 - s}{1 + r} \mathbb{E}_t G_{t+1} \\ = \mathcal{P}_t x_t - \Omega + (1 - \tau - \lambda_t) \frac{1 - s}{1 + r} \mathbb{E}_t [S_{\pi,t+1} - S_{g,t+1}] \end{aligned}$$

$$\mathcal{P}_t x_t = (1 - \delta) [v(x_t) + (1 - \eta_\sigma)\sigma(e)] + \delta\Omega + (1 - \delta)\lambda_t \frac{1 - s}{1 + r} \mathbb{E}_t G_{t+1} - (1 - \delta)^2 \lambda_t \frac{1 - s}{1 + r} \mathbb{E}_t G_{t+1}$$

$$\mathcal{P}_t x_t = (1 - \delta) [v(x_t) + (1 - \eta_\sigma)\sigma(e)] + \delta\Omega + (1 - \delta)\lambda_t \frac{1 - s}{1 + r} \mathbb{E}_t [\delta G_{t+1}]$$

Recall that $\delta G_t = D_{1,t} - D_{0,t}$, that from the optimal choice of effort $\frac{1}{1+r} \mathbb{E}_t [D_{1,t+1} - D_{0,t+1}] = \frac{\sigma'(e_t)}{\lambda_t}$, and that from the properties of the goods matching function we have that $\frac{\lambda_t}{\lambda_t} = \xi_t$.

$$\mathcal{P}_t x = (1 - \delta) [v(x_t) + (1 - \eta_\sigma)\sigma(e_t) + (1 - s)\sigma'(e_t)\xi_t] + \delta\Omega$$

Search Effort and disposable income

From now on, assume that

$$v(x_t) = \Phi x_t$$

Optimal search effort is determined by the condition. The optimal individual search effort is simply given by a condition equating the marginal cost of effort to the dis-

counted, expected benefit yielded by that marginal unit of effort:

$$\sigma'(\bar{e}_t) = \frac{\tilde{\lambda}_t}{1+r} \mathbb{E}_t [D_{1,t+1} - D_{0,t+1}]$$

Using the recursivity of the expression

$$D_{1,t} - D_{0,t} = (\Phi - \mathcal{P}_t) x_t + \sigma(e_t) - e_t \sigma'(e_t) + (1 - \tau) \frac{1-s}{1+r} \mathbb{E}_t L^{-1} [D_{1,t} - D_{0,t}]$$

where L^{-1} is the forward operator. Defining $\psi \equiv (1 - \tau) \frac{1-s}{1+r}$ a discount factor and $\tilde{\sigma}(e_t) \equiv \sigma(e_t) - e_t \sigma'(e_t)$, we have

$$\begin{aligned} D_{1,t} - D_{0,t} &= (\Phi - \mathcal{P}_t) x_t + \tilde{\sigma}(e_t) + (1 - \tau) \frac{1-s}{1+r} \mathbb{E}_t L^{-1} [D_{1,t} - D_{0,t}] \\ [D_{1,t} - D_{0,t}] [1 - \psi \mathbb{E}_t L^{-1}] &= (\Phi - \mathcal{P}_t) x_t + \tilde{\sigma}(e_t) \\ [D_{1,t} - D_{0,t}] &= \frac{1}{[1 - \psi \mathbb{E}_t L^{-1}]} [(\Phi - \mathcal{P}_t) x_t + \tilde{\sigma}(e_t)] \\ [D_{1,t} - D_{0,t}] &= \mathbb{E}_t \sum_{i=0}^{\infty} \psi^i [(\Phi - \mathcal{P}_{t+i}) x_{t+i} + \tilde{\sigma}(e_{t+i})] \end{aligned}$$

We know that a matched consumer will spend all disposable income Y_t^d on good $c_{1,t}$. Thus income Y_t^d can purchase $c_{1,t} = x_t = Y_t^d / \mathcal{P}_t$ goods and we have

$$\begin{aligned} [D_{1,t} - D_{0,t}] &= \mathbb{E}_t \sum_{i=0}^{\infty} \psi^i \left[(\Phi - \mathcal{P}_{t+i}) \frac{Y_{t+i}^d}{\mathcal{P}_{t+i}} + \tilde{\sigma}(e_{t+i}) \right] \\ [D_{1,t} - D_{0,t}] &= \mathbb{E}_t \sum_{i=0}^{\infty} \psi^i \left[\left(\frac{\Phi}{\mathcal{P}_{t+i}} - 1 \right) Y_{t+i}^d + \tilde{\sigma}(e_{t+i}) \right] \end{aligned}$$

This expression clearly ties the surplus to a consumption relationship for the consumer to the future expected paths of disposable income and the price. Plugging this expression in the condition for optimal consumer search effort:

$$\sigma'(\bar{e}_t) = \frac{\tilde{\lambda}_t}{1+r} \mathbb{E}_t \sum_{i=0}^{\infty} \psi^i \left[\left(\frac{\Phi}{\mathcal{P}_{t+1+i}} - 1 \right) Y_{t+1+i}^d + \tilde{\sigma}(e_{t+1+i}) \right]$$

C Allowing for savings or borrowing

C.1 Case 1

We have the new budget constraint per period, with r_s the rate of return on savings, assumed to be less than r :

$$\mathcal{P}_t c_{1,t} + c_{0,t} + s_t = Y_t^d + s_{t-1}(1 + r_s)$$

If the consumer expect a drop in price next period, he may not want to consume everything today in order to consume more of the numeraire next period. Hence, we have

$$\mathcal{P}_t c_{1,t} + s_t = \mathcal{P}_t x_t$$

Bellman equations for unmatched and matched consumers:

$$\begin{aligned} D_{0,t} &= c_{0,t} + s_{t-1}(1 + r_s) - \sigma(e_t) + \frac{1}{1+r} \mathbb{E}_t \left[e_t \tilde{\lambda}_t D_{1,t+1}(s_t) + (1 - \tilde{\lambda}_t e_t) D_{0,t+1}(s_t) \right] \\ &= Y_t^d + s_{t-1}(1 + r_s) - \sigma(e_t) + \frac{1}{1+r} \mathbb{E}_t \left[e_t \tilde{\lambda}_t D_{1,t+1}(s_t) + (1 - \tilde{\lambda}_t e_t) D_{0,t+1}(s_t) \right] \\ D_{1,t} &= v(c_{1,t}) + Y_t^d - \mathcal{P}_t c_{1,t} + s_{t-1}(1 + r_s) + \left(\frac{1-s}{1+r} \right) \mathbb{E}_t [\tau D_{0,t+1}(s_t) + (1 - \tau) D_{1,t+1}(s_t)] \\ &\quad + \frac{s}{1+r} \mathbb{E}_t D_{0,t+1}(s_t) \\ &= v(x_t - s_t/\mathcal{P}_t) + Y_t^d - \mathcal{P}_t x_t + s_{t-1}(1 + r_s) + \left(\frac{1-s}{1+r} \right) \mathbb{E}_t [\tau D_{0,t+1}(s_t) + (1 - \tau) D_{1,t+1}(s_t)] \\ &\quad + \frac{s}{1+r} \mathbb{E}_t D_{0,t+1}(s_t) \end{aligned}$$

The solution for the optimal choice of savings is a trade-off between sacrificing consumption today, which costs

$$\frac{v'(x_t - s_t/\mathcal{P}_t)}{\mathcal{P}_t}$$

and to raise consumption opportunities tomorrow, which yields additional utility depending on the future state. In particular, in state 0, we have

$$\frac{d\mathbb{E}_t D_{0,t+1}(s_t)}{ds_t} = 1 + r_s$$

and

$$\frac{d\mathbb{E}_t D_{1,t+1}(s_t)}{ds_t} = 1 + r_s$$

since he is constrained by the production tomorrow for his consumption next period. Hence without calculation, we know that the consumer does not want to save: what he gains tomorrow is only to consume more numeraire, which by definition costs him a fixed price. So at best, he gains some more numeraire tomorrow which brings $(1 + r_s)/(1 + r) < 1$ but sacrifices $\frac{v'(x_t - s_t/P_t)}{P_t}$ which is larger than 1: marginal utility is above 1, prices must be below 1.

C.2 Other cases

1. A slightly different reasoning applies if the agent could store good 1 in case he returned to stage 0. In this case the agent may have an interest to smooth consumptions. But good 1 was assumed to be non-storable.
2. By the same logic, one may want to know whether the agents could borrow in both stages at rate r_b , to consume more of the numeraire. Again, this is not possible given $(1 + r_b)/(1 + r) > 1$. The key insight here is that borrowing or savings is of no help because the agents are constrained to buy all the available goods.
3. Finally, a last possibility would be to have unconstrained agents, who would consume, when matched, both good 1 and the numeraire. In this case, they may want to reduce consumption of good 1 in periods of high prices and raise it in period of low price. However even in this case, this is not profitable: postponing consumption of good 1 today entails a risk: that of losing the good with probability τ . The gain is to have some more consumption of the good, by a marginal quantity equal deflation. In the numerical exercise, this marginal quantity has to be discounted by $(1 - \tau)/(1 + r)$ and compared to deflation. Given that deflation is typically at most 0.5% quarterly in our simulation and that τ is 1%, the consumer, in this case, would still not want to sacrifice consumption of good 1 today to buy more good 1 tomorrow.