Insurance and Optimal Growth*

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Abstract

The aim of this paper is to propose a theoretical connection between insurance and economic growth. We develop a simple model to insert microeconomically founded optimal insurance with moral hazard in a standard macroeconomic framework of optimal growth. We characterize the long-run equilibrium, the global dynamics and the evolution of insurance coverage with time. Three types of trajectories are identified: first, a simple dynamics with permanent partial insurance or full insurance, second, one with regime switching converging to a steady state with full or partial insurance and, finally, one with mixed equilibrium. **JEL codes:** O40, D81, G22, D91. **Keywords:** Optimal growth, Insurance, Moral hazard.

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1 Introduction

For half a century, growth and insurance have been among the star topics of economic literature. But this stardom has been obtained ignoring each other, in spite of numerous factual and theoretical arguments asserting strong links. In this paper, we propose to fill this gap. To this effect, we build a simple theoretical model to insert microeconomically founded insurance within a standard framework of optimal growth with prevention.

First, from a factual point of view, today, in the most developed countries, insurance has become a determinant and inevitable variable in the individual economic decisions, and, accordingly, it has a major macroeconomic weight. In effect, insurance expenditures, either private or public, represent a considerable fraction of the national income. According to OECD (2004), unemployment rates in EU approximate 9% against 6% in the US, with non negligible weights of unemployment benefits in the GDP (0.35% in average over the past 20 years in US according to the Congress Budget Office and roughly 1.7% in EU according to European commission); health insurance represents about 15% of GNP for US and between 7% and 10% for EU's countries (e.g., Colombo and Tapay, 2004 ; Huber, 1999). Moreover, the Insurance Information Institute (2005) estimates that US insurance industry accounts for 2,3 million jobs in 2004 and that over the last ten years, employment in the insurance industry amounts to 2,1% of the total employment. In 2004, its net income after taxes, at about 38,7$ billion, was at the highest level since 1998. Hence, insurance industry generates a significant fraction of gross domestic product. In addition, by alleviating subscribers' losses induced by various covered risks (unemployment, illness, casualties, etc), it sustains revenues, hence consumption and savings. It is even ascertained that the risk coverage has an influence on the macroeconomic amount of damages (e.g., Danzon and Pauly[1], 2002). As to the estimation of the statistical relation between economic growth and the development of the insurance industry, the notable study of OECD countries by Ward and Zurbruegg (2000) shows that the insurance industry Granger causes economic growth for some countries and the reverse is significant for others. The authors conclude that these differences across countries are probably due to "the nature of the cultural, regulatory, and legal environment [...] and the moral hazard effect of insurance".

Second, on theoretical grounds, individuals use insurance –either on a voluntary or compulsory basis– to face risk. In an intertemporal perspective, the

[1]The authors attribute "between one-fourth and one-half of the total growth in drug spending" to insurance coverage shifts over the 1990s in US.

(given) insurance coverage influences their will to smooth their intertemporal consumption paths and their prevention behavior[2]. By aggregation effect, on the macroeconomic side, insurance induces specific capital accumulation and growth rate, and conversely, the growth of income changes the individual trade-offs relative to risk. At last, it is shown that the governments can play an improving role in this relationship by controlling public insurance schemes and legislation[3].

As mentioned above, the literature seems to have neglected to study this "natural" link between growth and idiosyncrasic risk insurance. These two fields of economic research have developed separately. Earlier microeconomic research on insurance in an intertemporal framework divides in two trends: one without moral hazard (multi-period -Drèze and Modigliani, 1972 or Dionne and Eeckhoudt, 1984- or continuous time life cycle -Somerville, 2004 or Moore and Young, 2005- or continuous time infinite horizon -Bryis, 1986 or Gollier, 1994-), the other with moral hazard (Job search and optimal unemployment insurance -Eherenberg and Oaxaca, 1976 or Jovanovic, 1979 or Gruber, 1997 or Hopenhayn and Nicolini, 1997- or dynamic contracts with repeated games -Radner, 1985). The first trend aims to assess how the saving behaviors are modified in a context of partial insurance coverage. The authors find that insurance and intertemporal consumption are disconnected if full insurance is the rule and describe specific dynamics with *ad hoc* incomplete insurance. For the second trend, the main objective is to identify optimal dynamic contracts. But, both these microeconomic approaches rely on exogenous intertemporal prices, which elicits any connection between insurance and macroeconomic dynamics. Instead, in a macroeconomic intertemporal framework, prevention effort and savings are both influenced by expectations of intertemporal prices which are, in turn, explained by the capital accumulation and the individual risk levels.

The macroeconomic literature on this topic is not abundant, to say the least, despite notable exceptions. As an illustration, the macroeconomics with idiosyncratic shocks and moral hazard (for example, Banerjee and Newman, 1993 and 1994; Aghion and Bolton, 1997; Ghatak et al., 2001), by assuming the agents to be risk-neutral, generally avoids to deal with optimal demand for insurance. Other analyses have been developed where insurance plays a passive role with an *ad hoc* hypothesis of incomplete insurance

[2]For example, people will try to adopt diets which minimize the risk of coronarian disease, or systematically vaccinate their children or limit their smoking habit, or search jobs in a larger basin of employment.

[3]Theoretical arguments in favor of public insurance or public regulation can be found in Arrow (1963), Akerlof (1970) or Pauly (1974).

which is mainly a technical assumption to modelize the dynamics of wealth inequality. The dynamic general equilibrium is generally obtained by numerical methods (e.g. Koeniger, 2002). One notable exception is Young (2004) who goes further and presents an interesting study on optimal insurance with production and private savings. Although in an allusive manner, he points to the question of the link between optimal growth and the social planner's choice of replacement ratio. Due to incentive consideration, he obtains long-run optimality when the latter is nil. Another notable exception is Blanchard (1985) who proposes generalizations in dynamic general equilibrium framework of the model of Yaari (1965) with full life insurance in a life-cycle model with uncertain lifespan.

Formally, the paper develops a growth model with insurance and moral hazard. The framework is that of a benevolent planner which maximizes the intertemporal average welfare of agents. The agents have the same stucture of time and risk preferences, their life horizon is one period[4] and they are submitted to individual and endogenous stochastic damages. Time is continuous and each period's length is supposed to be of measure zero. The key factor of risk is the non observable discrete prevention effort of the agent which the planner wants to monitor via the insurance coverage of consumption loss. To determine the optimal trajectory of the economy, we use traditional optimal control theory. We describe the underpinnnings of the dynamics. First, we derive the first order conditions then we characterize the optimal path. We get a usual incentive constraint and a modified Euler condition of intertemporal consumption trade-off. In this paper, income effects on the optimal insurance contract are contemplated in a dynamic perspective. An obvious link exists between prevention and the development level. *A priori*, the dynamic characteristic of damage should determine the optimal insurance coverage degree. When the induced loss is weakly dependent on the economic growth, the latter may reduce the welfare benefit of prevention (difference between the utilities obtained with and without damage), which depends on the risk aversion parameter. In such a case, the prevention effort should decrease with economic development. In other configurations where the cost of damage depends strongly on wealth, the prevention effort can follow more complex dynamics. The optimal insurance and growth path should depend mainly on the characteristics of risk (notably value of damage, cost of prevention) and the public discount rate.

The paper is organized as follows. Section two describes the economy

[4]This assumption avoids to deal with issue about dynamic insurance penalties in case of repeated sinisters (see Radner, 1985).

and determines the economic tradeoffs for both the planner and the agents. Section three characterizes the long-run equilibrium, the global dynamics and the evolution of insurance coverage with time. Three types of trajectories are identified: first, a simple dynamics with permanent partial insurance or full insurance, second, one with regime switching converging to a steady state with full or partial insurance and, finally, one with mixed equilibrium. Section four concludes.

2 The model

We consider people living only one period and a planner with an infinite horizon of life. To simplify the identification of dynamic time path, we will assume time to be continuous (length of each period tends to zero) as in the standard model of Ramsey (1928).

2.1 The agents

People are risk averse. Their objective is to maximise their *ex ante* expected welfare. Each agent is given by the planner an endowment of goods to consume. This endowment depends on an i.i.d. perfectly observable state of nature experienced by the agent. The only choice left to anyone is his prevention effort, denoted hereafter e. The latter is assumed to take only two values, $e = 0$ (no prevention) or $e = 1$ (full prevention). Each agent uses a personal capital stock k (allocated by the planner) in his productive activity. The latter is risky: for a *per capita* stock k, the net production may be either $\overline{f}(k) = \underline{f}(k)$, in case of absence of damage with probability $1 - p(e)$, or $\underline{f}(k) < \overline{f}(k)$ with $\underline{f}(0) = 0$, in case of damage with probability $p(e)$. We denote $\mu(k) = \overline{f}(k) - \underline{f}(k)$ the production loss, i.e. the financial cost of damage. $\mu(0)$ is not necessary nil (for example in case of amount of damage independant of the level of capital). By construction, we have: $p(0) = p_0 > p(1) = p_1$. $\overline{f}$ and $\underline{f}$ have usual properties of decreasing returns production technologies: $\overline{f}' > 0$, $\underline{f}' > 0$ and $\overline{f}'' < 0$, $\underline{f}'' < 0$. Each agent chooses his individual effort level (*ex ante*) before knowing his actual production level (*ex post*) and implicitely, his consumption endowment.

The instantaneous level of welfare is measured by the following additively separable utility function: $v(\tilde{c}, e) = u(\tilde{c}) - \gamma \cdot e$ where u is the level of utility induced by stochastic consumption $\tilde{c}$, such that $u' > 0$ and $u'' < 0$ and where γ is the unit welfare cost of prevention effort.

The agent is VNM and maximizes his expected welfare. For a stochastic consumption bundle allocated by the planner and denoted $(\underline{c}; \overline{c})$, where $\underline{c}$

(resp. $\overline{c}$) is the level of consumption in case of damage (resp. no damage), he chooses prevention if and only if his benefit in terms of anticipated welfare is greater than his cost of prevention, i.e. when his individual incentive constraint is checked:

$$(p_0 - p_1) \cdot (u(\overline{c}) - u(\underline{c})) > \gamma. \tag{1}$$

2.2 The social planner

2.2.1 The static program: a standard optimal insurance problem with moral hazard

The social planner's information about the individual choices of prevention is imperfect since the individual efforts of prevention are not observable. Instead, the state of nature (high vs low production level) is perfectly observable. This configuration is the standard optimal insurance with moral hazard as described in the seminal paper of Arnott and Stiglitz (1990).

Her insurance policy relies on the representative agent's microeconomic tradeoffs and she applies the rule "bounty for lucky and penance for unlucky". The planner seeks to maximize the agents' anticipated welfare. To induce people to engage in greater prevention effort, she can use an imperfect insurance coverage in case of damage. We denote by $\varepsilon = \overline{c} - \underline{c}$ the gap in consumption inflicted by the planner when damage occurs.

Hereafter, we will denote $y = f(k) - \delta \cdot k - \dot{k}$ the potential average income allocated to agents before considering the cost of damages. It is defined as the difference between their production $f(k)$ net of the depreciation of capital (at rate δ) and the capital accumulation choice ($\dot{k}$). Moreover, the *expected* consumption for prevention effort $e = 0$ or 1 writes: $c = y - p_e \cdot \mu(k)$, with $\mu(k)$ being the production loss $\overline{f} - \underline{f}$ as defined earlier.

By using the individual choice rule and the resource constraint, the planner can encourage people to adopt prevention *iff*:

$$h(c, \varepsilon) = (p_0 - p_1) \cdot (u(c + p_1 \cdot \varepsilon) - u(c - (1 - p_1) \cdot \varepsilon)) \geq \gamma, \tag{2}$$

and welfare level with prevention will be maximized when the equality above is verified. Arnott-Stiglitz (1990) find a similar condition. The latter leads to a minimal cost incentive penalty rule which the existence is guaranteed according to the following proposition.

[P1] Existence of an incentive penalty rule:

For $\gamma < (p_0 - p_1)\left[u\left(\frac{c}{1-p_1}\right) - u(0)\right]$, *there exists a function* $\varepsilon(c)$ *such that* $h(c, \varepsilon) = \gamma$ *is verified with* $\varepsilon' > 0$.

Proof: See Appendix.

We will denote V_e the indirect utility levels of consumption associated to the effort level e such that:

$$\begin{cases} \text{For } e = 1,\ V_1(c) = E(u(\tilde{c})) = p_1 \cdot u(\underline{c}) + (1 - p_1) \cdot u(\overline{c}) \\ \qquad \text{with } \begin{cases} \overline{c} = c + p_1 \cdot \varepsilon(c) \\ \underline{c} = c - (1 - p_1) \cdot \varepsilon(c) \end{cases}, \\ \text{For } e = 0,\ V_0(c) = u(c). \end{cases}$$

The social planner's control variable, penalty ε, increases with the average level of consumption because the marginal disutility induced by uncertainty is decreasing with wealth. We have to stress that prevention is not seeked *per se*. The planner will opt for prevention only if it leads to a net positive welfare benefit. Formally, the condition writes at each period:

$$g(y, \mu) = V_1(y - p_1 \cdot \mu) - V_0(y - p_0 \cdot \mu) \geq \gamma. \tag{3}$$

This condition will be called hereafter the "positive welfare benefit condition" denoted PWBC.

2.2.2 The dynamic program

The choice of optimal insurance is contingent to the optimal choice of capital accumulation and *vice versa*: knowing the optimal value of y and the stock of capital k, the planner infers the corresponding level of optimal insurance; in turn, the adopted level of prevention determines the type of dynamics. This two-step maximization procedure is then equivalent to the one-step maximization of the whole program.

Time is assumed to be continuous. We call ρ the planner's time discounting value. The social planner's intertemporal program then writes:

$$\begin{cases} \max\limits_{\{k(t), y(t)\}} \int_0^{+\infty} \exp(-\rho t) \cdot \left[V_e\left(y(t) - p_{e(t)}\mu(k(t))\right) - \gamma \cdot e(t)\right] dt \\ s.t.\ \dot{k}(t) = f(k(t)) - \delta \cdot k(t) - y(t) \qquad (\lambda(t)) \end{cases}, \tag{4}$$

with $e = 1$ if $g(y, \mu(k)) \geq \gamma$ and $e = 0$ otherwise. $\lambda(t)$ denotes the costate variable for capital.

Let us form the current value Hamiltonian, omitting time t:

$$H_e = V_e\left(y - p_e \cdot \mu\left(k\right)\right) - \gamma \cdot e + \psi\left(f\left(k\right) - \delta \cdot k - y\right), \tag{5}$$

with $\psi\left(t\right) = \lambda\left(t\right) \cdot \exp\left(\rho t\right)$ the current value costate variable of the planner's program.

The first order conditions give:

$$\begin{cases} y : \frac{\partial H_e}{\partial y} = V_y - \psi = 0 \\ k : \dot{\psi} = -\frac{\partial H_e}{\partial k} + \rho \cdot \psi = -V_k - \psi \cdot \left(f'\left(k\right) - \delta\right) + \rho \cdot \psi \\ \lambda : \dot{k} = f\left(k\right) - \delta \cdot k - y \end{cases} \tag{6}$$

since $\dot{\psi} = \rho \cdot \exp\left(\rho t\right) \cdot \lambda + \exp\left(\rho t\right) \cdot \dot{\lambda}$ and $\dot{\lambda} = -\frac{\partial H_e}{\partial k}$.

When the level of prevention e is stationary, the intertemporal program of maximization is standard. So the usual transversality condition obtains. The first two equations of the FOC system rewrite as:

$$\begin{cases} \dot{\psi} = V_{yy} \cdot \dot{y} + V_{yk} \cdot \dot{k} \\ \dot{\psi} = -V_k - \psi \cdot \left(f'\left(k\right) - \delta - \rho\right) \end{cases}. \tag{7}$$

By using properties of $V\left(c\right)$, we have $V_y = V'\left(c\right)$, $V_k = -p \cdot \mu'\left(k\right) \cdot V_y$, $V_{yy} = V''\left(c\right)$, $V_{yk} = -p \cdot \mu'\left(k\right) \cdot V''\left(c\right)$ and $V_y / V_{yy} = \frac{u'(c + p_1 \varepsilon'(c))}{(1 + p_1 \varepsilon'(c)) u''(c + p_1 \varepsilon'(c))}$ if $g\left(y, \mu\left(k\right)\right) \geq \gamma$ or $V_y / V_{yy} = \frac{u'(c)}{u''(c)}$ if $g\left(y, \mu\left(k\right)\right) < \gamma$. (7) then leads to:

$$\begin{cases} \dot{\psi} = V_{yy} \cdot \left[\dot{y} - p\mu'\left(k\right)\dot{k}\right] \\ \dot{\psi} = -V_y \cdot \left[f'\left(k\right) - p\mu'\left(k\right) - \delta - \rho\right] \end{cases}. \tag{8}$$

Finally, we find:

$$\begin{cases} \dot{y} = -\frac{V_y}{V_{yy}}\left(f'\left(k\right) - p\mu'\left(k\right) - \left(\delta + \rho\right)\right) + p \cdot \mu'\left(k\right) \cdot \dot{k} \\ \dot{k} = f\left(k\right) - \delta \cdot k - y \end{cases}. \tag{9}$$

For e given, the expected consumption writes $c = y - p_e \cdot \mu\left(k\right)$. Also, we have $\dot{c} = \dot{y} - p \cdot \mu'\left(k\right) \cdot \dot{k}$ and the FODE system characterizes in the consumption and capital accumulation plane as follows:

$$\begin{cases} \dot{c} = C\left(c, k\right) = -\frac{V_y}{V_{yy}}\left(c\right) \cdot \left(f_e'\left(k\right) - \left(\delta + \rho\right)\right) \\ \dot{k} = K\left(c, k\right) = f_e\left(k\right) - \delta \cdot k - c \end{cases} \tag{10}$$

where $f_e\left(k\right) = f\left(k\right) - p_e \cdot \mu\left(k\right)$ is the expected production level for given e.

3 Study of the dynamics

3.1 Preliminary remark relative to the wealth effect on the incentive constraint

We now turn to the description of the dynamics which is similar to the standard optimal growth approach. The particularity here is that two *potential* dynamics can emerge. They depend on the value of the average income allocated to agents (y) and the cost of damage (μ). Let us stress that while they are given in the standard microeconomic approach, both variables obviously change over time in a macroeconomic dynamics approach, hence modifying the PWBC. This leads us to determine the two following subsets contingent to e in the (y,k) plane:

$$\Omega_{e=1\ (resp.\ 0)} = \{y \in \mathcal{D}_k \text{ and } k \in [0; k_{\max}] \,|\text{PWBC is (resp. not) checked}\},$$

where $k_{\max}$ is the stationary specific value of capital per head when the golden rule with prevention prevails ($\rho = 0$) *i.e.* $f_1'(k_{\max}) = \delta$ and for a given k, the domain of definition of the function $g(y,\mu)$ is the interval $\mathcal{D}_k = [y_{\min}(\mu(k)); y_{\max}(k)]$ with $y_{\min} = p_0.\mu$ and $y_{\max}(k) = f(k) + (1-\delta)k$.

To this effect, we need to identify a locus where agents are indifferent between prevention and no prevention. On the figures below, this frontier will be denoted $WB = 0$ for "zero welfare benefit". Its precise shape and position depend both on μ and y, with contradictory wealth effects. In a principal-agent moral hazard setting, Mookherjee (1997) evidences that an increase in wealth has two opposing effects on the optimal effort level: an incentive effect (cf. $\varepsilon' > 0$, i.e. stronger punishments) and a trade-off effect in terms of effort (the marginal disutility of loss decreases with respect to the agent's prevention cost). That means that the relationship between μ and y is complex.

Hereafter, for sake of simplicity, we will assume conditions on utility function to get a clearcut partition of these two continuous subsets in the plane of planner's choice variables (k,y). The frontier $WB = 0$ is expressed by an increasing separating relation between y and μ : $yt(\mu)$. This function gives the threshold value of y inducing a switch between $e = 1$ and $e = 0$. This increasing relation expresses formally the intuition that the threshold of gross income allocated to consumption and risk (y) will be higher the greater the financial cost of damage. The following proposition gives sufficient conditions for the existence of such a relation.

[P2] SC for the existence of an increasing separating relation $y = yt(\mu)$**.** *Sufficient conditions for* $g(y,\mu) = \gamma$ and $\frac{\partial y}{\partial \mu} > 0$ *are:*

$$\begin{array}{ll} [\text{C1}] & V_1(p_0\mu) - \gamma > V_0(0) \quad ; \\ [\text{C2}] & V_1(+\infty) - \gamma < V_0(+\infty) \quad ; \\ [\text{C3}] & (1 + p_1 \cdot \varepsilon') \cdot \frac{u'(y - p_1 \cdot \mu + p_1 \cdot \varepsilon)}{u'(y - p_0 \cdot \mu)} < 1 \quad . \end{array}$$

Proof: See Appendix.

The shape of $yt(\mu)$ depends on both real and psychological components. The former is the direct influence of capital accumulation (k) on the damage cost (μ). The latter lies with the consumers' psychological cost of prevention (γ).

The dependency of μ on the capital accumulation determines the slope of the curve in the plane (k, y). The stronger the correlation of μ with k, the greater the slope of $yt(\mu)$.

If μ is weakly correlated to k, the profile of this curve will be relatively flat. On contrary, if μ is strongly correlated to k, the curve will show an important slope. In our graphical representations, it will be assumed $\mu' \geq 0$.

Concerning the psychological cost of prevention, an increase in γ shifts the curve downward whenever [P2] applies: we remark first that $\frac{\partial \varepsilon}{\partial \gamma} > 0$; second, we have $\frac{\partial V_1}{\partial \gamma} = -p_1(1 - p_1)\frac{\partial \varepsilon}{\partial \gamma}(u'(\underline{c}) - u'(\bar{c})) - 1 < 0$. If [P2] applies, we deduce that: $\left(\frac{\partial y}{\partial \gamma}\right)_{g(y,\mu)=\gamma} < 0$. However, the level of γ influences also the slope of the $WB = 0$ curve but the global effect is not clear. It depends crucially on the explicit utility function chosen. Thereafter, for sake of simplicity, we will assume this "slope effect" is minor.

These two components allow to draw the $WB = 0$ curve. From the properties above, we can use γ as a parameter to control the reference level (in terms of y) of the curve and the slope of the function $\mu(k)$ plays an important role on $\frac{\partial yt}{\partial k}$. By modifying the relative importance of these components, it is possible to characterize different configurations of the trajectories followed by the economy.

3.2 Long run equilibrium

For e given, the two loci of stationarity $\dot{c} = 0$ ($cc_e(k)$ hereafter) –or equivalently $\dot{y} = 0$ ($yy_e(k)$ hereafter)– and $\dot{k} = 0$ ($kk_e(k)$ hereafter) satisfy respectively: $f'_e(k) - (\delta + \rho) = 0$ and $c = f_e(k) - \delta \cdot k$. On the figures thereafter, the loci $\dot{y} = 0$ are reported only for values of e respecting the PWBC. Let us stress, that the other loci $\dot{k} = 0$ is the same for both values of e in the plane (k, y) and has a positive intercept for $k = 0$ if we assume $\mu(0) > 0$ which is

the case on the figures. We will denote hereafter $\hat{x}_e$ the value taken by any variable x at the steady state e. The existence and unicity of the steady state for e *given* is established below (with PWBC not necessary checked).

[P3] Existence and unicity of the steady state for e given:
For each value of prevention effort, there exists only one steady state $\left(\hat{k}_e, \hat{y}_e\right)$*:*

$$\begin{cases} \hat{k}_e = f_e'^{-1}(\delta + \rho) \\ \hat{y}_e = f\left(\hat{k}_e\right) - \delta \cdot \hat{k}_e. \end{cases}.$$

Proof: See Appendix.

These two steady states are contingent to the satisfaction of the PWBC. In the long run (LR), four possibilities must be contemplated:

- LR1: if $g(\hat{y}_1, \hat{\mu}_1) > \gamma$ and $g(\hat{y}_0, \hat{\mu}_0) > \gamma$, the steady state *with prevention* obtains. It is denoted E_1 on Fig. 1a, Fig. 2a, Fig. 2b, Fig. 3 and Fig. 4.

- LR2: if $g(\hat{y}_0, \hat{\mu}_0) < \gamma$ and $g(\hat{y}_1, \hat{\mu}_1) < \gamma$, the steady state *without prevention* obtains. It is denoted E_0 on Fig. 1a, Fig. 2a, Fig. 2b, Fig. 3 and Fig. 4.

- LR3: if $g(\hat{y}_1, \hat{\mu}_1) > \gamma$ and $g(\hat{y}_0, \hat{\mu}_0) < \gamma$, the two previous equilibria are reachable. The planner must then compare the two welfare paths and choose the "best" one. This configuration is illustrated by Fig. 2c.

- LR4: if $g(\hat{y}_1, \hat{\mu}_1) < \gamma$ and $g(\hat{y}_0, \hat{\mu}_0) > \gamma$, none of the two previous equilibria respects the PWBC. We show in the next proposition that there exists an "intermediate" equilibrium which is a particular combination of the latter and which can be interpreted as a "randomized" insurance contract. The "intermediate" equilibrium corresponding to this situation is denoted E_{01} on Fig. 1b. This steady state is reachable in finite time contrarily to the two previous cases. For this reason, there is no need to derive the local stability condition. This steady state expresses a very particular situation where the planner arbitrarily splits the population in two groups by offering randomly two distinct menus in terms of insurance contracts and capital allocation. $1-q$ (resp. q) denotes the proportion of agents who opt for (resp. no) prevention. The average effort in the population is $\hat{e} = (1-q)\times 1 + q\times 0 = 1-q$.

[P4] Existence of an "intermediate" equilibrium $\left(\hat{k}_{01}, \hat{y}_{01}\right)$**:**
If $g(\hat{y}_1, \hat{\mu}_1) < \gamma$ *and* $g(\hat{y}_0, \hat{\mu}_0) < \gamma$*, there exists a unique scalar* $0 < q < 1$ *s.t.:*

$$
\begin{aligned}
&p_1 \cdot u\left(\hat{y}_{01} - p_1\hat{\mu}_1 - (1-p_1)\cdot\varepsilon\left(\hat{y}_{01} - p_1\hat{\mu}_1\right)\right) \\
&+ (1-p_1)\cdot u\left(\hat{y}_{01} - p_1\hat{\mu}_1 + p_1\cdot\varepsilon\left(\hat{y}_{01} - p_1\hat{\mu}_1\right)\right) \\
&= u\left(\hat{y}_{01} - p_0\hat{\mu}_0\right) + \gamma,
\end{aligned}
$$

with $\hat{y}_{01} = q\cdot\hat{y}_0 + (1-q)\cdot\hat{y}_1$.

Proof: See Appendix.

3.3 Local stability

In the neighborhood of the two "pure" steady states, the dynamics can be linearized as usual in the following way:

$$
\begin{cases}
\dot{c} = C_c\cdot(c-\hat{c}) + C_k\cdot\left(k-\hat{k}\right) \\
\dot{k} = K_c\cdot(c-\hat{c}) + K_k\cdot\left(k-\hat{k}\right)
\end{cases}. \tag{11}
$$

As seen before, $\begin{cases} C\left(\hat{c}_e,\hat{k}_e\right) = -\frac{V_y}{V_{yy}}\left(\hat{c}_e\right)\cdot\left(f_e'\left(\hat{k}_e\right) - (\delta+\rho)\right) \\ K\left(\hat{c}_e,\hat{k}_e\right) = f_e\left(\hat{k}_e\right) - \delta\cdot\hat{k}_e - \hat{c}_e \end{cases}$, then we have: $C_c = 0$, $C_k = -\frac{V_y}{V_{yy}}\left(\hat{c}_e\right)\cdot\left(f_e''\left(\hat{k}_e\right)\right)$, $K_c = -1$ and $K_k = \rho$. The characteristic equation of our dynamic system writes:

$$
\begin{vmatrix}
-\lambda & -\frac{V_y}{V_{yy}}\left(\hat{c}_e\right)\cdot\left(f_e''\left(\hat{k}_e\right)\right) \\
-1 & \rho-\lambda
\end{vmatrix} = \lambda^2 - \rho\lambda - \frac{V_y}{V_{yy}}\left(\hat{c}_e\right)\cdot\left(f_e''\left(\hat{k}_e\right)\right) = 0. \tag{12}
$$

The product of the two roots is equal to $-\frac{V_y}{V_{yy}}\left(\hat{c}_e\right)\cdot f_e''\left(\hat{k}_e\right)$. This term is negative. Hence, there will be one positive and one negative real root and, the same properties of saddle point as in a standard Ramsey problem obtain.

As for the "intermediate" steady state equilibrium, there is no standard local stability properties. Instead, the planner controls the trajectory so that the mixed equilibrium be reachable. At this equilibrium, she chooses a particular share q such that the stability is obtained both for consumption and capital accumulation.

3.4 Global dynamics and regime switching

Out of the neighborhood of the steady state equilibrium, the transitory dynamics cannot be sketched easily. However, in our case, the dynamic system

is relatively simple (*cf.* functions $C(c,k)$ and $K(c,k)$), so that we can describe the global dynamics in a good qualitative way (*cf.* Léonard and Van Long, 1992, p. 101). This heuristic resolution is endorsed by Gandolfo (1998, p. 407-408) who stresses that, as an alternative, "we may always try a numerical integration of the system (computers will do the job), but this possibility is not of great help to economic theorist. In economic theory one seeks general answers, independant of numerical analyses [...]". From a qualitative view point, a discussion on the value -high or low- of $\mu(k)$ or γ allows to distinguish different and relevant configurations of the $WB = 0$ curve. In all figures below, the stable branche of the different transitory dynamics is represented by a dashed upward-sloping curve. We distinguish between four situations :

- Weak correlation of μ w.r.t k (μ' weak) and relatively weak value of γ (case 1);
- Strong correlation of μ w.r.t k (μ' strong) and relatively strong value of γ (case 2);
- Strong correlation of μ w.r.t k (μ' strong) and relatively weak value of γ (case 3);
- Weak correlation of μ w.r.t k (μ' weak) and relatively strong value of γ (case 4).

- Case 1: Weak correlation of μ w.r.t k (μ' weak) and relatively weak value of γ.

That means in this configuration that: $yt(k_{\max}) < kk(k_{\max})$ and for initial value $k(0)$, $yt(k(0)) > kk(k(0))$. If the planner is too impatient, she opts for the unique trajectory converging to the saddle point with prevention (E_1). We know from the properties of the system that this trajectory describes a positive relation between capital and consumption. This trajectory is represented on Fig. 1a. That means that the economy will always be under partial insurance regime.

If the planner is sufficiently patient, her solution is straightforward when the initial value of capital per head ($k(0)$) and its associated value ($y(0)$) on the converging (to E_0) trajectory are located in the region of the plane compatible with the optimal choice of zero prevention. That means that the economy will always be under full insurance regime. Otherwise, the planner operates backward: knowing the end of the trajectory, she comes back until the point (k^{rs}, c_0^{rs}) where she is indifferent between full and zero prevention in terms of welfare. At this point, she can reach another point (k^{rs}, c_1^{rs}) giving equivalent welfare (she imposes a "regime switching", see Fig. 1a), and go back along the unique (unstable) trajectory reaching this same point until

the initial condition $k(0)$. That means that the economy will experience a partial insurance regime in finite time. In our illustrations, we draw paths with at most one regime switching. However, no condition exists to garantee the unicity of regime switching. Multi-regime switchings are realistic. That depends on the trajectory profiles as well as the $WB = 0$ curve.

If the planner is moderatly patient (intermediate case), knowing the long run equilibrium with mixed strategy, she chooses the (unstable) trajectory with prevention reaching this point in finite time. Then, she immediately adopts a strategy inducing a stable consumption and capital accumulation which relies on a mixed profile of prevention effort as described above. Knowing this trajectory, she tracks backward to the initial condition $k(0)$.

- Case 2: Strong correlation of μ w.r.t k (μ' strong) and relatively strong value of γ.

That means in this configuration that: $yt(k_{\max}) > kk(k_{\max})$ and (eventually) for initial value k_0, $yt(0) < kk(0)$. The results are the opposite of the previous ones. When sufficiently patient, the planner aims to reach a high capital accumulation with prevention (with eventual "regime switching", see Fig. 2a or Fig. 2b) while, when too impatient, she will lead the economy towards a full insurance (hence, zero prevention) equilibrium. With respect to case 1, the risk level crucially depends on the financial cost of damage, $\mu(k)$. The more the latter is linked to the level of economic development, the more likely the marginal benefit of prevention increases. The condition $yt(k_{\max}) > kk(k_{\max})$ suggests sufficiently high levels of development in the long run.

- Case 3: Strong correlation of μ w.r.t k (μ' strong) and relatively weak value of γ.

That means in this configuration that: $yt(k) > kk(k)\ \forall k \leq k_{\max}$. Financial cost of risk is always high enough and psychological cost of prevention is always low enough to guarantee the dominance of the equilibrium with prevention and partial insurance (see Fig. 3).

- Case 4: Weak correlation of μ w.r.t k (μ' strong) and relatively strong value of γ.

That means in this configuration that: $yt(k) < kk(k)\ \forall k \leq k_{\max}$. The opposite of previous case prevails. Financial cost of risk is always low enough and psychological cost of prevention is always high enough to guarantee the dominance of the equilibrium without prevention and full insurance (see Fig. 4).

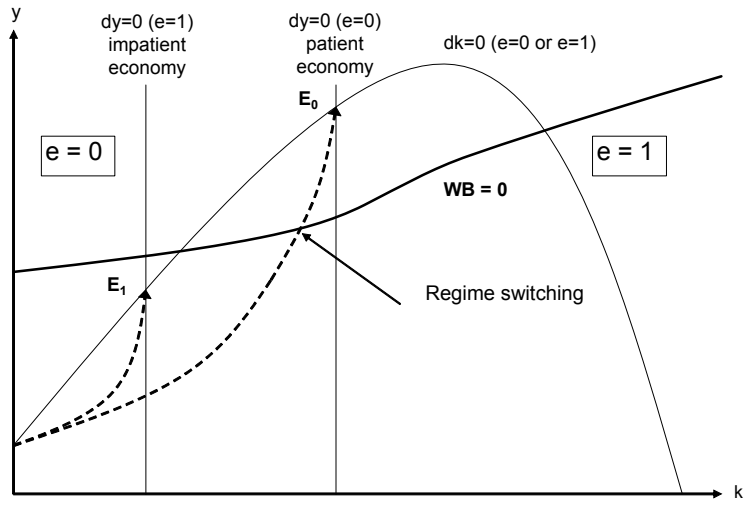


Fig. 1a – Weak correlation of μ w.r.t. k and relatively weak value of γ.

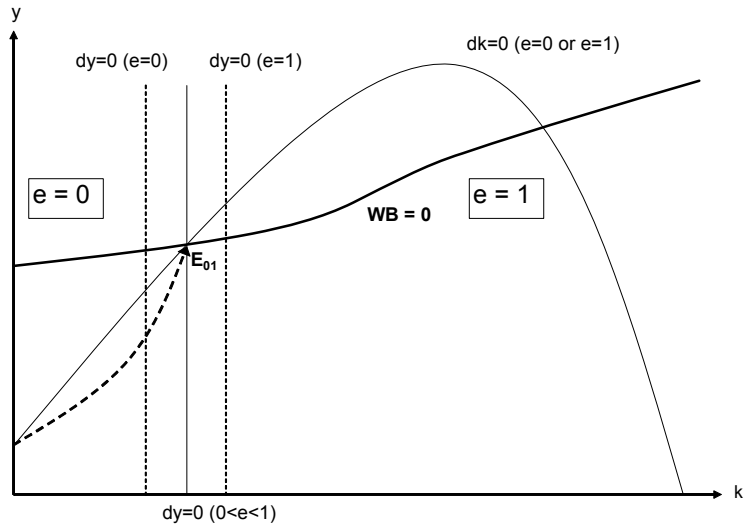


Fig. 1b – Case 1 with intermediate long run equilibrium.

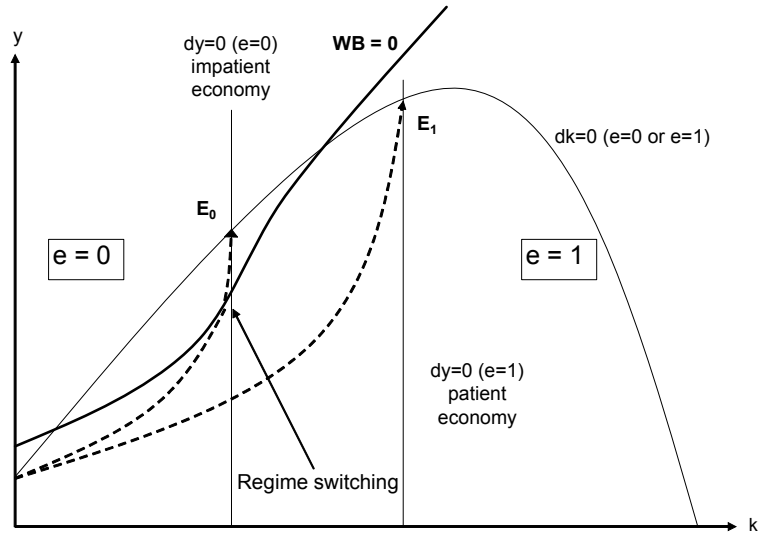


Fig. 2a – Strong correlation of μ w.r.t. k and relatively strong value of γ: (unicity of the long run equilibrium).

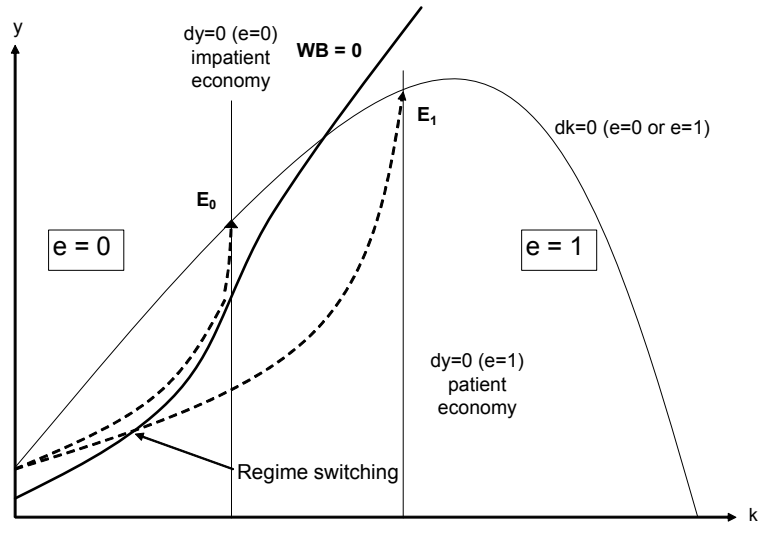


Fig. 2b – Other configuration showing "no prevention to prevention" trajectory.

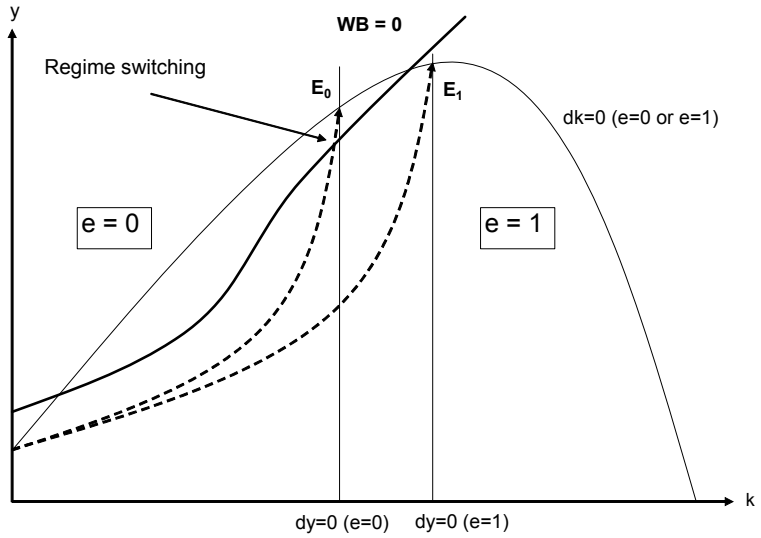


Fig. 2c – Case 2 with two long run equilibria

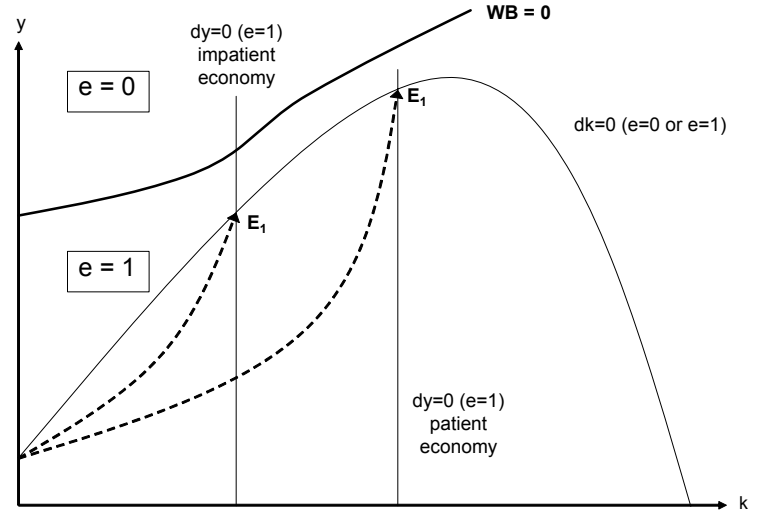


Fig. 3 – Strong correlation of μ w.r.t. k and relatively weak value of γ.

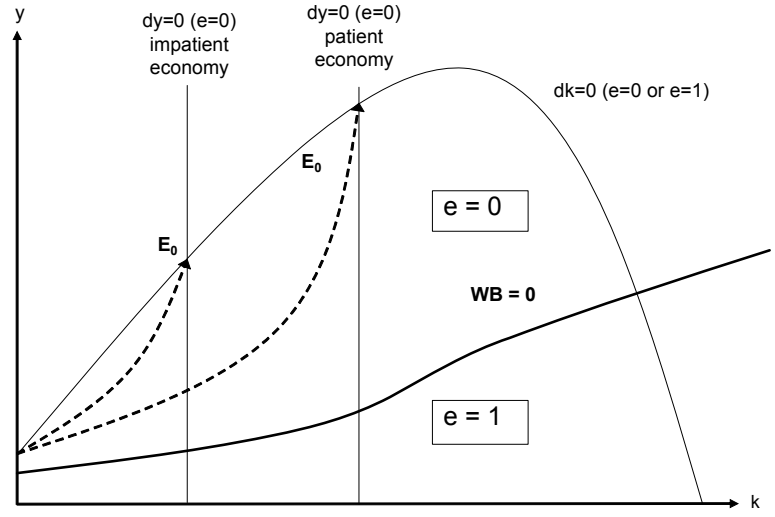


Fig. 4 – Weak correlation w.r.t. k and relatively strong value of γ.

4 Concluding remarks

This paper aims at proposing a very simple way to link insurance and economic growth in a Ramsey framework, hoping to contribute to fill strange vacuum in the literature on this topic. We identify and categorize not so trivial trajectories.

Globally, what determines the optimal trajectory of economy is the combined effect of the social net benefit of prevention effort and the public discount rate. The lower the latter and the higher the former, the more likely the economy will reach a full prevention with partial insurance steady state. The converse prevails.

We have evidenced three main configurations for the optimal dynamics with insurance:

- converging dynamics with permanent partial insurance, or inversely with full insurance;

- finite time dynamics with partial insurance then permanent full insurance regime, or inversely full insurance then permanent partial insurance;

- dynamics with partial insurance reaching (in finite time) a particular steady state with mixed insurance schemes (full for some and partial for the others) induced by the heterogeneity of both prevention choice and consumption allocation.

In the particular case where the social net benefit of prevention is not sufficiently high in the long run, our results show impatience puts brakes on

economic development and the society may never attain a sufficient wealth level to abandon prevention. That could offer one possible explanation *ceteris paribus* of the fact that some societies (the richest) can loose their prevention effort while others (the poorest) stick to it. Moreover, these results could be seen as an alternative theoritical interpretation of the Kuznets' curve (1955). Insurance is a good means to reduce inequality. In our framework, for incentive reasons, growth can induce an increase in the consumption gap deriving from risk. But, when developement is sufficient, the planner will switch to full insurance and will eliminate inequality in consumption due to hazard.

These results, notably the global properties of our dynamics, are obtained in a heuristic way. A useful prolongation of this work will consist in computational simulation to explore real solutions.

Appendix

[P1] Proof of the existence of an incentive penalty rule:
We have $h_c = (p_0 - p_1) \cdot (u'(\overline{c}) - u'(\underline{c})) < 0$ and $h_\varepsilon = (p_0 - p_1) \cdot ((1 - p_1) \cdot u'(\underline{c}) + p_1 \cdot u'(\overline{c})) > 0$. We can deduce that $h(c, \varepsilon) = \gamma$ admits only one solution, and we have $\frac{\partial \varepsilon}{\partial c} = -\frac{h_c}{h_\varepsilon} > 0$. QED. ■

[P2] Proof of the existence of SC for an increasing separating relation:
When the conditions $V_1(p_0\mu) - \gamma > V_0(0)$ [C1] and $V_1(+\infty) - \gamma < V_0(+\infty)$ [C2] are checked, the continuity of function g implies that there exist one or several solutions y for given μ such that $g(y, \mu) = \gamma$. If $g(y, \mu)$ for given μ is monotonic, there exists a unique solution $y(\mu)$. By construction, we have $\frac{\partial g}{\partial y} = V_1' - V_0'$. $\frac{\partial g}{\partial y} < 0$ *iff* $1 - (1 + p_1 \cdot \varepsilon') \cdot \frac{u'(y - p_1 \cdot \mu + p_1 \cdot \varepsilon)}{u'(y - p_0 \cdot \mu)} > 0$ [C3]. As $\frac{\partial g}{\partial \mu} = -p_1 \cdot \frac{\partial g}{\partial y} + (p_0 - p_1) \cdot u'(y - p_1 \cdot \mu)$, we deduce that if $\frac{\partial g}{\partial y} < 0$, then $\frac{\partial g}{\partial \mu} > 0$. [C3] is then a sufficient condition for $\left(\frac{\partial y}{\partial \mu}\right)_{g(y,\mu)=\gamma} = -\frac{\partial g}{\partial \mu} / \frac{\partial g}{\partial y} > 0$. QED. ■

[P3] Proof of the existence and unicity of the steady state for e given:
By construction $f_e'' < 0$ then f_e' is monotonically decreasing. There exists a solution $\hat{k}_e$ s.t. $\hat{k}_e = f_e'^{-1}(\delta + \rho)$. QED. ■

[P4] Proof of the existence of an "intermediate" equilibrium $(\hat{k}_{01}, \hat{y}_{01})$**:** We search a stationary solution such that:

$$\begin{aligned} &G(q) = p_1 \cdot u(\hat{y}_{01} - p_1\hat{\mu}_1 - (1-p_1) \cdot \varepsilon(\hat{y}_{01} - p_1\hat{\mu}_1)) \\ &+ (1-p_1) \cdot u(\hat{y}_{01} - p_1\hat{\mu}_1 + p_1 \cdot \varepsilon(\hat{y}_{01} - p_1\hat{\mu}_1)) \\ &- u(\hat{y}_{01} - p_0\hat{\mu}_0) - \gamma \\ &= 0 \end{aligned}$$

with $\hat{y}_{01}(q) = q \cdot \hat{y}_0 + (1-q) \cdot \hat{y}_1$
We have $G(0) > \gamma$ and $G(1) < \gamma$ with:

$$G'(q) = (\hat{y}_1 - \hat{y}_0) \cdot \left(u'(\hat{y}_{01} - p_0\hat{\mu}_0) - u'(\hat{y}_1 - p_0\hat{\mu}_1) - \frac{\partial g(\hat{y}_1, \hat{\mu}_1)}{\partial y} \right) > 0,$$

since $\hat{y}_{01} - p_0\hat{\mu}_0 < \hat{y}_1 - p_0\hat{\mu}_1$ and $\frac{\partial g(\hat{y}_1, \hat{\mu}_1)}{\partial y} < 0$. This function being continuous between $[0, 1]$, there exists necessarily only one scalar q s.t. $G(q) = \gamma$. QED. ■

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