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Abstract

Unemployment may depend on equilibrium in other markets than the labor markets. This paper adresses this old idea by introducing search frictions on several markets: in a model of credit and labor market imperfections as in Wasmer and Weil (2004), I further introduce search on the goods market. The model can be solved by blocks: on two of the three markets, the relevant "market tightness" is a constant of parameters. In particular, goods market tightness, expressed as the ratio of unmatched consumers to unmatched firms, is equal to 1 which corresponds to a stochastic Say's law: demand and supply are stochastically in equilibrium. Financial market tightness is also a function of three parameters related to financial frictions. Job creation and employment depend on the equilibrium in the other markets. Reciprocally, higher job destruction implies more volatility of income of individual consumers and higher destruction of consumption matches. This lowers profits and further reduces job creation. Finally, there are complementarities between frictions in each market: in particular, the marginal effect of financial frictions on equilibrium unemployment is amplified by goods market frictions and vice versa.

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1 Introduction

This paper is an attempt to model the steady-state of an economy with several search markets in general equilibrium interactions. Trade in every explicit market (in this paper, markets for labor, for credit and for final goods) is represented by a market-specific matching function. The goal is to study the properties of such an economy: we look for "theoretical regularities" such as the existence and uniqueness of equilibrium; block recursiveness in solutions, that is, whether the model can be solved first for some markets and then for the other markets independently; the size and determinants of accelerators in response to profit shocks; the impact of sequential bargaining on price/wage/interest rate determination and in particular strategic rent extraction.

The paper exhibits some similarities with the tradition of Walrasian general equilibrium theory. This theory and extensions to security markets provide a rich and elegant analysis of markets in complete (general equilibrium) interaction: as put in Tirole (1988), "*the best-developed and most aesthetically pleasing model in the field of economics is the competitive-equilibrium paradigm of Arrow-Debreu. (...) Weak assumptions about preferences and technological possibilities yield general results on competitive equilibrium*". The first and second welfare theorems are powerful, and most properties hold even in the presence of uncertainty, at least under the assumption of a sufficiently large number of security markets.

Yet, despite the general equilibrium theory's elegance and simplicity, it has fragile foundations. Many assumptions are unrealistic: agents are not all atomistic, they are far from being perfectly informed, transferring goods and production inputs from one place to another is often costly; and contrary to the Arrow-Debreu securitization assumptions, most future markets are inexistant. In spite of providing an inaccurate description of the real world, the normative implications of general equilibrium results have shaped policy debates for decades.

This paper deviates away from the general equilibrium theory in developing a stylized model of a "multi-frictional" economy with three markets (credit, labor and goods). An important assumption made here is that agents cannot consume alone their own output, as in Diamond (1982a), and Kiyotaki and Wright (1993), and instead rely on other agents to consume (consumer search) or to generate profits (firm search). Those "other agents" cannot be located, unless incurring some search effort. The goal is obviously not to be perfectly realistic: it is only to be more realistic than general equilibrium theory. We are motivated by the need of a simple alternative theory of general equilibrium, even if this paper is still a very preliminary step in this direction.

The equilibrium of this model can be easily characterized by three "market tightnesses" and three prices, solved by two free-entry conditions and one inelastic labor supply condition, and bargaining equations over the surplus or rigid wage/price. Whatever the assumptions about price/wage determination (exogeneity or bargaining), we find a convenient way of solving the model by blocks: credit and goods market tightness are expressed as a simple constant of parameters, as well as prices. Further, goods market tightness is even equal to 1 in steady-state. When it exists, we prove uniqueness of the equilibrium; we obtain that the economy is relatively more reactive to profit shocks when there are more market imperfections, through the spillovers from credit and goods market on the labor market; that the effects of frictions tend to be complement to each other; that credit frictions play quantitatively a larger role than the frictions on other markets; and that two sets of agents bargaining first extract a rent on the other agents.

In a companion paper, Petrosky-Nadeau and Wasmer explore the dynamic implications of a variant of the model, with some differences in particular in wage bargaining, in the specification of search effort by consumers and in the allocation of income in the economy. It turns out that the static and the dynamic properties of the model are very different from each other: in conclusion, we provide a discussion of the additional features brought by the dynamics of the model.

In **Section 2**, we review the literature and discuss the link between the early neo-keynesian models and matching models; in **Section 3**, we develop the labor and credit market, introduce free-entry conditions and derive some useful equilibrium conditions. In **Section 4**, we introduce the goods market, establish the equilibrium value of goods market tightness as the solution of a fixed point problem, and finally solve for price and wages. In **Section 5**, we study the comparative statics of the equilibrium and provide numerical exercises in order to measure the volatility of economies to shocks in different scenarios regarding market frictions. **Section 6** provides a parametrization and computes multipliers of profit shocks. **Section 7** then relaxes two assumptions, first that firms and bankers do not consume, second that the unemployed cannot consume. **Section 8** concludes.

2 Some related literature

Introducing deviations from the assumptions of competitive markets within a general equilibrium framework has been a central research agenda for many years. In an earlier work, Prescott and Townsend (1984) made an important step in expanding the general equilibrium theory with several asymmetries of information: their purpose was "*to explore the extent to which standard, general equilibrium analysis of Pareto optima and of competitive equilibria can be applied to economies with moral hazard and adverse selection problems.*" Introducing lotteries to smooth decision rules, they are able to show that moral hazard does not undo the optimality theorems, while in economies with adverse selection in insurance markets and signaling economies, the second theorem may not apply.

Another strand of research with a more explicitly macroeconomic focus is the Keynesian tradition under fixed prices and the neo-keynesian tradition under imperfectly competitive goods markets. Bénassy (1982, 1986) in particular developed the macroeconomic theory of disequilibrium in worlds of rigid prices. In Bénassy (1993), he showed a simple example of a fixed price/flexible wage economy where "*...unemployment in the labor market is due entirely to the malfunctioning of the goods market! Of course this is an example but we must keep in mind the extreme importance of the interactions across markets, which is a major reason why a full general equilibrium analysis is needed.*" (Bénassy 1993, p. 735). Subsequent contributions attempted to introduce endogenous price determination with imperfectly competitive product markets.

Another issue encountered in the Keynesian tradition was to provide a good description of who consumes and who does not consume, that is, an explicit rationing scheme at the microeconomic level. Indeed, under disequilibrium, not everybody can trade with each other. Some agents will consume, others will not. As shown in Bénassy (1993, pp. 736 to 747), in most cautious theoretical work of that tradition, some complexity arises from the need to precisely define nominal and effective demands and supplies of goods. In particular, there are two central questions that need answers: how can the numerous signals expressed by agents about their desired consumption levels be transformed into effective transactions, and how do these transformations make consistent the large set of nominal demands and supplies of each individual, through the assumption of an aggregate function of signals.

In our paper, the markets are non-walrasian, but for other reasons than imperfect price adjustment. We start by relaxing the central assumption of the existence of centralized markets and "*introduce costly delays in the process of finding trading partners and determining the terms of trade*" as stated in Mortensen and Pissarides (1999, p. 2569) for instance, along the lines of the search-and-matching literature. An important building block, following the pioneering work by Diamond (1981, 1982a, b), Mortensen (1982a,b), Pissarides (1984) is that the number of trading agents is governed by a smooth and well-behaved *aggregate matching function*, the input of which are the number of agents willing to trade.[1]

Quite interestingly, the way the early search-and-matching literature defines the aggregate function of signals of agents (Bénassy 1993) is very reminiscent of the way Pissarides (1990, chapter 1) rationalizes the aggregate matching function: he sees it as a technology that makes consistent the desired demand and desired supply side of markets. One can reinterpret Bénassy's signals of required consumption/supply as being the elements of the "vacancy" side of markets (that is how many and by how much agents are willing to trade) to link the neo-keynesian tradition and the general equilibrium matching approach.

The matching approach offers several advantages over the traditional neo-keynesian literature. It shares some virtues with competitive equilibrium theory (CET hereafter), i.e. simplicity of the solutions, elegance of the models allowing

[1] The other building blocks of search theory used here are: i) economic agents are not fully informed and in particular collect information about trading opportunities or quality of goods at random times; ii) a corollary of the previous block is that economic agents trade randomly, after irregular contacts; iii) a natural implication of the previous two blocks is that trade is priced differently from the competitive view of the world: subject to bilateral contacts, agents engage in a negotiation about how to split surpluses. Nash-bargaining solutions are therefore basic building blocks of this theory.

to express the main intuitions easily, convenient ways of introducing intertemporal trade-offs and investigation of the dynamics of macroeconomic variables. It also encompasses the frictionless general equilibrium theory as a limit case, when matching occurs at an infinitely high rate. Nesting CET, it must necessarily be the case that matching models are a *better* description of the real world than CET, over a range of matching parameters. Second, matching theory rationalizes the existence of vacancies, such as vacant job positions, vacant housing, unmatched capital units and unused money holdings.

It is therefore no surprise that the search and matching approach to labor markets has become central in both quantitative macroeconomics and in labor economics. What is somewhat a surprise is that it has not been used more to expand the neo-keynesian agenda, namely: i) to investigate the general equilibrium interactions between markets (what Bénassy calls "spillovers between markets"); ii) to identify causes of unemployment outside the labor markets.

One may thus expect matching theory to be a good starting point for the development of alternative general equilibrium theories. As a matter of fact; quite many contributions have attempted to model different markets with search or matching tools: for instance, Kiyotaki and Wright (1993) have modeled money in this way, Wheaton (1990) has adapted it for housing markets.[2] Other papers have explored similar assumptions with interactions between frictional markets. Bertensen, Menzio and Wright (2011) study a money-search economy and labor frictions. Their framework allows them to discuss monetary policy, inflation and interest rates and their effect on unemployment. Lehmann and Van der Linden (2010) have also introduced frictions in good and labor markets. They focus more on normative implications of search frictions and of inflation.

3 The structure and equilibrium of the model

The structure adapts and extends that of Wasmer and Weil (2004, WW hereafter). There are three types of agents: workers, who need to employed by a firm to produce, entrepreneurs/firms who need a worker to produce output, *but cannot hire and sell immediately* and private banks. Entrepreneurs need to pay their employees before selling, and for that need to borrow from the bank. The bank is a profit maximizing unit that is refinanced by a fourth (absent) agent, the Central Bank, at some discount factor r. The bank will deliver a particular financial contract to the entrepreneur; the important point is that the bank supplies liquidity to the entrepreneur as long as needed, and receives in exchange a predetermined part of future profits.

All agents are risk-neutral ; they have the same discount rate of the future r as the one defined by the Central Bank's interest rate (in continuous time). Another helpful assumption is that all contracts between agents can be enforced

[2] As stated in Rogerson et al. (2005), "*At the outset, it is important to point out that search theory constitutes a very large branch of economics. In addition to labor it has been used in many applications in both micro and macro, including monetary theory, industrial organization, finance, and the economics of the marriage market (...)*"

and they all expire after the arrival of a shock destroying the technology of the firm.

Hence, what we call a bank is actually only a (profit-maximizing) supplier of liquidity that allows entrepreneurs to finance two activities, first the hiring activities and second the search for consumers. This is a very stylized money market, and we do not enter the detail of its functioning, in contrast to the seminal paper by Kiyotaki and Wright (1993). Further, following Wasmer and Weil and in contrast to Kiyotaki and Wright (1993) or Diamond (1982a), we assume that agents are not able to store any value, even their own production: workers, who are also consumers, do not save (bank notes will burn spontaneously) and all produced goods fully depreciate when they are not sold: goods are like butter, not like guns. Therefore, banks are the only agents allowing to transmit value across time and thus permit the functioning of the economy.

3.1 Firms and the goods market

There are four periods in the lifetime of a firm: in period 0, it searches a banker. In period 1, it looks for a worker. In period 2, it produces but cannot store the good, and simultaneously looks for a consumer. In period 3, it sells the good, receives a price in return and finally pays a predetermined amount to the bank - that will have been negotiated between them in the beginning of period 1, at the time of their meeting.

Let E_0, E_1, E_2 and E_3 the respective steady-state asset values of a firm in each period. We have:

$$
\begin{aligned}
rE_0 &= -c + p(E_1 - E_0) && (1)\\
rE_1 &= -\gamma + \gamma + q(E_2 - E_1) && (2)\\
rE_2 &= -w + w + \lambda(E_3 - E_2) + s(E_0 - E_2) && (3)\\
rE_3 &= P - w - \rho + s(E_0 - E_3) + \tau(E_2 - E_3) && (4)
\end{aligned}
$$

where p is the Poisson rate at which a banker is found; γ is the hiring cost per period of time that is financed by the banker (hence the firm's cash-flow is zero in period 1; q is the Poisson rate at which the firm hires; w is the wage paid to the worker, financed by the bank in period 2; λ is the Poisson rate at which the firm finds a consumer; P is the selling price; ρ is the payment contracted with the bank in the beginning of stage 1; s is the rate of obsolescence of the good/technology, which strikes in both period 2 and 3 and τ is the rate at which the consumer stops buying the production, and measures the *versatility* of consumers; τ will actually not be a parameter, but the result of some exogenous income shock, specified later on. When the shock τ occurs, the firms revert to state 2: the firm preserves its past contractual arrangement and does survive, though at a lower value of expected profits. It only keeps its worker, and the bank pays for the wage.[3] Finally, c is the flow *disutility* of the firm in stage 0

[3] Note that the timing of events is important: the firm has to search for a bank first, then for a worker, then for a customer. We do not allow for a reversal in the order worker/customer.

and can be interpreted as the *effort* cost made to convince a banker to finance future negative cash-flows.

The parameter λ reflects frictions in the goods market, that is implicitly a combination of two factors: first, it reflects how heterogeneous the consumers are (see Petrongolo and Pissarides 2001 and Lagos 2000 for a discussion of how heterogeneity and dispersion in space provide foundations to matching), and second, it reflects the degree of imperfect competition on the goods market. We will examine later on what lies behind λ and its impact on wages and prices.[4]

3.2 Matching between banks and starting firms

We follow the matching literature and state that the total number of contacts is governed by a matching technology associating the number of banks in stage 0 denoted by $\mathcal{B}_0$ and the number of firms in stage 0 denoted by $\mathcal{N}_0$. Let $M_C(\mathcal{N}_0, \mathcal{B}_0)$ be the matching process in the credit market. We have that $p = M_C(\mathcal{N}_0, \mathcal{B}_0)/\mathcal{N}_0$. Symmetrically, the Poisson rate at which banks find a project they are willing to finance is $M_C(\mathcal{N}_0, \mathcal{B}_0)/\mathcal{B}_0 = \phi p$ where $\phi = \mathcal{N}_0/\mathcal{B}_0$. It follows that ϕ is a natural measure of the tightness of the credit market and, under the assumption of constant returns to scale of $M_C(\cdot,\cdot)$, that $p = p(\phi)$ with $p\prime(\phi) < 0$ and elasticity $\varepsilon(\phi) = -p'(\phi)/\phi p(\phi)$. We also make Inada assumptions that

$$\begin{aligned}\lim_{\phi\to 0} p(\phi) &= +\infty \\ \lim_{\phi\to +\infty} p(\phi) &= 0\end{aligned}$$

The first line states that, in the relative scarcity of competing firms relative to banks, matching with a banker is instantaneous, and the second line states that in the relative abundance of competing firms relative to banks, matching with a banker is infinitely slow.

3.3 Banks and liquidity

As said above, banks are refinanced by the Central Bank at a rate r. They are in one of four states: looking for projects to finance, financing job search and wages in stages 1 and 2 of the firm, finally enjoying the repayment in stage 3. Denoting by κ the cost of screening projects, we obtain the Bellman equations

It may indeed be possible or desirable for the customer to wait for production to occur. This is a situation which is encountered by consumers of specific good such as "Rolls Royce" or "Jaguar", that are ordered and paid and then only delivered after several months. The same is true for new housing. For most other goods, our sequencing assumption is more plausible. In contrast, reversing the sequencing with respect to credit is impossible, as firms need liquidity to proceed to subsequent stages, hiring or advertising the goods.

[4] This specification for search frictions in the goods market is a variant of the traditional money search literature, e.g. Kiyotaki and Wright (1993), where the probability to meet a trading partner is the square of the underlying probability for one side to agree on the exchange. The square reflects the double coincidence of goods.

for the banks in steady-state:

$$rB_0 = -\kappa + \phi p(B_1 - B_0) \tag{5}$$

$$rB_1 = -\gamma + q(B_2 - B_1) \tag{6}$$

$$rB_2 = -w + \lambda(B_3 - B_2) + s(B_0 - B_2) \tag{7}$$

$$rB_3 = \rho + s(B_0 - B_3) + \tau(B_2 - B_3) \tag{8}$$

In line 2 (stage 1), the bank finances the hiring costs and in line 3 (stage 2), the bank finances the wage, as long as the firm searches for a consumer. As previously, when the consumer quits the firm, the bank is back in stage 2 and still finances the wage costs. Banks are the key operator here: they transform the liquidity provided to them by the Central Bank into firm investment such as job creation and hiring.

3.4 Matching between firms and workers

Similarly to the credit market, we assume that the total number of contacts is governed by a matching technology associating the number of firms in stage 1 (that is, of vacancies) denoted by $\mathcal{V}$ and the total number of workers looking for a job. Since we rule out on-the-job search for simplicity, the number of workers looking for a job is simply the number of unemployed, denoted $\mathcal{U}$.

Let $M_L(\mathcal{V},\mathcal{U})$ be the matching process in the credit market. We have that $q = M_L(\mathcal{V},\mathcal{U})/\mathcal{V}$. Symmetrically, the Poisson rate at which workers would find a firm is $M_L(\mathcal{V},\mathcal{U})/\mathcal{U} = \theta q$ where $\theta = \mathcal{V}/\mathcal{U}$. Then, θ is a measure of the tightness of the labor market. We also assume constant returns to scale of $M_L(\cdot,\cdot)$, and therefore $q = p(\theta)$ with $q\prime(\theta) < 0$ and elasticity $\eta(\theta) = -q'(\theta)/\theta p(\theta)$. We also make Inada assumptions that

$$\lim_{\theta\to 0} q(\theta) = +\infty$$

$$\lim_{\theta\to +\infty} q(\theta) = 0$$

The first line states that, in the relative scarcity of competing vacancies relative to the unemployed workers, matching with a worker is instantaneous, and the second line states that in the relative abundance of competing vacancies relative to workers, matching with a worker is infinitely slow.

3.5 Implications of free entry

We assume free entry of firms and banks. That, is

$$B_0 = 0 \tag{9}$$

$$E_0 = 0 \tag{10}$$

This leads to the following useful intermediate steps: by difference of E_3 and E_2 we obtain that

$$(E_3 - E_2)(r + \lambda + \tau + s) = P - w - \rho \tag{11}$$

$$(B_3 - B_2)(r + \lambda + \tau + s) = \rho + w \tag{12}$$

An important notation for what follows is the ratio of transition rates α_λ where

$$\alpha_\lambda = \frac{r+s+\lambda}{r+s+\lambda+\tau}$$

This ratio goes from its maximum $(r+s)/(r+s+\tau)$ when λ tends to zero (infinite amount of frictions on the goods market) to 1 when λ tends to infinity (frictionless goods market) or alternatively when consumers never stop consuming (τ=0). In other words, when the goods market becomes unfriendly to firms (consumers difficult to locate or more versatile), α_λ gets closer to 0. It is thus a summary indicator of *fluidity* in the goods market.

We have the following useful equations:

$$\begin{aligned} B_1 &= \frac{\kappa}{\phi p(\phi)} && (13) \\ E_1 &= \frac{c}{p(\phi)} && (14) \\ B_3 &= \frac{\rho\alpha_\lambda - w(1-\alpha_\lambda)}{r+s} && (15) \\ E_3 &= \alpha_\lambda \frac{P-w-\rho}{r+s} && (16) \end{aligned}$$

The first two equations come from the free-entry of banks and firms and state that the value of a match for the bank (respectively the firm) in stage 1 is the expected value of search costs for the bank (respectively the firm). The next two equations combine the asset values of banks (respectively firms) in stage 2 and 3 and provide a calculation of the expected value of streams of revenues for each side (bank and firm).

We also report intermediate steps linking the asset values across different periods together, to simplify future derivations of the equilibrium. In particular:

$$\begin{aligned} B_2 &= \frac{1}{r+s+\lambda}\left[-w+\lambda B_3\right] && (17) \\ E_2 &= \frac{1}{r+s+\lambda}\left[0+\lambda E_3\right] && (18) \\ B_1 &= \frac{1}{r+q(\theta)}\left[-\gamma+qB_2\right] && (19) \\ E_1 &= \frac{1}{r+q(\theta)}\left[0+qE_2\right] && (20) \end{aligned}$$

3.6 Credit bargaining

The terms of repayment from the firm to the bank are denoted by the flow valueρ. It is the outcome of a bargaining process, and set so as to maximize

$$(B_1 - B_0)^\beta (E_1 - E_0)^{1-\beta} \tag{21}$$

where β is the bargaining power of the bank. We derive the outcome of bargaining under the important assumption that wages and prices, endogenously and determined later on, are however independent of the repayment ρ. This condition will actually be verified and proved ex-post, in Section 4.3 (Property 6). Thanks to this assumption, and to the free entry conditions, we easily obtain the values of B_1 and E_1, combining the free-entry conditions (13), (14) and the bargaining equation(21):

Property 1 (block recursiveness 1) (Wasmer and Weil, 2004). *The equilibrium credit market tightness is only a function of parameters of the financial market. It does not depend on goods market frictions:*

$$\phi^* = \frac{1-\beta}{\beta}\frac{\kappa}{c} \tag{22}$$

Now, thanks to the recursive structure, we also have the forward solutions to the Bellman equations in stage 1[5] allowing us to conveniently solve for ρ thanks again to equation (21):

Flow repayment ρ from the firm to the bank in profit stage 3:

$$\alpha_\lambda \frac{\rho}{r+s} = \beta\left[\frac{P-w}{r+s}\right] + (1-\beta)\left[\frac{\gamma}{q(\theta)}\frac{r+s+\lambda}{\lambda} + \frac{w}{\lambda} + \frac{w(1-\alpha_\lambda)}{r+s}\right] \tag{25}$$

The interpretation of equation (25) is as follows: the expected value of repayment is $\alpha_\lambda \frac{\rho}{r+s}$, and would be $\frac{\rho}{r+s}$ if consumers never changed taste ($\tau = 0$ implying $\alpha_\lambda = 1$). This expected repayment is simply a weighted average of future profits properly discounted $\frac{P-w}{r+s}$ and of costs borne by the bank: namely, the future value of start-up costs $\frac{\gamma}{q(\theta)}$ and all labor costs supported by the bank during stage 2.

The fact that goods market are imperfect (lower λ) lowers α_λ leading to higher financial repayment to the bank. In the limit case without frictions in the goods market, the repayment would then be, as in Wasmer and Weil (2004):

$$\frac{\rho}{r+s} = \beta\left[\frac{P-w}{r+s}\right] + (1-\beta)\left[\frac{\gamma}{q(\theta)}\right] \tag{26}$$

Overall, repayment here is higher than that for two reasons: a) there is a scale effect due to $\alpha_\lambda < 1$ in the left-hand side; and b) by definition of the role

[5] Intermediate steps are to replace the values of B_3 and E_3 into B_2 and E_2 and then replace them all into equations (19) and (20) to obtain:

$$E_1 = \frac{q(\theta)}{r+q(\theta)}\frac{\lambda}{r+s+\lambda}\alpha_\lambda\frac{P-w-\rho}{r+s} \tag{23}$$

$$B_1 = \frac{q(\theta)}{r+q(\theta)}\left[\frac{-\gamma}{q(\theta)} + \frac{\lambda}{r+s+\lambda}\left(-\frac{w}{\lambda} + \frac{\rho\alpha_\lambda - w(1-\alpha_\lambda)}{r+s}\right)\right] \tag{24}$$

of banks, they have to support labor costs during the search of consumers by firms. Hence the additional burden for banks described by $\frac{w}{\lambda} + \frac{w(1-\alpha_\lambda)}{r+s}$ which once again only disappears when λ goes to infinity. A more compact way of expressing ρ is to introduce a notation B_w:

$$\alpha_\lambda \frac{\rho}{r+s} = \beta \left[\frac{P-w}{r+s} \right] + (1-\beta) \left[\frac{\gamma}{q(\theta)} \frac{r+s+\lambda}{\lambda} + B_w \right] \tag{27}$$

where

$$B_w = \frac{w}{\lambda} + \frac{w(1-\alpha_\lambda)}{r+s}$$

Property 2 (Partial complementarity between financial and goods markets). *The lower λ (hence, the more frictions on the goods market), the higher the expected financial burden of the firm, represented by $\alpha_\lambda \rho$, because banks require a higher repayment if it takes more time for the firm to make profits. Note also that $\partial(\alpha_\lambda \rho)/\partial(1/\lambda) > 0$ is higher when the bargaining power of the bank $(1-\beta)$ is itself higher. Finally, a lower λ leads to a larger repayment ρ.*

Proof: Differentiate equation (27) using $\frac{\partial(1-\alpha_\lambda)}{\partial(1/\lambda)} = \frac{\tau\lambda^2}{(r+s+\lambda+\tau)^2}$ hence $\frac{\partial B_w}{\partial(1/\lambda)} = w\left(1 + \frac{\tau\lambda^2}{(r+s+\lambda+\tau)^2(r+s)}\right)$ and $\frac{\partial \frac{r+s+\lambda}{\lambda}}{\partial(1/\lambda)} = r+s$, hence the derivative calculated above. On the second point,

$$\frac{1}{r+s} \frac{\partial(\rho\alpha_\lambda)}{\partial(1/\lambda)} = (1-\beta) \left[\frac{\gamma(r+s)}{q(\theta)} + w \left(1 + \frac{\tau\lambda^2}{(r+s+\lambda+\tau)^2(r+s)} \right) \right]$$

On the last point, given that α_λ is itself increasing with λ, more goods market frictions are associated with a lower α_λ. Combined with the first part of the proposition, more goods market frictions imply higher repayment.

This happens to be the first of a relatively frequent result in the economics of "*multi-frictional economies*": imperfections in each market tend to reinforce each other. This is a partial equilibrium result, not a general equilibrium result yet. It has however an intuitive appeal: the present discounted value (PDV) of firm's future profits is reduced due to goods market imperfections since the firm will not sell immediately. In addition, there is a second indirect effect: since banks also suffer from the reduced future profits of the firm, they will bargain for higher repayment which further amplifies the effect on firms at the entry stage.

3.7 Equilibrium in θ and ϕ and its existence

One can easily plug ρ into the forward values of E_1 and B_1 to obtain two curves in the space (θ, ϕ). These two curves are shifted by all frictions, including by goods market frictions. They are determined by the following equations:

$$\text{(EE)} \quad : \quad \frac{c}{p(\phi)} = (1-\beta)\frac{q(\theta)}{r+q(\theta)}\left[\alpha_\lambda \frac{P-w}{r+s} - \left(\frac{\gamma}{q(\theta)}\frac{r+s+\lambda}{\lambda} + B_w\right)\right] \tag{28}$$

$$\text{(BB)} \quad : \quad \frac{\kappa}{\phi p(\phi)} = \beta\frac{q(\theta)}{r+q(\theta)}\left[\alpha_\lambda \frac{P-w}{r+s} - \left(\frac{\gamma}{q(\theta)}\frac{r+s+\lambda}{\lambda} + B_w\right)\right] \tag{29}$$

The interpretation of (EE) is the locus of free-entry combinations of (θ,ϕ) making the entry value of firms equal to zero. It is given by a decreasing relation between ϕ and θ: a higher θ reduces expected profits and must be compensated along EE by a less tight credit market for firms. As θ goes to zero, we obtain a limit denoted by ϕ_E

The interpretation of (BB) is the locus of free-entry combinations of (θ,ϕ) making the entry value of banks equal to zero. It is given by a increasing relation between ϕ and θ: a higher θ reduces expected profits and must be compensated along BB by a more tight credit market for firms (less tight from the banks' perspective). As θ goes to zero, we obtain a limit denoted by ϕ_B

Property 3 (existence). *A necessary condition for existence is that $\phi_E > \phi_B$ which reflects a condition on profits, entry costs κ and c and the efficiency of matching in the credit market.*

Proof: If we denote $\Lambda = \frac{P-w}{r+s} - \frac{w}{\lambda}$, we have that $\frac{c}{p(\phi_E)} = (1-\beta)\Lambda$ and $\frac{\kappa}{\phi_B p(\phi_B)} = \beta\Pi$. Under Cobb-Douglas assumptions $p(\phi) = p_0\phi^{-\varepsilon}$, we have

$$\begin{aligned} \ln\phi_E &= \frac{1}{\varepsilon}\ln[\frac{p_0}{c}(1-\beta)\Lambda] \\ \ln\phi_B &= -\frac{1}{1-\varepsilon}\ln[\frac{p_0}{k}\beta\Lambda] \end{aligned}$$

The existence condition amounts to $\ln\phi_E > \ln\phi_B$ that is $\frac{1}{\varepsilon}\ln[\frac{p_0}{c}(1-\beta)\Lambda] > -\frac{1}{1-\varepsilon}\ln[\frac{p_0}{k}\beta\Lambda]$ or after transformation,

$$\Lambda > \frac{\left(\frac{k}{\beta}\right)^{\varepsilon}\left(\frac{c}{1-\beta}\right)^{1-\varepsilon}}{p_0} \tag{30}$$

which states that profits have to be higher than a combination of entry costs weighted by β and ε and divided by the efficiency of the credit matching process p_0.

3.8 Special cases

We can easily revert to the special case of perfect goods markets. Unsurprisingly, the limit case when firms sell instantaneously (that is, when $\lambda \to \infty$) brings us back to Wasmer and Weil (2004) since $\alpha_\lambda \to 1$ and $B_w \to 0$:

$$(\text{EE}_0) \quad : \quad \frac{c}{p(\phi)} = (1-\beta)\frac{q(\theta)}{r+q(\theta)}\left[\frac{P-w}{r+s} - \frac{\gamma}{q(\theta)}\right] \tag{31}$$

$$(\text{BB}_0) \quad : \quad \frac{\kappa}{\phi p(\phi)} = \beta\frac{q(\theta)}{r+q(\theta)}\left[\frac{P-w}{r+s} - \frac{\gamma}{q(\theta)}\right] \tag{32}$$

We can also provide notations for the different equilibrium values of θ under different assumptions about market imperfections.

- let θ^* be the equilibrium labor market tightness defined by the combination of (28) and (29) (general case),
- let $\theta^{WW}_{g=0}$ be the Wasmer-Weil limit value of labor market tightness when goods market frictions disappear, defined by the intersection of (31) and (32) (no goods market imperfections),
- let $\theta^{P}_{c,g=0}$ be the Pissarides limit value of labor market tightness when both good ($\lambda \to \infty$) and credit market frictions (c, κ, p go to zero) disappear (labor constrained economy),
- finally, let $\theta_{c=0}$ be the limit value of labor market tightness when only credit frictions disappear but goods market frictions remain finite: $\lambda \geq 0$ (no credit market imperfections).

Denote by $K = \frac{c}{p(\phi^*)} + \frac{\kappa}{p(\phi^*)}$ which is a measure of financial frictions. The different equilibrium values for θ are defined in each case as the implicit solution of :

$$\text{(P):} \quad \frac{P-w}{r+s} = \frac{\gamma}{q(\theta^P_{c,g=0})} \tag{33}$$

$$\text{(WW):} \quad \frac{P-w}{r+s} = \frac{\gamma}{q(\theta^{WW}_{g=0})} + K\left(1+\frac{r}{q(\theta^{WW}_{g=0})}\right) \tag{34}$$

$$\text{(C0):} \quad \left(\alpha_\lambda \frac{P-w}{r+s} - B_w\right) = \frac{\gamma}{q(\theta_{c=0})}\left(1+\frac{r+s}{\lambda}\right)$$

$$\text{(ALL):} \quad \left(\alpha_\lambda \frac{P-w}{r+s} - B_w\right) = \frac{\gamma}{q(\theta^*)}\left(1+\frac{r+s}{\lambda}\right) + K\left(1+\frac{r}{q(\theta^*)}\right) \tag{35}$$

where the left-hand side terms of these equations represent the expected values of profits of the block constituted from the firm and the bank and the left hand-side represents the expected costs of labor and credit search. We can rank most of these values of labor market tightnesses. In particular, the following two inequalities hold:

$$\theta^P_{c,g=0} > \theta_{c=0} > \theta^* \tag{36}$$

$$\theta^P_{c,g=0} > \theta^{WW}_{g=0} > \theta^* \tag{37}$$

$$\theta_{c=0} \lesseqgtr \theta^{WW}_{g=0} \tag{38}$$

The first inequality states that goods market frictions slow down job creation because firms make less profit due to difficulties to sell, and that combining both frictions (goods and credit) further lowers equilibrium tightness. The second inequality states that credit market frictions slow down job creation as well because firms must pay higher entry costs to locate credit. The last line reflects

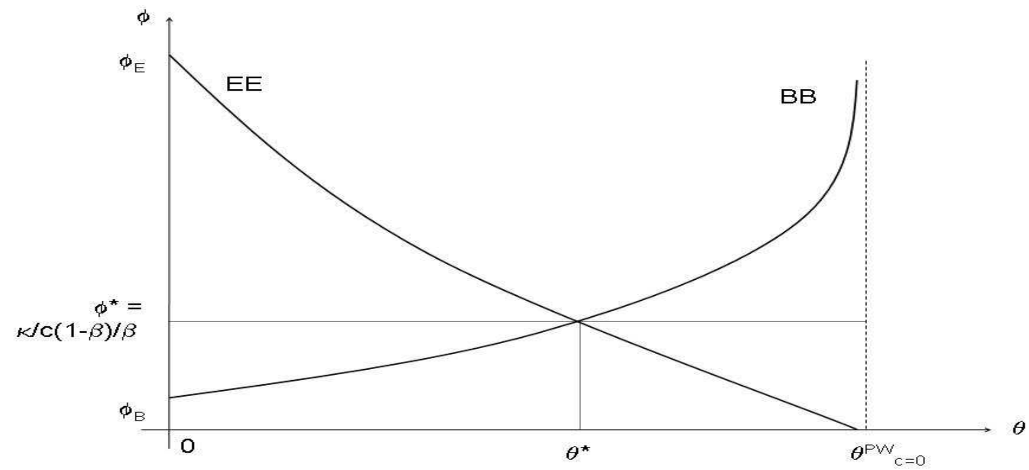

Fig. 1: Equilibrium with goods market frictions: free entry of firms (EE) and of banks (BB)

the fact there is no a priori ranking of labor market tightness in the two polar cases: perfect credit market and imperfect goods market vs. imperfect credit market and perfect goods market. There is no way to sign $\theta^{W}_{c=0} - \theta^{WW}_{g=0}$ in the general case.

4 The goods markets: consumers and prices

We assume the existence of two consumption goods. The first one is a numeraire good sold without frictions and produced with no labor and a linear technology (e.g. the fruits of a Lucas tree but the tree is known to everyone and not subject to search frictions). The second one is the manufactured good produced by firms in stage 2 and 3 and subject to search frictions. It is sold at price P : it is produced by firms and yields $1 + \Phi$ units of utility to consumers, with Φ being strictly positive. It is consumed inelastically, the second unit has marginal value inferior to its price. Think for instance of a micro-wave, the marginal utility of owning a second one is close to zero. Hence, at most individual consumers will consume 1 unit of the good.[6] Consistently with the assumption of credit

[6] Inelastic consumption of the manufactured good is a useful simplifying assumption. It makes the price a simpler solution to the Nash program between firms and consumers. In particular, the assumption implies that the worker's utility is (negatively) linear in price, and not in the inverse of price as this would be the case if consumption was equal to income over price.

market imperfections, consumers do not save.[7] The income is spent into the manufactured good, what is left is consumed in the numeraire.

4.1 Demand side

We assume that (net) profits are pooled and redistributed to workers as a lump-sum. Denote $\mathcal{B}_0$, $\mathcal{V}$, $\mathcal{N}_2$ and $\mathcal{N}_3$ respectively the number of banks in stage 0 and firms in stage 1 (that is, the number of vacancies), 2 (prospecting for consumers) and 3 (making profits).

The total net flows of profits Π in the economy are given by

$$\Pi = -k\mathcal{B}_0 - \gamma\mathcal{V} - w\mathcal{N}_2 + \mathcal{N}_3(P - w)$$

where the first, second and third are the negative cash-flows of the banks, and $\mathcal{N}_3(P - w)$ is the net profits of all banks and firms in stage 3. Consumers therefore receive per person and per period the transfer

$$T = \Pi + M$$

where M is the helicopter drop of money from the Central Bank insuring that the economy has a positive demand for goods.

The workers can be employed at a wage w or unemployed. The unemployed have no income except their share of profits and transfer T. We assume that the good is consumed inelastically at a price P. We further consider a regime in which

$$T < P < T + w$$

which implies that individual consumers can be in one of the 3 stages:

- unemployed, in which case they consume only the numeraire good and enjoy utility T;
- employed at a wage w and prospecting for the manufactured good, in that case they enjoy utility $w+T$;
- employed and consuming the manufactured good inelastically, at a price $P \leq w + T$.[8] In that case they enjoy utility $1 + \Phi$ plus the residual income $w + T - P$ which has marginal utility 1.

[7] It can be shown however that consumers, in this setup, do not necessarily want to save if they have access to the numeraire. This is because savings does not insure consumption of the manufactured good with certainty: the presence of search frictions for goods reduces the value of savings.

[8] We recognize that there are alternative specifications to the one studied in the text. One regime in which $T + w < P$ in which case there is no consumption and therefore a degenerate economy with only home production. And another, potentially less trivial one, in which $P < T$ in which the unemployed may consume too, thus crowding out the consumption market We explore this alternative specification in Section 7.2.

The Bellman equations describing each of these three stages in a steady-state are:

$$rU = T + \theta q(W_0 - U) \tag{39}$$

$$rW_0 = w + T + \eta(W_1 - W_0) + s(U - W_0) \tag{40}$$

$$rW_1 = (w + T - P) + (1 + \Phi) + s(U - W_1) \tag{41}$$

where U is the PDV of being unemployed, W_0 the PDV of being employed and consuming the essential good; W_1 the PDV of being employed and consuming both goods; and η the Poisson rate at which workers find a good to consume. In the last equation, the first parentheses represent the utility from consuming $w+T-P$ units of the numeraire, the second parentheses the utility of consuming one unit of the manufactured good.

We are now in a position to identify τ, the parameter reflecting the destruction of the firm-consumer relation. This is actually the outcome of a job separation from the firm, leading the worker to stop consuming the manufactured good. It follows that $\tau = s$.

Property 4 (Link between labor market and goods market). *When workers' income generating process is more volatile (higher job destruction s), consumers are more versatile on the perspective of firms. This destroys consumers-firms relationships and reduces job creation.*

An implication of Property 4 is to give a rationale for policies reducing job turnover, such as training policies or employment protection, or to raise benefits for a short period of time after layoff, to preserve consumption relations. This is not explored here any further but the model is an interesting starting point for such an analysis.

The other potential consumers are bankers in stage 3: they receive repayment ρ ; and firms in stage 3: they receive cash-flow $P - w - \rho$. In the benchmark case, we do not allow them to consume the manufactured good. We assumed so far that they are only concerned the utility from consuming the numeraire which has marginal utility 1. This simplifies a little the exposition of the model. We relax this assumption in Section 7.1.

4.2 Goods market frictions

Now, we can make explicit the frictions in the goods market. Let $M_G(\mathcal{C}_0, \mathcal{N}_2)$ be the number of contacts between $\mathcal{C}_0$ unmatched consumers and $\mathcal{N}_2$ firms producing and attempting to sell their product. We have then

$$\eta = \frac{M_G(\mathcal{C}_0, \mathcal{N}_2)}{\mathcal{C}_0} \tag{42}$$

$$\lambda = \frac{M_G(\mathcal{C}_0, \mathcal{N}_2)}{\mathcal{N}_2} \tag{43}$$

and therefore, the tension on the goods market (from the perspective of workers) is:

$$\xi = \frac{\mathcal{C}_0}{\mathcal{N}_2}$$

the higher ξ, the higher the demand from consumers relative to the production awaiting to be consumed. The matching properties of the goods market are:

$$\begin{aligned}
\eta(\xi) &= M_G(1,\xi) \text{ with } \eta'(\xi) \leq 0 \\
\lambda(\xi) &= M_G(\xi^{-1},) \text{ with } \lambda'(\xi) \geq 0 \\
\text{and } \lambda(\xi) &= \xi\eta(\xi)
\end{aligned}$$

The first equation states that the relative abundance of consumers leads them to compete more with each other in searching and consuming the good: this matching rate decreases with their number relative to unmatched firms. The second equation states that the relative abundance of consumers leads firms to compete less with each other to sell: this matching rate increases with the number of unmatched consumers relative to the number of unmatched firms.

Property 5 (block recursiveness 2). *Consumption tightness ξ in a steady-state is equal to 1: $\mathcal{C}_0 \equiv \mathcal{N}_2$.*

Proof: Start with some steady-state accounting relating $\mathcal{C}_0$, $\mathcal{C}_1$ the number of matched consumers, $\mathcal{V}$ the number of firms recruiting workers (e.g. the number of vacancies), $\mathcal{N}_3$ the number of firms matched to a consumer. A more general setup is provided in Appendix.

$$\begin{aligned}
\frac{d\mathcal{C}_0}{dt} &= \theta q(\theta)u - (\eta + s)\mathcal{C}_0 = 0 \\
\frac{d\mathcal{C}_1}{dt} &= \eta\mathcal{C}_0 - s\mathcal{C}_1 = 0 \\
\frac{d\mathcal{N}_2}{dt} &= q(\theta)\mathcal{V} - (\lambda + s)\mathcal{N}_2 = 0 \\
\frac{d\mathcal{N}_3}{dt} &= \lambda\mathcal{N}_2 - s\mathcal{N}_3 = 0 \\
1 - u &= \mathcal{C}_0 + \mathcal{C}_1 = \mathcal{N}_2 + \mathcal{N}_3
\end{aligned}$$

In steady-state, we thus have

$$\begin{aligned}
\eta\mathcal{C}_0 &= s\mathcal{C}_1 = \theta q(\theta)\frac{\eta}{\eta + s}u \\
\lambda\mathcal{N}_2 &= s\mathcal{N}_3 = q(\theta)\frac{\lambda}{\lambda + s}\mathcal{V}
\end{aligned}$$

Using

$$1 - u = \mathcal{C}_0 + \mathcal{C}_1 = \mathcal{C}_0\left(1 + \frac{\mathcal{C}_1}{\mathcal{C}_0}\right) = \mathcal{C}_0\left(1 + \frac{\eta}{s}\right) \quad (44)$$

$$1 - u = \mathcal{N}_2 + \mathcal{N}_3 = \mathcal{N}_2\left(1 + \frac{\mathcal{N}_3}{\mathcal{N}_2}\right) = \mathcal{N}_2\left(1 + \frac{\lambda}{s}\right) \quad (45)$$

hence

$$\xi = \frac{C_0}{N_2} = \frac{1+\frac{\lambda}{s}}{1+\frac{\eta}{s+\eta}} = \frac{s+\lambda(\xi)}{s+\eta(\xi)}$$

Define

$$g(\xi) = \frac{s+\lambda(\xi)}{s+\eta(\xi)}$$

Hence, the steady-state consumption tightness is given by the fixed point of function $g(\xi)$. It happens that this fixed point is 1: indeed, we have

$$\lambda(\xi) = \xi\eta(\xi)$$

and therefore:

$$g(\xi) = \frac{s+\lambda(\xi)}{s+\lambda(\xi)/\xi} = \frac{s+\xi\eta(\xi)}{s+\eta(\xi)}$$

First, $g(1) = 1$ so 1 is a fixed point. Second, $g(\xi)$ is uniformly increasing in ξ, from 0 to $+\infty$. Finally, it is easy to see that $g(\xi)$ is larger than 1 for $\xi > 1$ and smaller than 1 for $\xi < 1$ so that the only strictly positive fixed point is unique.

This property is at first intriguing. It is however very intuitive: this is simply Say's law, stating that supply creates its own demand. The law is extended to stochastic matching on the consumption market. Interestingly, while Say's law applies in a neoclassical world only when prices are fully flexible, the property that goods market tightness is equal to 1 is here true independently of the price adjustment mechanism.

It is noteworthy that the property depends on the fact that the price of the good does not affect the search behavior of consumers. If the price affected search effort for goods, then the tightness of the goods market would necessarily differ from unity. In Appendix, a calculation of ξ in a more general case is developed, with various alternative assumptions regarding preferences of consumers. We show that ξ can always be calculated as a fixed point, and that the only fixed point that is strictly positive is not necessarily equal to 1 in more general cases. It is however always a function of parameters. Hence the recursiveness property of the economy still holds under less specific assumptions about the timing and separation properties of consumers and firms.

Finally, Property 5 does not hold out of the steady-state. Instead, the interesting dynamics of goods market tightness are discussed in the conclusion of this paper.

4.3 Price and wage bargaining

We now proceed to the determination of prices and wages. A convenient assumption is to assume, as in Petrosky-Nadeau and Wasmer (2010), that the firm and bank form a bargaining block, both vis-à-vis the worker and vis-à-vis the consumer. This reduces complexity of the sequential bargaining game and avoids complex strategic interactions between firms, banks and workers (and here consumers) investigated in Wasmer and Weil (2004) for wage determination.

The price is here the solution of a bargaining process over the total consumption surplus, with bargaining strength δ and $1-\delta$, following the Nash program:

$$P = Arg\max(W_1 - W_0)^{\delta}(B_3 + E_3 - B_2 - E_2)^{1-\delta}$$

where $W_1 - W_0$ is the consumer's surplus.

The determination of wages is potentially more complex, because it requires further assumptions. Indeed, the wage may be bargained under different situations, it could be negotiated:

- at the entry into the firm, between the unmatched firm (stage 2) and the worker who is still an unmatched consumer (with value W_0)
- at some continuation stage, when the worker/consumer becomes matched to a goods to consume (with value W_0) while the firm is still unmatched with a consumer (stage 2)
- reciprocally, at some continuation stage, when the worker/consumer is still unmatched (value W_0) but the firm is now matched (stage 3)
- and finally when both the firm is matched to a consumer (stage 3) and its worker is matched with a good (with value W_1).

It is therefore a potentially complicated problem. We will simplify it is assuming long term contracts: the wage is bargained at the entry stage between the unmatched worker and the firm. Further, we involve the bank into the bargaining process, as in Petrosky-Nadeau and Wasmer (2010). We assume the wage to be the solution of a bargaining process over the total surplus of the firm-bank-worker in stage 2 of the life of the firm, with bargaining strength α and $1-\alpha$. The wage is therefore not renegotiated in period 3. The wage solves, in period 2:

$$w = Arg\max(W_0 - U)^{\alpha}(B_2 + E_2 - B_1 - E_1)^{1-\alpha} \tag{46}$$

Property 6 (Price and wage). *The price is a fraction of the marginal utility of consumption of the manufactured good and increases with goods market tightness. The wage and price are uniquely determined. Both are independent of the repayment ρ from the firm to the bank.*

Proofs: **Price.** We have from equation (41):

$$(r+s)(W_1 - W_0) = (w - P + 1 + \Phi) - rU - (r+s)(W_0 - U) \tag{47}$$

This implies that $\frac{\partial(W_1 - W_0)}{\partial P} = \frac{1}{r+s}$ while from equation (4) and (8), we have symmetrically:

$$(r+s+\tau)(B_3 + E_3 - B_2 - E_2) = P - w - (r+s)(B_2 + E_2)$$

The wage is predetermined, hence its partial derivative with respect to P is equal to zero.This implies that $\frac{\partial(B_3+E_3-B_2-E_2)}{\partial P} = \frac{1}{r+s+\tau}$. The bargaining process then leads to

$$(1-\delta)(r+s)(W_1 - W_0) \quad = \quad \delta(r+s+\tau)(B_3 + E_3 - B_2 - E_2)$$

Subtracting (41) from (40) we obtain

$$W_1 - W_0 = \frac{1 + \Phi - P}{r + s + \eta}$$

and adding up (11) and (12) we have

$$B_3 + E_3 - B_2 - E_2 = \frac{P}{r + s + \lambda + \tau} \tag{48}$$

leading to a convenient solution for the price:

$$(1 - \delta)(1 + \Phi - P)\frac{r + s}{r + s + \eta} \quad = \quad \delta P \frac{r + s + \tau}{r + s + \lambda + \tau}$$

or after simplification:

$$\text{Price: } P = \frac{1 + \Phi}{1 + \frac{\delta}{1-\delta}\frac{r+s+\tau}{r+s}\frac{r+s+\eta}{r+s+\lambda+\tau}} \tag{49}$$

The price is therefore a fraction of the marginal utility of consumption, and varies with goods market tightness: a higher goods market tightness leads to a higher λ and therefore firms sell faster, and to a lower η and therefore consumers locate goods at a slower rate. Therefore, the higher the consumption tightness, the higher the price will be .

Wage. On the wage side, we have from equation (40):

$$(r + s)(W_0 - U) = w + T + \eta(W_1 - W_0) - rU \tag{50}$$

$$= w + T - rU + \frac{\eta(1 + \Phi - P)}{r + s + \eta} \tag{51}$$

This implies $\frac{\partial(W_0 - U)}{\partial w} = \frac{1}{r+s}$. From equation (3) and (7), we have

$$(r + s)(B_2 + E_2) = -w + \lambda(B_3 - B_2 + E_3 - E_2)$$

$$= -w + \frac{\lambda P}{r + s + \lambda + \tau}$$

It follows that $\frac{\partial(B_2 + E_2 - B_1 - E_1)}{\partial w} = \frac{1}{r+s}$. The solution of the Nash-product for wages is thus:

$$\alpha(B_2 + E_2 - B_1 - E_1) = (1 - \alpha)(W_0 - U) \tag{52}$$

leading to

$$\alpha \left[w - \frac{\lambda P}{r + s + \lambda + \tau} + (r + s)(E_1 + B_1)\right] = (1 - \alpha)\left[w + T - rU + \frac{\eta(1 + \Phi - P)}{r + s + \eta}\right]$$

or

$$w = (1 - \alpha)\left(rU - T - \frac{\eta(1 + \Phi - P)}{r + s + \eta}\right) + \alpha\left(\frac{\lambda P}{r + s + \lambda + \tau} - (r + s)(E_1 + B_1)\right)$$

This equation has a standard form: the wage is a weighted average of reservation wage and of future profits. It increases notably with prices, for two reasons: the term weighted by α reflects future profits from the firm+bank entity, net of their threat point (returning to stage 1 with positive surplus value). The term weighted by $(1-\alpha)$ reflects future consumption of the good when it will be found. The higher that future price, the lower the consumption of the numeraire, and therefore the lower the gain from searching for the good thanks to the wage received, leading to a lower surplus from the worker and thus a need to bargain a higher wage.

Using first the equilibrium value of $E_1 + B_1$ as a function of tightness, and second the value for the reservation wage rU, using

$$rU = T + \theta q(\theta)(W_0 - U) = T + \theta q(\theta)\frac{\alpha(r+s+\lambda)(B_2+E_2)}{(1-\alpha)(r+s+\eta)}$$
$$= T + \theta q(\theta)\frac{\alpha(r+s+\lambda)\frac{-w+\frac{\lambda}{r+s+\tau+\lambda}P}{r+s}}{(1-\alpha)(r+s+\eta)}$$

we obtain a solution for the wage, solving:

$$w = (1-\alpha) \quad \left(\theta q(\theta)\tfrac{\alpha}{1-\alpha}\tfrac{r+s+\lambda}{r+s+\eta}\tfrac{-w+\frac{\lambda}{r+s+\tau+\lambda}P}{r+s} - \tfrac{\eta(1+\Phi-P)}{r+s+\eta}\right) \tag{53}$$
$$+\alpha\left(\tfrac{\lambda P}{r+s+\lambda+\tau} - (r+s)\left(\tfrac{c}{p(\phi)} + \tfrac{\kappa}{\phi p(\phi)}\right)\right) \tag{54}$$

which further simplifies moving the wage to the left:

$$w\left(1+\alpha\frac{\theta q(\theta)}{r+s}\frac{r+s+\lambda}{r+s+\eta}\right) = (1-\alpha)\left(-\frac{\eta(1+\Phi-P)}{r+s+\eta}\right)$$
$$+\,\alpha\left(\frac{\lambda P}{r+s+\lambda+\tau}\left(1+\frac{r+s+\lambda}{r+s+\eta}\frac{\theta q(\theta)}{r+s}\right) - (r+s)\left(\frac{c}{p(\phi)}+\frac{\kappa}{\phi p(\phi)}\right)\right) \tag{55}$$

Finally, using the fact that goods market tightness is equal to 1, we have in equilibrium: $\lambda = \eta$, leading to a simpler expression for the wage:

$$\text{Wage: } w\left(1+\alpha\frac{\theta q(\theta)}{r+s}\right) = (1-\alpha)\left(-\frac{\eta}{r+s+\eta}(1+\Phi-P)\right)$$
$$+\,\alpha\left(\frac{\lambda P}{r+s+\lambda+\tau}\left(1+\frac{\theta q(\theta)}{r+s}\right) - (r+s)\left(\frac{c}{p(\phi)}+\frac{\kappa}{\phi p(\phi)}\right)\right) \tag{56}$$

The uniqueness of (w, P) hinges on the fact that the wage equation (56) defines a wage uniquely defined and depending on P, itself determined through equation (49). This is another, nested, block-recursiveness property. Existence of a positive solution for the wage requires a condition on the bargaining power of workers (see Appendix).

Limit cases.

The price can be simplified under a specific case: when consumers have a very low versatility rate τ, then

$$P = \frac{1+\Phi}{1+\frac{\delta}{1-\delta}} = (1-\delta)(1+\Phi) \tag{57}$$

In this specific case, tightness of the goods market does not affect prices because consumers consume “forever”.

The wage can also be simplified when credit market imperfections disappear: as c and k jointly converge to zero (no financial frictions), the wage increases uniformly (as there is less rent to firms and banks in stage1), and the wage is then solution to

$$w\left(1+\alpha\frac{\theta q(\theta)}{r+s}\right)=(1-\alpha)\left(-\frac{\eta}{r+s+\eta}(1+\Phi-P)\right)$$
$$+\alpha\left(\frac{\lambda P}{r+s+\lambda+\tau}\left(1+\frac{\theta q(\theta)}{r+s}\right)\right)$$

Further, when goods market frictions disappear (η and λ go to infinity), the wage simplifies to

$$w\left(1+\alpha\frac{\theta q(\theta)}{r+s}\right)=-(1-\alpha)(1+\Phi-P)+\alpha\left(P\left(1+\frac{\theta q(\theta)}{r+s}\right)\right)$$
$$\text{or } w=P-\frac{(1-\alpha)(1+\Phi)}{1+\alpha\frac{\theta q(\theta)}{r+s}} \tag{58}$$

which is equal to the price net of a term going to zero when goods market friction disappear (competitive wage). The higher the bargaining power of worker α, the closer the wage to the marginal product. Conversely, the higher the marginal utility from consumption of the manufactured good, the better the bargaining position of the (firm+bank) block vis-à-vis the worker and hence, the lower the price.

4.4 Provisional conclusion: existence and uniqueness of the equilibrium

We have been able to derive the equilibrium in this world with 3 frictional markets. Formally, it is defined as follows.

Definition. *An equilibrium is a sextuplet (ϕ, θ, ξ, ρ, w, P) of market tightness and prices for each market. The model is solved by two free-entry conditions (banks and firms) and by the equality between the number of consumers and the number of workers, as well as three equations for "price" determination. Other relevant quantities (employment, unemployment, vacancies, prospecting consumers and prospecting firms) are easily derived from the market tightness variables, from stock-flow conditions and from the assumption of a fixed labor supply: employment + unemployment = 1.*

There is block-recursiveness in the solutions to the model, as follows: ϕ is fixed and equal to a simple function of parameters. Independently of the two other "price" determination rules (for the wage and for the price of the final good), ξ is also fixed and equal to 1.

Proposition 1. *Under market viability conditions, an equilibrium exists and is unique.*

Proof: Given that price, credit market tightness and goods market tightness are solved as functions of parameters, the only remaining set of equations relies labor market tightness and wages, namely the wage equation (56) and tightness equation (35). The wage equation positively links labor market tightness and wage. The tightness equation negatively links them. There is at most one solution in wage and tightness. The sextuplet (ϕ, θ, ξ, ρ, w, P) is thus uniquely determined when it exists.

5 Comparative statics and accelerators

To simplify the exposition of the next part devoted to comparative statics of the role of goods market frictions, we will now fix w and P. The role of goods market frictions is simple to study in this context. Having determined that $\lambda = \lambda(1)$ in a steady-state equilibrium in the goods market, we can vary λ exogenously: e.g., if $\lambda(\xi) = \lambda_0 \xi^{-\upsilon}$ where υ is a parameter, $\lambda(1) = \lambda_0$ is a free parameter that can be varied easily. So hereafter, we simply explore the role of shifts in λ induced by changes in λ_0.

5.1 Comparative statics of goods market imperfections λ^{-1}

An increase in λ causes the right-hand side of the equations (EE) and (BB) to increase, for a given θ. Hence, both curves are shifted towards the right in the space (θ, ϕ): the new equilibrium tightness of the labor market θ is therefore increased quite strongly. Reducing frictions in the goods markets improves both the entry of firms and the entry of banks, thus both curves shifts in a direction that favors job creation.

Here, as Figure 2 illustrates, a rise in λ^{-1} from 0 (no goods market imperfections) leads the vertical asymptote to shift leftward and the equilibrium market tightness is lower as a combination of the effect of the shift of both curves. Interestingly, the goods market imperfections also have an impact on the volatility of θ in response to other shocks. To see this, one can first calculate the slope of the right-hand side of the equations (EE) and (BB) with respect to θ, given by

$$\text{(EE)} \quad : \quad \beta \frac{q'(\theta)}{(r+q)^2} \left[\alpha_\lambda r \frac{P-w}{r+s} - \gamma \left(1 + \frac{r}{\lambda}\right) - rB_w \right] < 0$$

$$\text{(BB)} \quad : \quad (1-\beta) \frac{q'(\theta)}{(r+q)^2} \left[\alpha_\lambda r \frac{P-w}{r+s} - \gamma \left(1 + \frac{r}{\lambda}\right) - rB_w \right] < 0$$

The sign is negative because $q' < 0$ and the term in brackets must be positive in any viable markets, otherwise profits by firms would not cover the hiring and wage costs. Inspection of these two equations also reveals that these slopes in absolute values are closer to zero than in the absence of goods market imperfections: when λ increases, α_λ increases (hence the positive term inside the bracket is higher), and both negative parts within the bracket B_w and

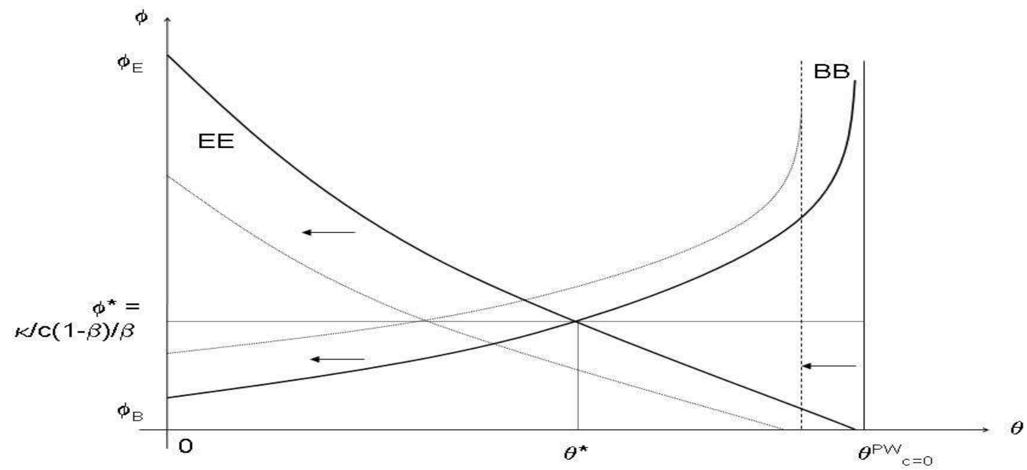

Fig. 2: Leftward shifts of EE and BB curves following a lower matching rate in the goods market λ

$\gamma\left(1+\frac{r}{\lambda}\right)$ decrease. Hence, denoting by $A(\lambda)$ the term within the brackets, we have $A(\lambda)>0$, $A'(\lambda)>0$, hence the slopes of the two curves are obtained as:

$$\text{(EE)} \quad : \quad \frac{d\phi}{d\theta}=\frac{1-\beta}{\frac{-cp'(\phi)}{p^2(\phi)}}\left(\frac{q'(\theta)}{(r+q)^2}\right)A(\lambda)<0$$

$$\text{(BB):} \quad \frac{d\phi}{d\theta}=\frac{\beta}{\frac{-k(1+\phi p'/p)}{\phi^2 p(\phi)}}\left(\frac{q'(\theta)}{(r+q)^2}\right)A(\lambda)>0$$

Property 7 (volatility of θ in response to goods market imperfection). *When the amount of goods market frictions increases (lower λ), $A(\lambda)$ is lower, and finally both (EE) and (BB) curves are flatter in (θ,ϕ). It follows that a shock on profit or productivity will have stronger effects of θ^* than it would have in the absence of goods market imperfections.*

5.2 Multiplier/accelerator

Fixing $r=0$ to simplify the analysis, we can calculate a multiplier of profit shocks.[9] Denoting by $\pi=P-w$ and by $\widehat{x}=dx/x$ the percentage variation of

[9] Wasmer and Weil (2004) derived the calculation of a multiplier of profits shocks. Petrosky-Nadeau and Wasmer (2010) reinterpret the multiplier as a variant of the small surplus assump-

any variable x , finally by $\eta = -\theta q'(\theta)/q(\theta)$, we have the following results:

$$\widehat{\theta} = \frac{1}{\eta}\widehat{\pi} \text{ as in Pissarides (2000)} \tag{59}$$

$$\widehat{\theta} = \frac{1+\mu_{g=0}^{WW}}{\eta}\widehat{\pi} = \frac{1}{\eta}\frac{q(\theta_{g=0}^{WW})}{q(\theta_{c,g=0}^{P})}\widehat{\pi} \text{ in Wasmer-Weil} \tag{60}$$

The quantity $\mu_{g=0}^{WW}$ is a variable quantifying the supplementary amplification of productivity shocks due to the existence of credit market imperfections, in absence of goods market imperfections. As revealed in equation (60), it is increased by the distance between the tightness of the labor market in a perfect capital market world ($\theta_{c,g=0}^{P}$) and the equilibrium tightness with credit market imperfections ($\theta_{g=0}^{WW}$). In Wasmer and Weil (2004), we estimated $\mu_{g=0}$ to be around 0.74 leading productivity shocks to be amplified by a factor 1.74.

The graphical insight of Property 7 can be quantified using the same methodology. Denote by μ the multiplier in a three imperfect markets economy. Rewrite (EE) and (BB) with $r = 0$. In this case, given $B_w = \frac{w}{\lambda} + \frac{w(1-\alpha_\lambda)}{s}$, we have

$$\text{(EE)} \quad : \quad \frac{c}{p(\phi)} = (1-\beta)\left[\frac{\alpha_\lambda P - w'}{s} - \frac{\gamma'}{q(\theta^*)}\right] \tag{61}$$

$$\text{(BB)} \quad : \quad \frac{\kappa}{\phi p(\phi)} = \beta\left[\frac{\alpha_\lambda P - w'}{s} - \frac{\gamma'}{q(\theta^*)}\right] \tag{62}$$

where $w' = w(1 + s/\lambda)$ and $\gamma' = \gamma(1 + s/\lambda)$. This implies that the multiplier can be calculated in the same way as in WW with simply larger labor costs and larger hiring costs, larger by an amount s/λ that would vanish when λ goes to infinity. We therefore have:

$$\begin{aligned}\widehat{\theta} &= \frac{1}{\eta}(1+\mu)\widehat{\pi} = \frac{1}{\eta}\frac{q(\theta^*)}{q(\theta_{c,g=0}^{P})}\widehat{\pi} \\ &= \frac{1}{\eta}\frac{q(\theta^*)}{q(\theta_{g=0}^{WW})}\frac{q(\theta_{g=0}^{WW})}{q(\theta_{c,g=0}^{P})}\widehat{\pi} = \frac{1+\mu_{g=0}^{WW}}{n}\frac{q(\theta^*)}{q(\theta_{g=0}^{WW})}\widehat{\pi}\end{aligned} \tag{63}$$

In the first line, the only difference with equation (60) is the top of the right-hand side: $\theta_{g=0}^{WW}$ is replaced by the equilibrium tightness of the model with three imperfect markets. This implies that there is a multiplier compared to the volatility of economies in a Pissarides world: $q(\theta^*)/q(\theta_{g=0}^{P}) > 1$.

The second line simply shows that the multiplier is larger than in the absence of goods market imperfections due to $q(\theta^*)/q(\theta_{g=0}^{WW}) > 1$.

Property 8. *The additional amplification of profit shocks due to goods market imperfections, expressed at the ratio of $\widehat{\theta}$ in equation (63) to the multiplier in the absence of goods market frictions in equation (60), is at least as large as the quantity $(1 + s/\lambda) \geq 1$.*

tion emphasized in Hagedorn and Manovskii (2008): when productive firms are closer to the margin of creation, productivity changes lead to large entries and exits of vacancies.

Finally, the next proposition summarizes the discussion:

Proposition 2:

1. *Frictions in the goods market $1/\lambda$ have no impact on equilibrium credit market tightness ϕ^* nor on equilibrium goods market tightness $\xi = 1$.*

2. *Frictions in the goods market $1/\lambda$ have a negative impact on equilibrium labor market tightness θ^* (with credit frictions) and a negative impact on the tightness of the labor market $\theta^W_{c=0}$ (in the absence of credit frictions).*

3. *Frictions in the goods market $1/\lambda$ have a positive impact on the multiplier μ of the economy defined by $\partial \ln \theta^* / \partial \ln \pi = \eta^{-1}(1+\mu)$.*

6 Numerical exercise

We calibrate the model using the same parameter values as in WW to ease the comparison: $c = k = 0.35$, $\gamma = 1.5$, $\beta = \eta = \varepsilon = 0.5$, $P = 1$, $w = 2/3$, $r = 5\%$, $s = 0.15$. We then vary credit, labor and goods market frictions . The results are presented in Table 1.

	Labor market→ Hiring time →	Some frictions (I) $q_0^{-1} = 1/1.5$ 2 months	Large frictions (II) $q_0^{-1} = 1/1.1$ ≃3 months
"Imperfect Labour Mkt" Perfect Credit Mkt Perfect Good Mkt	$1/p_0 = 0$ $1/\lambda = 0$	$u = 5.56$ $\mu = 0$ $\widehat{\theta}/\widehat{\pi} = 2$	$u = 9.86$ $\mu = 0.0$ $\widehat{\theta}/\widehat{\pi} = 2$
"Imperfect Labour Mkt" Imperfect Credit Mkt Perfect Good Mkt	$1/p_0 = 1$ $1/\lambda = 0$ Credit search takes a year	$u = 9.28$ $\mu = 0.74$ $\widehat{\theta}/\widehat{\pi} = 3.48$	$u = 16.0$ $\mu = 0.74$ $\widehat{\theta}/\widehat{\pi} = 3.48$
"Imperfect Labour Mkt" Imperfect Credit Mkt Imperfect Good Mkt	$1/p_0 = 1$ $1/\lambda = 1/3$ Selling time of a quarter	$u = 12.3$ $\mu = 1.38$ $\widehat{\theta}/\widehat{\pi} = 4.75$	$u = 20.6$ $\mu = 1.38$ $\widehat{\theta}/\widehat{\pi} = 4.75$
"Imperfect Labor Mkt" Perfect Credit Mkt Imperfect Good Mkt	$1/p_0 = 0$ $1/\lambda = 1/3$	$u = 6.72$ $\mu = 0.23$ $\widehat{\theta}/\widehat{\pi} = 2.45$	$u = 11.8$ $\mu = 0.23$ $\widehat{\theta}/\widehat{\pi} = 2.45$

Line 1 reveals the role of labor market imperfections in a Pissarides world with no other frictions: unemployment rises from 5.56 to 9.9 points. Line 2 reveals the amplification role of credit market imperfections. In column (I), unemployment rises from 5.56 to 9.3% by the moderate credit market frictions, and would additionally jump to 16% when the two market imperfections (credit and labor) are combined as incolumn II.

The role of goods market imperfections is underlined in line 3, where we impose moderate goods market imperfection: assuming $\lambda^{-1} = 3^{-1}$, it takes 4 months to sell the good while the sales (that is, the productive stage of the firm) lasts on average $1/s = 7$ years. In other words, goods market frictions prevent sales during 5% of the total life time of the firm. In this case, the rate of unemployment goes up by 2.5 percentage points in column I (moderate labor frictions), from 9.3 to 12.6 and by 4.6 percentage points in column II (large labor frictions), from 16 to 20.6%. Further, goods market frictions augment the unemployment rate and the volatility of the economy quite significantly. By setting λ to 3, the volatility of the economy $\widehat{\theta}/\widehat{\pi}$ which was $\eta^{-1} = 2$ in Pissarides with rigid wages, and $2 * 1.74 = 3.48$ with only credit market imperfections, is now $2 * (1 + 1.38) = 4.75$.

In Line 4, we impose goods market imperfections under perfect credit markets. The impact of goods market imperfections *per se*, i.e. independently of credit imperfections, is relatively modest: the increase in unemployment due to search frictions in the goods market only is 1.2 percentage point in Column II (difference between Line 1 and Line 4) and 1.9 percentage point in Column II (with larger labor frictions). The multiplier μ is merely 0.22 and $\widehat{\theta}/\widehat{\pi} = 2.45$, only 22% above the standard Pissarides volatility rate.

The conclusion is that *frictions in different markets are complements to each other*: for goods market to raise the volatility of the economy, we need credit market frictions.

7 Extensions

7.1 Extension 1: consumption by bankers and firms

So far we assumed for simplicity that only workers could consume thanks to the profits of banks and capitalists. We now relax this assumption and let workers, capitalists and banks to consume the manufactured good. We keep the wage and P exogenous here to avoid too complex bargaining interactions between producers in different consumption stages and consumers or workers.

For that, we need to change the timing and add a forth state for banks and firms: the stage in which they have a manufactured good to consume. Let E_4 be the value of the firms enjoying the manufactured good. We also assume that, as all consumers, the firm receives the cash transfer T in the third and forth states.

Let E_0, E_1, E_2 and E_3 the respective steady-state asset values of a firm in each period. We have:

$$rE_0 = -c + p(E_1 - E_0) \quad (64)$$

$$rE_1 = -\gamma + \gamma + q(E_2 - E_1) \quad (65)$$

$$rE_2 = -w + w + \lambda(E_3 - E_2) + s(E_0 - E_2) \quad (66)$$

$$rE_3 = P + T - w - \rho + s(E_0 - E_3) + \tau(E_2 - E_3) + \eta(E_4 - E_3) \quad (67)$$

$$rE_4 = (P + T - w - \rho - P) + (1 + \Phi) + s(E_0 - E_4) + \tau(E_2 - E_4) \quad (68)$$

In stage 3, the capitalist has the possibility to search and find a good with Poisson rate η. The fourth stage allows him to keep an amount $T - w - \rho$ of unitary good and enjoy the marginal utility of the good $1 + \Phi$.

The bank's problem is similar and we have

$$\begin{aligned} rB_0 &= -\kappa + \phi p(B_1 - B_0) && (69)\\ rB_1 &= -\gamma + q(B_2 - B_1) && (70)\\ rB_2 &= -w + \lambda(B_3 - B_2) + s(B_0 - B_2) && (71)\\ rB_3 &= T + \rho + s(B_0 - B_3) + \tau(B_2 - B_3) + \eta(B_4 - B_3) && (72)\\ rB_4 &= T + \rho - P + 1 + \Phi + s(B_0 - B_4) + \tau(B_2 - B_4) && (73) \end{aligned}$$

After intermediate steps described in Appendix, we finally obtain the set of equilibrium conditions determining labor market tightness and credit market tightness:

$$(\text{EE}_4) \quad : \quad \frac{c}{p(\phi)} = \frac{q(\theta)(1-\beta)}{r+q(\theta)}\left[\alpha_\lambda \frac{T+P-w+B_\eta}{r+s} - \left(\frac{\gamma}{q(\theta)}\frac{r+s+\lambda}{\lambda} + B_w\right)\right]$$

$$(\text{BB}_4) \quad : \quad \frac{\kappa}{\phi p(\phi)} = \frac{q(\theta)\beta}{r+q(\theta)}\left[\alpha_\lambda \frac{T+P-w+B_\eta}{r+s} - \left(\frac{\gamma}{q(\theta)}\frac{r+s+\lambda}{\lambda} + B_w\right)\right]$$

where the difference with the benchmark model (equations (28) and (29)) arises from additional term $B_\eta = \eta\frac{1+\Phi-P}{r+s+\eta+\tau}$. This term reflects the slow transition for banks and firm to the final consumption stage 4. We still obtain the same qualitative insights as before, except that these two curves are now shifted rightwards: since capitalists and bankers can now enjoy more utility through money, they face higher incentives to enter the market.

We also have more competition for goods from firms and banks consuming, changing the structure of matching externalities. Indeed, let $M_G(\mathcal{C}, \mathcal{N}_2)$ be the number of contacts between $\mathcal{C}$ unmatched consumers and $\mathcal{N}_2$ firms producing and attempting to sell their product. We have now

$$\mathcal{C} = \mathcal{C}_0 + \mathcal{N}_3 + \mathcal{B}_3$$

and

$$\begin{aligned} \eta &= \frac{M_G(\mathcal{C}, \mathcal{N}_2)}{\mathcal{C}} && (74)\\ \lambda &= \frac{M_G(\mathcal{C}, \mathcal{N}_2)}{\mathcal{N}_2} && (75) \end{aligned}$$

and therefore, the new tension in the goods market (on the perspective of consumers) is now:

$$\xi = \frac{\mathcal{C}_0 + \mathcal{N}_3 + \mathcal{B}_3}{\mathcal{N}_2}$$

the higher ξ, the higher the demand from consumers relative to the production awaiting to be consumed. The steady-state tightness is the fixed point of a function $h(\xi)$ described in Appendix. It is still a function of parameters, however it is not necessarily unique.

7.2 Extension 2: the unemployed consume

We now relax the hypothesis that stated that the unemployed do not consume because the transfer T is lower than the price. We now assume instead that $T > P$ thanks to an appropriate Central Bank drop of money, in this case, the demand for goods is

$$\mathcal{C} = \mathcal{C}_0 + u = (1 - u - \mathcal{C}_1) + u = 1 - \mathcal{C}_1$$

We now have

$$\frac{d\mathcal{C}_1}{dt} = \eta(1 - \mathcal{C}_1) - s\mathcal{C}_1 = 0$$

therefore in a steady-state, we have

$$\mathcal{C}_1 = \frac{\eta}{s + \eta}$$

and therefore the demand for goods

$$\mathcal{C} = \frac{s}{s + \eta}$$

On the other side of the market, we still have $\mathcal{N}_2$. Now, the relevant transition rates is still

$$\eta = \frac{M_G(\mathcal{C}, \mathcal{N}_2)}{\mathcal{C}} = \eta(\xi) \text{ with } \eta'(\xi) < 0 \tag{76}$$

and from the benchmark case in equation (45), we still have

$$\mathcal{N}_2 = \frac{1 - u}{1 + \frac{\lambda}{s}}$$

Finally, since

$$1 - u = \frac{\theta q(\theta)}{s + \theta q(\theta)}$$

we obtain an equality for the consumption tightness

$$\xi = \frac{\mathcal{C}}{\mathcal{N}_2} = \frac{s(s + \theta q(\theta))}{(s + \eta(\xi))\theta q(\theta)} = j(\xi, \theta)$$

where the right-hand side, denoted by $j(\xi, \theta)$, is such that

$$\frac{\partial j}{\partial \xi} > 0, \; \frac{\partial^2 j}{\partial \xi^2} < 0, \; \frac{\partial j}{\partial \theta} < 0$$

The concavity of j insures the uniqueness of the fixed point in consumption tightness and that fixed point is decreasing in θ: the intuition is that a higher θ has no impact on demand for consumption as all workers consume, and has positive impact on the production side.

8 Conclusion

We have built a model of a multi-frictional economy with imperfections in three markets. The uniqueness of the equilibrium in the relevant space of three market tightness (credit, good and labor) is fully preserved. In the main equilibrium, goods market tightness has to be equal to 1 in a steady state, a generalization of Say's law. Since credit market tightness is also a function of entry costs and bargaining parameters, this implies a full block recursiveness making the model easy to solve. Finally, the volatility of this economy is significantly increased via the labor market, due to complementarity between frictions. This raises volatility by a factor 2 to 3 according to our calibration exercise. This work is a first attempt to derive a simple search model of a generalized search economy and its main steady-state properties. It can be developed in three directions.

First, we have a simple setup that allows for a proper calibration of different economies with various degrees of imperfections in different markets. A proper calibration of different OECD countries in order to identify the main bottlenecks to full employment is therefore a natural next step.

Second, alternative timing assumptions may be made. In particular, whether goods should be produced first or whether consumers should be searched first depends on the types of good produced and the alternative timing may lead to interesting insights.

Third, the dynamic properties may provide interesting insights. In a companion paper (Petrosky-Nadeau and Wasmer, 2011), we precisely investigate the dynamic implications of an adaptation of the model, with several different features. First, we introduce a consumer search effort which depends on an arbitrage between costs of effort and returns to effort, itself depending on income. Second, the consumption side is modified: here we assumed that only the employed workers consume the manufactured good, because transfers to the unemployed by helicopter drop is insufficient to let them consume this inelastically supplied good. In Petrosky-Nadeau and Wasmer, we assume instead, following Merz (1996), that all income is pooled and redistributed to the consumers : everybody has access to all consumption goods, with however an intensity that varies with income thanks to the first assumption of endogenous search effort of consumers. We found in Petrosky-Nadeau and Wasmer (2011) that these specific goods market frictions drastically change the qualitative and quantitative dynamics of labor market variables, as compared to the steady-state properties and notably the size of the multipliers in the steady-state. The main reason is that amplification arises from the effort made by consumers: after a positive productivity shocks, firms make more profit, and even though prices fall, they redistribute more income to consumers. Those in turn will make more effort to search and therefore will raise the incentives for firms to enter the market. Overall this leads to a positive feedback and to a slower return to the equilibrium path after the shock.

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Appendix

The consumption tightness property in a more general case

There are two types of consumers: those in a firm in stage 2 (without consumer) and those in a firm in stage 3 (selling its good). Denote them by $\mathcal{C}_i^2$ and $\mathcal{C}_i^3$ for $i = 0, 1$ with $\mathcal{C}_i^2 + \mathcal{C}_i^3 = \mathcal{C}_i$, where i is the consumption status: 0 means no consumption of a manufactured good, 1 means some consumption. In this Appendix, we introduce some more transitions rates between states. In particular, define τ_c as the Poisson rate at which a consumer in consumption state 1 is subject to a taste shock and dislikes the good he used to consume, and by τ_f the rate at which a firm selling (in stage 3) stops being liked by its consumer. Although it seems natural to assume from the start $\tau_f = \tau_c$, we keep separate notations to clarify the economics. We have, as represented on Figure 3, the following relationships:

$$\begin{aligned}
\frac{d\mathcal{C}_0^2}{dt} &= \theta q(\theta)u + (s+\tau_c)\mathcal{C}_1^2 + (\tau_f + s)\mathcal{C}_0^3 - (\eta+\lambda+s)\mathcal{C}_0^2 = 0 \\
\frac{d\mathcal{C}_0^3}{dt} &= \lambda\mathcal{C}_0^2 + (s+\tau_c)\mathcal{C}_1^3 - (\eta+\tau_f+2s)\mathcal{C}_0^3 = 0 \\
\frac{d\mathcal{C}_1^2}{dt} &= \eta\mathcal{C}_0^2 + (\tau_f+s)\mathcal{C}_1^3 - (\tau_c+\lambda+2s)\mathcal{C}_1^2 = 0 \\
\frac{d\mathcal{C}_1^3}{dt} &= \eta\mathcal{C}_0^3 + \lambda\mathcal{C}_1^2 - (\tau_c+\tau_f+2s)\mathcal{C}_1^3 = 0 \\
\frac{d\mathcal{N}_2}{dt} &= \theta q(\theta)u + (s+\tau_f)\mathcal{N}_3 - (\lambda+s)\mathcal{N}_2 = 0 \\
\frac{d\mathcal{N}_3}{dt} &= \lambda\mathcal{N}_2 - (2s+\tau_f)\mathcal{N}_3 = 0 \\
1-u &= \mathcal{C}_0 + \mathcal{C}_1 = \mathcal{N}_2 + \mathcal{N}_3
\end{aligned} \tag{77}$$

To simplify the equations, re-define by $a = \mathcal{C}_0^2$, $b = \mathcal{C}_0^3$, $c = \mathcal{C}_1^2$, $d = \mathcal{C}_1^3$ and $H = \theta q(\theta)u$ the various unknown above. We then have:

$$H + (s+\tau_c)c + (s+\tau_f)b = (\eta+\lambda+s)a \tag{78}$$
$$\lambda a + (s+\tau_c)d = (\eta+\tau_f+2s)b \tag{79}$$
$$\eta a + (\tau_f+s)d = (\tau_c+\lambda+2s)c \tag{80}$$
$$\eta b + \lambda c = (\tau_c+\tau_f+2s)d \tag{81}$$
$$H + (s+\tau_f)\mathcal{N}_3 = (\lambda+s)\mathcal{N}_2 \tag{82}$$
$$\lambda\mathcal{N}_2 = (2s+\tau_f)\mathcal{N}_3 \tag{83}$$

The last two equations (82) and (83) give:

$$\frac{\lambda}{2s+\tau_f}\mathcal{N}_2 = \mathcal{N}_3 \tag{84}$$
$$H = \left((\lambda+s) - \lambda\frac{s+\tau_f}{2s+\tau_f}\right)\mathcal{N}_2 \tag{85}$$
$$\text{or} \tag{86}$$
$$\mathcal{N}_2 = \left(s\frac{\lambda+2s+\tau_f}{2s+\tau_f}\right)^{-1} H \tag{87}$$

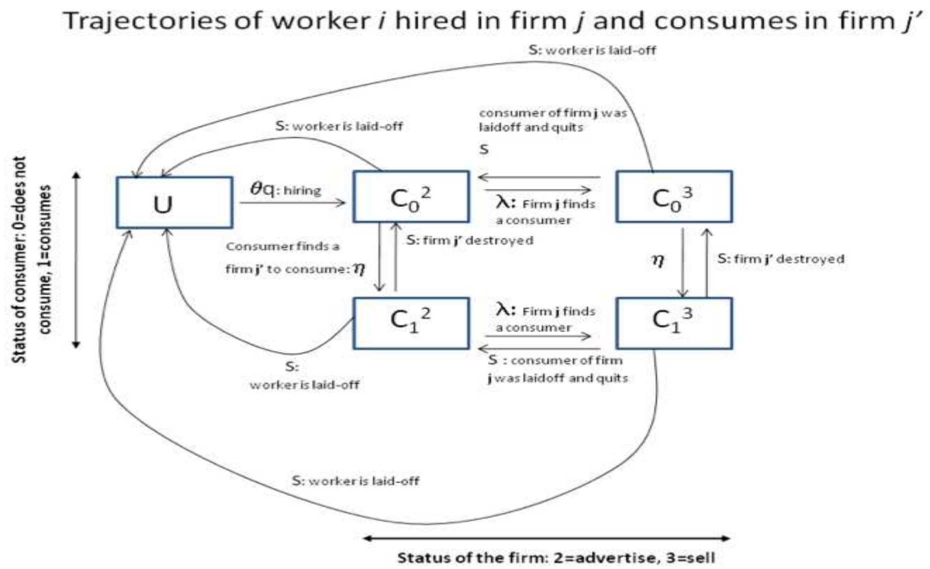

Fig. 3: Consumptions patterns under alternative assumptions

Summing up the 4 first equations (78) to (81) we obtain

$$\begin{aligned} H &= \theta q(\theta) u = s(a+b+c) \\ &= s(\mathcal{C}_0^3 + \mathcal{C}_1^3) + s(\mathcal{C}_0^2 + \mathcal{C}_1^2) \end{aligned} \tag{88}$$

stating that inflows of consumers (potential and matched with a good) are those coming from unemployment while outflows are those laid-off in firms in stage 3 with rate s_3 and laid-off in stage 2 with rate s_2.

To calculate goods market tightness, we need to calculate

$$\xi = \mathcal{C}_0/\mathcal{N}_2 = (a+b)/\mathcal{N}_2$$

So, to obtain $a+b$, we proceed as follows. First, summing up equations (80) and (81):

$$\eta(a+b) = (\tau_c + s)c + (\tau_c + s)d \tag{89}$$

Equation (89) becomes a simpler expression linking $a+b$ and $c+d$, that is the total of consumers in state 0 and in state 1. This simplification arises from the fact that we do not need to follow consumers in stage $i = 0, 1$ in each type of firm 2 or 3 when

$$c + d = \frac{\eta}{\tau_c + s}(a+b) \tag{90}$$

while, from equation (88):

$$H = s(a+b+c+d) \tag{91}$$

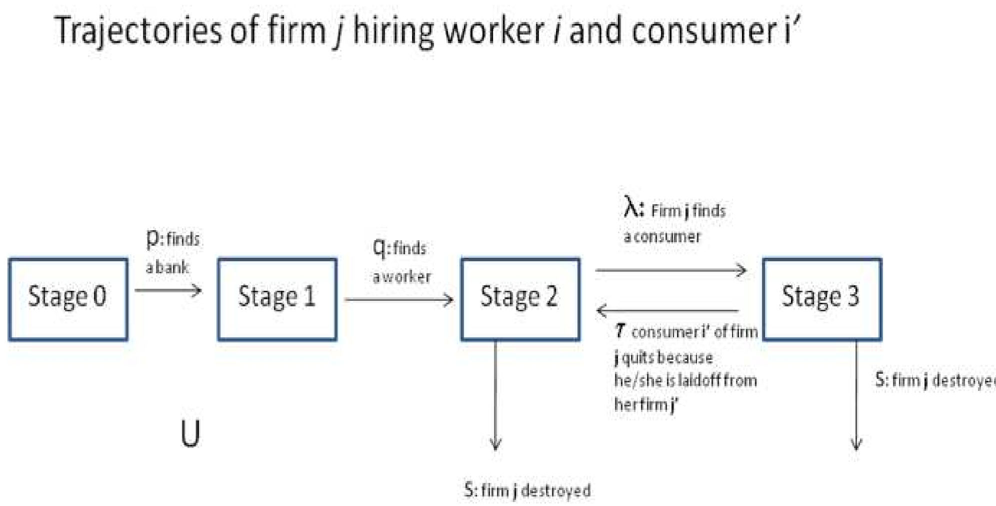

Fig. 4: Consumptions patterns under alternative assumptions

Combining (90) and (91) leads to

$$a + b = \mathcal{C}_0 = \frac{H}{s} \frac{1}{1 + \frac{\eta}{s+\tau_c}} \tag{92}$$

Goods market tightness is then equal to

$$\begin{aligned} \xi &= \frac{\mathcal{C}_0}{n\mathcal{N}_2} = \frac{\frac{H}{s}\frac{1}{1+\frac{\eta}{s+\tau_c}}\left(\frac{\lambda s}{s+\tau_f} + s\right)}{H} \\ \xi &= \frac{1 + \frac{\lambda}{s+\tau_f}}{1 + \frac{\eta}{s+\tau_c}} \end{aligned}$$

Returning to the notations $\lambda(\xi) = \xi\eta(\xi)$, we further obtain

$$\xi = \frac{\xi\eta(\xi) + s + \tau_c}{\eta(\xi) + s + \tau_f}$$

We then see that the level of consumption tightness is a fixed point depending only on parameters of the matching function M_G, of layoff rates and of switching rate and versatility of consumers. The right-hand side is monotonically increasing in ξ from 0 when $\xi = 0$ to $+\infty$ when ξ goes to infinity. It follows that the steady-state consumption tightness is a fixed point. It exists and is a unique solution of parameters:

$$\xi^* = \xi^*(M_G(.), s, \tau_c, \tau_f)$$

To see uniqueness, define

$$\begin{aligned} g(\xi) &= \xi - \frac{\xi\eta(\xi) + s + \tau_c}{\eta(\xi) + s + \tau_f} \\ &= \frac{(s+\tau_f)(\xi - 1) + \tau_f - \tau_c}{\eta(\xi) + s + \tau_f} \end{aligned}$$

The denominator is of constant positive sign and the numerator is linear and increasing in ξ, so there can be at most one zero in $g(\xi)$. This actually determines

$$\xi^* = 1 + \frac{\tau_f - \tau_c}{s + \tau_f}$$

Finally, we retrieve the result of the text: to the extent that $\tau_f = \tau_c = \tau$ as suggested as the natural assumption in the beginning of this Appendix section, 1 is the only fixed point for goods market tightness.

On Properties 7 and 8

The multiplier calculated in Section (5.2) depends on the gap between $q(\theta^*)$ and $q(\theta^{WW}_{g=0})$ in the absence of frictions in the goods market. We can approximate by how much it is larger due to goods market frictions, by calculating $q(\theta^{WW}_{g=0})/q(\theta^*)$. Using $\frac{k}{\phi^* p(\phi^*)} = \frac{B_1}{\beta}$ with $\lambda^{-1} > 0$ or (second line) with $\lambda^{-1} = 0$, we have the four cases where (ALL) means our most general equilibrium, (WW) means no goods market frictions, (C=0) means no credit market frictions but goods market frictions and finally P is the Pissarides equilibrium with only labor market frictions. Recall that $w' = w(1 + s/\lambda)$ and $\gamma' = \gamma(1 + s/\lambda)$ defined in Section (5.2), we have:

$$\text{(ALL)} \quad : \quad \frac{1}{\beta}\frac{k}{\phi^* p(\phi^*)} = \alpha_\lambda \frac{P - w'}{s} - \frac{\gamma'}{q(\theta^*)} \tag{93}$$

$$\text{(WW)} \quad : \quad \frac{1}{\beta}\frac{k}{\phi^* p(\phi^*)} = \frac{P - w}{s} - \frac{\gamma}{q(\theta^{WW}_{g=0})} \tag{94}$$

$$\text{(P)} \quad : \quad 0 = \frac{P - w}{s} - \frac{\gamma}{q(\theta^{P}_{c,g=0})} \tag{95}$$

$$\text{(C=0)}: \quad 0 = \alpha_\lambda \frac{P - w}{s} - \frac{w'}{s} - \frac{\gamma'}{q(\theta_{c=0})} \tag{96}$$

or

$$\text{(ALL)} \quad : \quad \frac{1}{q(\theta^*)} = \frac{\alpha_\lambda \frac{P-w'}{s} - \frac{1}{\beta}\frac{k}{\phi^* p(\phi^*)}}{\gamma'} \tag{97}$$

$$\text{(WW)} \quad : \quad \frac{1}{q(\theta^{WW}_{g=0})} = \frac{\frac{P-w}{s} - \frac{1}{\beta}\frac{k}{\phi^* p(\phi^*)}}{\gamma} \tag{98}$$

$$\text{(P)} \quad : \quad \frac{1}{q(\theta^P)} = \frac{\frac{P-w}{s}}{\gamma} \tag{99}$$

$$\text{(C=0)}: \quad \frac{1}{q(\theta_{c=0})} = \frac{\alpha_\lambda \frac{P-w'}{s}}{\gamma'} \tag{100}$$

leading to:

$$\frac{q(\theta^{WW}_{g=0})}{q(\theta^*)} = \frac{\gamma}{\gamma'} \frac{\alpha_\lambda \frac{P-w'}{s} - \frac{1}{\beta}\frac{k}{\phi^* p(\phi^*)}}{\frac{P-w}{s} - \frac{1}{\beta}\frac{k}{\phi^* p(\phi^*)}} < 1$$

The last inequality is the combination of three effects going in the same direction: with positive search frictions in the goods markets, γ' is higher relative to γ, expected profits are lower due to $\alpha_\lambda < 1$ and w' is higher.

Condition for an equilibrium with positive price and wage

The price is always positive thanks to equation (49). The wage rewrite as:

$$w\left(1+\alpha\frac{\theta q(\theta)}{r+s}\right) = (1-\alpha)\left(-a_\eta(1+\Phi-P)\right) + \alpha\left(a_\lambda P\left(1+\frac{\theta q(\theta)}{r+s}\right)-(r+s)K\right) \quad (101)$$

where $K = \left(\frac{c}{p(\phi)}+\frac{\kappa}{\phi p(\phi)}\right)$; $a_\eta = \frac{\eta}{r+s+\eta}$; $a_\lambda = \frac{\lambda}{r+s+\lambda+\tau}.A = (1-\delta)(1+\Phi)$. Note first that $1+\Phi-P$ is positive given the price equation. Second, a necessary and sufficient condition for a positive wage is

$$\frac{1-\alpha}{\alpha}\left(a_\eta(1+\Phi-P)\right)+(r+s)K \leq a_\lambda P\left(1+\frac{\theta q(\theta)}{r+s}\right) \quad (102)$$

which requires: sufficently small search frictions in the financial sector (low K) and a sufficiently large bargaining power of worker α. Interestingly, for all values of K and for the other parameters, there always exists a value of α which insure that condition (102) is satisfied.

Appendix to Section 7

Firms and banks consume: Section 7.1.

Differencing the expression for rB_4 and for rB_3, we obtain first

$$B_4 - B_3 = \frac{1+\Phi-P}{r+s+\eta+\tau}$$

The same procedures on the firm's side leads to

$$E_4 - E_3 = \frac{1+\Phi-P}{r+s+\eta+\tau}$$

Then, differencing the expression for rB_3 and for rB_2, we obtain

$$B_3 - B_2 = \frac{T+\rho+w+\eta(B_4-B_3)}{r+s+\lambda+\tau} = \frac{T+\rho+w+\eta\frac{1+\Phi-P}{r+s+\eta+\tau}}{r+s+\lambda+\tau}$$

and similarly for firms,

$$E_3 - E_2 = \frac{T+P-w-\rho+\eta(E_4-E_3)}{r+s+\lambda+\tau} = \frac{T+P-w-\rho+\eta\frac{1+\Phi-P}{r+s+\eta+\tau}}{r+s+\lambda+\tau}$$

Then, we have

$$\begin{aligned} B_1 &= \frac{-\gamma + qB_2}{r+q} = \frac{-\gamma + qB_2}{r+q} \\ (r+s)B_2 &= -w + \lambda(B_3 - B_2) = -w + \lambda \frac{T + \rho + w + \eta \frac{1+\Phi-P}{r+s+\eta+\tau}}{r+s+\lambda+\tau} \\ \text{therefore} &: \quad B_1 = \frac{-\gamma + qB_2}{r+q} = \frac{-\gamma + q\frac{-w+\lambda \frac{T+\rho+w+\eta\frac{1+\Phi-P}{r+s+\eta+\tau}}{r+s+\lambda+\tau}}{r+s}}{r+q} \end{aligned}$$

and similarly,

$$E_1 = \frac{q\frac{\lambda \frac{T+P-w-\rho+\eta\frac{1+\Phi-P}{r+s+\eta+\tau}}{r+s+\lambda+\tau}}{r+s}}{r+q}$$

We therefore find ρ as the solution of

$$E_1\beta = B_1(1-\beta)$$

which leads to

$$\left[-\gamma(r+s) + q\left(-w + \alpha_\lambda \left(T + \rho + w + \eta \frac{1+\Phi-P}{r+s+\eta+\tau}\right)\right)\right](1-\beta)$$
$$= \beta q \alpha_\lambda \left(T + P - w - \rho + \eta \frac{1+\Phi-P}{r+s+\eta+\tau}\right)$$

or

$$\frac{\alpha_\lambda \rho}{r+s} = \beta\alpha_\lambda \left(\frac{T + P - w + \eta\frac{1+\Phi-P}{r+s+\eta+\tau}}{r+s}\right) + (1-\beta)\left[\frac{\gamma}{q} + \frac{\left(w + \alpha_\lambda \left(T + w + \eta\frac{1+\Phi-P}{r+s+\eta+\tau}\right)\right)}{r+s}\right]$$

From here, we easily obtain the new (EE4) and (BB4) in the text. To obtain an expression for goods market tightness, we proceed as follows. First, notice that we still have:

$$\begin{aligned} \eta(\xi) &= M_G(1,\xi) \text{ with } \eta'(\xi) \leq 0 \\ \lambda(\xi) &= M_G(\xi^{-1},) \text{ with } \lambda'(\xi) \geq 0 \\ \text{and } \lambda(\xi) &= \xi\eta(\xi) \end{aligned}$$

Second, we need to calculate $\mathcal{N}_3$ and $\mathcal{B}_3$, knowing that for $i = 2,3,4$, we have $\mathcal{N}_i = \mathcal{B}_i$. Focusing only on firms and workers, we have:

$$\begin{aligned} \frac{d\mathcal{C}_0}{dt} &= \theta q(\theta)u - (\eta+s)\mathcal{C}_0 = 0 \\ \frac{d\mathcal{C}_1}{dt} &= \eta\mathcal{C}_0 - s\mathcal{C}_1 = 0 \\ \frac{d\mathcal{N}_2}{dt} &= q(\theta)\mathcal{V} - (\lambda+s)\mathcal{N}_2 + \tau\mathcal{N}_3 + \tau\mathcal{N}_4 = 0 \\ \frac{d\mathcal{N}_3}{dt} &= \lambda\mathcal{N}_2 - (s+\lambda+\tau+\eta)\mathcal{N}_3 = 0 \\ \frac{d\mathcal{N}_4}{dt} &= \eta\mathcal{N}_3 - (s+\tau)\mathcal{N}_4 \\ 1-u &= \mathcal{C}_0 + \mathcal{C}_1 = \mathcal{N}_2 + \mathcal{N}_3 + \mathcal{N}_4 \end{aligned}$$

Hence, we have

$$\begin{aligned}
\mathcal{N}_4 &= \frac{\eta}{s+\tau}\mathcal{N}_3 \\
\mathcal{N}_3 &= \frac{\lambda}{s+\lambda+\tau+\eta}\mathcal{N}_2 \\
q(\theta)\mathcal{V} &= (\lambda+s)\mathcal{N}_2+\tau\mathcal{N}_3+\tau\frac{\eta}{s+\tau}\mathcal{N}_3
\end{aligned}$$

Going back to ξ, and using $\mathcal{N}_3 \equiv \mathcal{B}_3$, we have

$$\begin{aligned}
\xi &= \frac{\mathcal{C}_0+\mathcal{N}_3+\mathcal{B}_3}{\mathcal{N}_2} \\
&= \frac{\mathcal{C}_0}{\mathcal{N}_2}+\frac{2\lambda}{s+\lambda+\tau+\eta}
\end{aligned}$$

Using

$$\begin{aligned}
1-u &= \mathcal{C}_0+\mathcal{C}_1=\mathcal{C}_0\left(1+\frac{\mathcal{C}_1}{\mathcal{C}_0}\right)=\mathcal{C}_0\left(1+\frac{\eta}{s}\right) \\
1-u &= \mathcal{N}_2+\mathcal{N}_3+\mathcal{N}_4=\mathcal{N}_2\left[1+\frac{\lambda}{s+\lambda+\tau+\eta}+\frac{\eta}{s+\tau}\frac{\lambda}{s+\lambda+\tau+\eta}\right]
\end{aligned}$$

hence

$$\begin{aligned}
\xi &= \frac{1+\frac{\lambda}{s+\lambda+\tau+\eta}+\frac{\eta}{s+\tau}\frac{\lambda}{s+\lambda+\tau+\eta}}{1+\frac{\eta}{s}}+\frac{2\lambda}{s+\lambda+\tau+\eta} \\
&= \frac{1}{s+\lambda+\tau+\eta}\left[s+4\lambda+\tau+\eta+\frac{\eta\lambda}{s+\tau}+2\frac{\eta\lambda}{s}\right]
\end{aligned}$$

Define now

$$h(\xi)=\frac{1}{s+\lambda(\xi)+\tau+\eta(\xi)}\left[s+4\lambda(\xi)+\tau+\eta(\xi)+\frac{\eta(\xi)\lambda(\xi)}{s+\tau}+2\frac{\eta(\xi)\lambda(\xi)}{s}\right]$$

The solution to the fixed point problem $h(\xi)=\xi$ is therefore only a function of parameters. The fixed point may not be unique.

Technical Appendix: Summary of the main equations of the benchmark model

$$
\begin{aligned}
rE_0 &= -c + p(E_1 - E_0) && (103)\\
rE_1 &= -\gamma + \gamma + q(E_2 - E_1) && (104)\\
rE_2 &= -w + w + \lambda(E_3 - E_2) + s(E_0 - E_2) && (105)\\
rE_3 &= P - w - \rho + s(E_0 - E_3) + \tau(E_2 - E_3) && (106)
\end{aligned}
$$

$$
\begin{aligned}
rB_0 &= -\kappa + \phi p(B_1 - B_0) && (107)\\
rB_1 &= -\gamma + q(B_2 - B_1) && (108)\\
rB_2 &= -w + \lambda(B_3 - B_2) + s(B_0 - B_2) && (109)\\
rB_3 &= \rho + s(B_0 - B_3) + \tau(B_2 - B_3) && (110)
\end{aligned}
$$

Useful intermediate steps: by difference of E_3 and E_2

$$
\begin{aligned}
(E_3 - E_2)(r + \lambda + \tau + s) &= P - w - \rho && (111)\\
(B_3 - B_2)(r + \lambda + \tau + s) &= \rho + w && (112)
\end{aligned}
$$

$$\alpha_\lambda = \frac{r + s + \lambda}{r + s + \lambda + \tau}$$

Free-entry equations:

$$
\begin{aligned}
B_1 &= \frac{\kappa}{\phi p(\phi)}; && (113)\\
E_1 &= \frac{c}{p(\phi)}; && (114)\\
B_3 &= \frac{\rho\alpha_\lambda - w(1 - \alpha_\lambda)}{r + s}; && (115)\\
E_3 &= \alpha_\lambda \frac{P - w - \rho}{r + s} && (116)\\
&\text{and recursively,}\\
B_2 &= \frac{1}{r + s + \lambda}\left[-w + \lambda B_3\right]; && (117)\\
E_2 &= \frac{1}{r + s + \lambda}\left[0 + \lambda E_3\right]; && (118)\\
B_1 &= \frac{1}{r + q(\theta)}\left[-\gamma + qB_2\right]; && (119)\\
E_1 &= \frac{1}{r + q(\theta)}\left[0 + qE_2\right]. && (120)
\end{aligned}
$$

$$
\begin{aligned}
rU &= T + \theta q(W_0 - U) && (121)\\
rW_0 &= w + T + \eta(W_1 - W_0) + s(U - W_0) && (122)\\
rW_1 &= (w + T - P) + (1 + \Phi) + s(U - W_1) && (123)
\end{aligned}
$$

$$\phi^* = \frac{1 - \beta}{\beta}\frac{\kappa}{c} \quad (124)$$

$$B_w = \frac{w}{\lambda} + \frac{w(1 - \alpha_\lambda)}{r + s}$$

The price

$$P = Arg\max(W_1 - W_0)^{\delta}(B_3 + E_3 - B_2 - E_2)^{1-\delta}$$

leads to

$$P = \frac{1+\Phi}{1 + \frac{\delta}{1-\delta}\frac{r+s+\tau}{r+s}\frac{r+s+\eta}{r+s+\lambda+\tau}} \tag{125}$$

The wage

$$w = Arg\max(W_0 - U)^{\alpha}(B_2 + E_2 - B_1 - E_1)^{1-\alpha} \tag{126}$$

leads to

$$\begin{aligned} w\left(1 + \alpha\frac{\theta q(\theta)}{r+s}\right) &= (1-\alpha)\left(-\frac{\eta}{r+s+\eta}(1+\Phi-P)\right) \\ &+ \alpha\left(\frac{\lambda P}{r+s+\lambda+\tau}\left(1+\frac{\theta q(\theta)}{r+s}\right) - (r+s)\left(\frac{c}{p(\phi)} + \frac{\kappa}{\phi p(\phi)}\right)\right) \end{aligned} \tag{127}$$